**Final Exam**

Question 1

1. If the control points are treated as known (constants), then define the total number of observations (n), the number of unknowns (u), and the redundancy (r) of the adjustment.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Observations** | | | **Unknowns** | | |
| 3D GCP | 1 x 3 x 2 | 6 | Images | 6 x 8 | 48 |
| HCP | 2 x 2 x 3 | 12 | Tie points | 27 x 3 | 81 |
| VCP | 2 x 2 x 2 | 8 | HCP | 2 x 1 | 2 |
| Tie Points | 15 x 2 x 2 | 60 | VCP | 2 x 2 | 4 |
| Tie Points | 10 x 3 x 2 | 60 | **Total Unknowns (u)** | | **135** |
| Tie Points | 2 x 4 x 2 | 16 | **Redundancy (r = n-u)** | | **27** |
| **Total Observations (n)** | | **162** |  |  |  |

1. Recompute n, u, and r if we have GPS estimates of the perspective centers at image exposure, and both the GPS PC coordinates and the control points are treated as weighted coordinate parameter observations.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Observations** | |  | **Unknowns** |  |  |
| 3D GCP | 1 x 3 x 2 | 6 | Images | 6 x 8 | 48 |
| HCP | 2 x 3 x 2 | 12 | Tie points | 27 x 3 | 81 |
| VCP | 2 x 2 x 2 | 8 | HCP | 2 x 3 | 6 |
| Tie Points | 15 x 2 x 2 | 60 | VCP | 2 x 3 | 6 |
| Tie Points | 10 x 3 x 2 | 60 | 3D GCP | 1 x 3 | 3 |
| Tie Points | 2 x 4 x 2 | 16 | **Total Unknowns (u)** | | **144** |
| 3D GCP | 1 x 3 | 3 | **Redundancy (r = n-u)** | | **51** |
| HCP | 2 x 2 | 4 |  |  |  |
| VCP | 1 x 2 | 2 |  |  |  |
| GPS PCs | 3 x 8 | 24 |  |  |  |
| **Total Observations (n)** |  | **195** |  |  |  |

Question 2

1. R = R3\*R2\*R1

|  |  |
| --- | --- |
| **R = R3\*R2\*R1** | |
| ω (°) | -38.6244 |
| ϕ (°) | 35.003 |
| κ (°) | -152.6317 |

1. R = R1\*R2\*R3

|  |  |
| --- | --- |
| **R = R1\*R2\*R3** | |
| ω (°) | 28.5601 |
| ϕ (°) | -43.2307 |
| κ (°) | -176.761 |

1. R = R3\*R1\*R2

|  |  |
| --- | --- |
| **R = R3\*R1\*R2** | |
| ω (°) | -30.7509 |
| ϕ (°) | 41.8718 |
| κ (°) | -177.2544 |

Question 3

1. Determine the most appropriate calibration model for the camera from the options

available in MATLAB (do NOT use the skew correction). The most appropriate model

should be the minimum amount of parameters without sacrificing accuracy. Justify your

selection of the final distortion model. Provide the distortion parameters AND their

estimated errors. Transform the principal point solution from a digital image to a fiducial

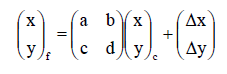
coordinate system.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Camera Model #1** | | **3 Coefficients** | | |
| **Radial Distortion** | | **Tangential Distortion** | | |
| K0 | -0.20376 ± 0.00704 | P1 | -0.00200 ± 0.00014 | |
| K1 | 0.12836 ± 0.08631 | P2 | -0.00062 ± 0.00014 | |
| K2 | 0.42631 ± 0.31554 |  | |  |
| Mean Reprojection Error | | 0.54178 | | |
|  |  |  |  | |
| **Camera Model #2** | | **2 Coefficients** | | |
| **Radial Distortion** | | **Tangential Distortion** | | |
| K0 | -0.21248 ± 0.00282 | P1 | -0.00200 ± 0.00014 | |
| K1 | 0.24302 ± 0.01561 | P2 | -0.00062 ± 0.00014 | |
| Mean Reprojection Error | | 0.54241 | | |
|  |  |  |  | |
| Removed 2 Images | | | | |
| **Camera Model # 3** | | **3 Coefficients** | | |
| **Radial Distortion** | | **Tangential Distortion** | | |
| K0 | -0.20190 ± 0.00673 | P1 | -0.00203 ± 0.00013 | |
| K1 | 0.11866 ± 0.08167 | P2 | -0.00074 ± 0.00013 | |
| K2 | 0.43733 ± 0.29608 |  | |  |
| Mean Reprojection Error | | 0.50064 | | |
|  |  |  |  | |
| **Camera Model # 4** | | **2 Coefficients** | | |
| **Radial Distortion** | | **Tangential Distortion** | | |
| K0 | -0.21102 ± 0.00268 | P1 | -0.00203 ± 0.00013 | |
| K1 | 0.23732 ± 0.00013 | P2 | -0.00073 ± 0.00013 | |
| Mean Reprojection Error | | 0.50138 | | |

