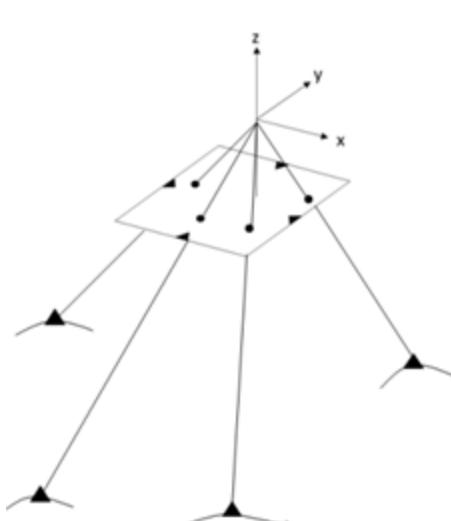
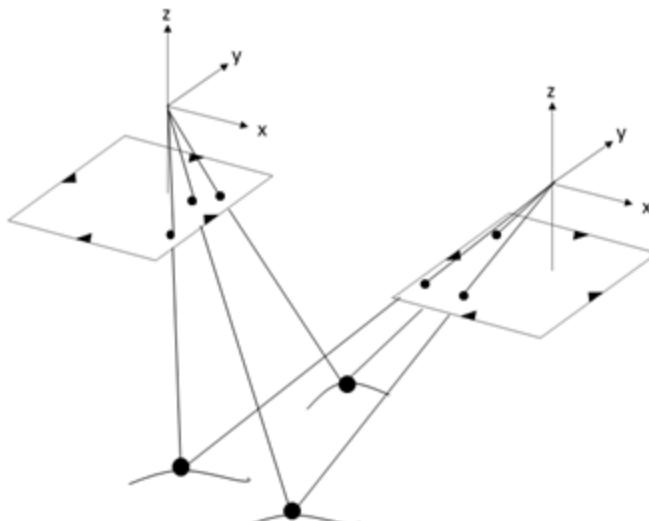


# Indirect Image Orientation



Single image



Stereo image pair

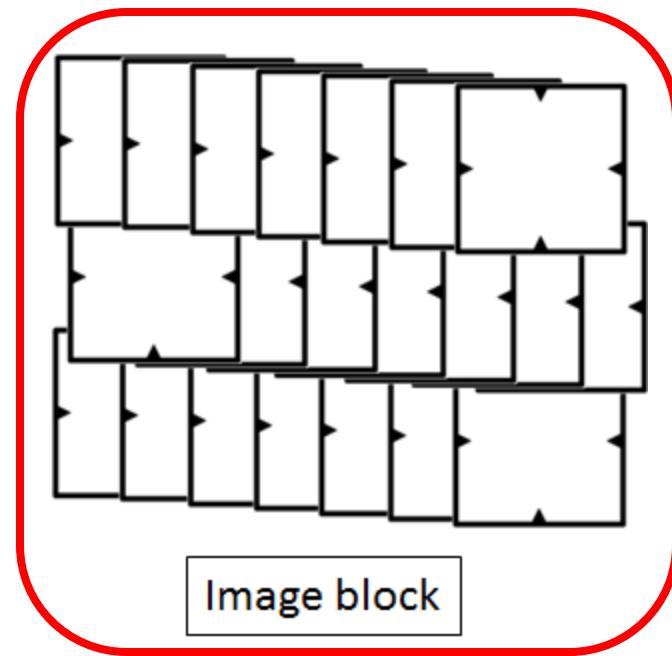
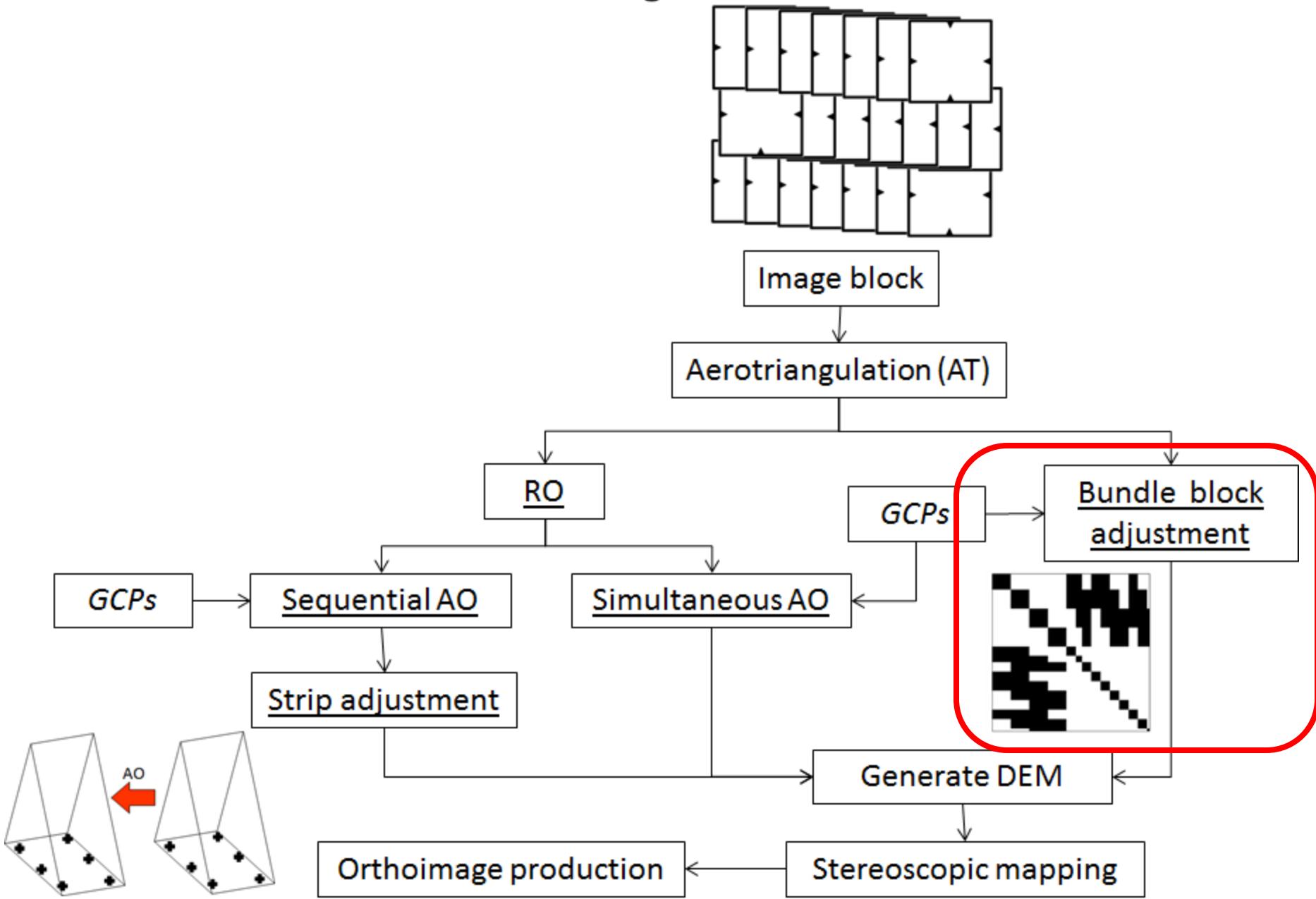


Image block

All Slides Courtesy of:

**Dr. Derek Lichti**  
University of Calgary

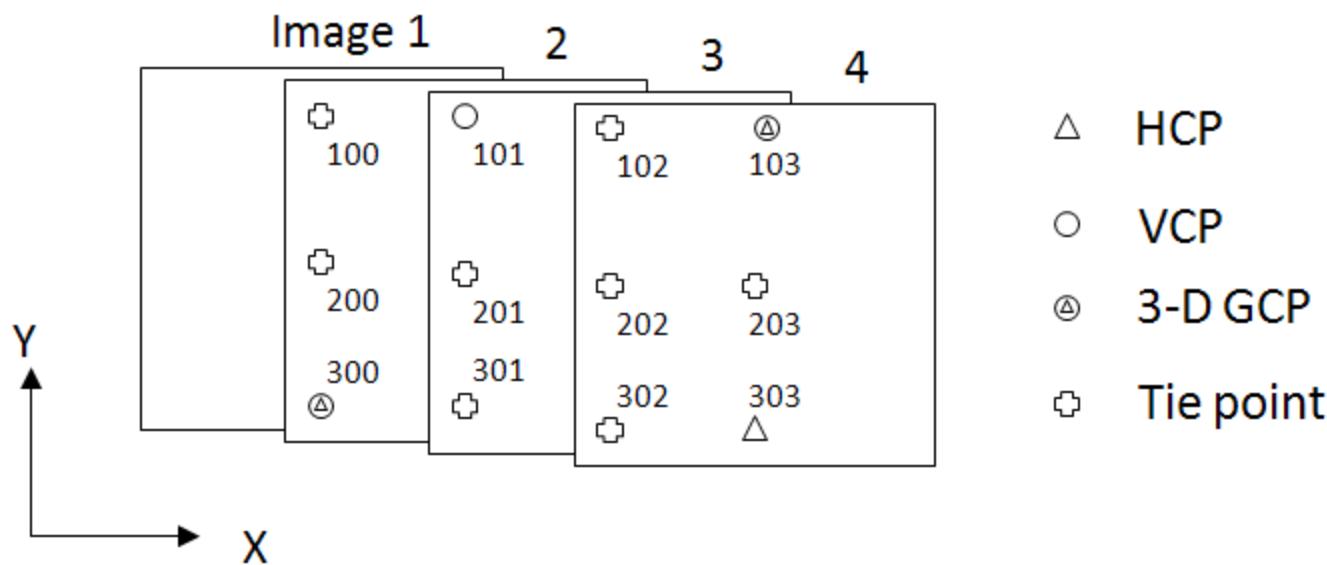
# Indirect image orientation



# Matrix Structure Example

## ► Block layout

- How many observations, unknowns and degrees-of-freedom are there?



