Study of the energy resolution and uncertainties of Germanium detectors

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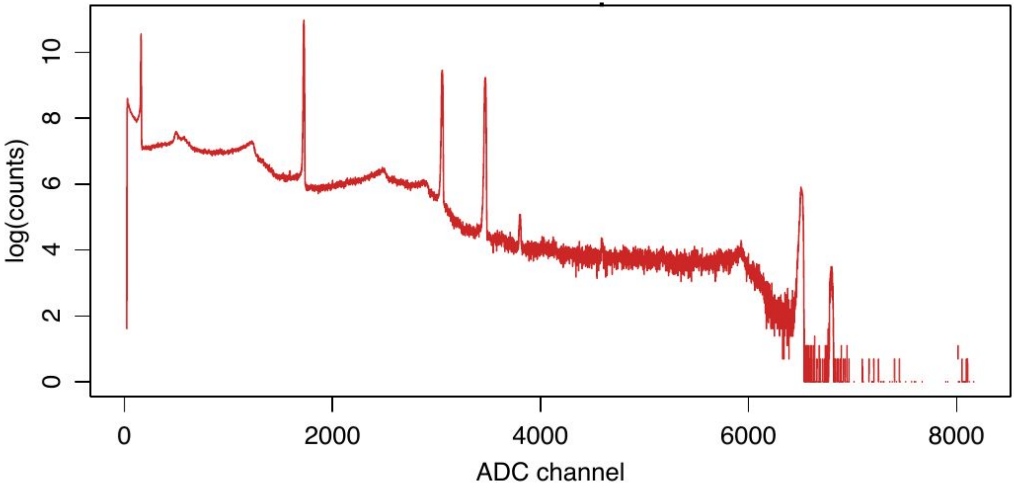
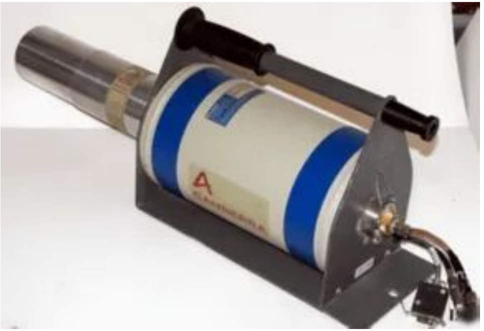
Introduction

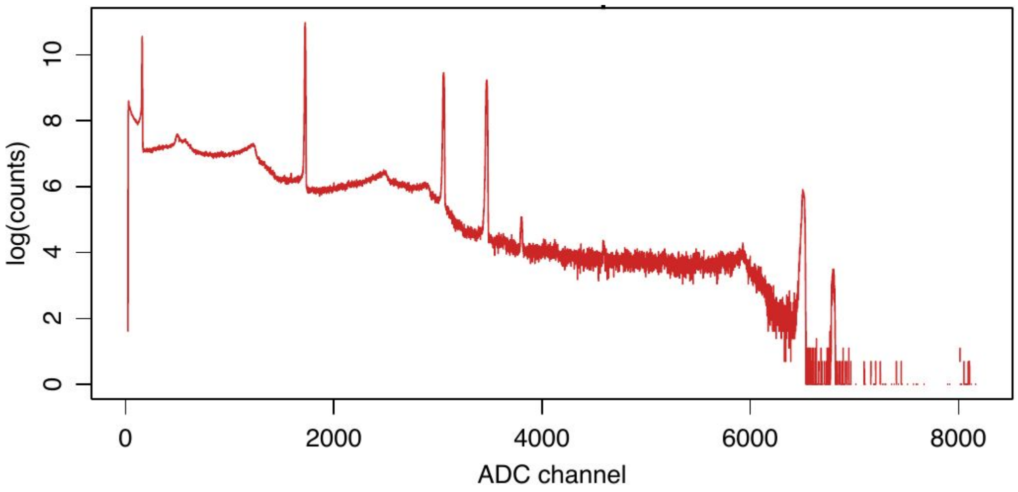
Germanium detectors have wide fields of applications for gamma and X-ray spectrometry thanks to their excellent energy resolution.





Germanium detector [3]

Introduction



Energy calibration

Energy resolution

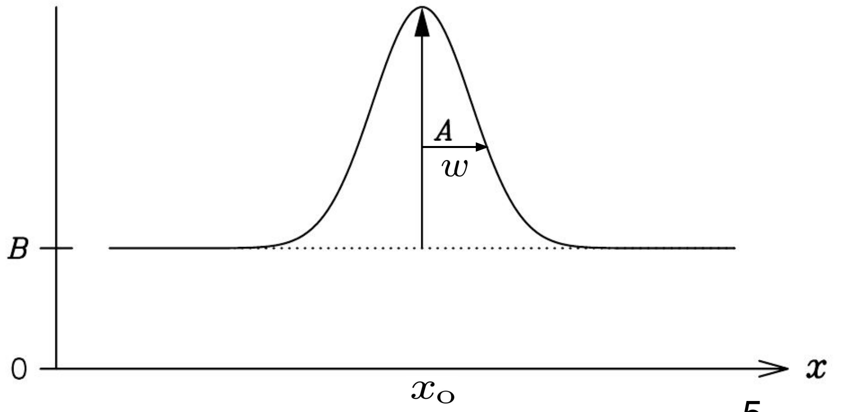
Peaks characterization

Peak characterization: Model

Given a set of counts {N_k} measured at the channels {x_k}, what is our best estimate of the peak position and peak width?

