Problem 1

Define Variables for equations

```
syms q1(t) q2(t) q3(t) L1 L2 X1(t) Y1(t) X1_dot(t) Y1_dot(t) X2(t) Y2(t) X2_dot(t) Y2_dot(t);
syms m1 m2 h1 h2 g
```

Define x and y position for the center of mass of each link

```
q1_dot = diff(q1,t);
q2_dot = diff(q2,t);
q3_dot = diff(q3,t);

Iz1 = m1/12*(L1^2+h1^2)
```

```
Iz1 =
```

$$\frac{m_1\left(L_1^2+h_1^2\right)}{12}$$

```
Iz2 = m2/12*(L2^2+h2^2)
```

```
Iz2 =
```

$$\frac{m_2\left(L_2^2+h_2^2\right)}{12}$$

```
X1 = (L1+q2)/2 * cos(q1)
```

```
X1(t) =
```

$$\cos(q_1(t))\left(\frac{L_1}{2}+\frac{q_2(t)}{2}\right)$$

```
X2 = (L1+q2)/2 * sin(q1)
```

```
X2(t) =
```

$$\sin(q_1(t))\left(\frac{L_1}{2}+\frac{q_2(t)}{2}\right)$$

```
X1_dot = (q2_dot/2)*cos(q1)-(L1+q2)/2 * q1_dot * sin(q1)
```

```
X1_dot(t) =
```

$$\frac{\cos(q_1(t))\frac{\partial}{\partial t}\,q_2(t)}{2}-\sin(q_1(t))\left(\frac{L_1}{2}+\frac{q_2(t)}{2}\right)\frac{\partial}{\partial t}\,q_1(t)$$

```
Y1_dot = (q2_dot/2)*sin(q1)-(L1+q2)/2 * q1_dot * cos(q1)
```

```
Y1_dot(t) =
```

$$\frac{\sin(q_1(t))\,\frac{\partial}{\partial t}\,q_2(t)}{2} - \cos(q_1(t))\,\left(\frac{L_1}{2}+\frac{q_2(t)}{2}\right)\frac{\partial}{\partial t}\,q_1(t)$$

```
X2 = (L1+q2)*cos(q1)+(L2/2)*cos(q1+q3)
```

X2(t) =

$$\cos(q_1(t))\,(L_1+q_2(t)) + \frac{L_2\cos(q_1(t)+q_3(t))}{2}$$

```
Y2 = (L1+q2)*sin(q1)+(L2/2)*sin(q1+q3)
```

Y2(t) =

$$\sin(q_1(t))\,(L_1+q_2(t)) + \frac{L_2\sin(q_1(t)+q_3(t))}{2}$$

```
X2_dot = diff(X2,t)
```

X2_dot(t) =

$$\cos(q_1(t))\frac{\partial}{\partial t}\,q_2(t) - \sin(q_1(t))\,(L_1+q_2(t))\frac{\partial}{\partial t}\,q_1(t) - \frac{L_2\sin(q_1(t)+q_3(t))\,\left(\frac{\partial}{\partial t}\,q_1(t)+\frac{\partial}{\partial t}\,q_3(t)\right)}{2}$$

```
Y2_dot = diff(Y2,t)
```

Y2_dot(t) =

$$\sin(q_1(t))\frac{\partial}{\partial t}\,q_2(t) + \frac{L_2\cos(q_1(t)+q_3(t))\,\left(\frac{\partial}{\partial t}\,q_1(t)+\frac{\partial}{\partial t}\,q_3(t)\right)}{2} + \cos(q_1(t))\,(L_1+q_2(t))\frac{\partial}{\partial t}\,q_1(t)$$

```

```

Kinetic Energy Potential Energy and Lagrangian

```
K = (1/2)*m1*(X1_dot^2+Y1_dot^2)+(1/2)*Iz1*q1_dot^2+(1/2)*m2*(X2_dot^2+Y2_dot^2)+(1/2)*Iz2*(q1_
```

