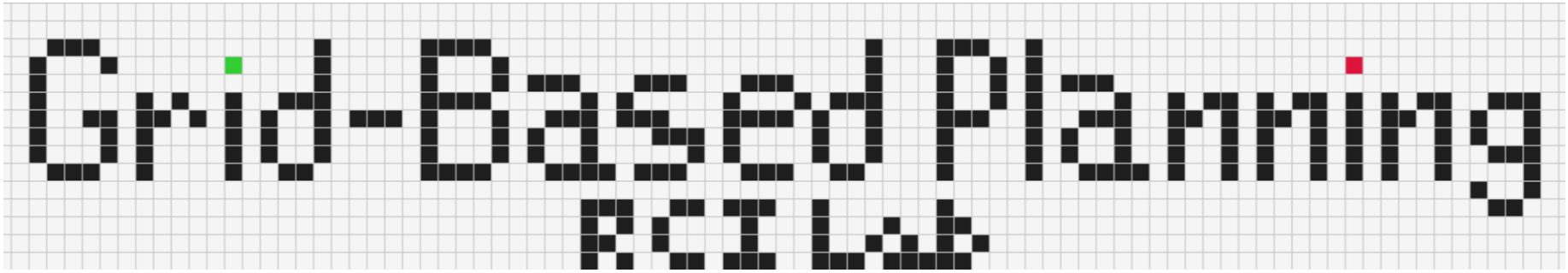
Introduction of Grid-Based Planning



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KHU

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01 What is the Grid?

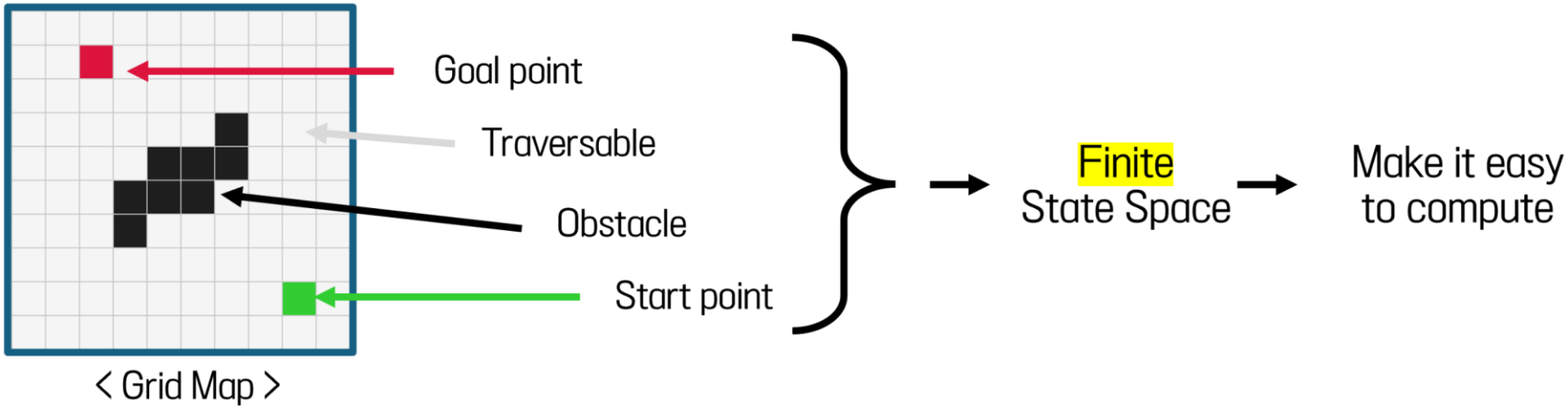
01 What is the Grid?

Grid

A model that simplifies a complex continuous space into a set of uniform cells that are easy for a computer to understand.

Key Features of Grid

- **Discrete Space**: Represents the world as a set of distinct cells.
- **Simplicity**: Each cell holds simple information, such as 'traversable' or 'obstacle'.
- **Coordinates**: Every cell has a unique address, like (x, y).



< Grid Map >

01 What is the Grid?

- Simplicity: Each cell holds simple information, such as 'traversable' or 'obstacle'.
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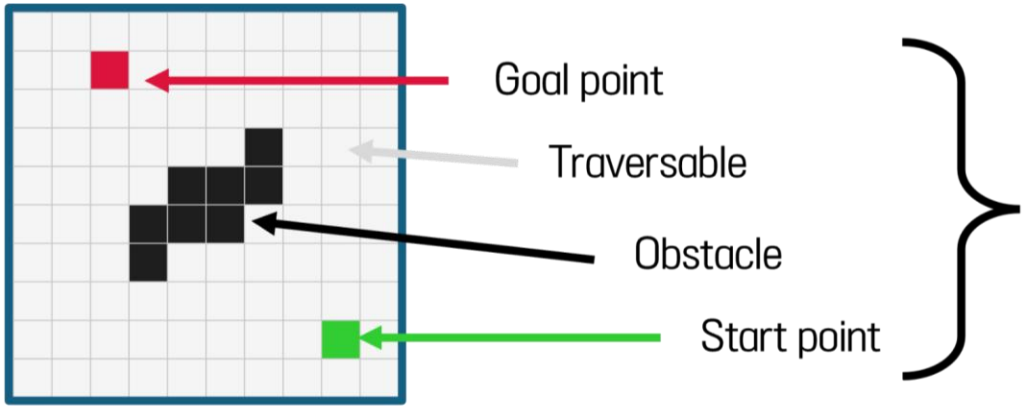
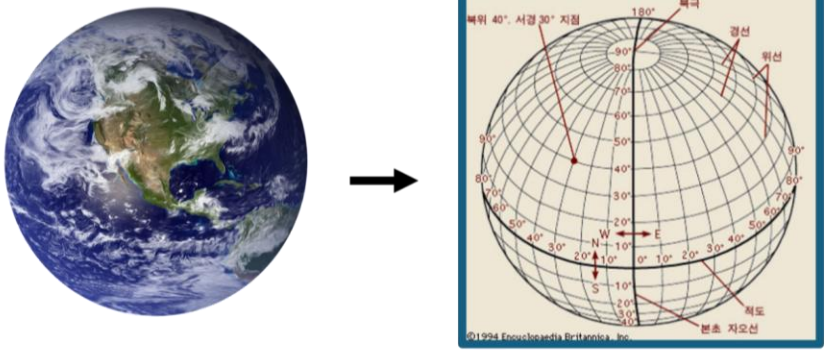


< Grid Map >

Finite State Space → Make it easy to compute



< Finite State Space of Earth >

I'm located at 37.5665 degrees north latitude and 126.9780 degrees east longitude.

