Genova, Joshua 2005332 | ECE 6397: Fuzzy Logic | 03/06/2023

P1.1) $\chi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$

$\chi_{A\cup B}(x) = \max\{\chi_A(x), \chi_B(x)\}$
$\chi_{A\cap B}(x) = \min\{\chi_A(x), \chi_B(x)\}$
$\chi_{\bar{A}}(x) = 1 - \chi_A(x)$

Consider the sets:

~~$\chi_A(x)$ $\chi_B(x)$ $\chi_{A\cup B}(x)$ $\chi_{A\cap B}$~~

$C = \{c \mid c \in A \cap B\}$
$D = \{d \mid d \in A \cap \bar{B}\}$
$E = \{e \mid e \in \bar{A} \cap B\}$
$F = \{f \mid f \in \bar{A} \cap \bar{B}\}$

| | $\chi_A(x)$ | $\chi_B(x)$ | $\max\{\chi_A(x), \chi_B(x)\}$ | $\min\{\chi_A(x), \chi_B(x)\}$ | $1-\chi_A(x)$ |
|---|---|---|---|---|---|
| $x=c$ | 1 | 1 | 1 | 1 | 0 |
| $x=d$ | 1 | 0 | 1 | 0 | 0 |
| $x=e$ | 0 | 1 | 1 | 0 | 1 |
| $x=f$ | 0 | 0 | 0 | 0 | 1 |

$\max\{\chi_A(x), \chi_B(x)\} = \begin{cases} 1 & \text{if } x \in (A\cap B)\cup(A\cap\bar{B})\cup(\bar{A}\cap B) \\ 0 & \text{if } x \in \bar{A}\cap\bar{B} \end{cases}$

$\therefore (A\cap B)\cup(A\cap\bar{B})\cup(\bar{A}\cap B) = (A\cap(\cancel{B\cup\bar{B}}))\cup(\bar{A}\cap B)$
$= A\cup(\bar{A}\cap B)$
$= \cancel{(A\cup\bar{A})}\cap(A\cup B) = A\cup B$

$\bar{A}\cap\bar{B} = \overline{A\cup B}$

$\therefore \max\{\chi_A(x), \chi_B(x)\} = \begin{cases} 1 & \text{if } x \in A\cup B \\ 0 & \text{if } x \in \overline{A\cup B} \end{cases} = \begin{cases} 1 & \text{if } x \in A\cup B \\ 0 & \text{if } x \notin A\cup B \end{cases} = \chi_{A\cup B}(x)$

---

$\min\{\chi_A(x), \chi_B(x)\} = \begin{cases} 1 & \text{if } x \in (A\cap B) \\ 0 & \text{if } x \in (A\cap\bar{B})\cup(\bar{A}\cap B)\cup(\bar{A}\cap\bar{B}) \end{cases}$

$\therefore (A\cap\bar{B})\cup(\bar{A}\cap B)\cup(\bar{A}\cap\bar{B}) = (A\cap\bar{B})\cup(\bar{A}\cap\cancel{(B\cup\bar{B})})$
$= (A\cap\bar{B})\cup(\bar{A})$
$= \cancel{(A\cup\bar{A})}\cap(\bar{A}\cup\bar{B}) = \overline{A\cap B}$

$\therefore \min\{\chi_A(x), \chi_B(x)\} = \begin{cases} 1 & \text{if } x \in A\cap B \\ 0 & \text{if } x \in \overline{A\cap B} \end{cases} = \begin{cases} 1 & \text{if } x \in A\cap B \\ 0 & \text{if } x \notin A\cap B \end{cases} = \chi_{A\cap B}(x)$

---

$1-\chi_A(x) = \begin{cases} 1 & \text{if } x \in (\bar{A}\cap B)\cup(\bar{A}\cap\bar{B}) \\ 0 & \text{if } x \in (A\cap B)\cup(A\cap\bar{B}) \end{cases}$

$\therefore (\bar{A}\cap B)\cup(\bar{A}\cap\bar{B}) = \bar{A}\cap\cancel{(B\cup\bar{B})} = \bar{A}$
$(A\cap B)\cup(A\cap\bar{B}) = A\cap\cancel{(B\cup\bar{B})} = A$

$\therefore 1-\chi_A(x) = \begin{cases} 1 & \text{if } x \in \bar{A} \\ 0 & \text{if } x \in A \end{cases} = \begin{cases} 1 & \text{if } x \in \bar{A} \\ 0 & \text{if } x \notin \bar{A} \end{cases} = \chi_{\bar{A}}(x)$