K(t) =

$$\frac{m_2 \left(\left(\sigma_3 + \frac{L_2 \cos(\sigma_6)\, \sigma_2}{2} + \cos(q_1(t))\ (L_1 + q_2(t)) \frac{\partial}{\partial t}\ q_1(t) \right)^2 + \left(-\sigma_1 + \sin(q_1(t))\ (L_1 + q_2(t)) \frac{\partial}{\partial t}\ q_1(t) + \frac{L_2}{2} \right.}$$

where

$$\sigma_1 = \cos(q_1(t)) \frac{\partial}{\partial t}\ q_2(t)$$

$$\sigma_2 = \frac{\partial}{\partial t}\ q_1(t) + \frac{\partial}{\partial t}\ q_3(t)$$

$$\sigma_3 = \sin(q_1(t)) \frac{\partial}{\partial t}\ q_2(t)$$

$$\sigma_4 = \frac{L_1}{2} + \frac{q_2(t)}{2}$$

$$\sigma_5 = \left(\frac{\partial}{\partial t}\ q_1(t) \right)^2$$

$$\sigma_6 = q_1(t) + q_3(t)$$

```
P = m1*g*Y1 + m2*g*Y2
```

P(t) =

$$g\, m_1\, Y_1(t) + g\, m_2 \left(\sin(q_1(t))\ (L_1 + q_2(t)) + \frac{L_2 \sin(q_1(t) + q_3(t))}{2} \right)$$

```
L = K-P
```

L(t) =

$$\frac{m_2 \left(\left(\sigma_3 + \frac{L_2 \cos(\sigma_6)\, \sigma_2}{2} + \cos(q_1(t))\ (L_1 + q_2(t)) \frac{\partial}{\partial t}\ q_1(t) \right)^2 + \left(-\sigma_1 + \sin(q_1(t))\ (L_1 + q_2(t)) \frac{\partial}{\partial t}\ q_1(t) + \frac{L_2}{2} \right.}$$

where

$$\sigma_1 = \cos(q_1(t)) \frac{\partial}{\partial t}\ q_2(t)$$

$$\sigma_2 = \frac{\partial}{\partial t}\ q_1(t) + \frac{\partial}{\partial t}\ q_3(t)$$

$$\sigma_3 = \sin(q_1(t)) \frac{\partial}{\partial t}\ q_2(t)$$

$$\sigma_4 = \frac{L_1}{2} + \frac{q_2(t)}{2}$$

$$\sigma_5 = \left(\frac{\partial}{\partial t}\ q_1(t) \right)^2$$

$$\sigma_6 = q_1(t) + q_3(t)$$

Dynamic equation formation

```
pL_pq1_dot = diff(L,q1_dot)
```

```
pL_pq1_dot(t) =
```

$$\frac{m_2 \left(2 \left(\cos(q_1(t))\ (L_1 + q_2(t)) + \frac{L_2 \cos(\sigma_5)}{2}\right) \left(\sigma_4 + \frac{L_2 \cos(\sigma_5)\, \sigma_3}{2} + \cos(q_1(t))\ (L_1 + q_2(t)) \frac{\partial}{\partial t}\ q_1(t)\right) + \left(\mathrm{s}\right.}{2}$$

where

$$\sigma_1 = \frac{L_1}{2} + \frac{q_2(t)}{2}$$

$$\sigma_2 = \cos(q_1(t)) \frac{\partial}{\partial t}\ q_2(t)$$

$$\sigma_3 = \frac{\partial}{\partial t}\ q_1(t) + \frac{\partial}{\partial t}\ q_3(t)$$

$$\sigma_4 = \sin(q_1(t)) \frac{\partial}{\partial t}\ q_2(t)$$

$$\sigma_5 = q_1(t) + q_3(t)$$

```
pL_pq2_dot = diff(L,q2_dot)
```

```
pL_pq2_dot(t) =
```

