RBE501 Week 2 Assignment
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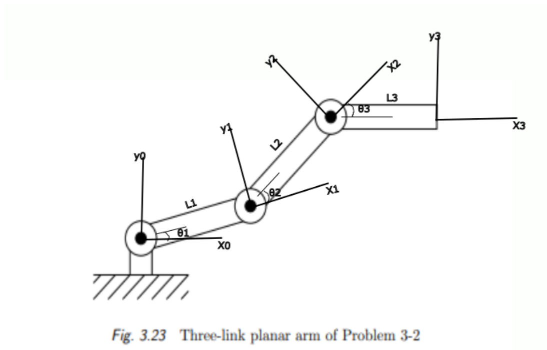

Fig. 1. Diagram for Problem 3-2 [1]

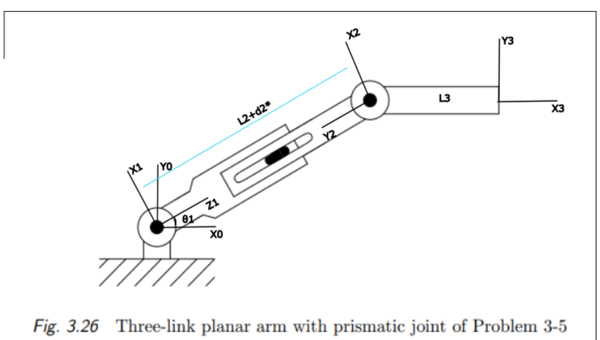

Fig. 2. Diagram for Problem 3-5 [1]

I. Introduction

The assignment consisted of two problems: 3-2, 3-5. The objective of problem 3-2 was to determine the forward kinematics equations of the robot seen in figure 1, without using DH parameters.
The objective of problem 3-5 was to determine the forward kinematic equations of the robot seen in figure 2 while using the DH parameters and convention.

II. Materials and Methods

A. Problem 3-2

In figure 1 the Z axes all point out of the page, and the orientation of the frames only changes with a rotation around the Z axis. Therefore the forward kinematics can easily be determined by a Z rotation (eq 1), and a translation (eq 2) multiplied together as seen in is eq 3. Then for each link you can replace θ with the equivalent variable (i.e. $\theta_1\theta_2\theta_3$) and L with the appropriate link length (i.e. L1,L2,L3). Then you multiply each transformation matrix together as seen in eq 4.

$$H_z = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

$$H_{trans} = \begin{pmatrix} 1 & 0 & 0 & L\cos(\theta) \\ 0 & 1 & 0 & L\sin(\theta) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

$$H_j^i = H_{trans} * H_z = \begin{pmatrix} \cos(\theta_j) & -\sin(\theta_j) & 0 & L_j\cos(\theta_j) \\ \sin(\theta_j) & \cos(\theta_j) & 0 & L_j\sin(\theta_j) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

$$H_3^0 = H_1^0 * H_2^1 * H_3^2 \tag{4}$$

B. Problem 3-5

DH parameters are a convention for notating the transformation between two frames. These can be defined by four parameters: θ, d, a, α. Then they can be substituted into a standard matrix (eq **??**) to create the transformation between frames. The difficulty of this problem is the proper frame assignments. As seen in figure 2, The Z axis changes orientation between every frame. For the prismatic joint, the z axis has to be along the axis of motion for the DH parameters to work. To simplify the number of frames. Frame 1 stays at the same origin as frame 0, and then frame 2 contains both the length of link 2 and the actuation parameter d2. The DH parameters can be seen in table I. These can be inserted into the homogeneous transformation matrix (equation 5), and multiplied together as seen in equation 4 for the forward kinematics.

TABLE I
DH PARAMETERS FOR PROBLEM 3-5

θ	d	a	α
θ_1+ 90°	0	0	90°
0	$d_2 + L_2$	0	-90°
θ_3−90°	0	L_3	0

$$\mathrm{H}_i^j = \begin{pmatrix} \cos(\theta) & -\cos(\alpha)\sin(\theta) & \sin(\alpha)\sin(\theta) & a\cos(\theta) \\ \sin(\theta) & \cos(\alpha)\cos(\theta) & -\sin(\alpha)\cos(\theta) & a\sin(\theta) \\ 0 & \sin(\alpha) & \cos(\alpha) & d \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{5}$$

III. RESULTS

A. Problem 3-2

The results were acquired by entering the equations, previously described, into MATLAB using symbolic variables. The results can be seen in equation 6.

$$\begin{pmatrix} \sigma_1 & -\cos(\theta_3)\,\sigma_3 - \sin(\theta_3)\,\sigma_2 & 0 & L_1\cos(\theta_1) + L_3\cos(\theta_3)\,\sigma_2 - L_3\sin(\theta_3)\,\sigma_3 + L_2\cos(\theta_1)\cos(\theta_2) - L_2\sin(\theta_1)\sin(\theta_2) \\ \cos(\theta_3)\,\sigma_3 + \sin(\theta_3)\,\sigma_2 & \sigma_1 & 0 & L_1\sin(\theta_1) + L_3\cos(\theta_3)\,\sigma_3 + L_3\sin(\theta_3)\,\sigma_2 + L_2\cos(\theta_1)\sin(\theta_2) + L_2\cos(\theta_2)\sin(\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \cos(\theta_3)\,\sigma_2 - \sin(\theta_3)\,\sigma_3$$

$$\sigma_2 = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$$

$$\sigma_3 = \cos(\theta_1)\sin(\theta_2) + \cos(\theta_2)\sin(\theta_1) \tag{6}$$

B. Problem 3-5

Similarly to problem 3-2, problem 3-5 was solved by using MATLAB's symbolic math functionality. The results can be seen in equation 7.

$$\begin{pmatrix} \sigma_1 & -\sigma_5\,\sigma_2 - \sigma_3\,\sigma_4 & 0 & \sigma_2\,(L_2 + d_2) + L_3\,\sigma_5\,\sigma_3 - L_3\,\sigma_4\,\sigma_2 \\ \sigma_5\,\sigma_2 + \sigma_3\,\sigma_4 & \sigma_1 & 0 & L_3\,\sigma_5\,\sigma_2 - \sigma_3\,(L_2 + d_2) + L_3\,\sigma_3\,\sigma_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \sigma_5\,\sigma_3 - \sigma_4\,\sigma_2$$

$$\sigma_2 = \sin\left(\theta_1 + \frac{\pi}{2}\right)$$

$$\sigma_3 = \cos\left(\theta_1 + \frac{\pi}{2}\right)$$

$$\sigma_4 = \sin\left(\theta_1 - \frac{\pi}{2}\right)$$

$$\sigma_5 = \cos\left(\theta_1 - \frac{\pi}{2}\right) \tag{7}$$

IV. DISCUSSION

This problem set continued to help me develop my skills using Matlab's live scripting and converting it to LaTeX format. Solving the first problem without DH parameters helped foster a greater understanding for the transformation matrices, and how the DH conventions and matrices are formed. The second problem helped reinforce the idea that the frames can be placed separate from the joints, and still result in correct solutions. This practice is important for analyzing new, non-standard, robotic manipulators.

REFERENCES

[1] Mark W. Spong, Seth Hutchinson, and M. Vidyasagar. *Robot modeling and control*. Vol. 26. 2006. DOI: 10.1109/MCS.2006.252815.