01 What is the Grid?

How many meters per cell?

Resolution (r)

- Definition: how much real-world distance a single grid cell represents.
- Unit: meters per cell [m/cell]

※ There can be another length scale(or unit) instead of meters

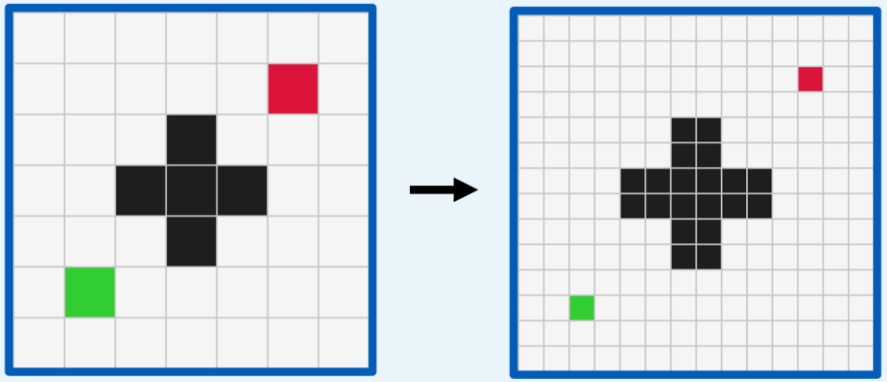
- World (x, y) → Grid (i, j), total counts of cells N

$$i = \left\lfloor \frac{y - y_{\min}}{r} \right\rfloor, \quad j = \left\lfloor \frac{x - x_{\min}}{r} \right\rfloor, \quad N \approx \frac{L_x L_y}{r^2} \quad \begin{cases} L_x : \text{length of width} \\ L_y : \text{length of height} \end{cases}$$

- It directly controls **detail, computation, and memory.**

< Low Resolution >

- Fewer cells
- Detail ↓
- Computation ↓
- Memory ↓



< High Resolution >

- More cells
- Detail ↑
- Computation ↑
- Memory ↑

01 What is the Grid?

How many meters per cell?

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< Low Resolution >
- Fewer cells
- Detail ↓
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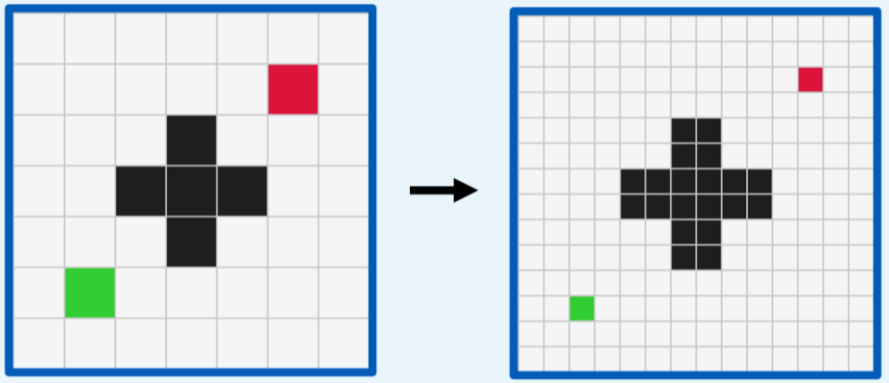


< High Resolution >
- More cells
- Detail ↑
- Computation ↑
- Memory ↑

Example)

- Map: Width(L_x) = 1m, Height(L_y): 1m
- Resolution: r_{low} = 1/7 [m/cell], r_{high} = 1/14 [m/cell]

$$\rightarrow \begin{cases} N_{\text{low}} = 49 \text{ cells} \\ N_{\text{high}} = 196 \text{ cells} \end{cases}$$

- The computational time complexity of Dijkstra/A* in the worst case: $O(N \log N)$

02 Occupancy Grid

02 Occupancy Grid

Occupancy Grid

A map that encodes whether each grid cell is occupied by an obstacle.

Representation

State	Integer Convention (ROS)	Probabilistic	Meaning
Free space	0	Low p_{occ}	Traversable
Occupied space	100	High p_{occ}	Non-traversable (collision)
Unknown space	-1	Undetermined p_{occ}	Unobserved or Uncertain area

- For probabilistic maps, choose a threshold τ, declare occupied if $p_{occ} \geq \tau$.
- We should choice the **policy** for unknown area :
  1. **Conservative**: treat as blocked or assign very high cost (avoid exploration into unknown).
  2. **Aggressive**: treat as free (faster paths but higher collision risk).
  3. **Compromise**: assign medium-to-high cost and let the planner decide.