$$\frac{m_2 \left(2 \sin(q_1(t)) \left(\sigma_3 + \frac{L_2 \cos(\sigma_5)\, \sigma_2}{2} + \cos(q_1(t))\ (L_1 + q_2(t)) \frac{\partial}{\partial t}\ q_1(t)\right) - \cos(q_1(t)) \left(-\sigma_1 + \sin(q_1(t))\ (L_1\right.}{2}$$

where

$$\sigma_1 = \cos(q_1(t)) \frac{\partial}{\partial t}\ q_2(t)$$

$$\sigma_2 = \frac{\partial}{\partial t}\ q_1(t) + \frac{\partial}{\partial t}\ q_3(t)$$

$$\sigma_3 = \sin(q_1(t)) \frac{\partial}{\partial t}\ q_2(t)$$

$$\sigma_4 = \frac{L_1}{2} + \frac{q_2(t)}{2}$$

$$\sigma_5 = q_1(t) + q_3(t)$$

```
pL_pq3_dot = diff(L,q3_dot)
```

```
pL_pq3_dot(t) =
```

$$\frac{m_2 \left(L_2 \cos(\sigma_2) \left(\sin(q_1(t)) \frac{\partial}{\partial t}\ q_2(t) + \frac{L_2 \cos(\sigma_2)\ \sigma_1}{2} + \cos(q_1(t))\ (L_1 + q_2(t)) \frac{\partial}{\partial t}\ q_1(t)\right) + L_2 \sin(\sigma_2) \left(-\cos(\right.\right.}{2}$$

where

$$\sigma_1 = \frac{\partial}{\partial t}\ q_1(t) + \frac{\partial}{\partial t}\ q_3(t)$$

$$\sigma_2 = q_1(t) + q_3(t)$$

```
pL_pq1 = diff(L,q1)
```

```
pL_pq1(t) =
```

$$\frac{m_1\ ((\sigma_4 + \sigma_1)\ (\sigma_3 - \sigma_2)\ 2 - (\sigma_3 + \sigma_2)\ (\sigma_4 - \sigma_1)\ 2)}{2} - g\ m_2 \left(\cos(q_1(t))\ (L_1 + q_2(t)) + \frac{L_2 \cos(q_1(t) + q_3(t))}{2}\right)$$

where

$$\sigma_1 = \sin(q_1(t))\ \left(\frac{L_1}{2} + \frac{q_2(t)}{2}\right) \frac{\partial}{\partial t}\ q_1(t)$$

$$\sigma_2 = \cos(q_1(t))\ \left(\frac{L_1}{2} + \frac{q_2(t)}{2}\right) \frac{\partial}{\partial t}\ q_1(t)$$

$$\sigma_3 = \frac{\sin(q_1(t)) \frac{\partial}{\partial t}\ q_2(t)}{2}$$

$$\sigma_4 = \frac{\cos(q_1(t)) \frac{\partial}{\partial t}\ q_2(t)}{2}$$

```
pL_pq2 = diff(L,q2)
```

```
pL_pq2(t) =
```

$$\frac{m_2 \left(2\cos(q_1(t))\left(\sigma_3+\frac{L_2\cos(\sigma_5)\,\sigma_2}{2}+\cos(q_1(t))\,(L_1+q_2(t))\frac{\partial}{\partial t}\,q_1(t)\right)\frac{\partial}{\partial t}\,q_1(t)+2\sin(q_1(t))\left(\sin(q_1(t))\right.\right.}{2}$$

where

$$\sigma_1=\cos(q_1(t))\frac{\partial}{\partial t}\,q_2(t)$$

$$\sigma_2=\frac{\partial}{\partial t}\,q_1(t)+\frac{\partial}{\partial t}\,q_3(t)$$

