HOMEWORK 6

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11/15/21

Section 1

Solution 1.1

We are told that the diagonal entries of K are equal to 1 while any element off diagonal equals 0. This is an identity matrix so $v^T K v = v^T I v = v^T v = |v|^2$ since we are squaring the norm of v it must be greater than or equal to 0. Therefore $v^T K v \geq 0$ must be true.

Solution 1.2

Again we know that our kernel matrix K is equal to identity. Since we are told that
$f(z) = \sum_{i=1}^{n} \alpha_i y_i k(z_i, z) + b$
If we pick $\alpha = 1$ and $b = 0$ then we can simplify to $f(z) = y_i$.
This means that for each point y_i the decision boundary is $sgn(f(z)) = sgn(y_i)$.
So, each point in the set has its own decision boundary implying k creates an n dimension space.

Solution 1.3

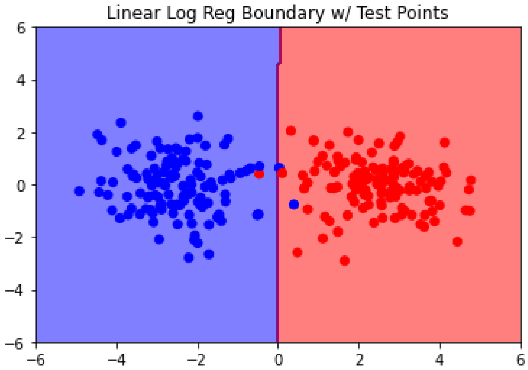
For z not in the dataset we know our kernel will take the value of 0 so $f(z) = b$.
This means that untrained test points will be classified based upon the classifier's bias term.

Section 2

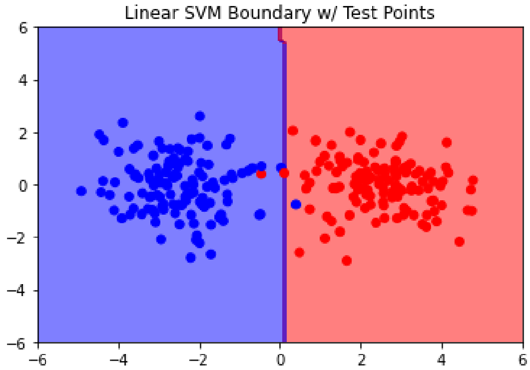
Solution 2.1

Yes implementing linear SVM using a kernel is possible. In the dual problem for linear kernel SVM we find that our kernel $k(x, x') = \phi(x)\phi(x') = x^T x$ so as long our kernel is the dot product between x and itself we can implement linear SVM with a kernel.

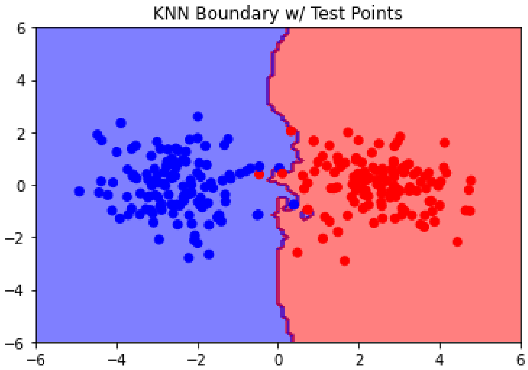
Solution 2.2.1



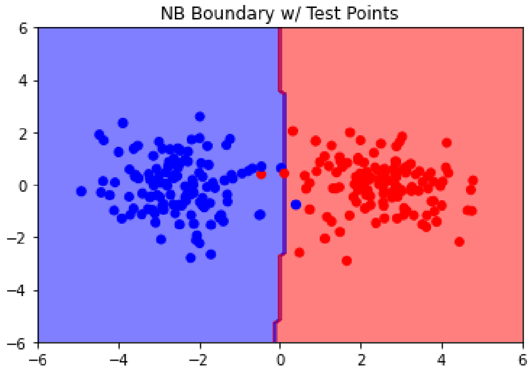
(a) Logistic Regression: Error Percent = 1.21



(b) Linear SVM: Error Percent = 0.81



(c) KNN k=3: Error Percent = 0.81



(d) Naive Bayes: Error Percent = 1.21

Figure 1: Decision Boundaries for Linearly Separable Data

These decision boundaries all use the same train data and have the test data scatter plot on top. $\mu = 2.4$