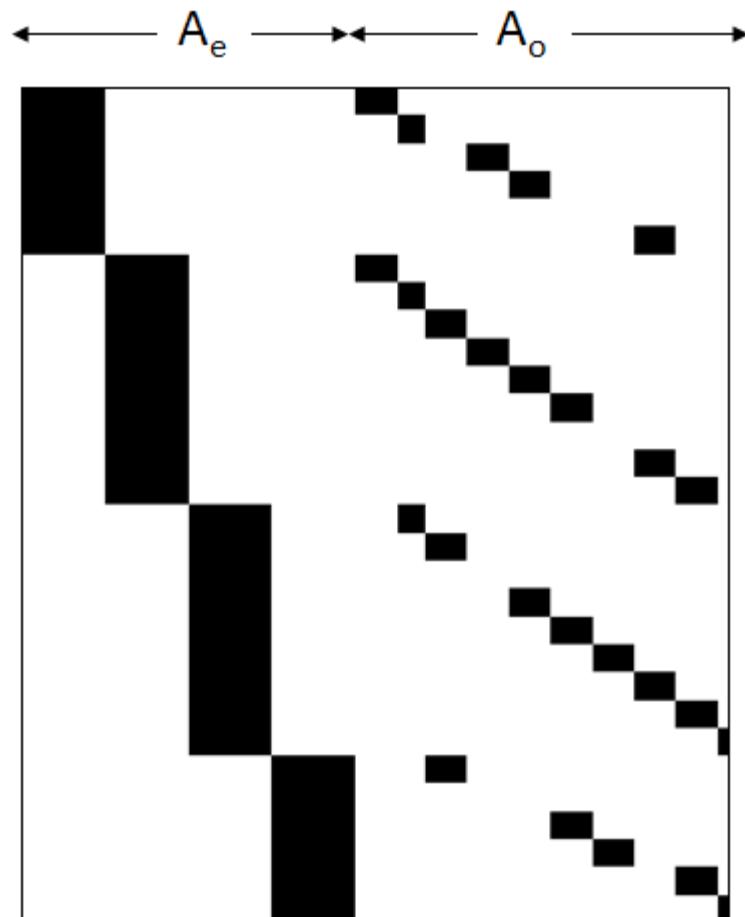
# Matrix Structure Example (cont'd)

## ► General structure of the design matrix

	EO Parameters ( $A_e$ )				Object Point Parameters ( $A_o$ )									
	Image 1 (6)	Image 2 (6)	Image 3 (6)	Image 4 (6)	100 (3)	101 (2)	102 (3)	200 (3)	201 (3)	202 (3)	203 (3)	301 (3)	302 (3)	303 (1)
Image 1 (12)														
Image 2 (18)														
Image 3 (18)														
Image 4 (12)														

# Matrix Structure Example (cont'd)

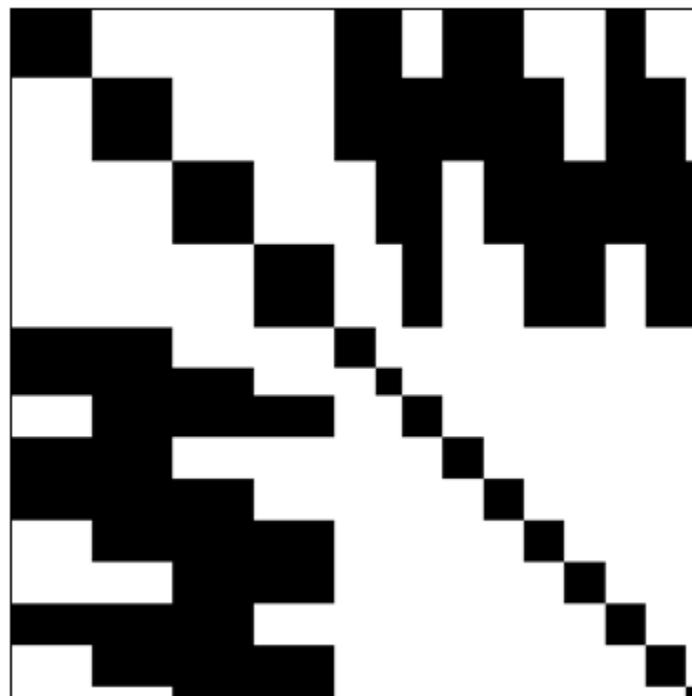
## ► Design matrix structure



# Matrix Structure Example (cont'd)

---

- ▶ Normal equations matrix structure



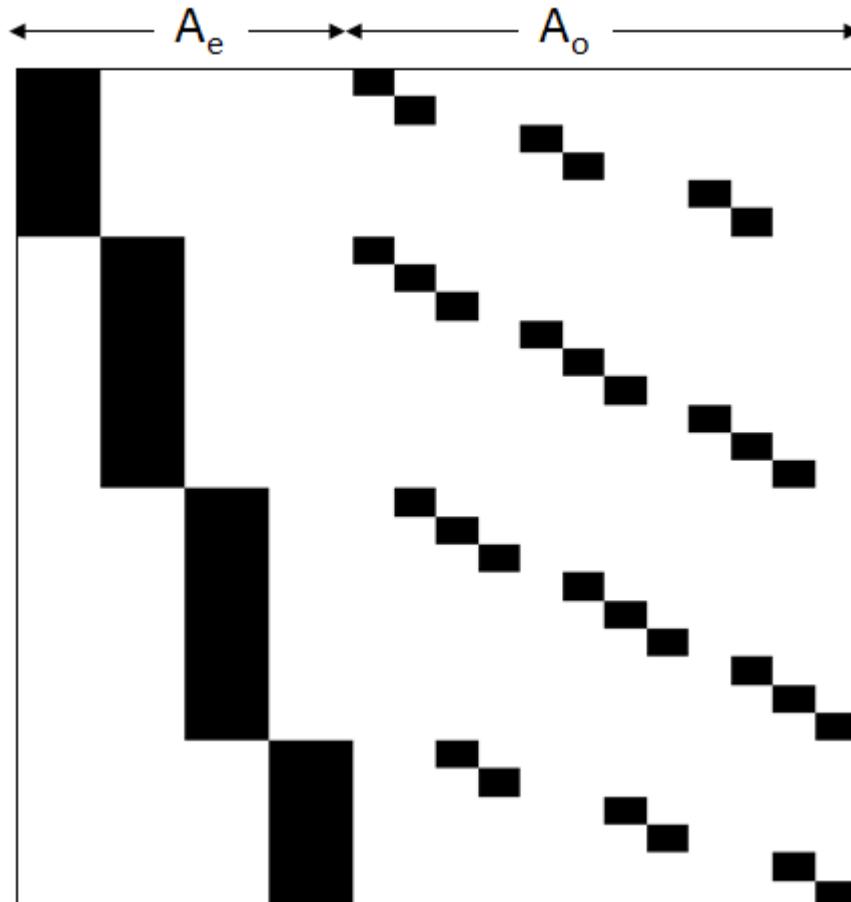
# Matrix Structure Example (cont'd)

- ▶ General structure of the design matrix—unified least squares case

	EO Parameters ( $A_e$ )				Object Point Parameters ( $A_o$ )										
	Image 1 (6)	Image 2 (6)	Image 3 (6)	Image 4 (6)	100 (3)	101 (3)	102 (3)	103 (3)	200 (3)	201 (3)	202 (3)	203 (3)	300 (3)	301 (3)	302 (3)
Image 1 (12)															
Image 2 (18)															
Image 3 (18)															
Image 4 (12)															

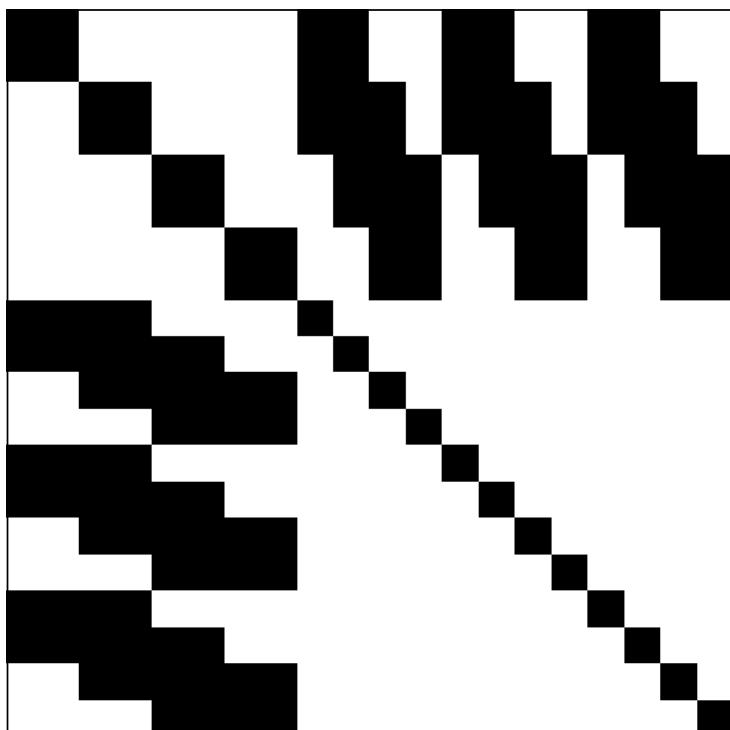
# Matrix Structure Example (cont'd)

- ▶ Design matrix structure—unified least squares case



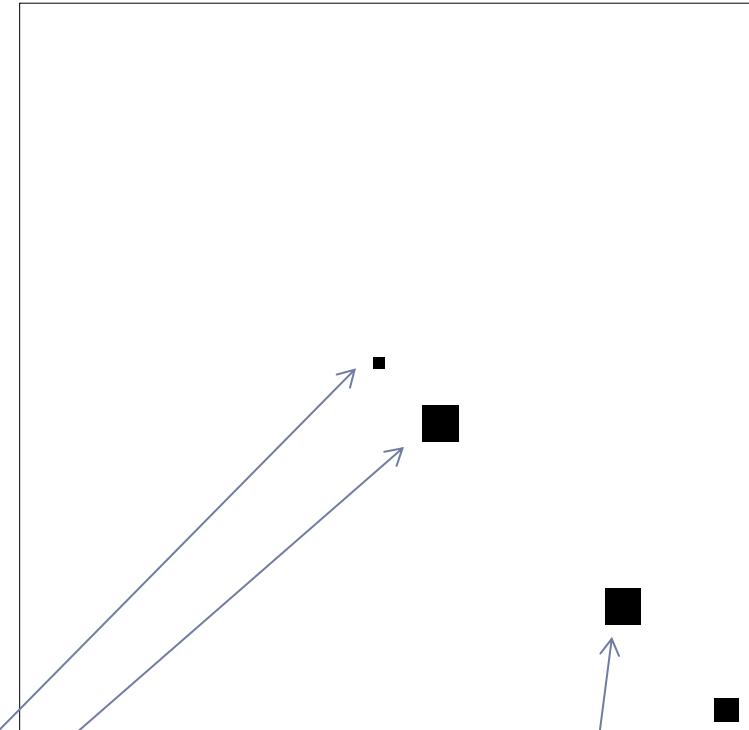
# Matrix Structure Example (cont'd)