P1.2.) $x = [1,3] \quad y = [3,5]$

- $x+y = [1+3,\ 3+5] = [4,8]$
- $x-y = [1-5,\ 3-3] = [-4,0]$
- $x \times y = [\underline{p},\ \bar{p}]\ \cancel{\phantom{00}} = [3,15]$

  $p = \{1\cdot 3,\ 1\cdot 5,\ 3\cdot 3,\ 3\cdot 5\} = \{3, 5, 9, 15\}$

  $\underline{p} = \min\{p\} = 3 \qquad \bar{p} = \max\{p\} = 15$

  $x \div y = x y^{-1}$

  $y^{-1} = [1/5,\ 1/3]$
- $x y^{-1} = [1,3] \cdot [1/5,\ 1/3] = [1/5, 1]$

  $p = \{1/5,\ 1/3,\ 3/5,\ 1\}$

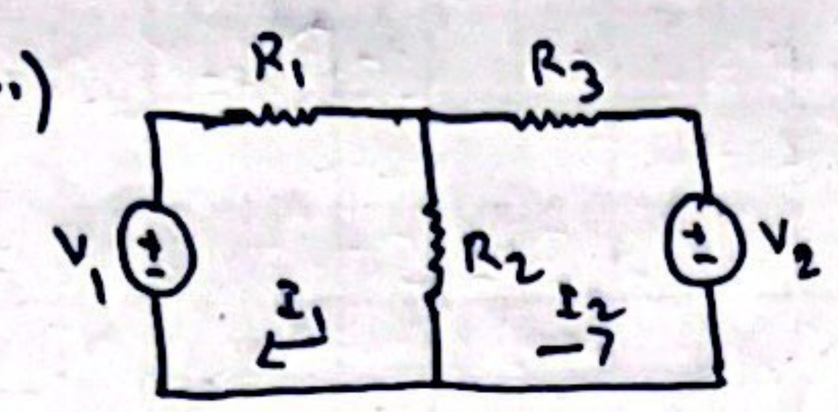
  $\underline{p} = \min\{p\} = 1/5 \qquad \bar{p} = \max\{p\} = 1$
- $\max\{x,y\} = [\underline{p},\ \bar{p}] = [3,5]$

  $\underline{p} = \max\{\underline{x},\ \underline{y}\} = \max\{1,3\} = 3 \qquad \bar{p} = \max\{\bar{x},\ \bar{y}\} = \max\{3,5\} = 5$
- $\min\{x,y\} = [\underline{p},\ \bar{p}] = [1,3]$

  $\underline{p} = \min\{\underline{x},\ \underline{y}\} = \min\{1,3\} = 1 \qquad \bar{p} = \max\{\bar{x},\ \bar{y}\} = \min\{3,5\} = 3$

$$\boxed{\begin{aligned} x+y &= [4,8] \\ x-y &= [-4,0] \\ x \times y &= [3,15] \\ x \div y &= [1/5, 1] \\ \max\{x,y\} &= [3,5] \\ \min\{x,y\} &= [1,3] \end{aligned}}$$

P1.3.)




$$\begin{bmatrix} R_1+R_2 & R_2 \\ R_2 & R_2+R_3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$R_1 = R_2 = R_3 = [0.8, 1.2]$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [R_I]^{-1}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{adj[R_I]}{det[R_I]}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$det[R_I] = (R_1+R_2)(R_2+R_3) - R_2^2$

$R_I = \begin{bmatrix} R_1+R_2 & R_2 \\ R_2 & R_2+R_3 \end{bmatrix}$

$adj[R_I] = \begin{bmatrix} R_2+R_3 & -R_2 \\ -R_2 & R_1+R_2 \end{bmatrix}$

$\cancel{det[R_I] = ([0.8,1.2]+[0.8,1.2])([0.8,1.2])}$

$det[R_I] = ([0.8,1.2]+[0.8,1.2])([0.8,1.2]+[0.8,1.2]) - [0.8,1.2]^2 = \underline{[1.12, 5.12]}$

$[0.8,1.2]+[0.8,1.2] = [1.6, 2.4]$

$[0.8,1.2]\cdot[0.8,1.2] = [0.64, 0.96, 0.96, 1.44] = [0.64, 1.44]$

$[1.6,2.4]\cdot[1.6,2.4] = [2.56, 3.84, 3.84, 5.76] = [2.56, 5.76]$

$[2.56, 5.76] - [0.64, 1.44] = [1.12, 5.12]$

$\cancel{adj[R_I] = \begin{bmatrix} [1.6,2.4] & [0.8,1.2] \\ [0.8,1.2] & [1.6,2.4] \end{bmatrix}}$

$adj[R_I] = \begin{bmatrix} [1.6,2.4] & [-1.2,-0.8] \\ [-1.2,-0.8] & [1.6,2.4] \end{bmatrix}$