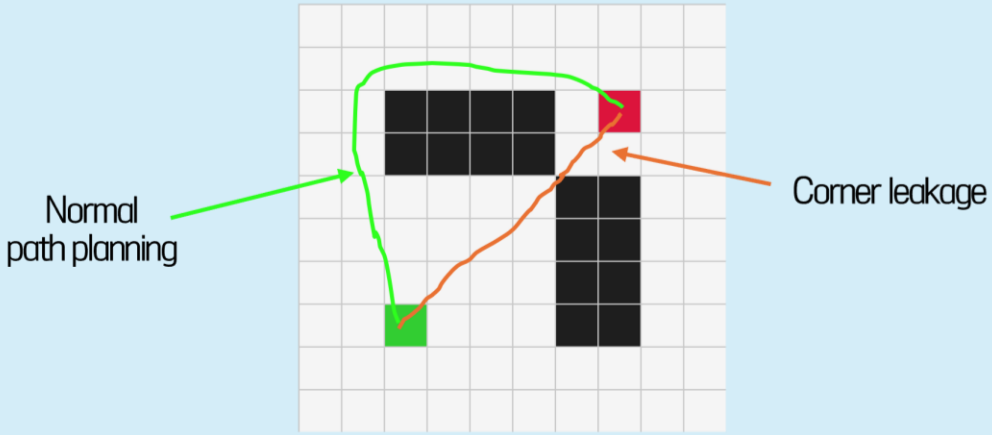
02 Occupancy Grid

Occupancy Grid

A map that encodes whether each grid cell is occupied by an obstacle.

Limitations

- Occpancy grid only represent whether robot can pass.
- Corner leakage on diagonals:



→ Solution: Combine occupancy with connectivity rules and inflation.

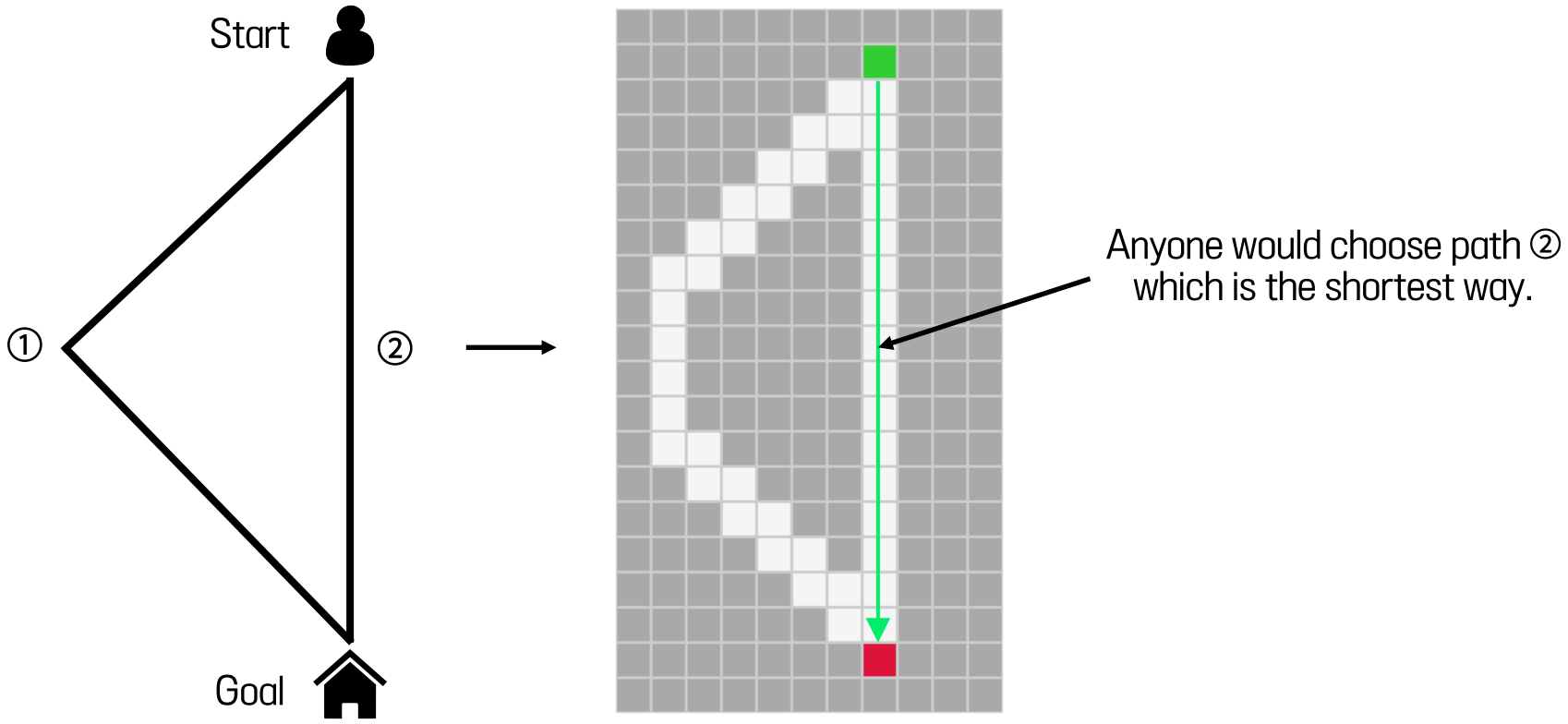
03 Costmap

03 Costmap

Cost

A scalar score for undesirability.

Smaller is better.