$$\sigma_3=\sin(q_1(t))\frac{\partial}{\partial t}\,q_2(t)$$

$$\sigma_4=\frac{L_1}{2}+\frac{q_2(t)}{2}$$

$$\sigma_5=q_1(t)+q_3(t)$$

```
pL_pq3 = diff(L,q3)
```

```
pL_pq3(t) =
```

$$-\frac{m_2 \left(L_2\sin(\sigma_2)\,\sigma_1\left(\sin(q_1(t))\frac{\partial}{\partial t}\,q_2(t)+\frac{L_2\cos(\sigma_2)\,\sigma_1}{2}+\cos(q_1(t))\,(L_1+q_2(t))\frac{\partial}{\partial t}\,q_1(t)\right)-L_2\cos(\sigma_2)\,\sigma_1\right.}{2}$$

where

$$\sigma_1=\frac{\partial}{\partial t}\,q_1(t)+\frac{\partial}{\partial t}\,q_3(t)$$

$$\sigma_2=q_1(t)+q_3(t)$$

```
tau1 = diff(pL_pq1_dot,t)-pL_pq1
```

```
tau1(t) =
```

$$m_2 \left(\left(\sin(q_1(t)) \ (L_1 + q_2(t)) + \frac{L_2 \sin(\sigma_{11})}{2} \right) \left(-\sigma_6 + \frac{L_2 \sin(\sigma_{11}) \, \sigma_{10}}{2} + 2 \sin(q_1(t)) \frac{\partial}{\partial t} \ q_2(t) \frac{\partial}{\partial t} \ q_1(t) + \cos(q_1($$

where

$$\sigma_1 = \left(\frac{\partial}{\partial t} \ q_1(t) \right)^2$$

$$\sigma_2 = \frac{\sin(q_1(t)) \frac{\partial}{\partial t} \ q_2(t)}{2}$$

$$\sigma_3 = \frac{\cos(q_1(t)) \frac{\partial}{\partial t} \ q_2(t)}{2}$$

$$\sigma_4 = \left(\frac{\partial}{\partial t} \ q_1(t) + \frac{\partial}{\partial t} \ q_3(t) \right)^2$$

$$\sigma_5 = \cos(q_1(t)) \ (L_1 + q_2(t)) + \frac{L_2 \cos(\sigma_{11})}{2}$$

$$\sigma_6 = \cos(q_1(t)) \, \sigma_{13}$$

$$\sigma_7 = \sin(q_1(t)) \, \sigma_{12} \frac{\partial}{\partial t} \ q_1(t)$$

$$\sigma_8 = \cos(q_1(t)) \, \sigma_{12} \frac{\partial}{\partial t} \ q_1(t)$$

$$\sigma_9 = \sin(q_1(t)) \, \sigma_{13}$$

$$\sigma_{10} = \sigma_{14} + \frac{\partial^2}{\partial t^2} \ q_3(t)$$

$$\sigma_{11} = q_1(t) + q_3(t)$$

$$\sigma_{12} = \frac{L_1}{2} + \frac{q_2(t)}{2}$$

$$\sigma_{13} = \frac{\partial^2}{\partial t^2} \ q_2(t)$$

$$\sigma_{14} = \frac{\partial^2}{\partial t^2} \ q_1(t)$$

```
tau2 = diff(pL_pq2_dot,t)-pL_pq2
```

```
tau2(t) =
```

$$\frac{m_1\ (\sigma_5+\sigma_4)}{2}+\frac{m_2\ \left(-\cos(q_1(t))\ \left(-\sigma_6+\frac{L_2\sin(\sigma_{10})\,\sigma_8}{2}+2\sin(q_1(t))\frac{\partial}{\partial t}\ q_2(t)\frac{\partial}{\partial t}\ q_1(t)+\cos(q_1(t))\ (L_1+q_2(\right.\right.}{}$$