- ▶ Normal equations matrix structure and parameter weight matrix—unified least squares case



1x1 matrix for point 101  
3x3 matrix for point 103

+



3x3 matrix for point 300  
2x2 matrix for point 303

# Bundle Adjustment Example 1

---

- ▶ Partial solution
- ▶ All the same input data as in the resection and intersection examples
  - ▶ 2 images, 2 tie points (72, 127), 4 3D control points
  - ▶ Control points treated as constants
  - ▶  $n=24$ ,  $u=18$ ,  $r=6$
- ▶ Final solution after 4 iterations

Other quantities	
RMS <sub>vx</sub> (mm)	0.008
RMS <sub>vy</sub> (mm)	0.009
Redundancy	6
Variance factor	1.283

# Bundle Adjustment Example 1 (cont'd)

---

Residuals (mm)		
	Left	Right
$x_{30}$	-0.010	0.001
$y_{30}$	0.024	0.000
$x_{40}$	0.023	-0.002
$y_{40}$	-0.014	-0.001
$x_{50}$	-0.012	0.003
$y_{50}$	0.001	0.000
$x_{112}$	-0.001	-0.003
$y_{112}$	-0.011	0.002
$x_{72}$	0.000	0.000
$y_{72}$	0.000	0.000
$x_{127}$	0.000	0.000
$y_{127}$	0.001	-0.001

# Bundle Adjustment Example 1 (cont'd)

Parameters and standard deviations				
	Left		Right	
X <sup>c</sup> (m)	6349.495	0.320	7021.896	0.377
Y <sup>c</sup> (m)	3965.305	0.412	3775.658	0.408
Z <sup>c</sup> (m)	1458.094	0.154	1466.700	0.151
ω (°)	0.98672	0.01501	1.87411	0.01583
ϕ (°)	0.40715	0.01385	1.67510	0.01657
κ (°)	-18.90545	0.00569	-15.74806	0.00494

Parameters and standard deviations		
X <sub>72</sub> (m)	6869.170	0.111
Y <sub>72</sub> (m)	3844.538	0.115
Z <sub>72</sub> (m)	283.205	0.319

Parameters and standard deviations		
X <sub>170</sub> (m)	6316.138	0.142
Y <sub>170</sub> (m)	3934.680	0.126
Z <sub>170</sub> (m)	283.228	0.341

# Bundle Adjustment Example 1 (cont'd)

Correlation coefficient matrix of the parameters—left image						
	$X^c$	$Y^c$	$Z^c$	$\omega$	$\phi$	$\kappa$
$X^c$	1					
$Y^c$	-0.12	1				
$Z^c$	0.71	0.22	1			
$\omega$	0.20	-0.98	0.29	1		
$\phi$	0.97	-0.21	0.80	0.29	1	
$\kappa$	-0.13	-0.66	-0.02	0.59	-0.06	1

# Questions for Discussion

---

- ▶ Comment on the RMS of residuals
- ▶ Comment on the precision of the tie point co-ordinates relative to the intersection solution
- ▶ Comment on the precision of the EO parameters relative to the resection solution

# Bundle Adjustment Example 2

---

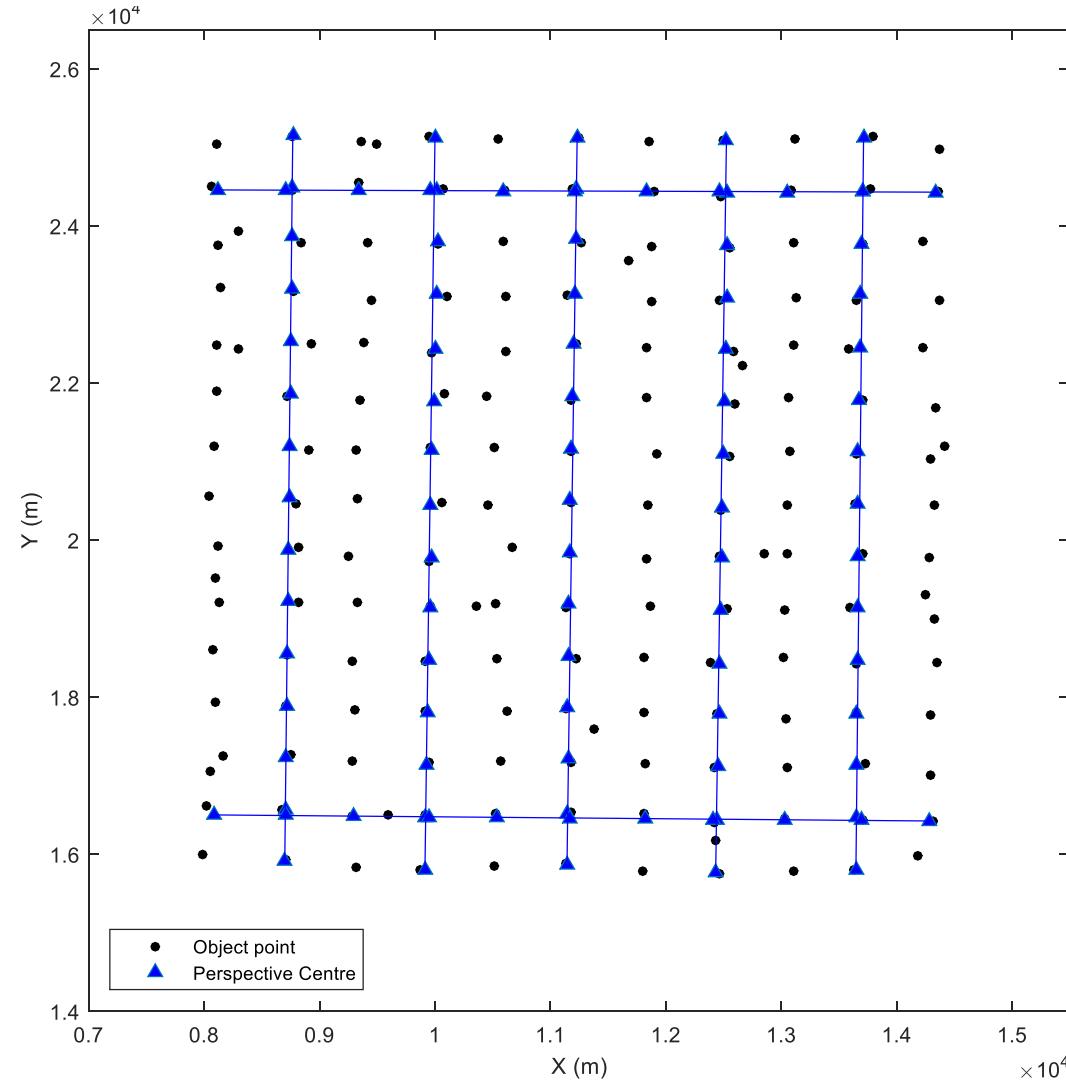
- ▶ Ellenbrook block
  - ▶ 97 images
  - ▶ 5 strips and 2 cross-strips
  - ▶ 60% end lap and 20% side lap
  - ▶ Scale 1:7500
  - ▶ 4 weighted control points (co-ordinate parameter observations)
  - ▶ 175 pass/tie points
  - ▶ 878 image point observations
  - ▶ 17 check points (solved as tie points)

## Example 2 Adjustment Cases

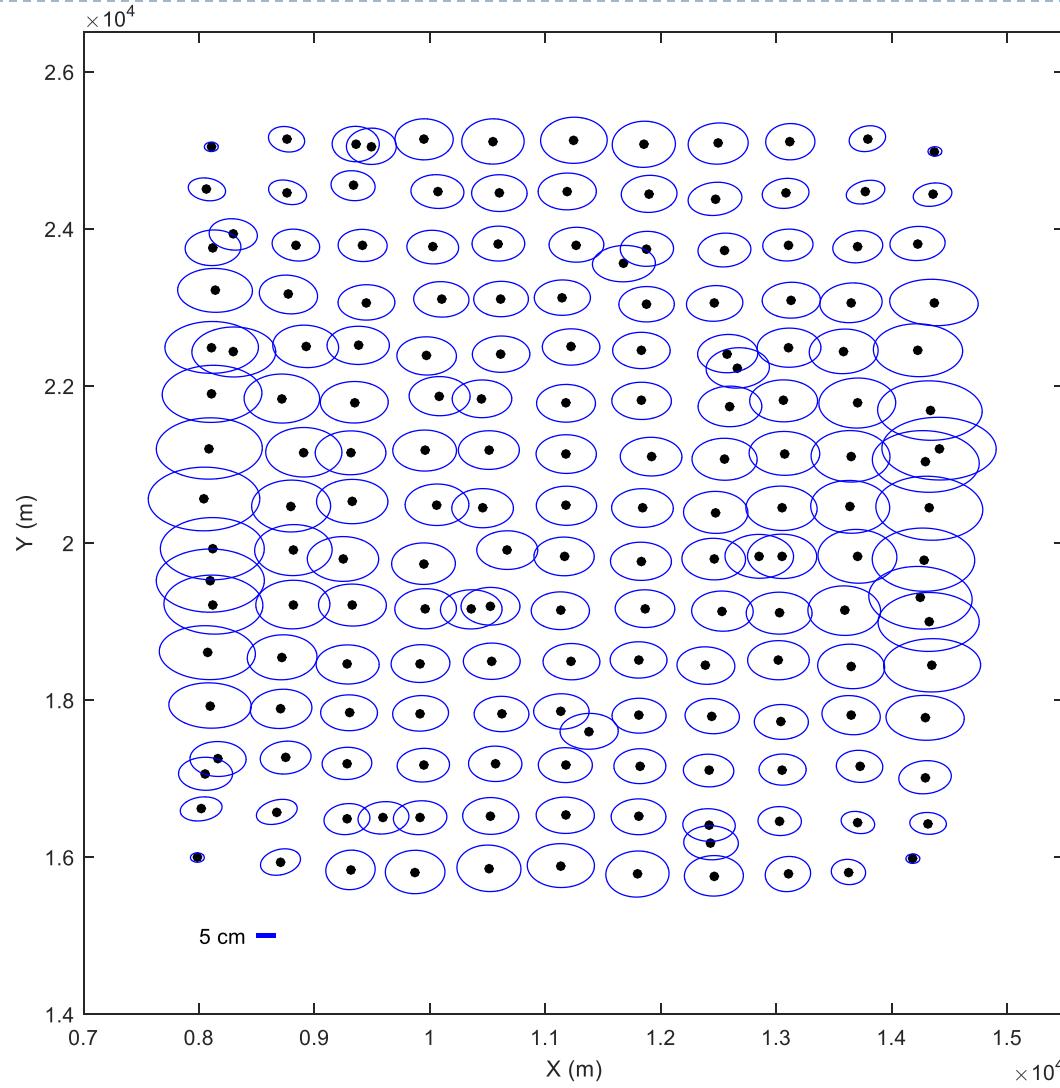
---

1. Minimal ground control: 4 3D GCPs in the corners
2. Full ground control: 21 3D GCPs throughout the block
3. Minimal ground control + airborne GPS observations of each PC