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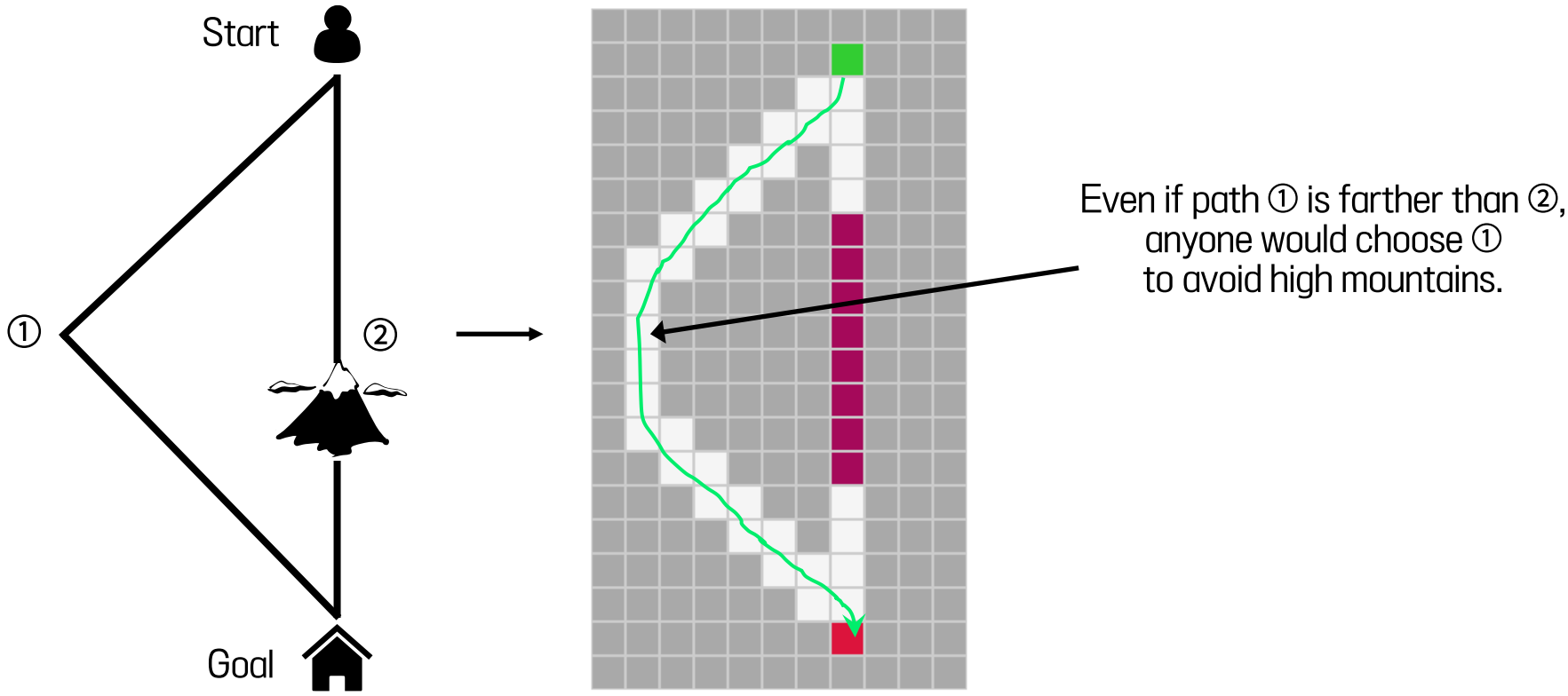

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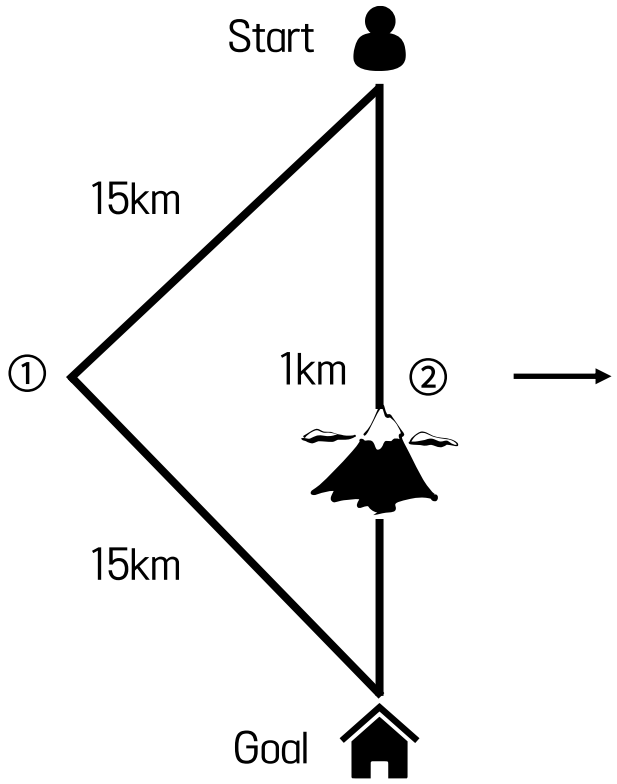

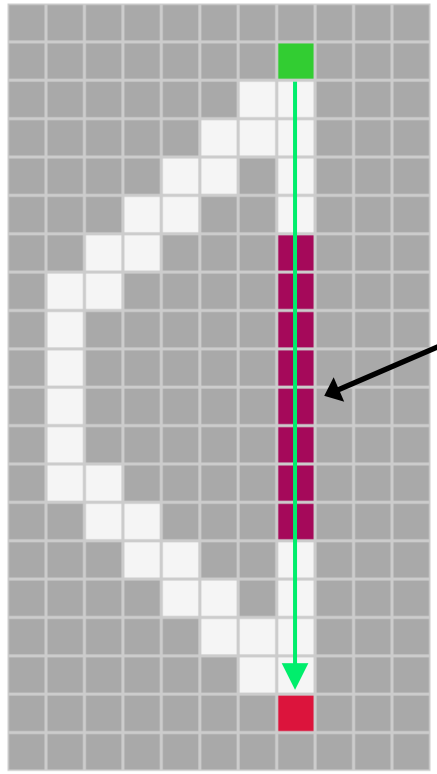

If path ① is too far,
it may be better to climb the mountains.