where

$$\sigma_1=\left(\frac{\partial}{\partial t}\ q_1(t)\right)^2$$

$$\sigma_2=2\sin(q_1(t))\ \left(\sin(q_1(t))\ (L_1+q_2(t))\frac{\partial}{\partial t}\ q_1(t)-\sigma_{11}+\frac{L_2\sin(\sigma_{10})\,\sigma_9}{2}\right)\frac{\partial}{\partial t}\ q_1(t)$$

$$\sigma_3=2\cos(q_1(t))\ \left(\sigma_{13}+\frac{L_2\cos(\sigma_{10})\,\sigma_9}{2}+\cos(q_1(t))\ (L_1+q_2(t))\frac{\partial}{\partial t}\ q_1(t)\right)\frac{\partial}{\partial t}\ q_1(t)$$

$$\sigma_4=\sin(q_1(t))\ \left(\frac{\sigma_{11}}{2}-\sin(q_1(t))\,\sigma_{12}\frac{\partial}{\partial t}\ q_1(t)\right)\frac{\partial}{\partial t}\ q_1(t)$$

$$\sigma_5=\cos(q_1(t))\ \left(\frac{\sigma_{13}}{2}-\cos(q_1(t))\,\sigma_{12}\frac{\partial}{\partial t}\ q_1(t)\right)\frac{\partial}{\partial t}\ q_1(t)$$

$$\sigma_6=\cos(q_1(t))\,\sigma_{14}$$

$$\sigma_7=\sin(q_1(t))\,\sigma_{14}$$

$$\sigma_8=\sigma_{15}+\frac{\partial^2}{\partial t^2}\ q_3(t)$$

$$\sigma_9=\frac{\partial}{\partial t}\ q_1(t)+\frac{\partial}{\partial t}\ q_3(t)$$

$$\sigma_{10}=q_1(t)+q_3(t)$$

$$\sigma_{11}=\cos(q_1(t))\frac{\partial}{\partial t}\ q_2(t)$$

$$\sigma_{12}=\frac{L_1}{2}+\frac{q_2(t)}{2}$$

$$\sigma_{13}=\sin(q_1(t))\frac{\partial}{\partial t}\ q_2(t)$$

$$\sigma_{14}=\frac{\partial^2}{\partial t^2}\ q_2(t)$$

$$\sigma_{15}=\frac{\partial^2}{\partial t^2}\ q_1(t)$$

```
tau3 = diff(pL_pq2_dot,t)-pL_pq3
```

```
tau3(t) =
```

$$\frac{m_2\ (L_2 \sin(\sigma_{14})\,\sigma_{13}\,\sigma_5 - L_2 \cos(\sigma_{14})\,\sigma_{13}\ (-\sigma_2 + \sigma_4 + \sigma_9))}{2} + \frac{m_2\ \left(-\cos(q_1(t))\ \left(-\sigma_6 + \frac{L_2 \sin(\sigma_{14})\,\sigma_8}{2} + 2\sin(q\right.\right.}{}$$

where

$$\sigma_1 = \frac{L_1}{2} + \frac{q_2(t)}{2}$$

$$\sigma_2 = \cos(q_1(t))\frac{\partial}{\partial t}\ q_2(t)$$

$$\sigma_3 = \left(\frac{\partial}{\partial t}\ q_1(t)\right)^2$$

$$\sigma_4 = \sin(q_1(t))\ (L_1 + q_2(t))\frac{\partial}{\partial t}\ q_1(t)$$

$$\sigma_5 = \sigma_{10} + \frac{L_2 \cos(\sigma_{14})\,\sigma_{13}}{2} + \cos(q_1(t))\ (L_1 + q_2(t))\frac{\partial}{\partial t}\ q_1(t)$$