The model:

$$D_k = A \exp\left(-\frac{(x_k - x_0)^2}{2\omega^2}\right) + B$$



Peak characterization

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Bayesian Inference

$$P(x_0, \omega, A, B | \{N_k\}, I) \propto P(\{N_k\} | x_0, \omega, A, B) \times P(x_0, \omega, A, B | I)$$

The likelihood:

$$D_k = A \exp\left(-\frac{(x_k - x_0)^2}{2\omega^2}\right) + B$$

$$P(N_k | x_0, \omega, A, B, I) = \frac{D_k^{N_k} e^{-D_k}}{N_k!}$$

$$P(\{N_k\} | x_0, \omega, A, B, I) = \prod_{k=1}^{M} P(N_k | x_0, \omega, A, B, I)$$

The priors:

$$P(A, B|I) = \begin{cases} \text{constant} & \text{for } A \geq 0 \text{ and } B \geq 0, \\ 0 & \text{otherwise} \end{cases}$$

$$P(x_0|I) = \begin{cases} \text{constant} & x_{0\,min} \leq x_0 \leq x_{0\,max}, \\ 0 & \text{otherwise} \end{cases}$$

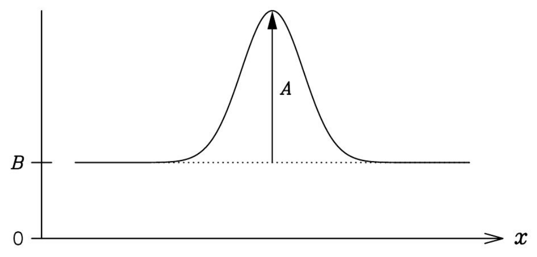
$$P(\omega|I) = \begin{cases} \text{constant} & 1 \leq \omega \leq \omega_{max}, \\ 0 & \text{otherwise} \end{cases}$$

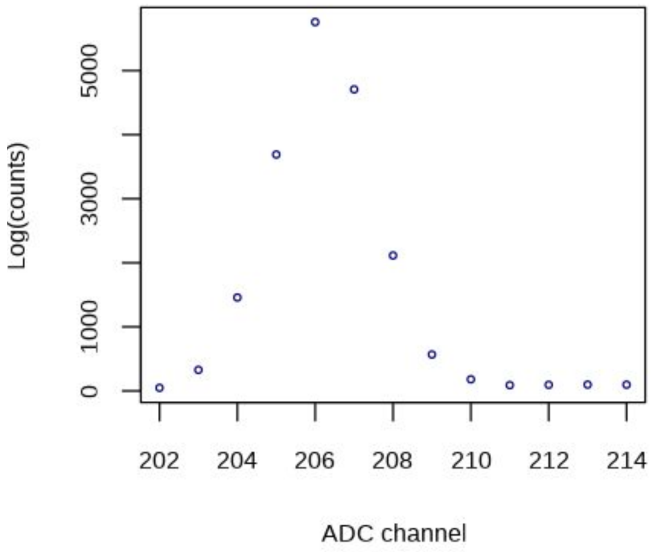
Peak characterization: JAGS

```
model{
     #The likelihood:

    for (i in 1:length(x)){
    S[i] <- (A * exp((-(x[i]-x0)^2) / ( 2 *w^2)) + B)
    y[i] ~ dpois(S[i])
    }

          # Priors for  A, B, x0, w

          A  ~ dunif(0,50000)
          B  ~ dunif(0,50000)
          x0 ~ dunif(",x0.min,",",x0.max,")
          w  ~ dunif(1,20)",

}
```

Peak characterization: JAGS


```
init <- NULL;
init$B <- B.max;  init$A  <- A.max-B.max
init$w <- 5;      init$x0 <- (x0.max-x0.min)/2 + x0.min

init <- list(c(init,.RNG.seed  = 3712,.RNG.name = "base::Wichmann-Hill"),
             c(init,.RNG.seed  = 4021,.RNG.name = "base::Wichmann-Hill"),
             c(init,.RNG.seed  = 1532,.RNG.name = "base::Wichmann-Hill"))
```



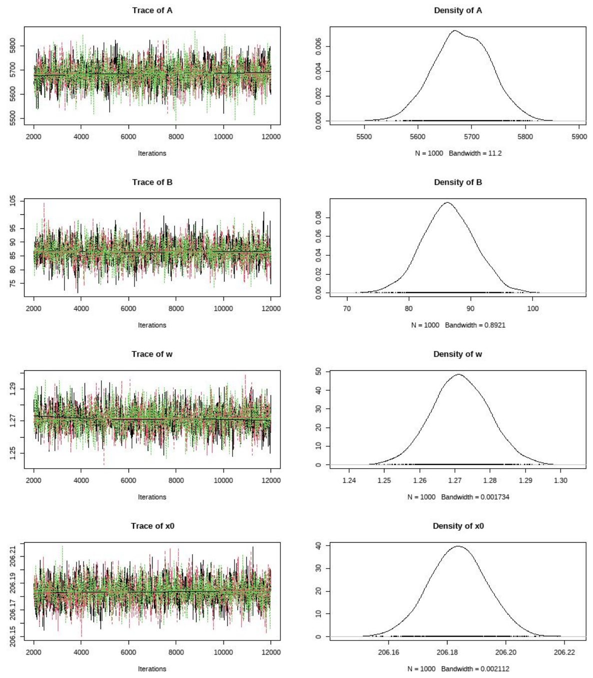


Peak characterization: JAGS

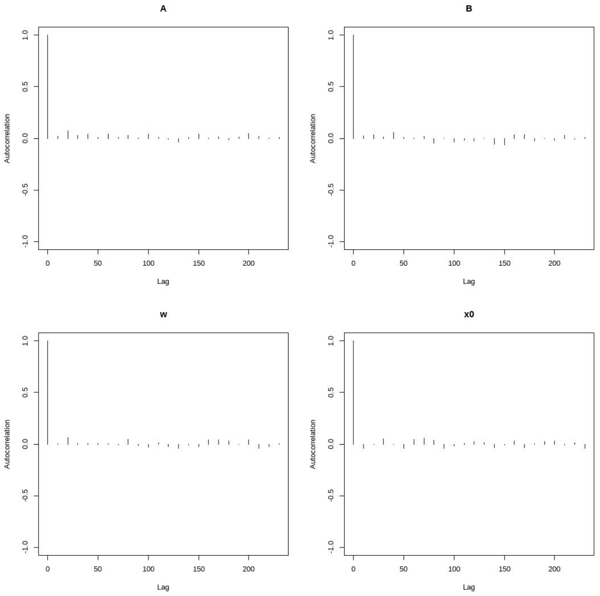

```
chain_number <- 3
burnin       <- 1000
iterations   <- 1.e4
thining      <- 10

jm <- jags.model(model,
                 pk,
                 inits=init,
                 n.chains=chain_number,
                 quiet=TRUE)

#Update the Markov chain (Burn-in)
update(jm, burnin)

chain <- coda.samples(jm, c("A","B","w","x0"), n.iter=iterations,thin=thining)
```