03 Costmap

Cost

$$\text{PathCost} = \sum_{(u \to v \in \text{path})} \text{Cost}(u \to v)$$

$$\text{where} \quad \text{Cost}(u \to v) = \underbrace{\text{move\_cost}(u, v)}_{\text{Length}} + \beta \cdot \underbrace{\text{sample}(c(u), c(v))}_{\text{Safety}}$$

Trade-off between length and safety

※ Reference of the distance

	Manhattan	Octile	Chebyshev
Definition	$\lvert x_1 - x_2 \rvert + \lvert y_1 - y_2 \rvert$	$\sqrt{2} \times \min(\Delta x, \Delta y) + \lvert \Delta x - \Delta y \rvert$	$\min(\lvert \Delta x_1 - \Delta x_2 \rvert, \lvert \Delta y_1 - \Delta y_2 \rvert)$
Characteristic	Used when robots can only travel in certain directions	Particularly useful for path planning of diagonal-moving robots in grid based planning	Suitable when assuming diagonal travel costs equal to straight travel costs

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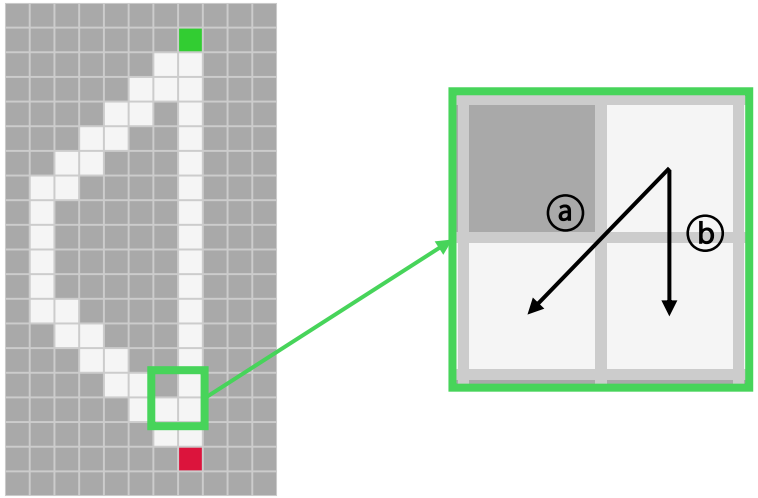
Trade-off between length and safety



In this tutorial,
Manhattan & Octile distance will be used.

ⓐ Diagonal : $\text{move\_cost}(u, v) = \sqrt{2}$

ⓑ Straight : $\text{move\_cost}(u, v) = 1$

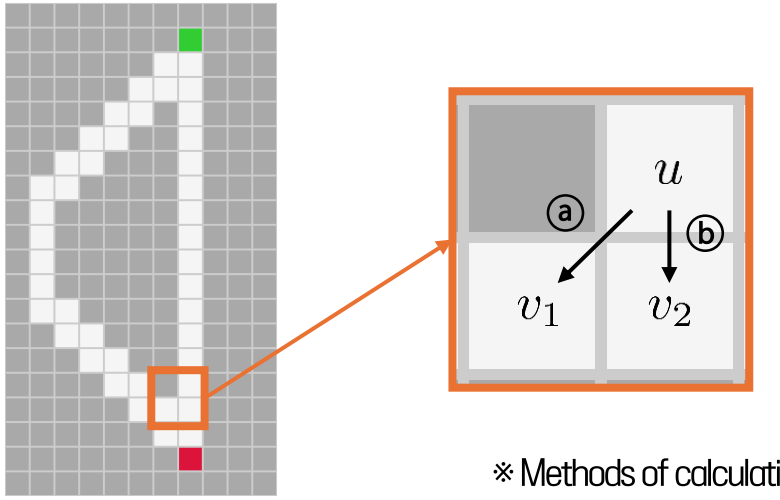
03 Costmap

Cost

$$\text{PathCost} = \sum_{(u \to v \in \text{path})} \text{Cost}(u \to v)$$

$$\text{where} \quad \text{Cost}(u \to v) = \underbrace{\text{move\_cost}(u, v)}_{\text{Length}} + \beta \cdot \underbrace{\text{sample}(c(u), c(v))}_{\text{Safety}}$$

Trade-off between length and safety



ⓐ : $\text{sample}(c(u), c(v_1)) = \dfrac{c(u) + c(v_1)}{2}$

ⓑ : $\text{sample}(c(u), c(v_2)) = \dfrac{c(u) + c(v_2)}{2}$

※ Methods of calculating costs are determined by trade-offs such as speed, safety, and complexity.

03 Costmap

Cost Convention in ROS

Cost uses an 8-bit integer(0 ~ 255) scale in ROS Navigation stack(ROS1 costmap_2d, ROS2 Nav2).

Value	Meaning	Note
0	Free space	Traversable
1 ~ 252	Inflation/Potential	The closer to the obstacle, the larger cost. (Detailed values are determined by Inflation layer.)
253	Inscribed inflated obstacle	Substantial collision range. Treat it an evasive/prohibitive level when planning.
254	Lethal obstacle	Actual obstacle core. Treat it a prohibitive level when planning.
255	No information (Unknown)	Unobserved/uncertain areas. According to policy, it is blocked or set high cost.

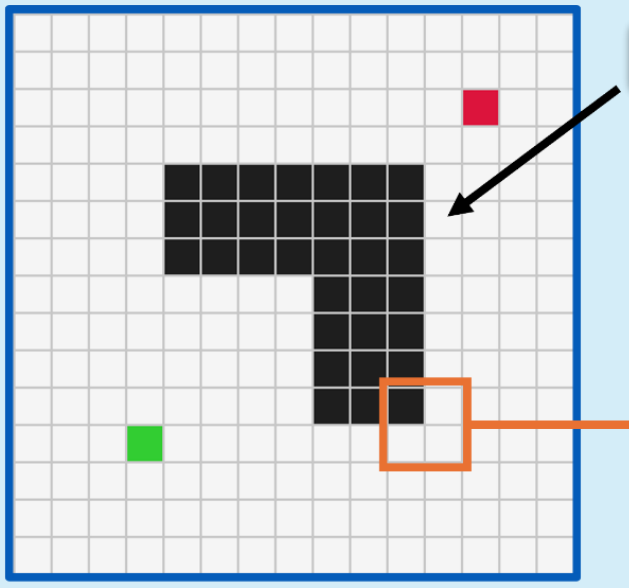
03 Costmap

Costmap

A scalar cost field built from occupancy (and optionally other layers).