$$\sigma_6 = \cos(q_1(t))\,\sigma_{11}$$

$$\sigma_7 = \sin(q_1(t))\,\sigma_{11}$$

$$\sigma_8 = \sigma_{12} + \frac{\partial^2}{\partial t^2}\ q_3(t)$$

$$\sigma_9 = \frac{L_2 \sin(\sigma_{14})\,\sigma_{13}}{2}$$

$$\sigma_{10} = \sin(q_1(t))\frac{\partial}{\partial t}\ q_2(t)$$

$$\sigma_{11} = \frac{\partial^2}{\partial t^2}\ q_2(t)$$

$$\sigma_{12} = \frac{\partial^2}{\partial t^2}\ q_1(t)$$

$$\sigma_{13} = \frac{\partial}{\partial t}\ q_1(t) + \frac{\partial}{\partial t}\ q_3(t)$$

$$\sigma_{14} = q_1(t) + q_3(t)$$

Problem 2

```
syms q1(t) q2(t) q3(t) L1 L2 L3 X1(t) Y1(t) Z1(t) X1_dot(t) Y1_dot(t) Z1_dot(t) X2(t) Y2(t) Z2(
syms X3(t) Y3(t) Z3(t) X3_dot(t) Y3_dot(t) Z3_dot(t) m1 m2 m3 h1 h2 h3 w1 w2 w3 g


q1_dot = diff(q1,t);
q2_dot = diff(q2,t);
q3_dot = diff(q3,t);

Iz1 = m1/12*(L1^2+h1^2)
```

Iz1 =

$$\frac{m_1\left(L_1^2+h_1^2\right)}{12}$$

```
Iz2 = m2/12*(L2^2+h2^2)
```

Iz2 =

$$\frac{m_2\left(L_2^2+h_2^2\right)}{12}$$

```
Iz3 = m3/12*(L3^2+h3^2)
```

Iz3 =

$$\frac{m_3\left(L_3^2+h_3^2\right)}{12}$$

```
Ix1 = m1/12*(L1^2+w1^2)
```

Ix1 =

$$\frac{m_1\left(L_1^2+w_1^2\right)}{12}$$

```
Ix2 = m2/12*(L2^2+w2^2)
```

Ix2 =

$$\frac{m_2\left(L_2^2+w_2^2\right)}{12}$$

```
Ix3 = m3/12*(L3^2+w3^2)
```

Ix3 =

$$\frac{m_3\left(L_3^2+w_3^2\right)}{12}$$

```
Iy1 = m1/12*(w1^2+h1^2)
```

Iy1 =

$$\frac{m_1 \left(h_1^2 + w_1^2\right)}{12}$$

```
Iy2 = m2/12*(w2^2+h2^2)
```

Iy2 =

$$\frac{m_2 \left(h_2^2 + w_2^2\right)}{12}$$

```
Iy3 = m3/12*(w3^2+h3^2)
```

Iy3 =

$$\frac{m_3 \left(h_3^2 + w_3^2\right)}{12}$$

```
X1 = (L1/2)*cos(q1)
```

X1(t) =

$$\frac{L_1 \cos(q_1(t))}{2}$$

```
Y1 = (L1/2)*sin(q1)
```

Y1(t) =

$$\frac{L_1 \sin(q_1(t))}{2}$$

```
Z1 = 0
```

Z1 = 0

```
X1_dot = diff(X1,t)
```

X1_dot(t) =

$$-\frac{L_1 \sin(q_1(t)) \frac{\partial}{\partial t} q_1(t)}{2}$$

```
Y1_dot = diff(Y1,t)
```

Y1_dot(t) =

$$\frac{L_1 \cos(q_1(t)) \frac{\partial}{\partial t} q_1(t)}{2}$$

```
Z1_dot = 0
```

Z1_dot = 0

```
X2 = L1*cos(q1)+(L2/2)*cos(q1)*cos(q2)
```

X2(t) =

$$L_1 \cos(q_1(t)) + \frac{L_2 \cos(q_1(t)) \cos(q_2(t))}{2}$$

```
Y2 = L1*sin(q1)+(L2/2)*sin(q1)*sin(q2)
```

