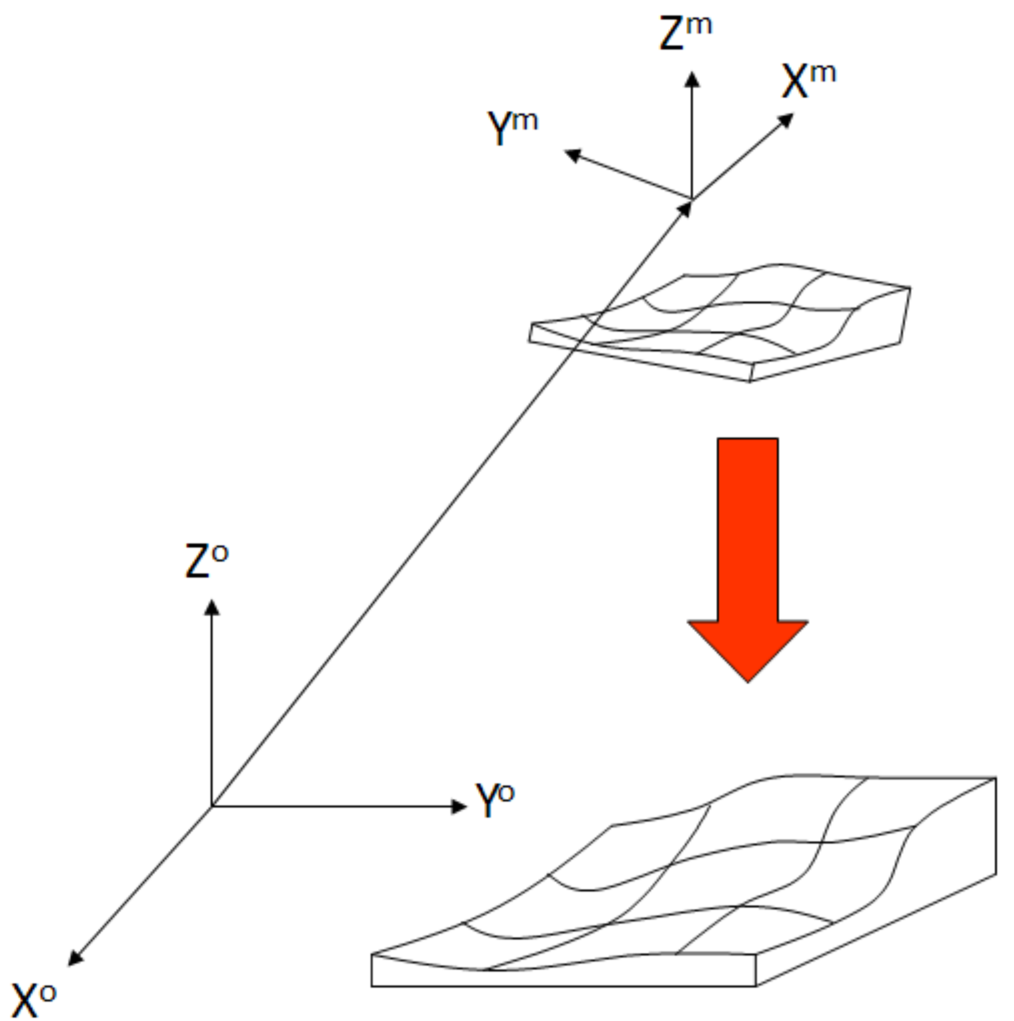
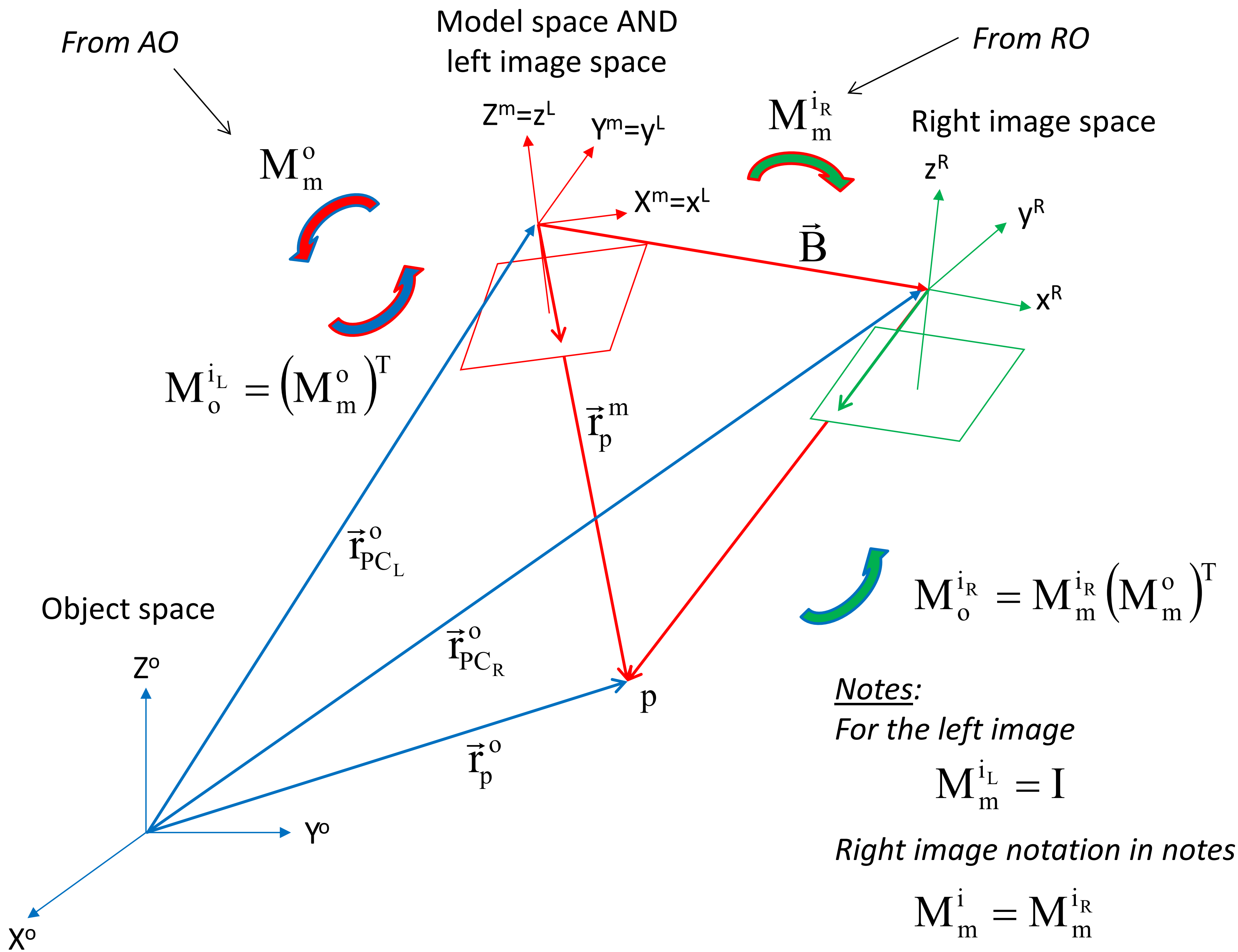
Absolute Orientation