Occupancy grid tells only can/can't pass.
Costmap adds how preferable each traversable cell is, combining shortness vs. safety.

Components of costmap



Obstacle core

Occupied cells are non-traversable and should receive a very large cost C_{obs} in your chosen scale.

254	0
0	0

ex) 255 scale: fixed to 254

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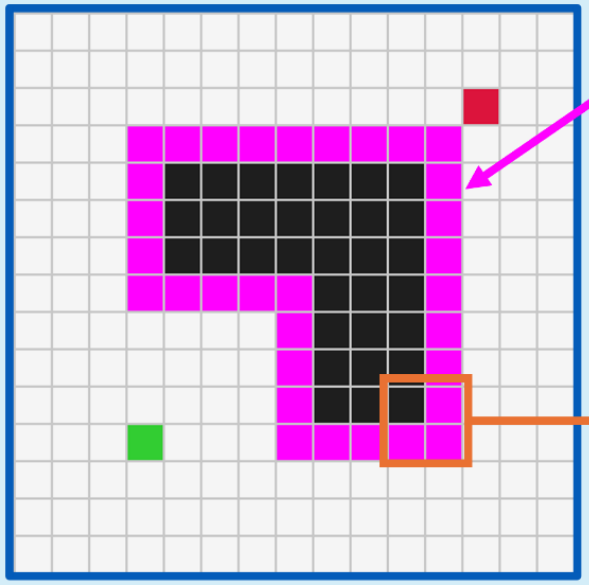

Inflation ring

Around each obstacle, assign a distance-dependent penalty.

The closer a free cell is to an obstacle, the higher its cost.

This prevents corner-cutting paths that graze obstacles.

254	200
200	180

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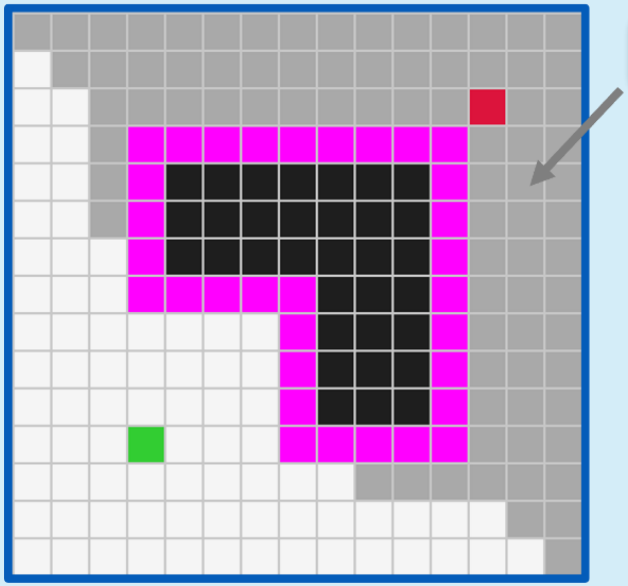

Unknown

If you decided not to hard-block Unknown in the occupancy layer, you can encode caution here by assigning a high but finite cost to unknown cells.

This keeps exploration possible yet discouraged compared to well-observed free space.

03 Costmap

Costmap

A scalar cost field built from occupancy (and optionally other layers).

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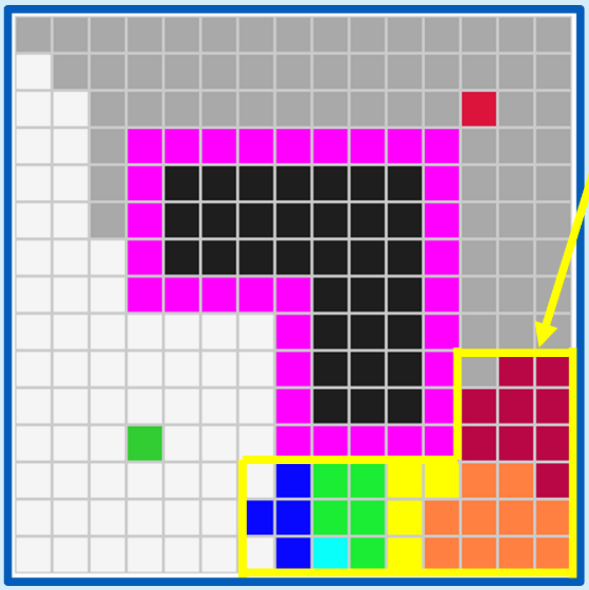


Additional layers

Real systems may add layers for slope, sound, radiation, etc., then combine layer via a weighted sum.

For this tutorial, stick to a clean baseline:
obstacle core + inflation (+ optional unknown)

04 Inflation

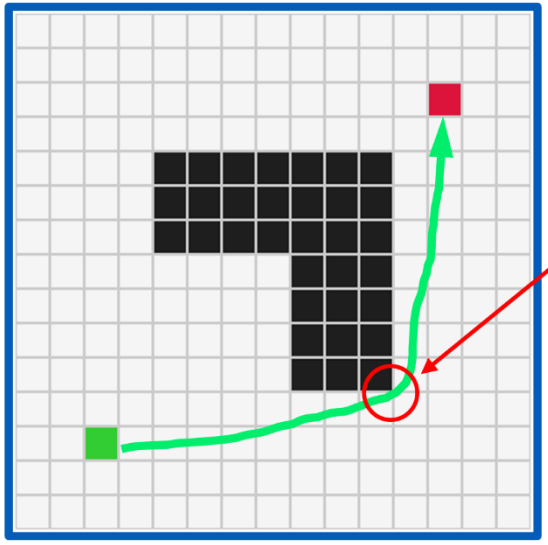
04 Inflation

Why do we need inflation?

