Problems

P1.1 For a set A, the *characteristic function* of A is defined by

$$X_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$

For any two sets A and B and for any element $x \in S$, verify that

$$X_{A \cup B}(x) = \max\{ X_A(x), X_B(x) \},$$

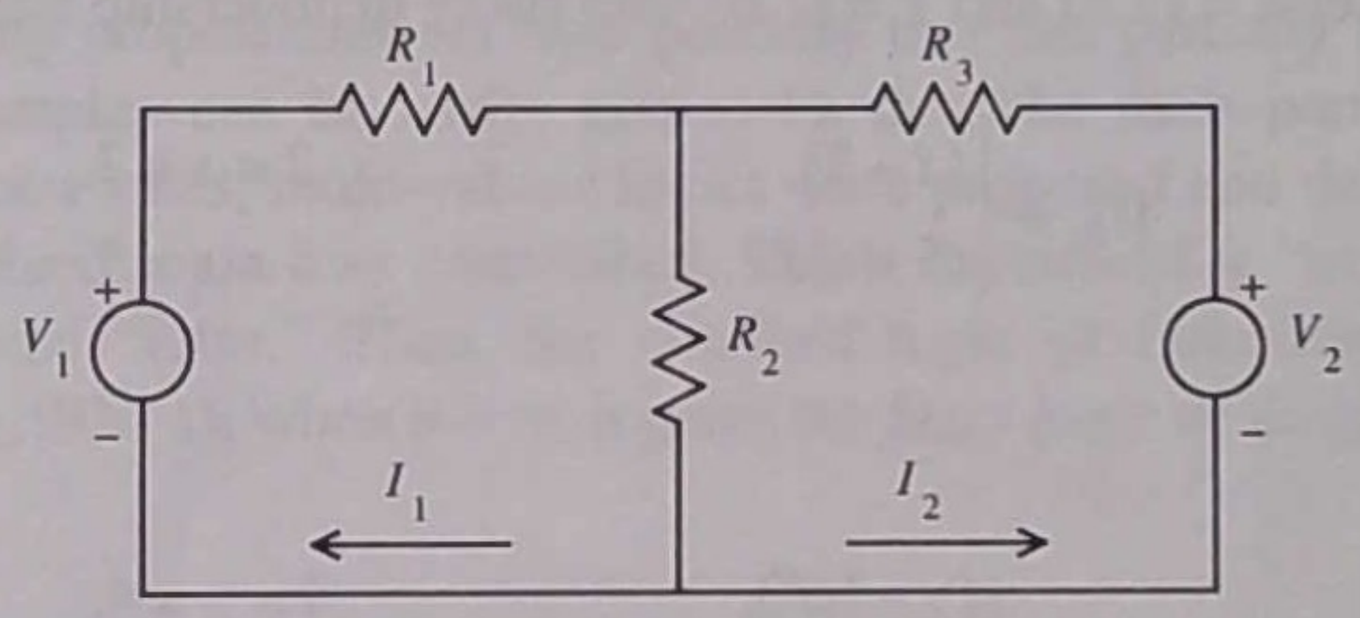
$$X_{A \cap B}(x) = \min\{ X_A(x), X_B(x) \},$$

$$X_{\bar{A}}(x) = 1 - X_A(x).$$

P1.2 For $X = [1,3]$ and $Y = [3,5]$, calculate $X * Y$ for all six operations

$$* \in \{+, -, \times, \div, \max, \min\}.$$

P1.3 Consider the circuit



In the normal case one has

$$\begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}.$$

Now, assume that $V_1 = 1$, $V_2 = 0$, but all the resistors have uncertainties, so that

$$R_1 = R_2 = R_3 = [0.8, 1.2].$$

Find the interval solution of the current vector by using the following formula (accurate to 2 decimal points):

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [R_{\mathbf{I}}]^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{\text{adj}[R_I]}{\det[R_I]} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix},$$

where $\det[R_{\mathbf{I}}] = (R_1 + R_2)(R_2 + R_3) - R_2^2$ and

$$R_{\mathbf{I}} = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix},$$

$$\text{adj}[R_{\mathbf{I}}] = \begin{bmatrix} R_2 + R_3 & -R_2 \\ -R_2 & R_1 + R_2 \end{bmatrix}.$$

P1.4 Find the best solutions for the following expressions:

(a) $Y = 1 + X + X^2 + X^3 + X^4 + X^5$ with $X = [2,3]$

(b) $Y = \dfrac{X^3 - 1}{1 - X}$ with $X = [1,5]$

P1.5 Let $X = [2,5]$ and $Y = [1,6]$ with fuzzy membership functions

$$\mu_X = \begin{cases} (x-2), & 2 \le x \le 3, \\ (5-x)/2, & 3 \le x \le 5, \end{cases}$$

and

$$\mu_Y = \begin{cases} (y-1)/3, & 1 \le y \le 4, \\ (6-y)/2, & 4 \le y \le 6, \end{cases}$$

respectively. Compute each of the following operations to obtain Z and μ_Z:

$$z = F(x,y) = x * y \quad \text{for} \quad * \in \{ +, -, \max, \min \}.$$