Peaks characterization: Statistics

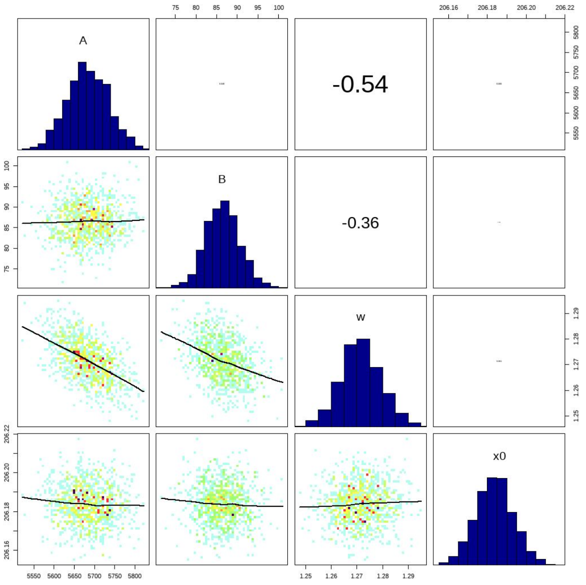
Chain progress



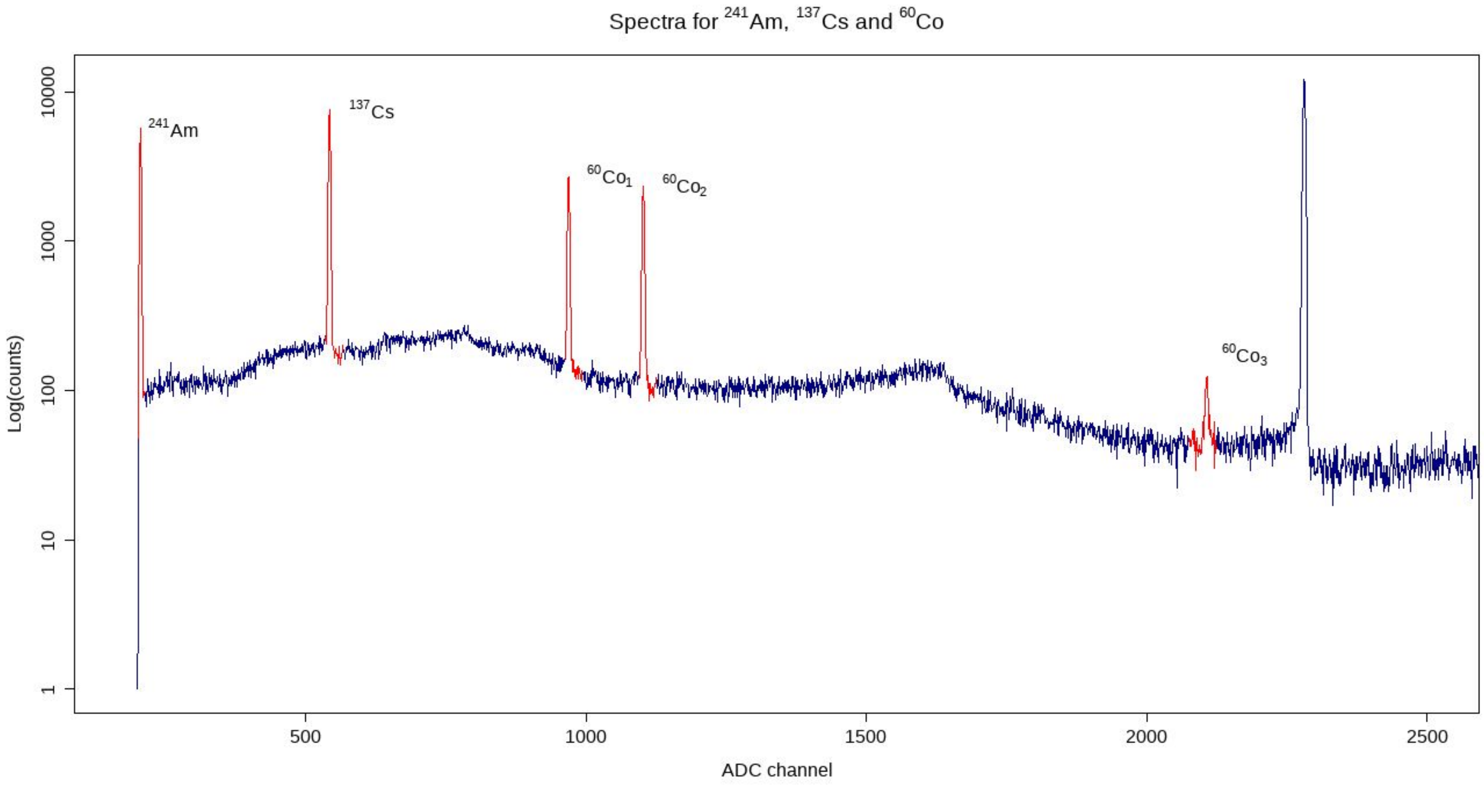
Autocorrelations



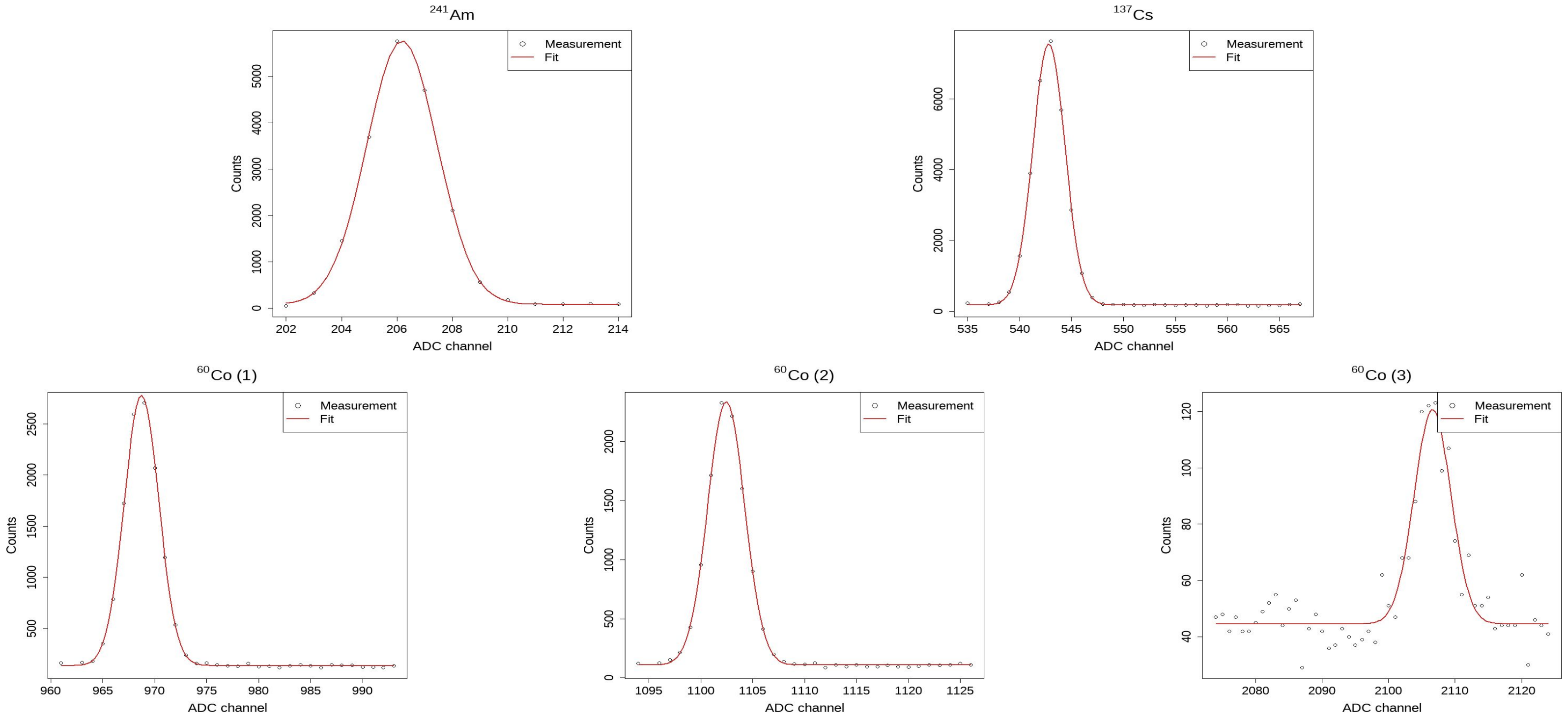
Correlations



Peaks characterization: DATA1



Peaks characterization: DATA1

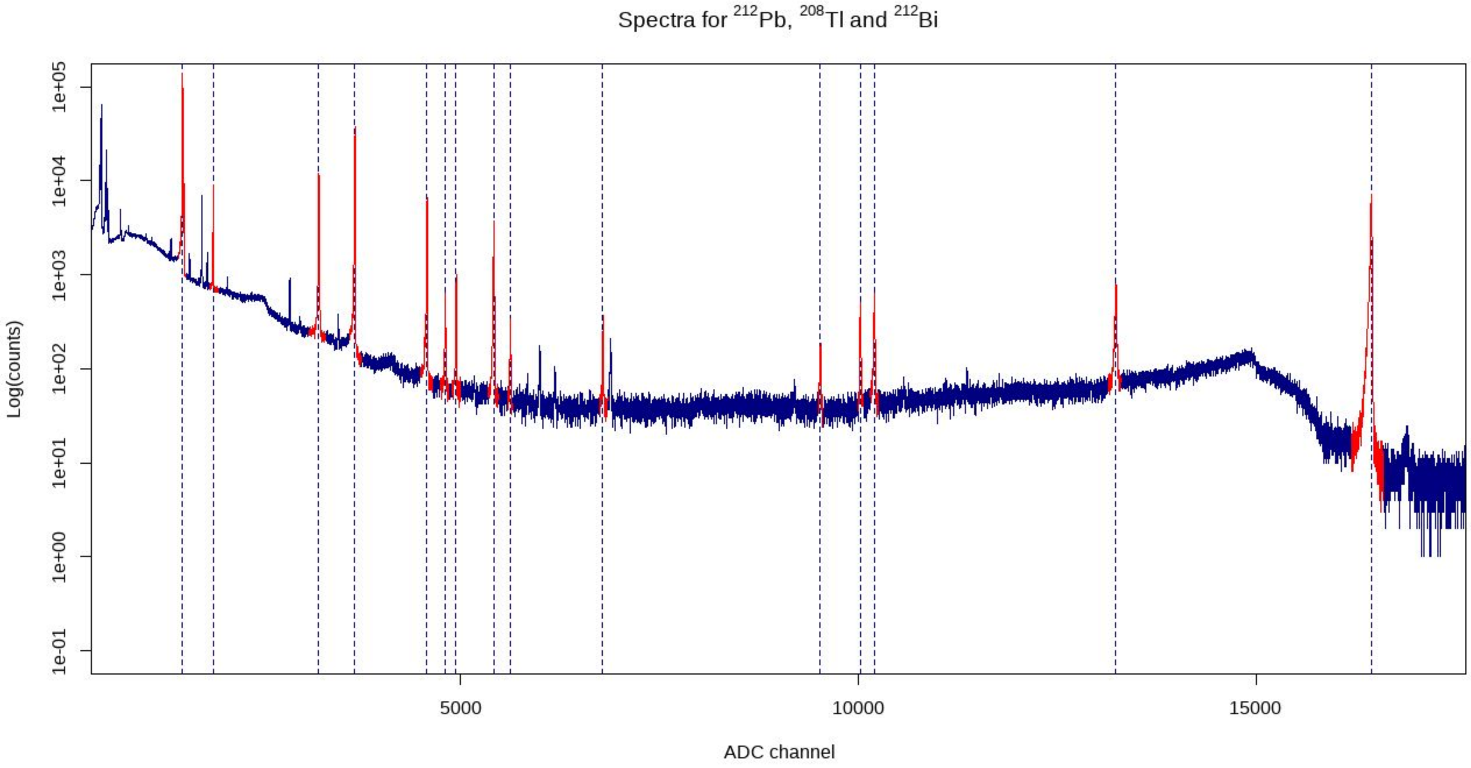


Peaks characterization: DATA1

FHWM	σ_{FHWM}	95% CI
3.763	0.024	[3.716, 3.810]
4.583	0.023	[4.538, 4.628]