Model space AND
left image space
From AO
From RO
$M_m^{i_R}$
Right image space
$Z^m=z^L$
$Y^m=y^L$
$X^m=x^L$
M_m^o
z^R
y^R
x^R
$\vec{B}$
$M_o^{i_L} = (M_m^o)^T$
$\vec{r}_p^{\,m}$
$\vec{r}_{PC_L}^{\,o}$
Object space
$\vec{r}_{PC_R}^{\,o}$
$M_o^{i_R} = M_m^{i_R}(M_m^o)^T$
Z^o
p
Notes:
For the left image
$M_m^{i_L} = I$
$\vec{r}_p^{\,o}$
Y^o
Right image notation in notes
$M_m^i = M_m^{i_R}$
X^o

AO Example

- Model and known object space co-ordinates

id	Xm (mm)	Ym (mm)	Zm (mm)	Xg (m)	Yg (m)	Zg (m)
30	108.9302	92.5787	-155.7696	7350.27	4382.54	276.42
40	19.5304	96.0258	-156.4878	6717.22	4626.41	280.05
72	71.8751	4.9657	-154.1035	6869.09	3844.56	283.11
127	-0.9473	-7.4078	-154.8060	6316.06	3934.63	283.03
112	9.6380	-96.5329	-158.0535	6172.84	3269.45	248.10
50	100.4898	-63.9177	-154.9389	6905.26	3279.84	266.47