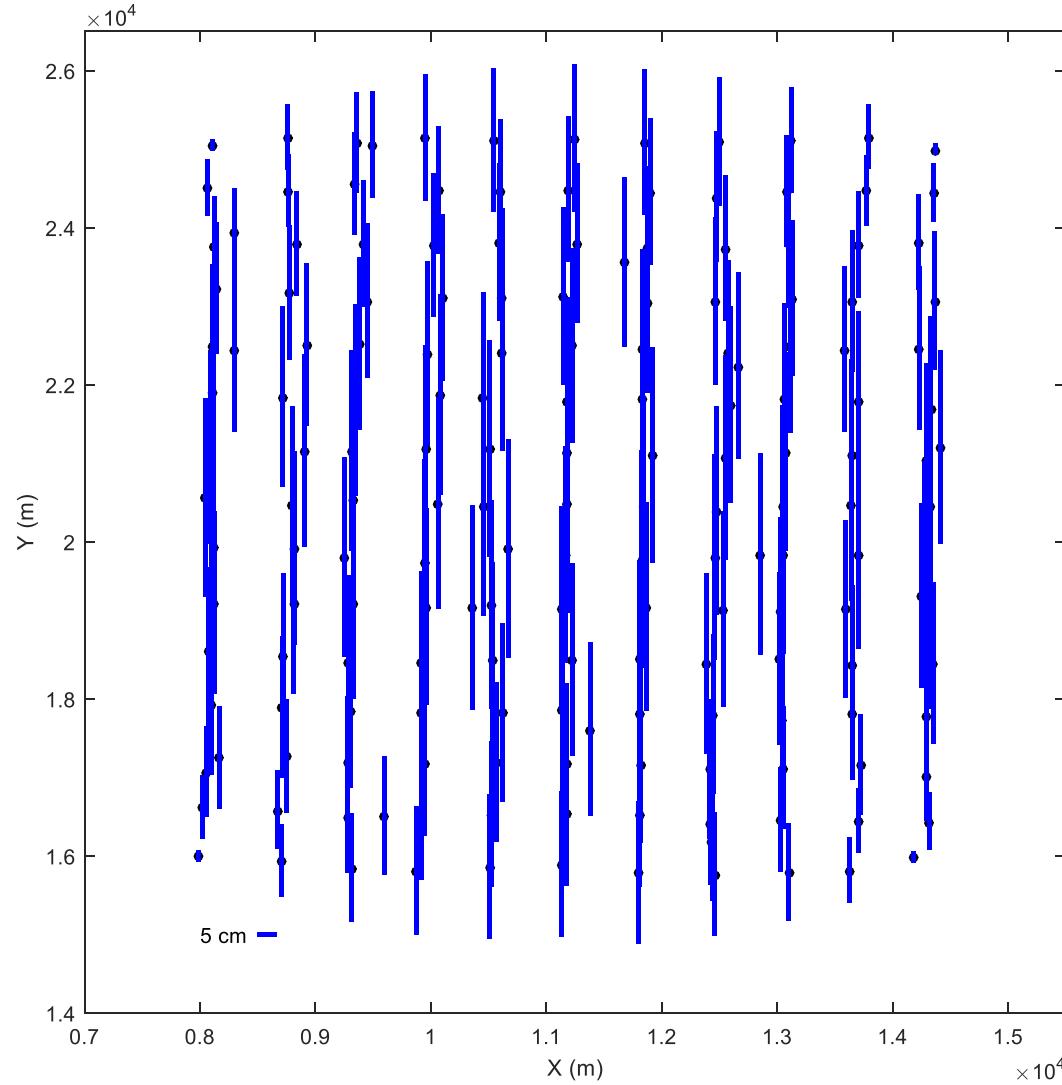
# Example 2 Block Configuration



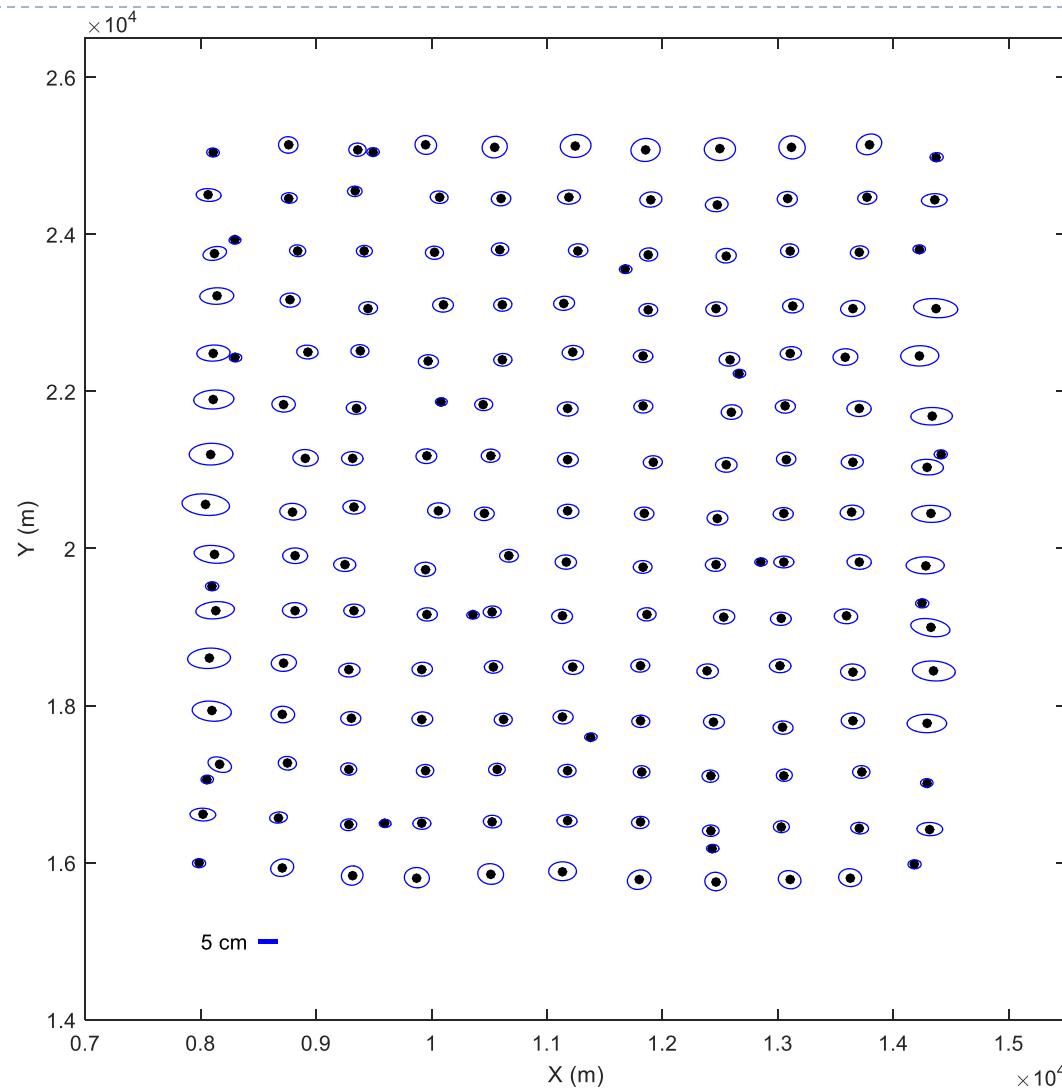
# Case 1—Minimum Ground Control Planimetric Precision