4.938	0.046	[4.848, 5.028]
5.267	0.051	[5.167, 5.367]
8.204	0.715	[6.803, 9.605]

x_o	σ_{x_o}	95% CI
206.184	0.010	[206.164, 206.204]
542.816	0.010	[542.796, 542.836]
968.728	0.019	[968.691, 968.765]
1102.446	0.020	[1102.407, 1102.485]
2106.612	0.217	[2106.187, 2107.037]

Peaks characterization: DATA2



Peaks characterization: DATA2

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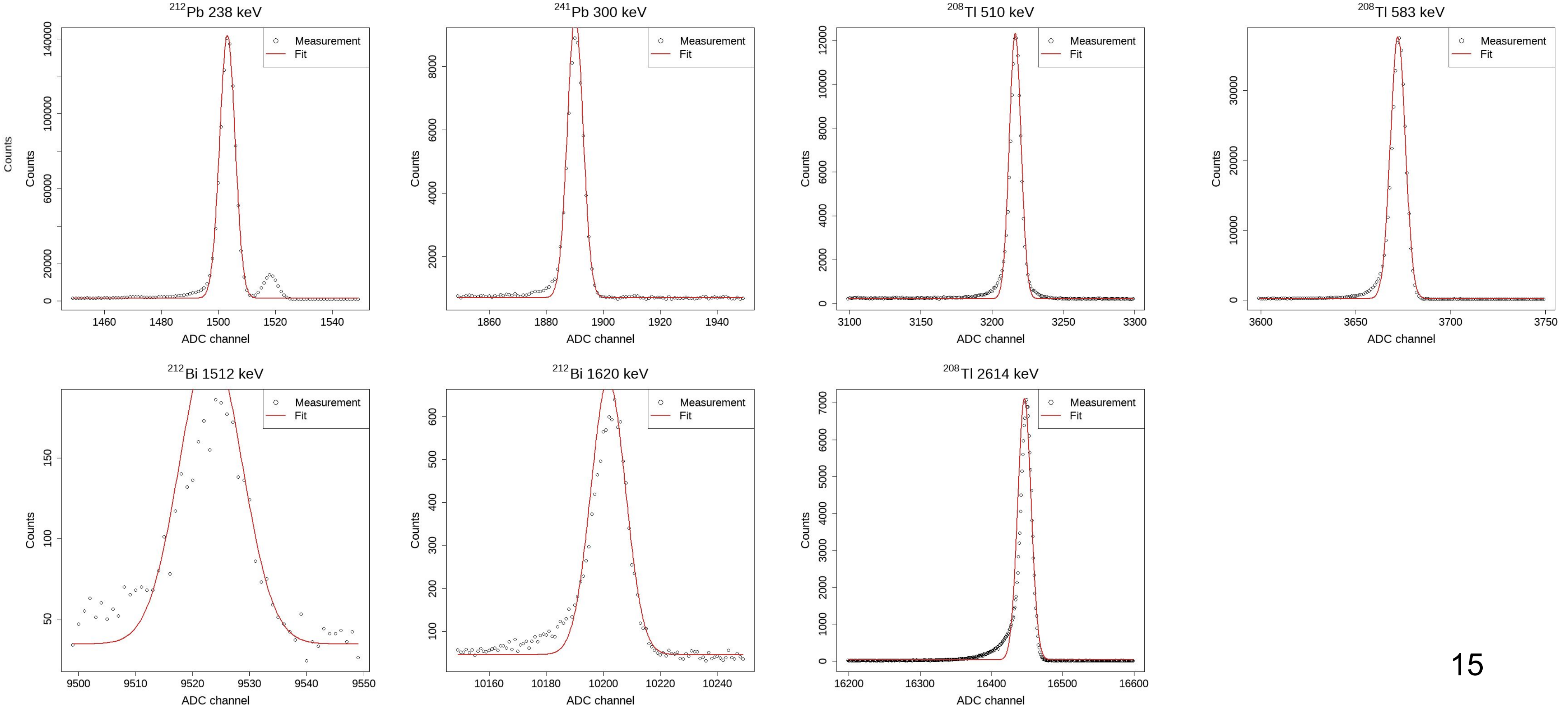

Peaks characterization: DATA2

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FHWM	σ_{FHWM}	95% CI
1.0123	0.0007	[1.0109, 1.0137]
1.0312	0.0035	[1.0251, 1.0389]
1.5339	0.0033	[1.5274, 1.5404]
1.4706	0.0017	[1.4673, 1.4739]
...	...	...

x_o	σ_{x_o}	95% CI
1503.164	0.003	[1503.158, 1503.170]
1890.172	0.014	[1890.145, 1890.199]
3216.230	0.013	[3216.205, 3216.255]
3672.191	0.007	[3672.177, 3672.205]
...	...	...

