ECE 561 Hw 5

1a) scaling does not impact mean
offset does not impact variance

$X \sim \mathcal{N}(\mu, \sigma^2) \quad Y = aX + b$

$E[Y] = \mu + b$

$Var(Y) = a^2 Var(x) = (a\sigma)^2$

1b) If $X \sim \mathcal{N}(\mu, \sigma^2)$ then $aX + b \sim \mathcal{N}(0,1)$
when $b = -\mu$

$a = \frac{1}{\sigma}$

1c) see code

2a) $E[X] = \mu_x \quad Var(X) = \Sigma_x$

$Y = A(X+c) \quad \boxed{E[Y] = A(\mu_x + c)}$

2b) $Var(X) = \sigma^2 \quad Var(aX) = (a\sigma)^2 \quad Var(X+b) = \sigma^2$

so $Var(A(X+c)) = Var(Y) = \boxed{A^T \Sigma_x A = \Sigma_Y}$

3a) we know each feature can take 2 values. Since we have 784 features we get $2 \cdot 784$ total outcomes $\quad 2 \cdot 784 = 1568$

3b) It is not a reasonable estimate as many of the pixels have a high dependence on others for certain digits. For example if the label is a 1 it is likely most pixels with a value of 1 will have a 1 above or below them.