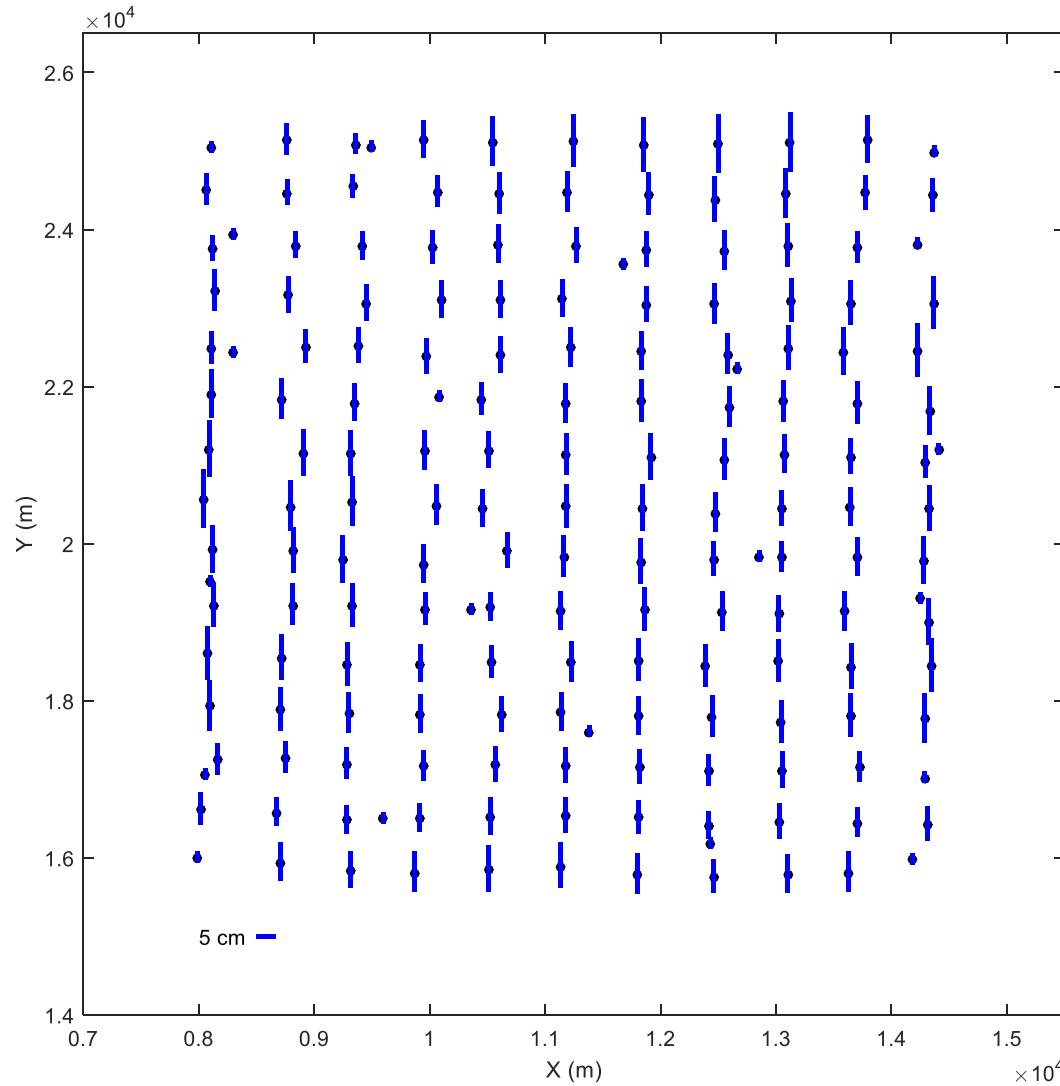
# Case 1—Minimum Ground Control Vertical Precision



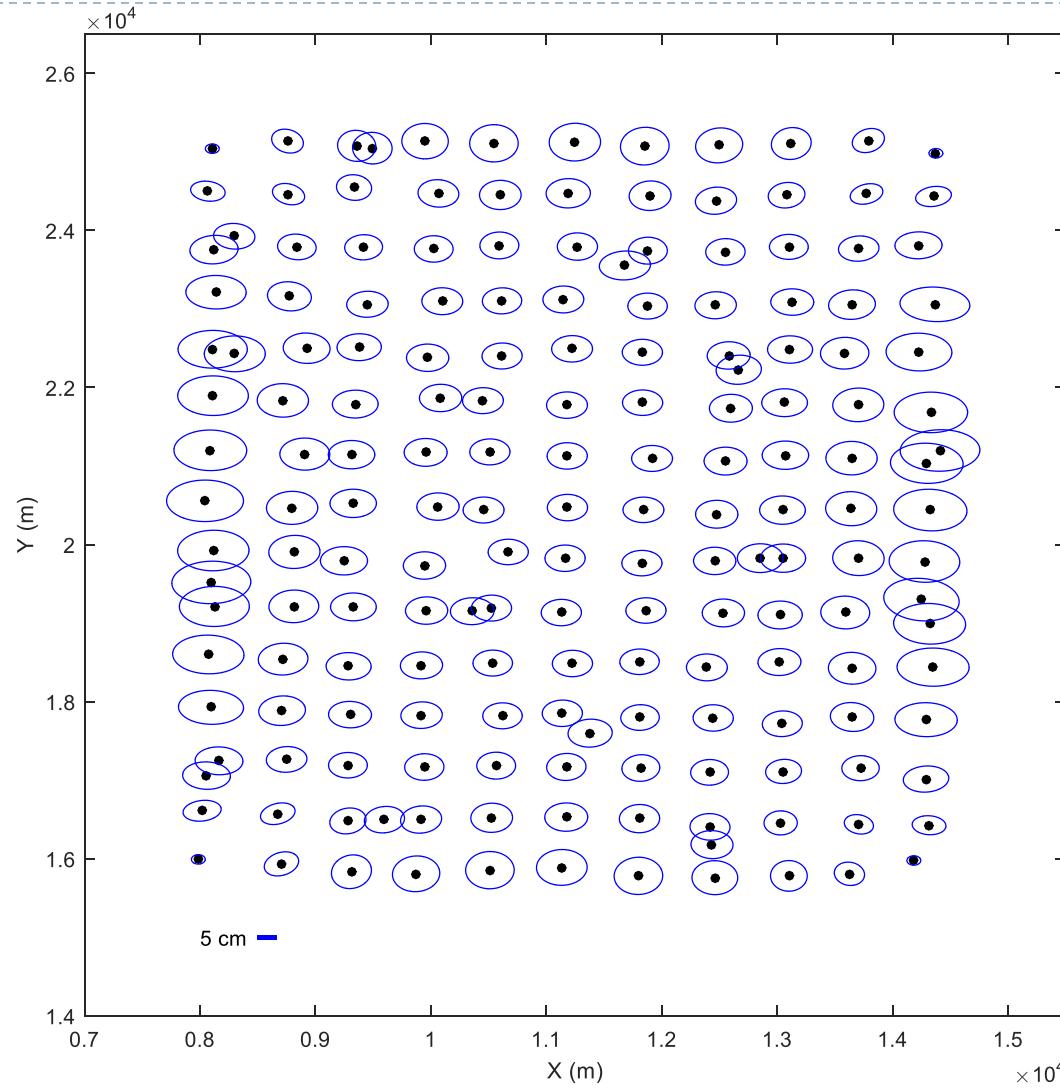
# Case 2—Full Ground Control Planimetric Precision



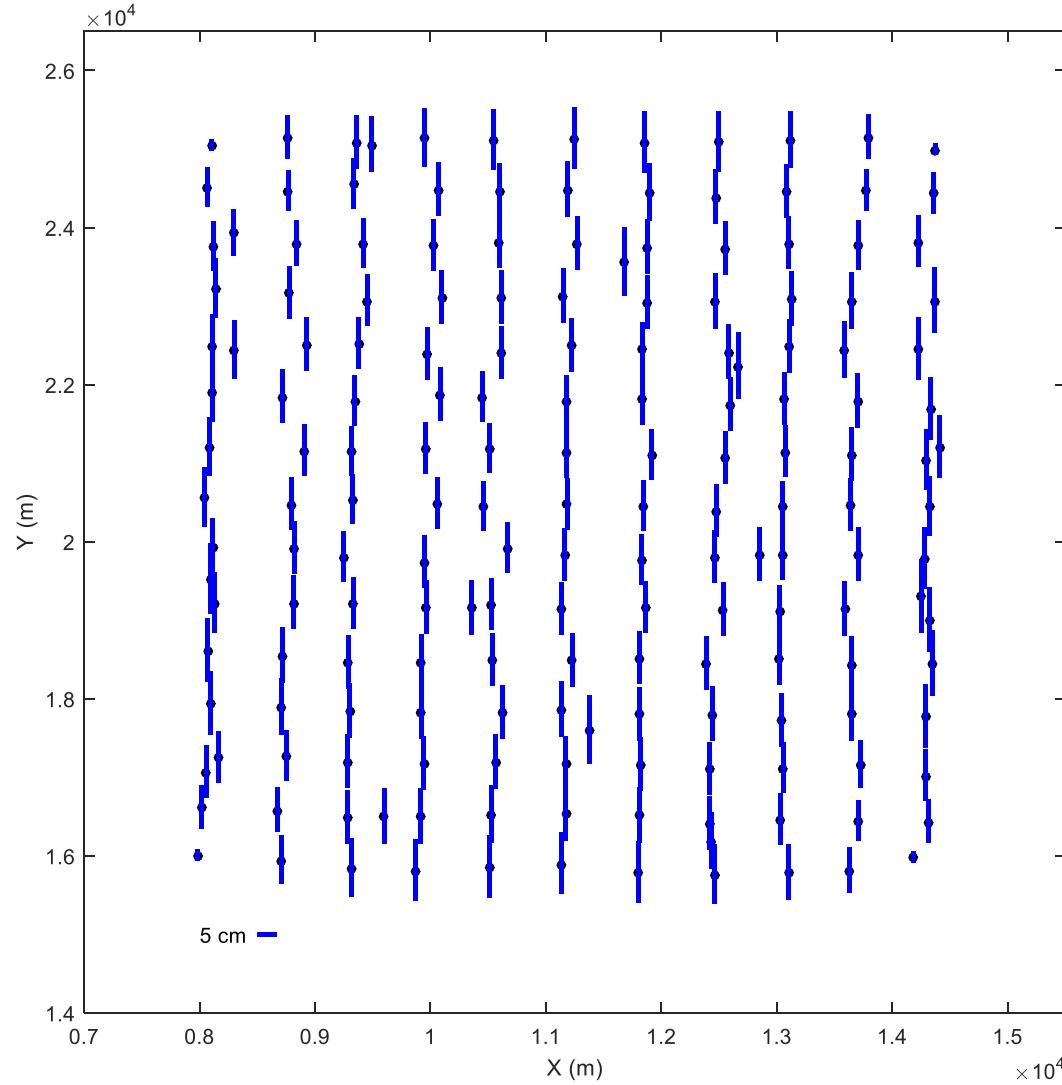
# Case 2—Full Ground Control Vertical Precision



# Case 3—Min Ground Control+Airborne GPS Planimetric Precision



# Case 3—Min Ground Control+Airborne GPS Vertical Precision



# Image Orientation Methods

## Indirect Georeferencing

1960s-1970s

- Georeferencing done by ground control points (GCPs)
- Control extended across the block by aerotriangulation (AT)