Y2(t) =

$$L_1 \sin(q_1(t)) + \frac{L_2 \sin(q_1(t)) \sin(q_2(t))}{2}$$

```
Z2 = (L2/2)*sin(q2)
```

Z2(t) =

$$\frac{L_2 \sin(q_2(t))}{2}$$

```
X2_dot = diff(X2,t)
```

X2_dot(t) =

$$-L_1 \sin(q_1(t)) \frac{\partial}{\partial t} q_1(t) - \frac{L_2 \cos(q_2(t)) \sin(q_1(t)) \frac{\partial}{\partial t} q_1(t)}{2} - \frac{L_2 \cos(q_1(t)) \sin(q_2(t)) \frac{\partial}{\partial t} q_2(t)}{2}$$

```
Y2_dot = diff(Y2,t)
```

Y2_dot(t) =

$$L_1 \cos(q_1(t)) \frac{\partial}{\partial t} q_1(t) + \frac{L_2 \cos(q_1(t)) \sin(q_2(t)) \frac{\partial}{\partial t} q_1(t)}{2} + \frac{L_2 \cos(q_2(t)) \sin(q_1(t)) \frac{\partial}{\partial t} q_2(t)}{2}$$

```
Z2_dot = diff(Z2,t)
```

Z2_dot(t) =

$$\frac{L_2 \cos(q_2(t)) \frac{\partial}{\partial t} q_2(t)}{2}$$

```
X3 = (L3/2)*cos(q1)*cos(q2+q3)+L2*cos(q1)*cos(q2)+L1*cos(q1)
```

X3(t) =

$$L_1 \cos(q_1(t)) + L_2 \cos(q_1(t)) \cos(q_2(t)) + \frac{L_3 \cos(q_1(t)) \cos(q_2(t) + q_3(t))}{2}$$

```
Y3 = (L3/2)*sin(q1)*cos(q2+q3)+L2*sin(q1)*cos(q2)+L1*sin(q1)
```

Y3(t) =

$$L_1 \sin(q_1(t)) + L_2 \cos(q_2(t)) \sin(q_1(t)) + \frac{L_3 \sin(q_1(t)) \cos(q_2(t) + q_3(t))}{2}$$

```
Z3 = (L2)*sin(q2)+(L3)*sin(q2+q3)
```

Z3(t) = $L_2 \sin(q_2(t)) + L_3 \sin(q_2(t) + q_3(t))$

```
X3_dot = diff(X3,t)
```

X3_dot(t) =

$$-L_1 \sin(q_1(t)) \frac{\partial}{\partial t}\, q_1(t) - L_2 \cos(q_2(t)) \sin(q_1(t)) \frac{\partial}{\partial t}\, q_1(t) - L_2 \cos(q_1(t)) \sin(q_2(t)) \frac{\partial}{\partial t}\, q_2(t) - \frac{L_3 \sin(q_1(t)) \cos(}{2}$$

where

$$\sigma_1 = q_2(t) + q_3(t)$$

```
Y3_dot = diff(Y3,t)
```

Y3_dot(t) =

$$L_1 \cos(q_1(t)) \frac{\partial}{\partial t}\, q_1(t) - L_2 \sin(q_1(t)) \sin(q_2(t)) \frac{\partial}{\partial t}\, q_2(t) + \frac{L_3 \cos(q_1(t)) \cos(\sigma_1) \frac{\partial}{\partial t}\, q_1(t)}{2} - \frac{L_3 \sin(q_1(t)) \sin(\sigma_1)}{}$$

where

$$\sigma_1 = q_2(t) + q_3(t)$$

```
Z3_dot = diff(Z3,t)
```

Z3_dot(t) =

$$L_3 \cos(q_2(t) + q_3(t)) \left(\frac{\partial}{\partial t}\, q_2(t) + \frac{\partial}{\partial t}\, q_3(t)\right) + L_2 \cos(q_2(t)) \frac{\partial}{\partial t}\, q_2(t)$$