$(det[R_I])^{-1} = ([1.12, 5.12])^{-1} = [0.2, 0.89]$

$[1.6,2.4][0.2,0.89] = [0.32, 1.42, 0.48, 2.14] = [0.32, 2.14]$

$[-1.2,-0.8]\cdot[0.2,0.89] = [-0.24, -1.07, -0.16, -0.712] = [-1.07, -0.16]$

$$adj[R_I](det[R_I])^{-1} = \begin{bmatrix} [0.32,2.14] & [-1.07,-0.16] \\ [-1.07,-0.16] & [0.32,2.14] \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [R_I]^{-1}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} [0.32,2.14] & [-1.07,-0.16] \\ [-1.07,-0.16] & [0.32,2.14] \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} [0.32,2.14] \\ [-1.07,-0.16] \end{bmatrix}$$

$$\boxed{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} [0.32,2.14] \\ [-1.07,-0.16] \end{bmatrix}}$$

P1.4.) (a.) $y = 1 + x + x^2 + x^3 + x^4 + x^5 \qquad x = [2,3]$

$$y = 1 + x(1 + x(1 + x(1 + x(1 + x))))$$

$1 + x$

$[1,1] + [2,3] = [3,4]$

$y = 1 + x(1 + x(1 + x(1 + x([3,4])$

$(1 + [2,3][3,4]$

$[2,3][3,4] = [6,8,9,12] = [6,12]$

$y = 1 + x(1 + x(1 + x(1 + [6,12])))$

$1 + x(1 + x(1 + x([7,13])))$

$[2,3]\cdot[7,13] = [14,26,21,39] = [14,39]$

$y = 1 + x(1 + x(1 + [14,39])))$

$1 + x(1 + x([15,40]))$

$[2,3][15,40] = [30,80,45,120] = [30,120]$

$y = 1 + x(1 + [30,120])$

$y = 1 + x([31,121])$

$[2,3][31,121] = [62,242,93,363] = [62,363]$

$y = 1 + [62,363]$

$\boxed{y = [63,364]}$

(b.) $y = \dfrac{x^3 - 1}{1 - x} \qquad x = [1,5]$

$x^2 \quad [1,5][1,5] = [1,5,5,25] = [1,25]$

$x^3 \quad [1,5][1,25] = [1,25,5,125] = [1,125]$

$x^3 - 1 = [1,125] - [1,1] = [0,124]$

$1 - x = [1,1] - [1,5] = [-4,0]$

$(1-x)^{-1} = [-4,0]^{-1}$

with 0 $\boxed{\text{undefined}}$

P1.5.) $X = [2,5]$ $\quad Y = [1,6]$

$$\mu_X(x) = \begin{cases} x-2 & 2 \le x \le 3 \\ \frac{5-x}{2} & 3 \le x \le 5 \end{cases} \qquad \mu_Y(y) = \begin{cases} \frac{y-1}{3} & 1 \le y \le 4 \\ \frac{6-y}{2} & 4 \le y \le 6 \end{cases}$$

$X+Y = [3,11]$ $\quad X-Y = [-4,4]$

$(X)_\alpha = [x_1, x_2]$ $\qquad (Y)_\alpha = [y_1, y_2]$

$\alpha = x_1 - 2$ $\quad \alpha = \frac{5-x_2}{2}$ $\qquad \alpha = \frac{y_1-1}{3}$ $\quad \alpha = \frac{6-y_2}{2}$

$x_1 = \alpha + 2$ $\quad x_2 = -2\alpha + 5$ $\qquad 3\alpha = y_1 - 1$ $\quad 2\alpha = 6 - y_2$

$(X)_\alpha = [\alpha+2, -2\alpha+5]$ $\qquad y_1 = 3\alpha + 1$ $\quad y_2 = -2\alpha + 6$ $\quad (Y)_\alpha = [3\alpha+1, -2\alpha+6]$

$$\begin{aligned} (Z)_\alpha &= (X)_\alpha + (Y)_\alpha \\ &= [\alpha+2, -2\alpha+5] + [3\alpha+1, -2\alpha+6] \\ &= [4\alpha+3, -4\alpha+11] = [z_1, z_2] \end{aligned}$$

$z_1 = 4\alpha + 3$ $\qquad z_2 = -4\alpha + 11$

$\alpha = -\frac{z_2}{4} + \frac{11}{4}$

$\alpha = \frac{z_1 - 3}{4}$

$0 = \frac{z_1}{4} - \frac{3}{4}$ $\quad z_1 = 3$ $\qquad 1 = -\frac{z_2}{4} + \frac{11}{4}$ $\quad z_2 = 7$