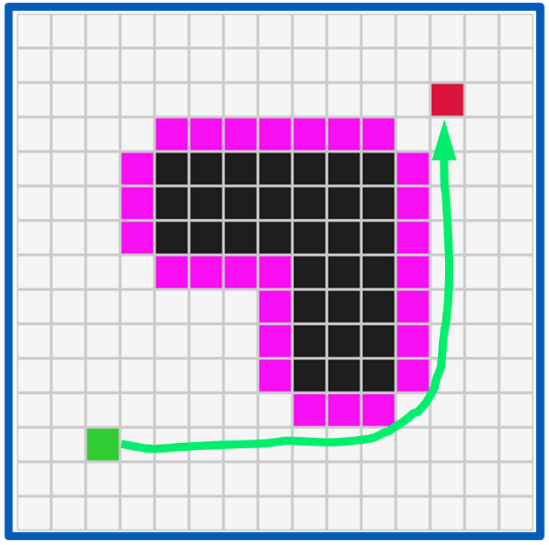
Occupancy grid tells only can/can't pass.

Real routes must also consider how far robots keep from obstacles.

Inflation creates a high-cost halo around obstacles so that a planner naturally maintains clearance and reduces unrealistic corner-cutting using cost only.



< without inflation >

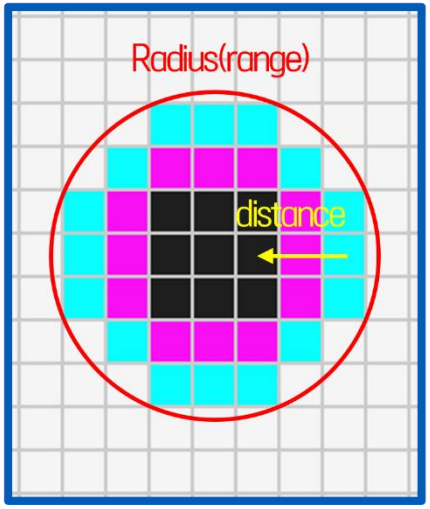


< inflation applied >

04 Inflation

Core Intuitions of Inflation

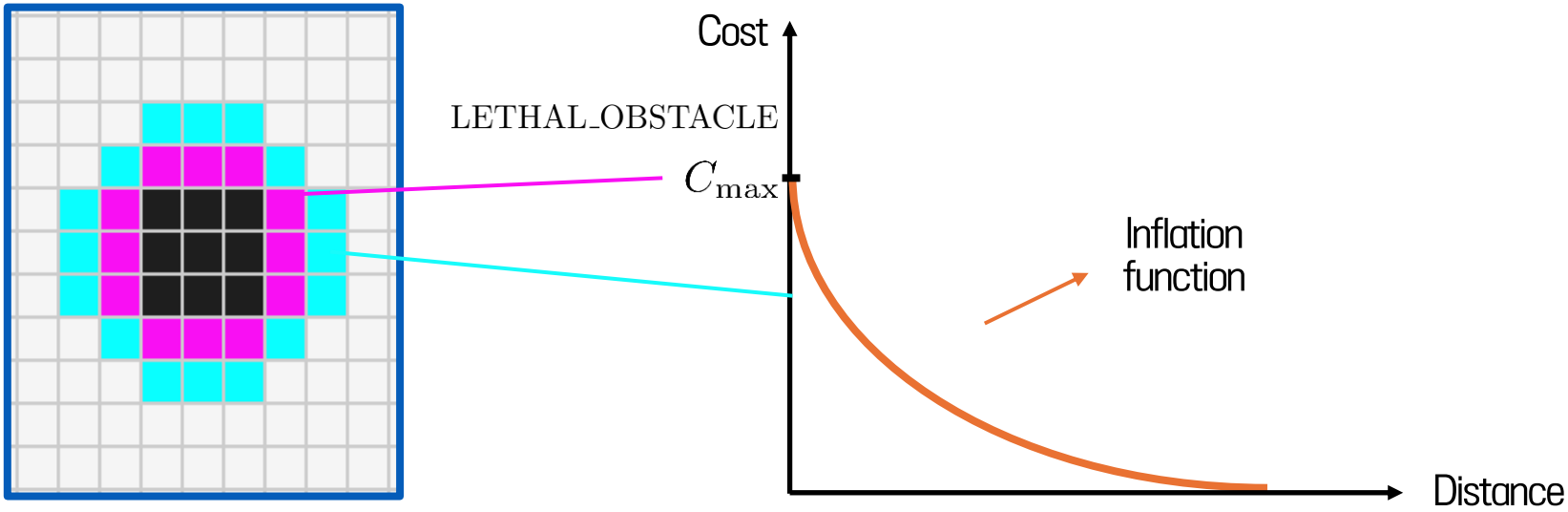
- Distance-based cost
  → Each cell's cost depends on its nearest-obstacle distance d.
- Closer → Higher cost; Farther → Lower cost
  ($d = 0$ marks the non-traversable core.)
- Radius R : the effective range of inflation. → Choose from robot size + margin.
  → For $d > R$, set cost to zero(or very small) so far-away cells remain free.