## GPS-AT

1980s-1990s

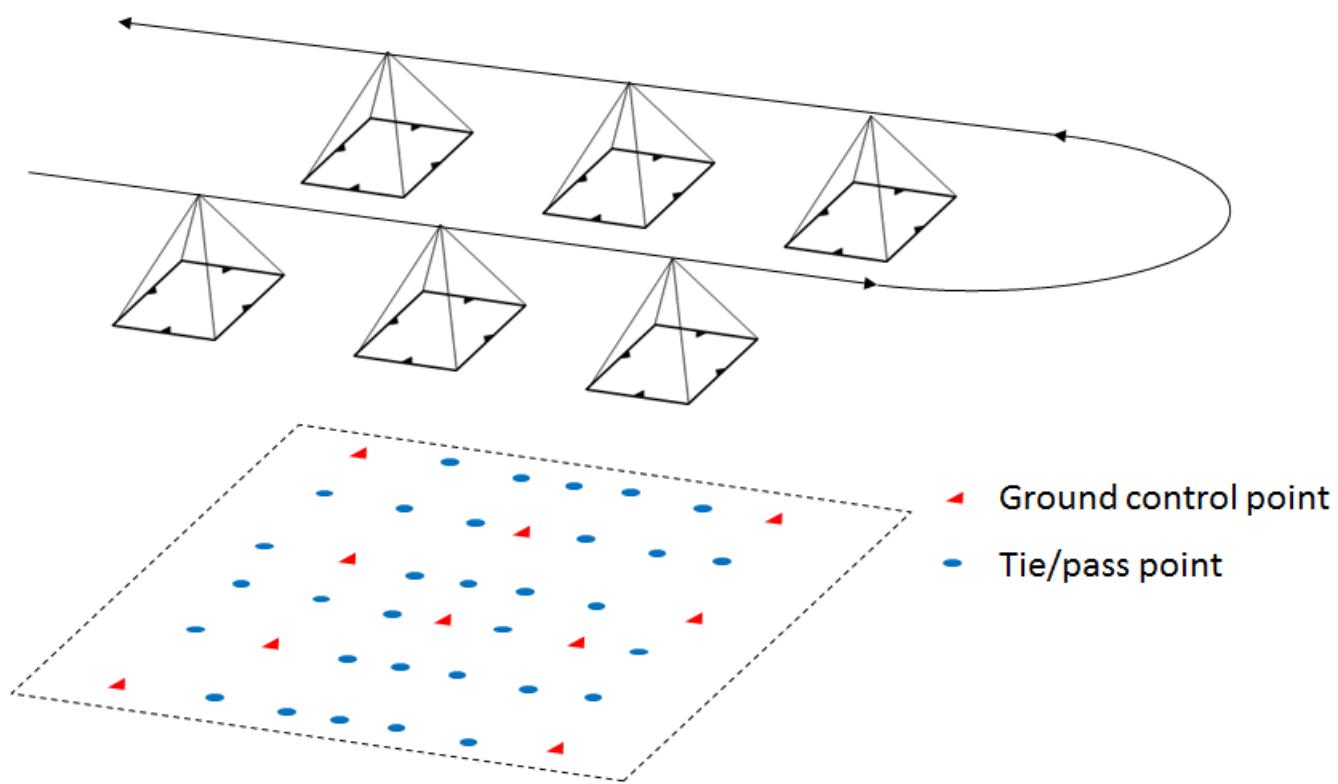
- The beginning of direct measurement of exterior orientation parameters
- GPS antenna used to observe the PC
- GCP requirements reduced to 4 per block

## Direct Georeferencing

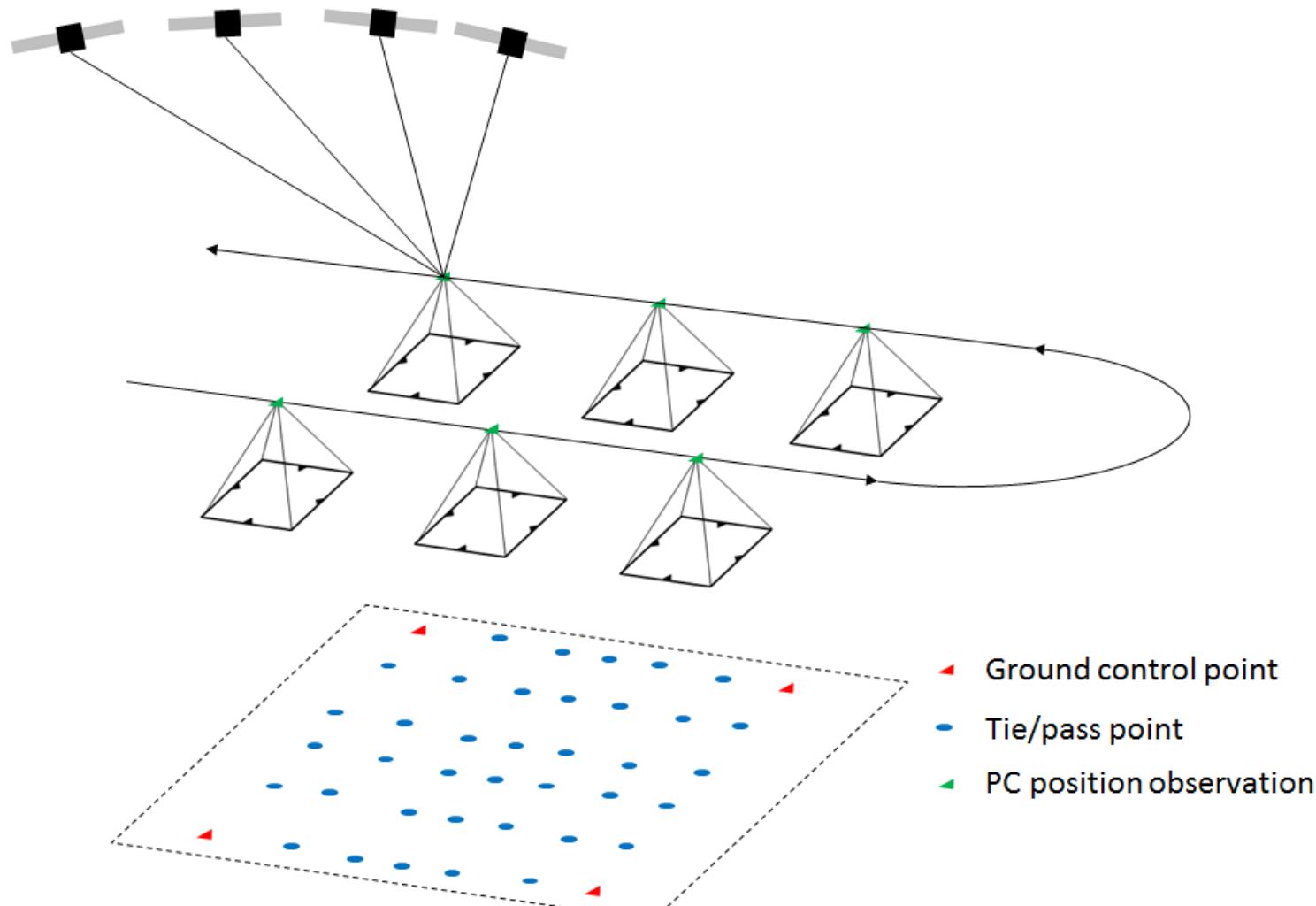
1990s onward

- Integrated GNSS/IMU systems to determine sensor position and attitude
- At least 2 modes
- Integrated sensor orientation (ISO)
  - Sensor position and attitude observations
  - Image point observations
  - Some ground control
  - Self-diagnostic quality control
  - Self-calibration possible
- Direct sensor orientation (DSO)
  - Sensor position and attitude observations only
  - Accuracy relies on GNSS/IMU alone