- Residuals and estimated transformation parameters

id	vX (m)	vY (m)	vZ (m)
30	-0.015	-0.205	0.048
40	-0.109	0.307	-0.158
72	0.063	-0.145	-0.044
127	0.044	-0.073	0.278
112	0.067	-0.002	-0.151
50	-0.050	0.117	0.027
RMS	0.065	0.172	0.147

tX (m)	6349.551
tY (m)	3964.645
tZ (m)	1458.114
omega (dd)	-0.824127
phi (dd)	-0.717738
kappa (dd)	18.891137
lambda	7.585632

AO Example (cont'd)

- Transformed object points

id	Xo (m)	Yo (m)	Zo (m)
30	7350.255	4382.335	276.468
40	6717.111	4626.717	279.892
72	6869.153	3844.415	283.066
127	6316.104	3934.557	283.308
112	6172.907	3269.448	247.949
50	6905.210	3279.957	266.497

- Transformed PCs

Image	Xm (mm)	Ym (mm)	Zm (mm)	Xo (m)	Yo (m)	Zo (m)
Left	0.00	0.00	0.00	6349.551	3964.645	1458.114
Right	92.0000	5.0455	2.1725	7022.302	3774.625	1466.399

AO Example (cont'd)

- Angles and matrices $\left(M_m^i\right)$ from RO

Left	w (dd)	0
	p (dd)	0
	k (dd)	0

M	1	0	0
	0	1	0
	0	0	1

Right	w (dd)	0.4392
	p (dd)	1.5080
	k (dd)	3.1575

M	0.9981	0.0553	-0.0259
	-0.0551	0.9984	0.0091
	0.0263	0.0077	0.9996

- Matrices $\left(M_m^o\right)$ from AO

M	0.9461	0.3239	0.0072
	-0.3237	0.9460	-0.0177
	-0.0125	0.0144	0.9998

M trans	0.9461	-0.3237	-0.0125
	0.3239	0.9460	0.0144
	0.0072	-0.0177	0.9998

AO Example (cont'd)

- Resultant matrix products $\left(M_o^i\right)$

Left M	0.9461	-0.3237	-0.0125
	0.3239	0.9460	0.0144
	0.0072	-0.0177	0.9998

Right M	0.9620	-0.2704	-0.0376
	0.2714	0.9622	0.0242
	0.0296	-0.0334	0.9990

- Extracted angles

Left	w (dd)	1.0121
	p (dd)	0.4122
	k (dd)	-18.8999

Right	w (dd)	1.9164
	p (dd)	1.6966
	k (dd)	-15.7533

Summary of AO Equations

- Observation equations

$$\vec{r}_i^{\,o} = \Lambda M \vec{r}_i^{\,m} + \vec{t}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_i^o = \Lambda \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_i^m + \begin{pmatrix} t_X \\ t_Y \\ t_Z \end{pmatrix} \qquad M = R_3(K) R_2(\Phi) R_1(\Omega)$$

- Point transformation $\vec{r}_i^{\,o} = \hat{\Lambda} \hat{M} \vec{r}_i^{\,m} + \hat{\vec{t}}$
- PC transformation $\vec{r}_{PC}^{\,o} = \hat{\Lambda} \hat{M} \vec{r}_{PC}^{\,m} + \hat{\vec{t}}$
  - Left $\vec{r}_{PC}^{\,o} = \hat{\vec{t}}$
  - Right $\vec{r}_{PC}^{\,o} = \hat{\Lambda} \hat{M} \vec{B} + \hat{\vec{t}}$
- Rotation matrices

$$\begin{aligned} M_o^i &= M_m^i M_o^m \\ &= M_m^i {M_m^o}^T \end{aligned}$$

$$\begin{aligned} M_m^i &= R_3(\kappa_R) R_2(\phi_R) R_1(\omega_R) \\ M_m^o &= R_3(K) R_2(\Phi) R_1(\Omega) \end{aligned}$$