$1 = \frac{z_1}{4} - \frac{3}{4}$ $\quad z_1 = 7$ $\qquad 0 = -\frac{z_2}{4} + \frac{11}{4}$ $\quad z_2 = 11$

$$\boxed{\mu_Z(z) = \begin{cases} \frac{z-3}{4} & 3 \le z \le 7 \\ \frac{-z+11}{4} & 7 \le z \le 11 \end{cases}} \qquad \boxed{Z = [3,11]}$$

$$\begin{aligned} (Z)_\alpha &= (X)_\alpha - (Y)_\alpha \\ &= [\alpha+2, -2\alpha+5] - [3\alpha+1, -2\alpha+6] \\ &= [3\alpha-4, -5\alpha+4] \end{aligned}$$

$z_1 = 3\alpha - 4$ $\qquad z_2 = -5\alpha + 4$

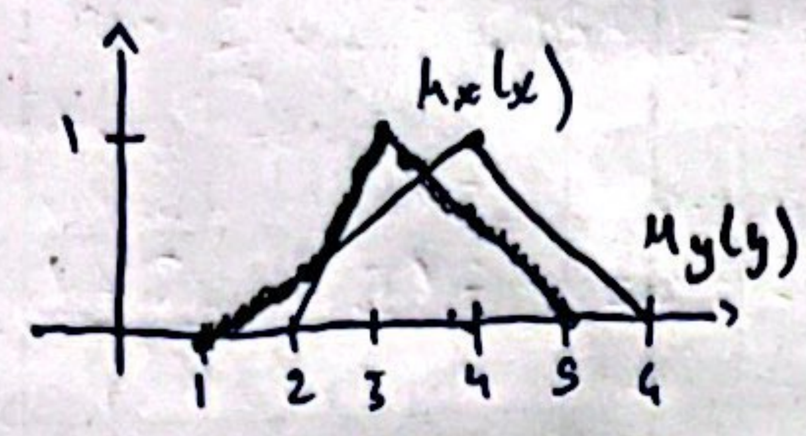
$\alpha = \frac{z_1+4}{3}$ $\qquad \alpha = \frac{-z_2+4}{5}$

$0 = \frac{z_1+4}{3}$ $\quad z_1 = -4$ $\qquad 1 = \frac{-z_2+4}{5}$ $\quad z_2 = -1$

$1 = \frac{z_1+4}{3}$ $\quad z_1 = -1$ $\qquad 0 = \frac{-z_2+4}{5}$ $\quad z_2 = 4$

$$\boxed{\mu_Z(z) = \begin{cases} \frac{z+4}{3} & -4 \le z \le -1 \\ \frac{-z+4}{5} & -1 \le z \le 4 \end{cases}} \qquad \boxed{Z = [-4,4]}$$

$\min(z) = X \wedge y = [1, 5]$




$$\boxed{\mu_z(z) = \begin{cases} \frac{z-1}{3} & 1 \le z \le 2.5 \\ z-2 & 2.5 \le z \le 3 \\ \frac{5-z}{2} & 3 \le z \le 5 \end{cases}}$$

$$\boxed{Z = [1, 5]}$$

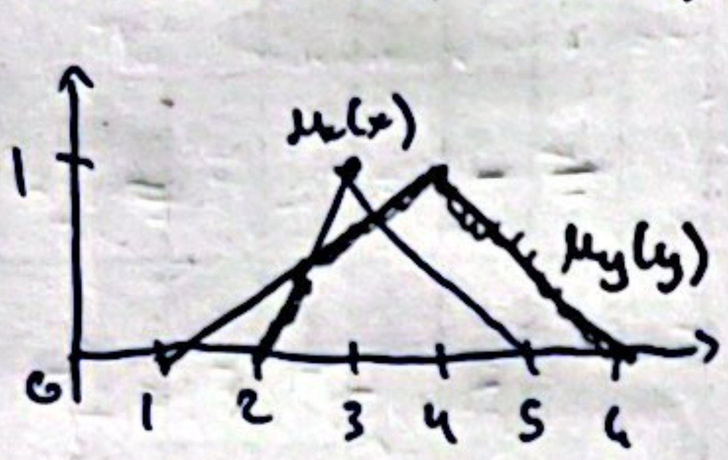
$z - 2 = \frac{z-1}{3}$

$3z - 6 = z - 1$

$2z = 5$

$z = 2.5$

$\max(z) = X \vee y = [2, 6]$




$$\boxed{\mu_z(z) = \begin{cases} z-2 & 2 \le z \le 2.5 \\ \frac{z-1}{3} & 2.5 \le z \le 4 \\ \frac{6-z}{2} & 4 \le z \le 6 \end{cases}}$$

$$\boxed{Z = [2, 6]}$$