Maximum FHWM: 3.6910 ± 0.0233 in Thallium (6)
Minimum FHWM: 1.0123 ± 0.0007 in Lead (1)

Detector calibration

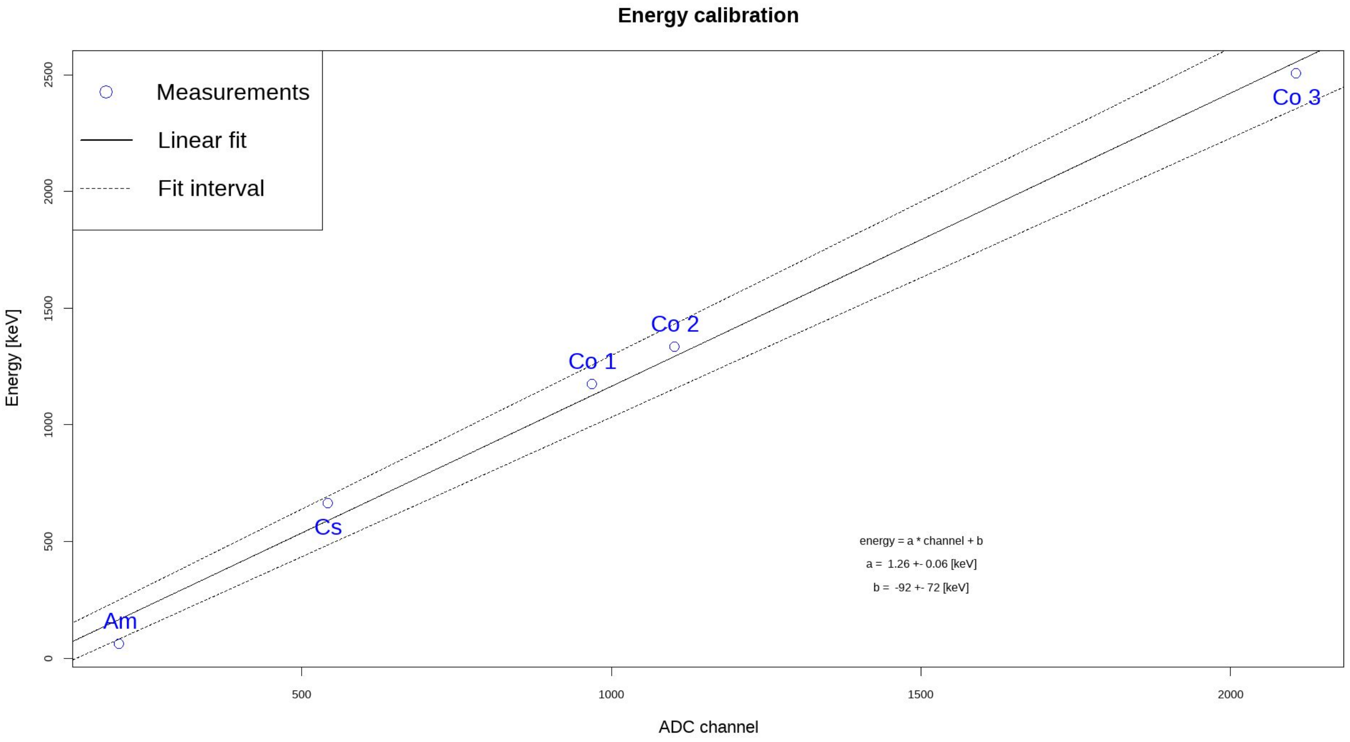
Energy = a * channel + b

Element	Energies	$\overline{x_o}$
^{241}Am	59.5409	206.1838
^{137}Cs	661.6570	542.8155
^{60}Co (1)	1173.2280	968.7283
^{60}Co (2)	1332.4920	1102.4458
^{60}Co (3)	2505.6900	2106.6115

```
linear_fit <- lm(df_ACC$ACC_energies[1:5] ~ df_ACC$x0_mean[1:5] )

intercept     <- summary(linear_fit)$coefficients[1,1]
std_intercept <- summary(linear_fit)$coefficients[1,2]
slope         <- summary(linear_fit)$coefficients[2,1]
std_slope     <- summary(linear_fit)$coefficients[2,2]
```