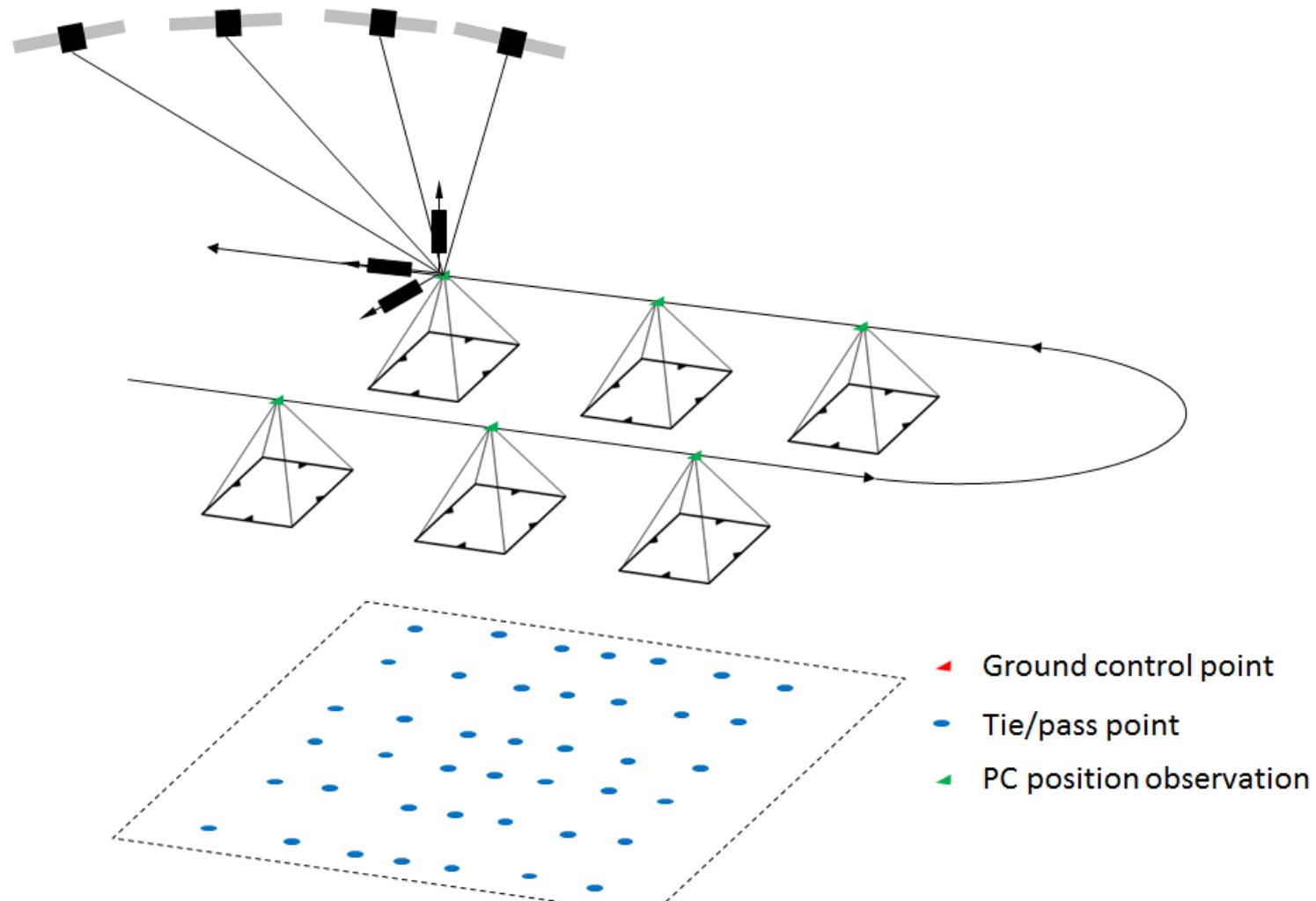
# Indirect Georeferencing



# GPS-AT



# Direct Georeferencing

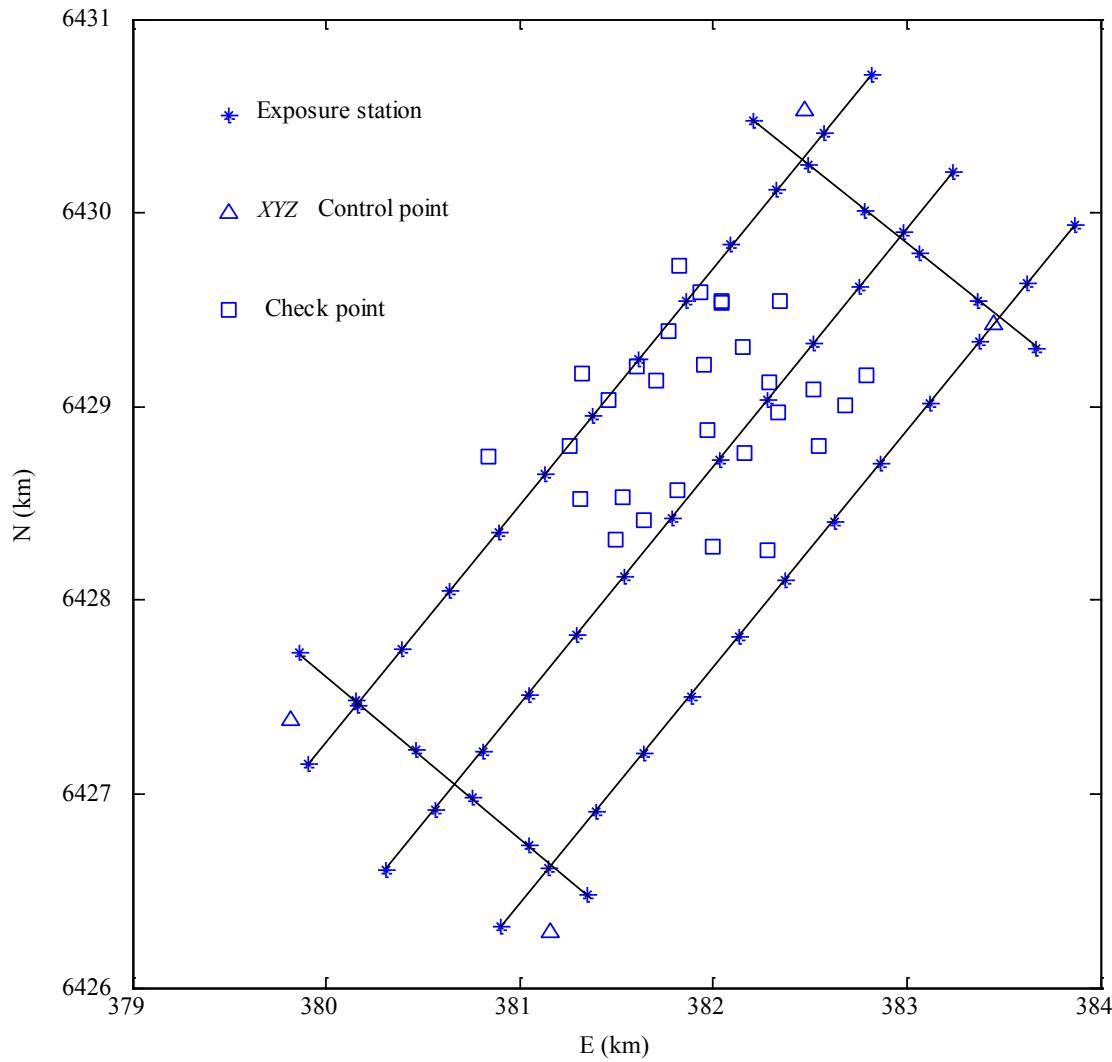


# Bundle Adjustment Example 3

---

- ▶ Rockingham block (GPS-AT example)
  - ▶ 51 images observed with GPS (EO parameter observations)
  - ▶ 3 strips and 2 cross-strips
  - ▶ 60% end lap and 20% side lap
  - ▶ Scale 1:5000
  - ▶ 4 weighted control points (co-ordinate parameter observations)
  - ▶ 238 pass/tie points
  - ▶ 934 image point observations
  - ▶ 29 check points (solved as tie points)
  - ▶ 5 sets of drift parameters to model errors in GPS positions (caused by incorrect integer ambiguities)