Camera Model # 4 is the most appropriate calibration model for the camera since it uses only 2 coefficients for radial distortion. Simply comparing 2 vs 3 coefficients had no change in the Mean Projection Error so accuracy is not sacrificed by removing a coefficient. In addition, removing 2 images that produced the largest pixel mean error in the Camera Calibrator app decreased the Mean Projection Error. Therefore, Camera Model #4 is the most appropriate model.

|  |  |  |  |
| --- | --- | --- | --- |
| **PrincipalPoint** | | **Fiducial Coordinate System** | |
| **xp (cm)** | **yp (cm)** | **xf (cm)** | **yf (cm)** |
| 1277.369254 | 909.1294 | 15.2081 | 8.99 |

From the Camera Calibrator App, we export the cameraParams and used the WorldPoints as fiduciary coordinates and one of the image’s ReprojectedPoints as image points. We then use Affine Transformation for 54 points from each set to calculate a, b, c, d, delta x, and delta y in:



Then, we use the PrincipalPoints from cameraParams for xc and yc, to obtain the fiducial coordinate system.

1. Are there any calibration images that appear to be outliers? If so, why do you think those image(s) are causing larger measurement errors? Do the camera calibration parameters estimated change significantly if you remove the outlier image(s)? Justify why you would characterize the change in parameters as significant/not significant (Hint: the significance level may be different for different distortion parameters).

Img\_0371.JPG seems like an outlier since the Mean Error in Pixels on the Reprojection Error graph of this image is extremely high compared to the other images. The larger measurements error may be due to the quality of the image, the blurrier the image, the higher the error. The corners of the squares are more pronounced on the clearer images. Image\_0367.JPG is also removed for Camera Models #3 and #4 with a total of 2 images removed for a smaller Mean Reprojection Error. Removing the 2 images did not change the tangential distortion significantly since the changes are within range of the estimated errors of the original tangential distortion parameters. Even though there is some change in parameters in radial distortion when removing these 2 images, they are still not significant because the new parameters are still relatively close to the original parameters and its estimated errors.

1. The Matlab calibration model returns two values for the focal length. Briefly discuss why two values are returned, and how these would be used in the collinearity equations (this may require some literature review).

**Focal Length = [26462 ± 1.4383, 26462 ± 1.4375]**

The two focal lengths are relatively the same, which just their estimated errors differing. Two focal length values can mean that this camera uses two focal lengths for the horizontal and vertical axes, which causes non-square pixels. However, even though there are two focal lengths, the value is the same, which means this is still a square-pixel camera.

Question 4

Using a bundle adjustment, derive exterior orientation parameters for each of the cameras and estimate ground coordinates for the pass points. Compute estimated accuracy for both EOPs and pass point coordinates.

Additional Questions:

1. Are any of the image measurements outliers? If yes, rerun the adjustment with those image observation(s) removed. Does it make a difference in the final solution?
2. Is the estimated accuracy given for the image measurements correct? Based on the adjustment results, is the actual image measurement accuracy lower or higher? Justify your answer.
3. The estimated standard deviation for the final coordinates of one of the pass points is significantly higher than the others. Which point is it? Why do you think the estimated accuracy of this point is worse than the others?