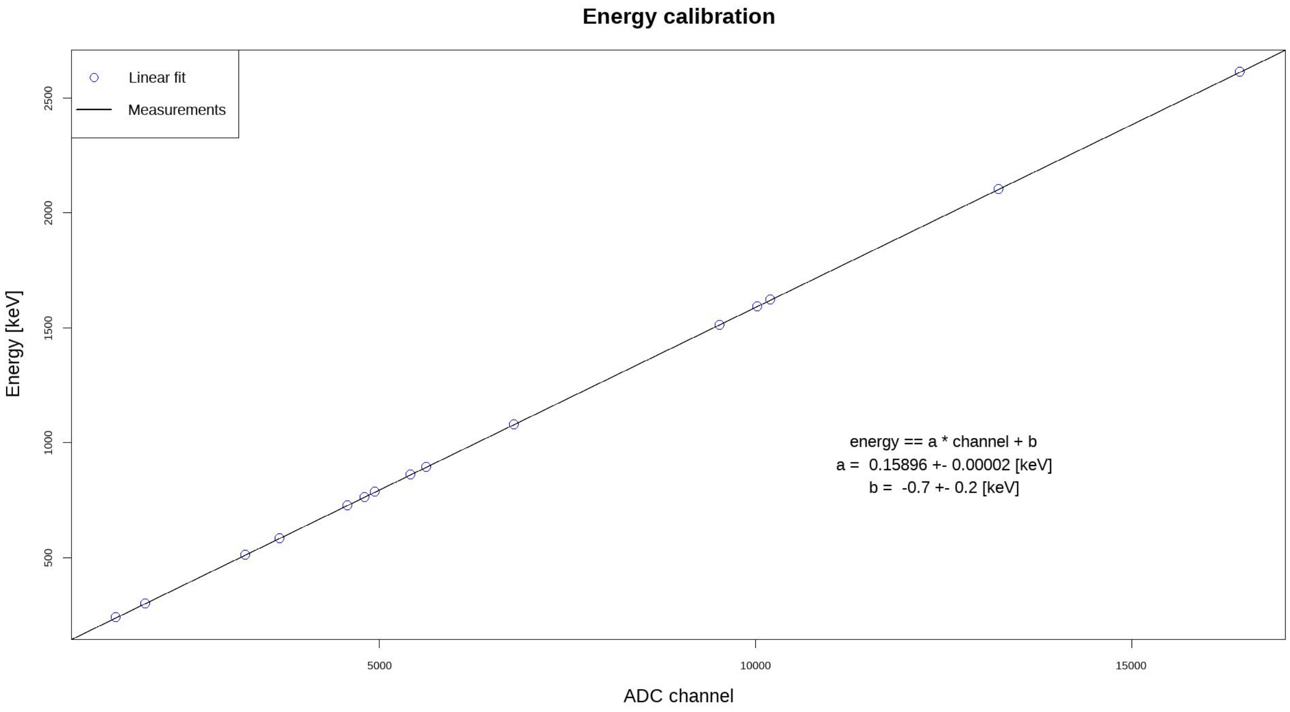
Detector calibration: DATA1



Slope: 1.26 ± 0.06 keV

y-intercept: -92 ± 72 keV

Detector calibration: DATA2



Slope: 0.15896 ± 0.00002 keV

y-intercept: -0.7 ± 0.2 keV

Energy resolution: Model

Given a set of measurements of peak widths {y_i} at energies {E_gamma_i}, what is our best estimate of w and w_e?

Generative model:

$$FWHM(E_\gamma) = \sqrt{8\ln(2)\, F\, E_\gamma\, w + w_e^2}$$

Noise model:

$$y = FWHM(E_\gamma) + \epsilon$$

$$\epsilon = y - FWHM(E_\gamma)$$

Normal distribution with zero mean and standard deviation sigma

Energy resolution

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Bayesian Inference

$$P(w, w_e, \sigma | \{y_i\}, I) \propto P(\{y_i\} | w, w_e, \sigma, I) \times P(w, w_e, \sigma | I)$$

The likelihood:

$$P(y_i | E_{\gamma\, i}, w, w_e) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(y_i - FWHM(E_{\gamma\, i}))^2}{2\sigma^2} \right]$$

$$P(\{y_i\} | w, w_e, \sigma, I) = \prod_{i=1}^{M} P(y_i | w, w_e, \sigma, I)$$

The priors:

$$P(w, w_e, \sigma | I) = P(w|I)\, P(w_e|I)\, P(\sigma|I)$$

$$P(w|I) = \begin{cases} \text{constant} & 1.e-3 \leq \omega \leq 1, \\ 0 & \text{otherwise} \end{cases}$$

$$P(w_e|I) = \begin{cases} \text{constant} & 0 \leq \omega \leq 5, \\ 0 & \text{otherwise} \end{cases}$$

$$P(\sigma|I) = Norm(0, 1)$$

Peak characterization: JAGS

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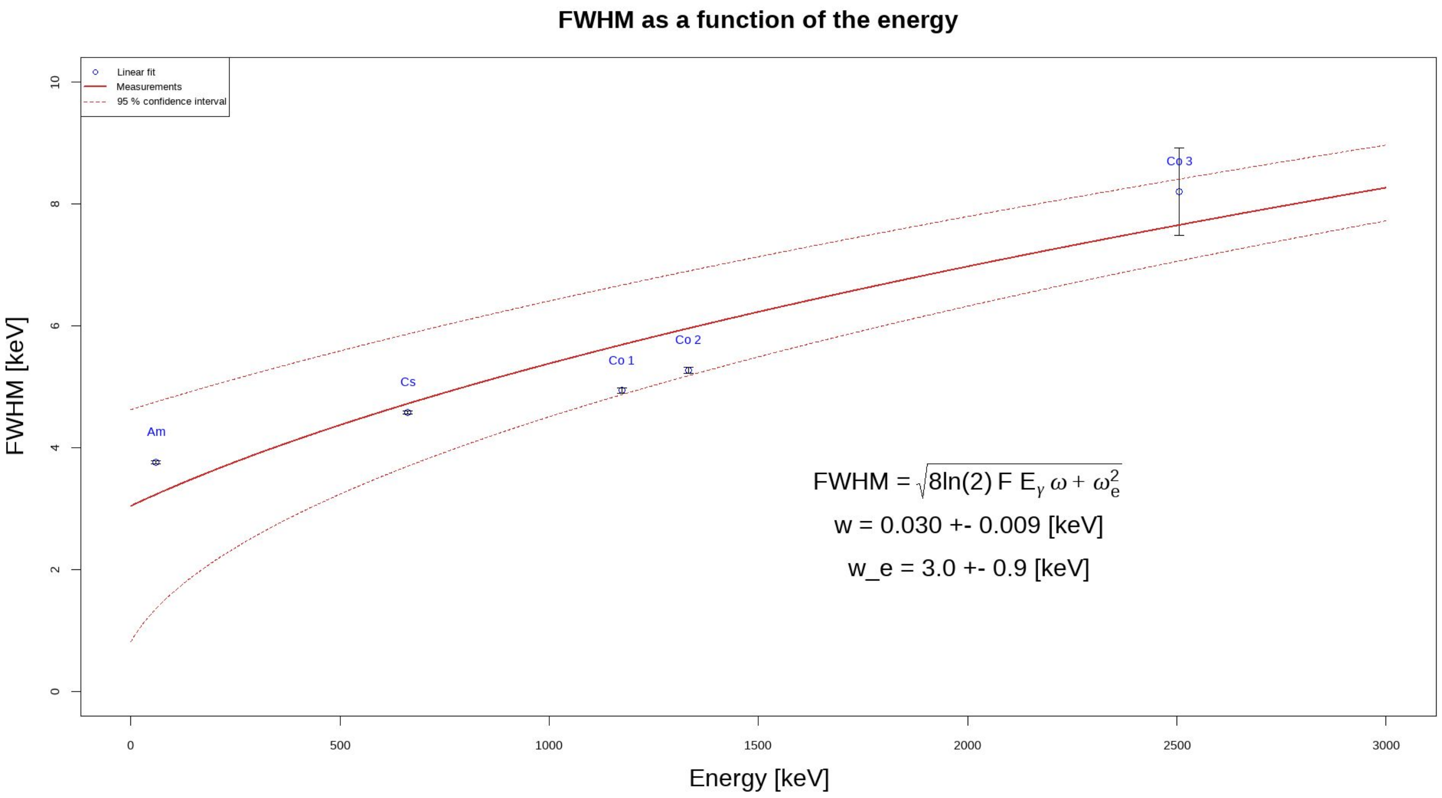
```
model{

    #The likelihood:

    F = 0.113 #Fano factor

    for (i in 1:length(x)){
        f[i] <- sqrt(4*2*log(2) * F *x[i] * w + we**2);

        y[i] ~ dnorm(f[i],sigma);
    }

    # Priors for
    w    ~ dunif(1.e-3,1);
    we   ~ dunif(0,10);
    sigma ~ dnorm(0,1);
    }
```