# Bundle Adjustment Example 3 (cont'd)

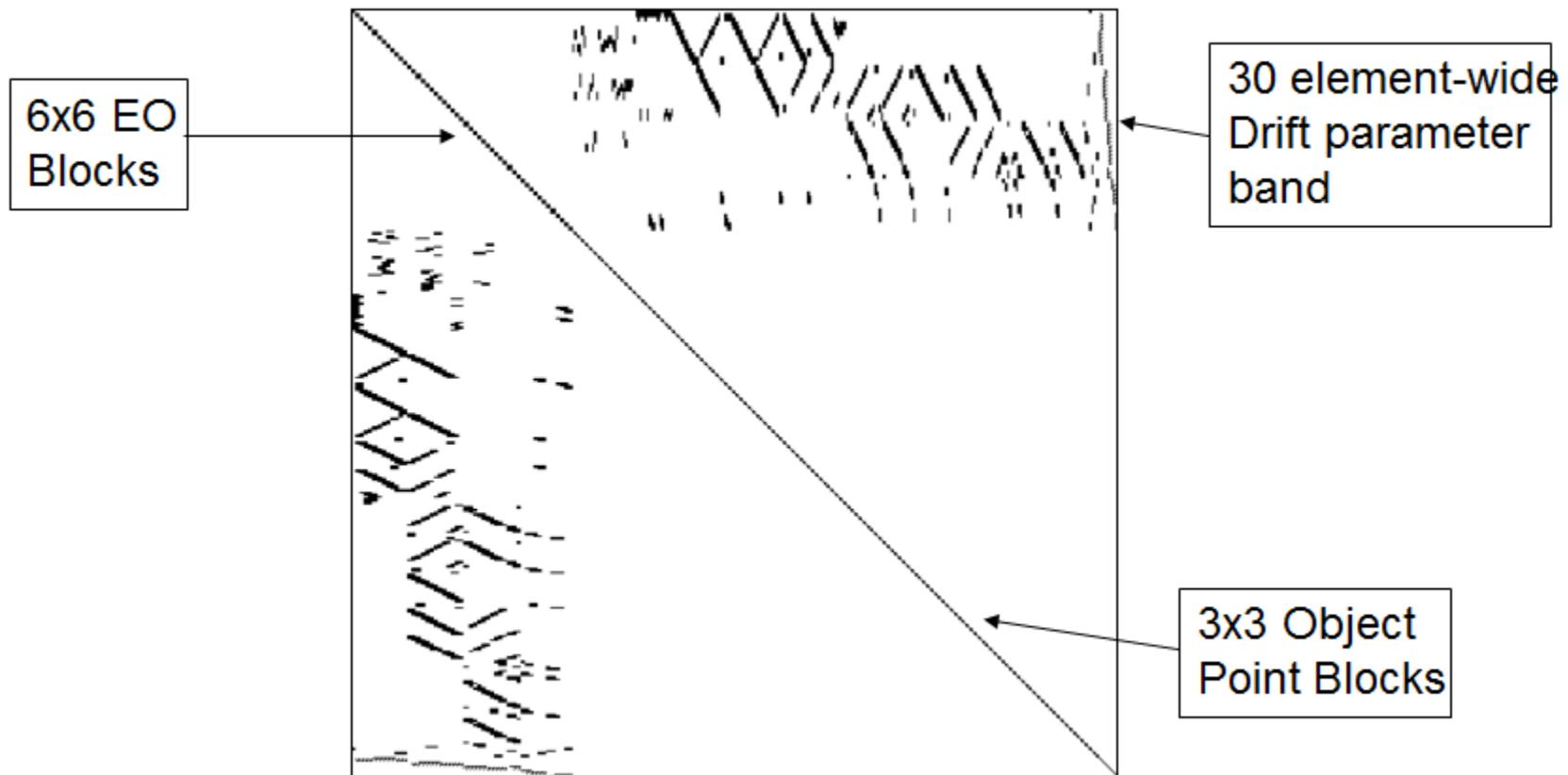


# Bundle Adjustment Example 3 (cont'd)

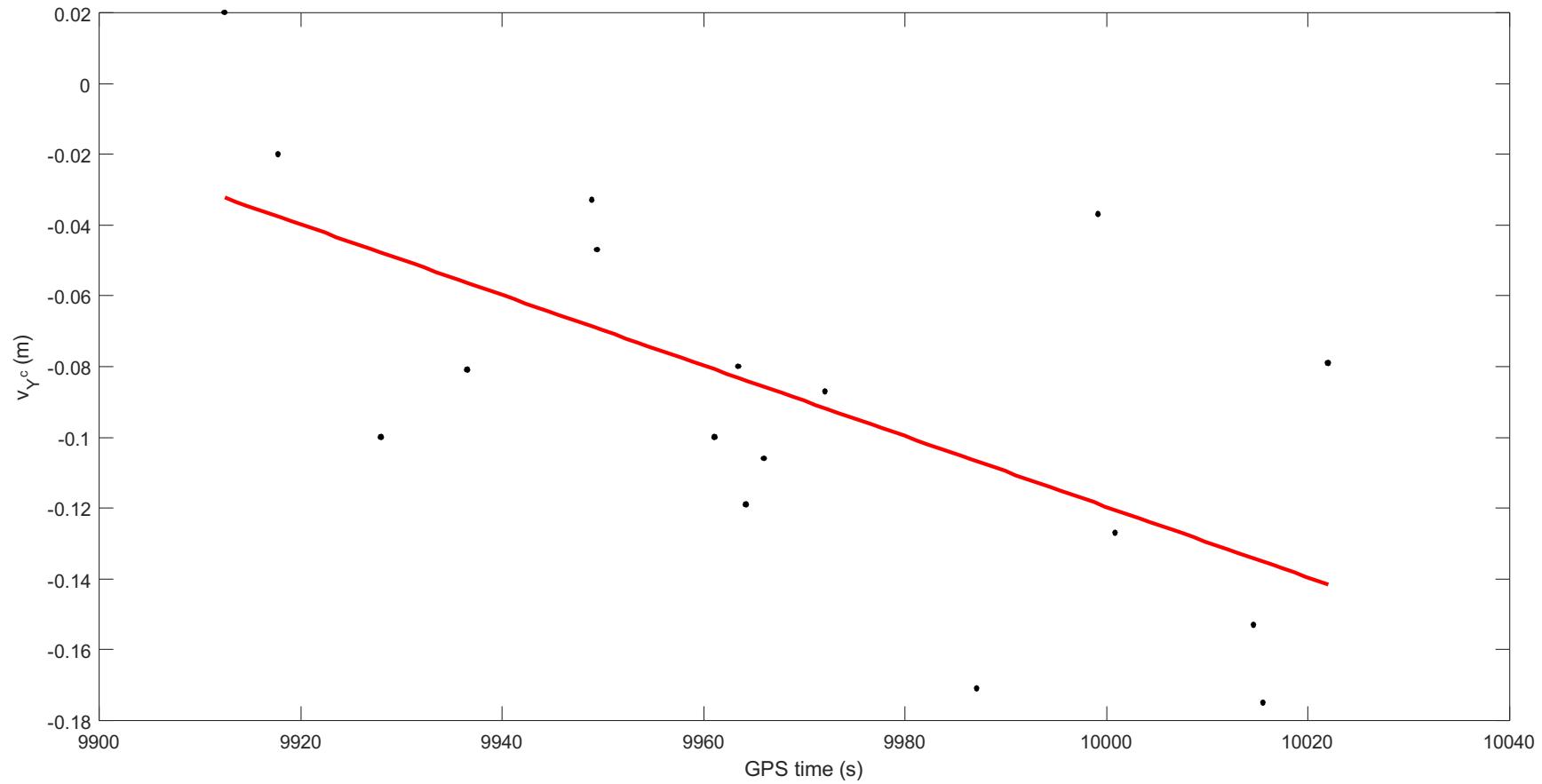
<b>Observations</b>			
GPS PCs	$\sigma = \pm 0.05 \text{ m}$	3 x 51	153
Image points	$\sigma = \pm 6 \mu\text{m}$	2 x 934	1868
Control points	$\sigma = \pm 0.05 \text{ m}$	3 x 4	12
<b>Total observations (n)</b>			<b>2033</b>
<b>Unknowns</b>			
PCs		6 x 51	306
Tie/pass points		3 x 238	714
Control points		3 x 4	12
Drift parameters		6 x 5	30
<b>Total Unknowns (u)</b>			<b>1062</b>
<b>Redundancy (r=n-u)</b>			<b>971</b>

# Bundle Adjustment Example 3 (cont'd)

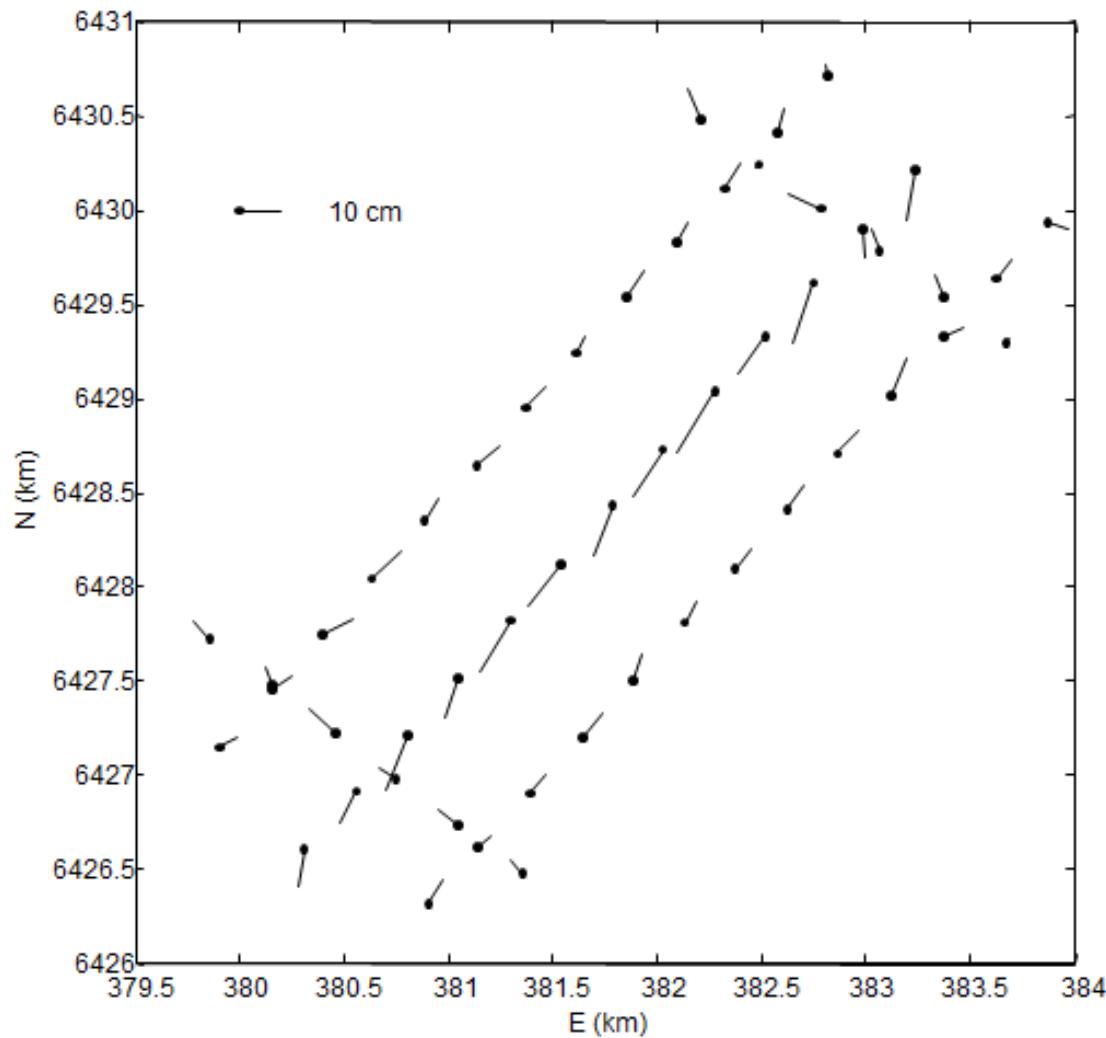
- ▶ Normal equations matrix ( $1062 \times 1062$ )—note sparse structure



# Drift vs Time (Single Strip)



# Drift Behaviour—Horizontal Component



# Bundle Adjustment Example 3 (cont'd)

---

## ▶ Results

### ▶ Mean object point precision

$$\sigma_x = \sigma_y = \pm 0.05m$$

$$\sigma_z = \pm 0.07m$$

### ▶ Check point difference RMS

$$RMS_x = \pm 0.03m$$

$$RMS_y = \pm 0.04m$$

$$RMS_z = \pm 0.09m$$

# Summary of Equations

---

## ▶ Observation equations

$$[A_e \quad A_o] \begin{bmatrix} \hat{\delta}_e \\ \hat{\delta}_o \end{bmatrix} + [w] = [\hat{v}]$$

$$\underset{n,u_e}{A_e} \underset{u_e,l}{\hat{\delta}_e} + \underset{n,u_o}{A_o} \underset{u_o,l}{\hat{\delta}_o} + w = \underset{n,l}{\hat{v}}$$

$$A_e = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots \\ \dots & \frac{\partial f_{x_{ij}}}{\partial X^c_j} & \frac{\partial f_{x_{ij}}}{\partial Y^c_j} & \frac{\partial f_{x_{ij}}}{\partial Z^c_j} & \frac{\partial f_{x_{ij}}}{\partial \omega_j} & \frac{\partial f_{x_{ij}}}{\partial \phi_j} & \frac{\partial f_{x_{ij}}}{\partial \kappa_j} \\ \dots & \frac{\partial f_{y_{ij}}}{\partial X^c_j} & \frac{\partial f_{y_{ij}}}{\partial Y^c_j} & \frac{\partial f_{y_{ij}}}{\partial Z^c_j} & \frac{\partial f_{y_{ij}}}{\partial \omega_j} & \frac{\partial f_{y_{ij}}}{\partial \phi_j} & \frac{\partial f_{y_{ij}}}{\partial \kappa_j} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$A_o = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \dots \\ \dots & \frac{\partial f_{x_{ij}}}{\partial X_i} & \frac{\partial f_{x_{ij}}}{\partial Y_i} & \frac{\partial f_{x_{ij}}}{\partial Z_i} & \dots \\ \dots & \frac{\partial f_{y_{ij}}}{\partial X_i} & \frac{\partial f_{y_{ij}}}{\partial Y_i} & \frac{\partial f_{y_{ij}}}{\partial Z_i} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

## ▶ Normal equations

$$A^T P A \hat{\delta} + A^T P w = 0$$

$$\begin{bmatrix} A_e^T P A_e & A_e^T P A_o \\ \text{sym.} & A_o^T P A_o \end{bmatrix} \begin{bmatrix} \hat{\delta}_e \\ \hat{\delta}_o \end{bmatrix} + \begin{bmatrix} A_e^T P w \\ A_o^T P w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} A_e^T P A_e + P_e & A_e^T P A_o \\ \text{sym.} & A_o^T P A_o + P_o \end{bmatrix} \begin{bmatrix} \hat{\delta}_e \\ \hat{\delta}_o \end{bmatrix} + \begin{bmatrix} A_e^T P w + P_e w_e \\ A_o^T P w + P_o w_o \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$