AO Partial Derivatives

- Partial derivatives of X°

$$\frac{\partial X^o}{\partial \Omega} = \lambda^0 Y^m\left(-\sin\Omega^0 \sin K^0 + \cos\Omega^0 \sin\Phi^0 \cos K^0\right) + \lambda^0 Z^m\left(\cos\Omega^0 \sin K^0 + \sin\Omega^0 \sin\Phi^0 \cos K^0\right)$$

$$\frac{\partial X^o}{\partial \Phi} = -\lambda^0 X^m \sin\Phi^0 \cos K^0 + \lambda^0 Y^m \sin\Omega^0 \cos\Phi^0 \cos K^0 - \lambda^0 Z^m \cos\Omega^0 \cos\Phi^0 \cos K^0$$

$$\frac{\partial X^o}{\partial K} = -\lambda^0 X^m \cos\Phi^0 \sin K^0 + \lambda^0 Y^m\left(\cos\Omega^0 \cos K^0 - \sin\Omega^0 \sin\Phi^0 \sin K^0\right) + \lambda^0 Z^m\left(\sin\Omega^0 \cos K^0 + \cos\Omega^0 \sin\Phi^0 \sin K^0\right)$$

$$\frac{\partial X^o}{\partial \lambda} = X^m m_{11}^0 + Y^m m_{12}^0 + Z^m m_{13}^0$$

$$\frac{\partial X^o}{\partial t_X} = 1$$

$$\frac{\partial X^o}{\partial t_Y} = 0$$

$$\frac{\partial X^o}{\partial t_Z} = 0$$

AO Partial Derivatives (cont'd)

- Partial derivatives of Y°

$$\frac{\partial Y^o}{\partial \Omega} = \lambda^0 Y^m\left(-\sin\Omega^0\cos K^0 - \cos\Omega^0\sin\Phi^0\sin K^0\right) + \lambda^0 Z^m\left(\cos\Omega^0\cos K^0 - \sin\Omega^0\sin\Phi^0\sin K^0\right)$$

$$\frac{\partial Y^o}{\partial \Phi} = \lambda^0 X^m\sin\Phi^0\sin K^0 - \lambda^0 Y^m\sin\Omega^0\cos\Phi^0\sin K^0 + \lambda^0 Z^m\cos\Omega^0\cos\Phi^0\sin K^0$$

$$\frac{\partial Y^o}{\partial K} = -\lambda^0 X^m\cos\Phi^0\cos K^0 + \lambda^0 Y^m\left(-\cos\Omega^0\sin K^0 - \sin\Omega^0\sin\Phi^0\cos K^0\right) + \lambda^0 Z^m\left(-\sin\Omega^0\sin K^0 + \cos\Omega^0\sin\Phi^0\cos K^0\right)$$

$$\frac{\partial Y^o}{\partial \lambda} = X^m m_{21}^0 + Y^m m_{22}^0 + Z^m m_{23}^0$$

$$\frac{\partial Y^o}{\partial t_X} = 0$$

$$\frac{\partial Y^o}{\partial t_Y} = 1$$

$$\frac{\partial Y^o}{\partial t_Z} = 0$$

AO Partial Derivatives (cont'd)

- Partial derivatives of Z^o

$$\frac{\partial Z^o}{\partial \Omega} = -\lambda^0 Y^m \cos\Omega^0 \cos\Phi^0 - \lambda^0 Z^m \sin\Omega^0 \cos\Phi^0$$

$$\frac{\partial Z^o}{\partial \Phi} = \lambda^0 X^m \cos\Phi^0 + \lambda^0 Y^m \sin\Omega^0 \sin\Phi^0$$
$$- \lambda^0 Z^m \cos\Omega^0 \sin\Phi^0$$

$$\frac{\partial Z^o}{\partial K} = 0$$

$$\frac{\partial Z^o}{\partial \lambda} = X^m m_{31}^0 + Y^m m_{32}^0 + Z^m m_{33}^0$$

$$\frac{\partial Z^o}{\partial t_X} = 0$$

$$\frac{\partial Z^o}{\partial t_Y} = 0$$

$$\frac{\partial Z^o}{\partial t_Z} = 1$$