Peak characterization: JAGS

```
chain_number <- 10
burnin       <- 1000
iterations   <- 1.e4
thining      <- 10

jm <- jags.model(model,
             data,
             inits=init,
             n.chains=chain_number,
             quiet=TRUE)

update(jm, burnin)



chain <- coda.samples(jm, c("w","we","sigma"), n.iter=iterations,thin=thining)
```

Energy resolution: DATA1



Energy resolution: DATA2

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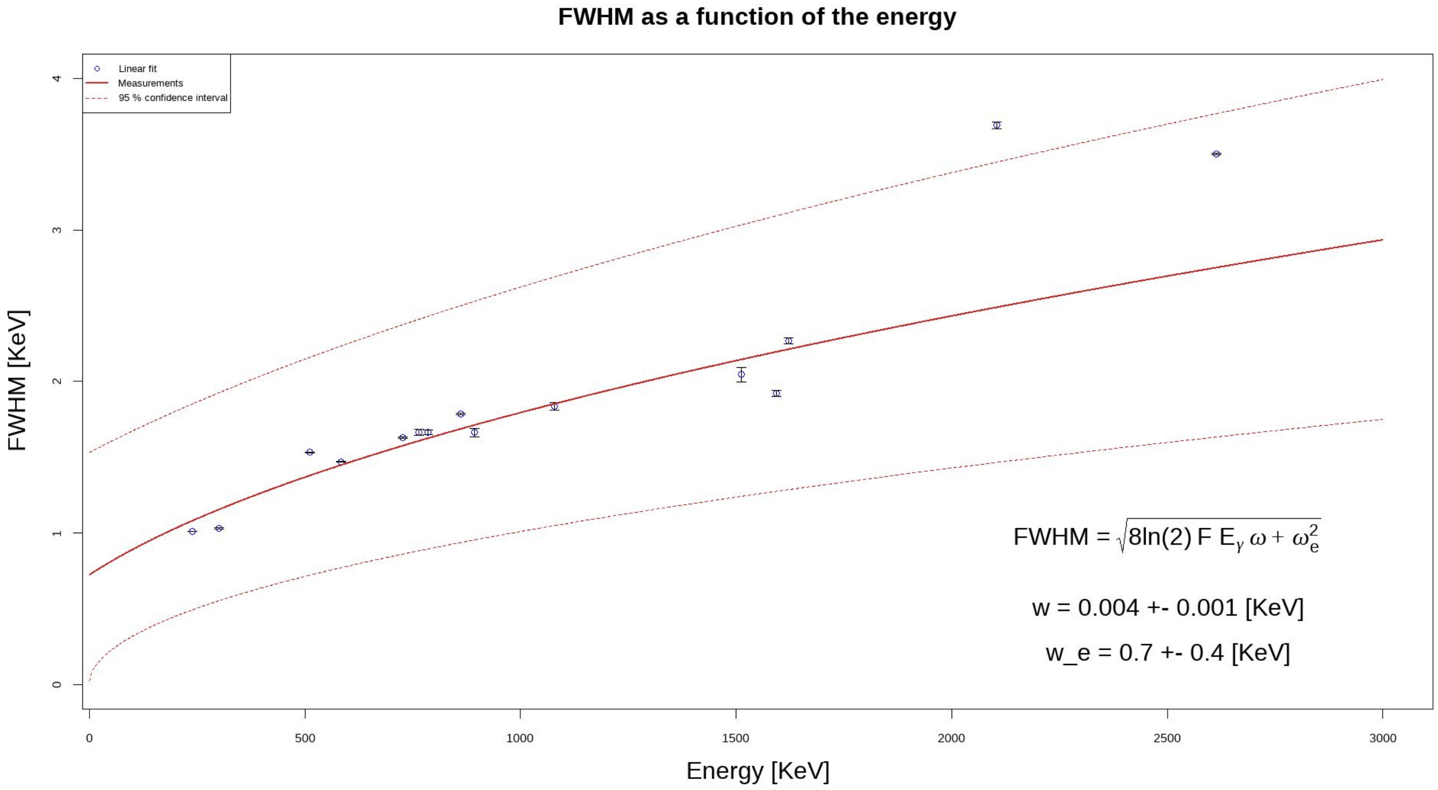

Conclusions

- Energy calibration for Dataset 1
- Energy resolution for Dataset 1
- Energy calibration for Dataset 2
- Energy resolution for Dataset 2

Thank you for your attention

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References

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Data Analysis a Bayesian Tutorial.

[2] L.Baudis,G. Benato, P. Carconi, C. Cattadori, P. De Felice, K. Eberhardt, R. Eichler, A. Petrucci, M. Tarka and M. Walter
Production and Characterization of 228Th Calibration Sources with Low Neutron Emission for GERDA
Journal of Instrumentation, volume 10, Issue 12, article id. P12005, 2015

[3] radiation dosimetry

[4] Nucleid.org