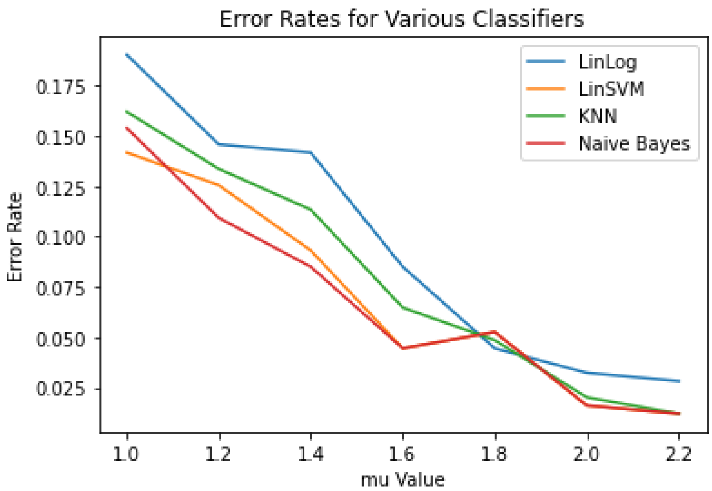
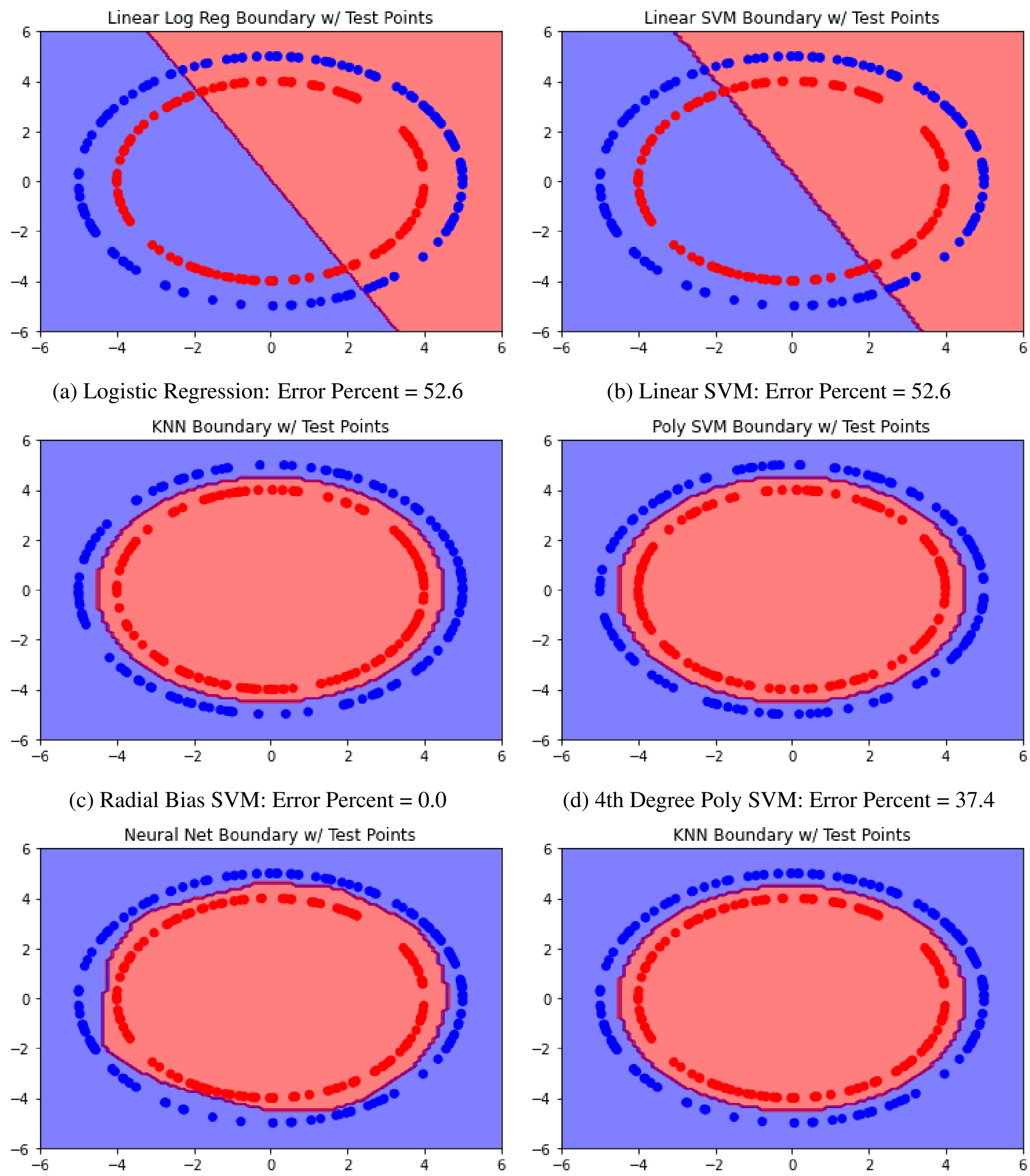


Figure 2: Classifier error rates for various μ values

We can clearly see that as the value of μ increases the accuracy of the classifiers increase. This is because there is less overlap in data points and the classes become more separated.

Solution 2.2.2



(a) Logistic Regression: Error Percent = 52.6

(b) Linear SVM: Error Percent = 52.6

(c) Radial Bias SVM: Error Percent = 0.0

(d) 4th Degree Poly SVM: Error Percent = 37.4

(e) Neural Network: Error Percent = 0.0

(f) KNN k=1: Error Percent = 0.0

Figure 3: Decision Boundaries for Nonlinear Data

In order to effectively classify data that is no longer linearly separable in 2 dimensions we must use the kernel trick to transform the data into a feature space that allows for linear separability.
I believe that these graphs also show how powerful a simple learning algorithm like KNN can be even when compared to something as complex as a linear SVM.

Solution 2.3

Classifier	Error Rate	Parameter
Linear SVM	0.0106	C=10
Linear LR	0.0532	N/A
Poly SVM	0.0106	Degree=2, C=10000
Poly SVM	0.0212	Degree=3, C=1000
Poly SVM	0.0319	Degree=4, C=1000
RBF SVM	0.0212	Sigma=1, C=1000
KNN	0.0638	k=9
Neural Net	0.0319	Hidden Size=64

Table 1: Error Rate Table

I then went and added L1 regularization to my SVM and trained the Linear SVM with various values for the regularization constant 'C'. I used the validation set to find the optimal value and then trained a final model. Looking at the weight coefficients of the trained model we know that any non-zero weights account for some portion of the variance in the data as L1 promotes sparsity. Checking these against the feature names I was able to extract that the following features are the most important in determining the tumor type.

- mean smoothness
- mean compactness
- mean concavity
- mean concave points
- mean symmetry
- worst compactness
- worst concave points
- worst symmetry