CIVE 6393 – Geostatistics
Linear Least Squares
Module 7

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- Suppose m independent equally weighted measurements z_1, z_2, z_3, …. z_n of the same quantity that has a most probable value (MPV) of M. The residuals are:

- The residual errors are normally distributed. Recall normal distribution defined as:

- Probabilities are represented by areas under the normal distribution curve. Thus the probability of each residual is zero and must be multiplied by some small (infinitesimal) increment of v (Δv) to generate an area, which gives us probabilities of:

- Probability of two or more independent events occurring is a product of their individual probabilities, therefore residual probability is:

- Going back to z measurements, we want MPV of M. Alternatively, is also finding MPV of residuals (which is a maximum value for P).

- To maximize P, we must minimize

- This is the sum of squares of the residuals → Principle of Least Squares

- i.e. the MPV for a quantity obtained from repeated measurements of equal weight is the value that renders the sum of squared residuals to a minimum.

- Recall we can minimize S.O.S. by taking first derivative w.r.t. unknown value (M) and setting equal to zero:

- Example:

- Example Continued:
  - We need to minimize f(x,y) by taking the derivatives and setting equal to zero. We have two unknowns, so we end up with two equations:

- Recap:
- We showed how maximizing the probabilities of the residuals leads to a minimization of the S.O.S. residuals (mean of several measurements).

- Recap:
- We solved a system of 3 linear equations by manually computing the min. of the S.O.S. of residuals.

- NEXT UP: How can we replace manual equations with matrices and linear algebra?

- Consider a generic set of 3 equations with two unknowns:

- Put the original equations in Matrix and Vector Form:

- Consider our original example:

- Generic Form of our Linear System of Equations

- Why are They Called The Normal Equations?