04 Inflation

Mathematical Models for Inflation Costs

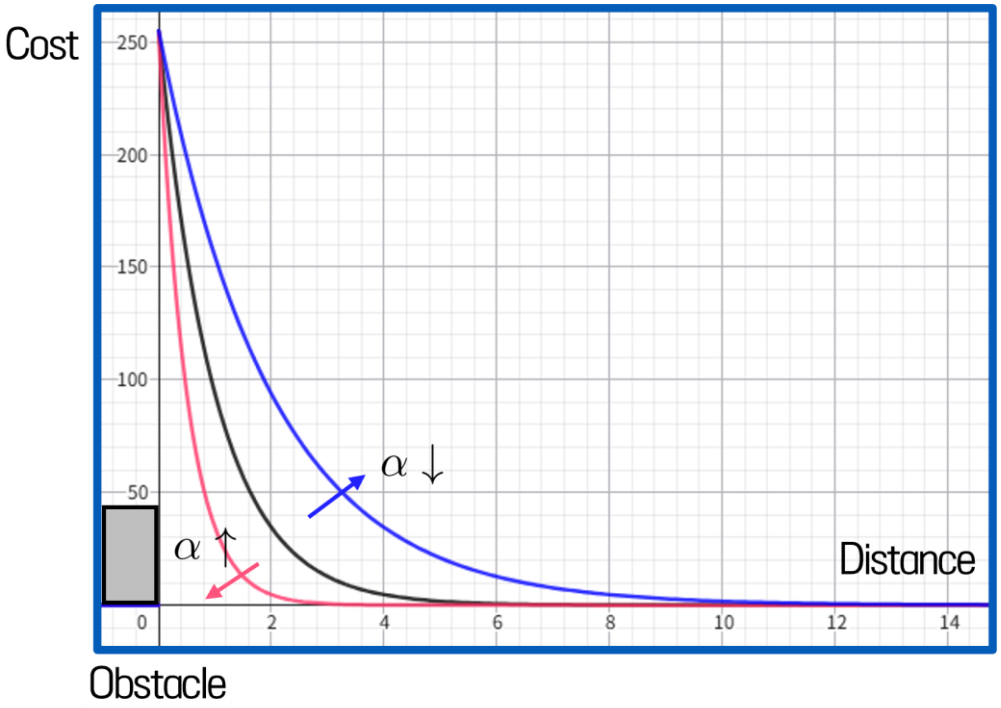
- Design the inflation function to match your intent. → Not a rule!
- Shape the inflation curve to encode your desired clearance and detour preference.
- The essentials are higher cost near obstacles, lower cost father away. → depending on d



04 Inflation

Example Models – Exponential Decay

$$C_{\text{infl}}(d) = \begin{cases} \text{Obstacle core} & d = 0 \\ C_{\max} e^{-\alpha d} & 0 < d \leq R \\ 0 & d > R \end{cases}$$

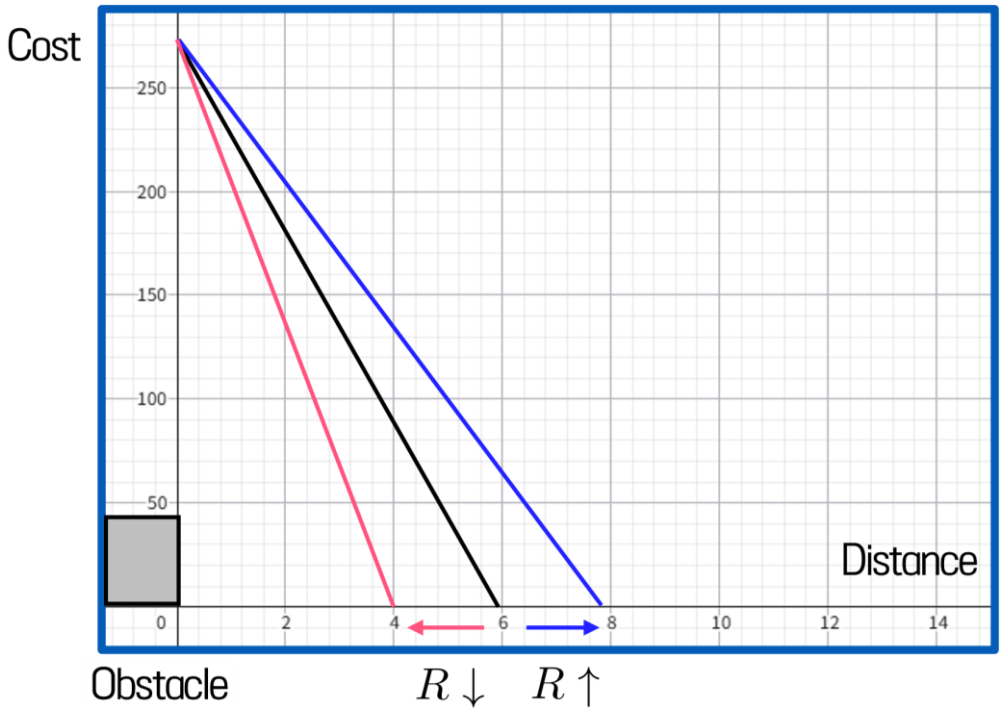


- Strong near the core → drops quickly just a bit away.
- Tuning Parameter α : strong near-core penalty, fast falloff.
- Exponential method is mainly used.

04 Inflation

Example Models – Linear Lamp

$$C_{\text{infl}}(d) = C_{\max} \max(0,\, 1 - \frac{d}{R})$$



- Linear decrease → very intuitive and predictable.

04 Inflation

Application of Inflation in a Real ROS/Nav2 Environment

Location: /opt/ros/humble/include/nav2_costmap_2d/nav2_costmap_2d/inflation_layer.hpp

```
144
145  /** @brief  Given a distance, compute a cost.
146   * @param  distance The distance from an obstacle in cells
147   * @return A cost value for the distance */
148  inline unsigned char computeCost(double distance) const
149  {
150    unsigned char cost = 0;
151    if (distance == 0) {
152      cost = LETHAL_OBSTACLE;
153    } else if (distance * resolution_ <= inscribed_radius_) {
154      cost = INSCRIBED_INFLATED_OBSTACLE;
155    } else {
156      // make sure cost falls off by Euclidean distance
157      double factor =
158        exp(-1.0 * cost_scaling_factor_ * (distance * resolution_ - inscribed_radius_));
159      cost = static_cast<unsigned char>((INSCRIBED_INFLATED_OBSTACLE - 1) * factor);
160    }
161    return cost;
162  }
```

254

253

d

α

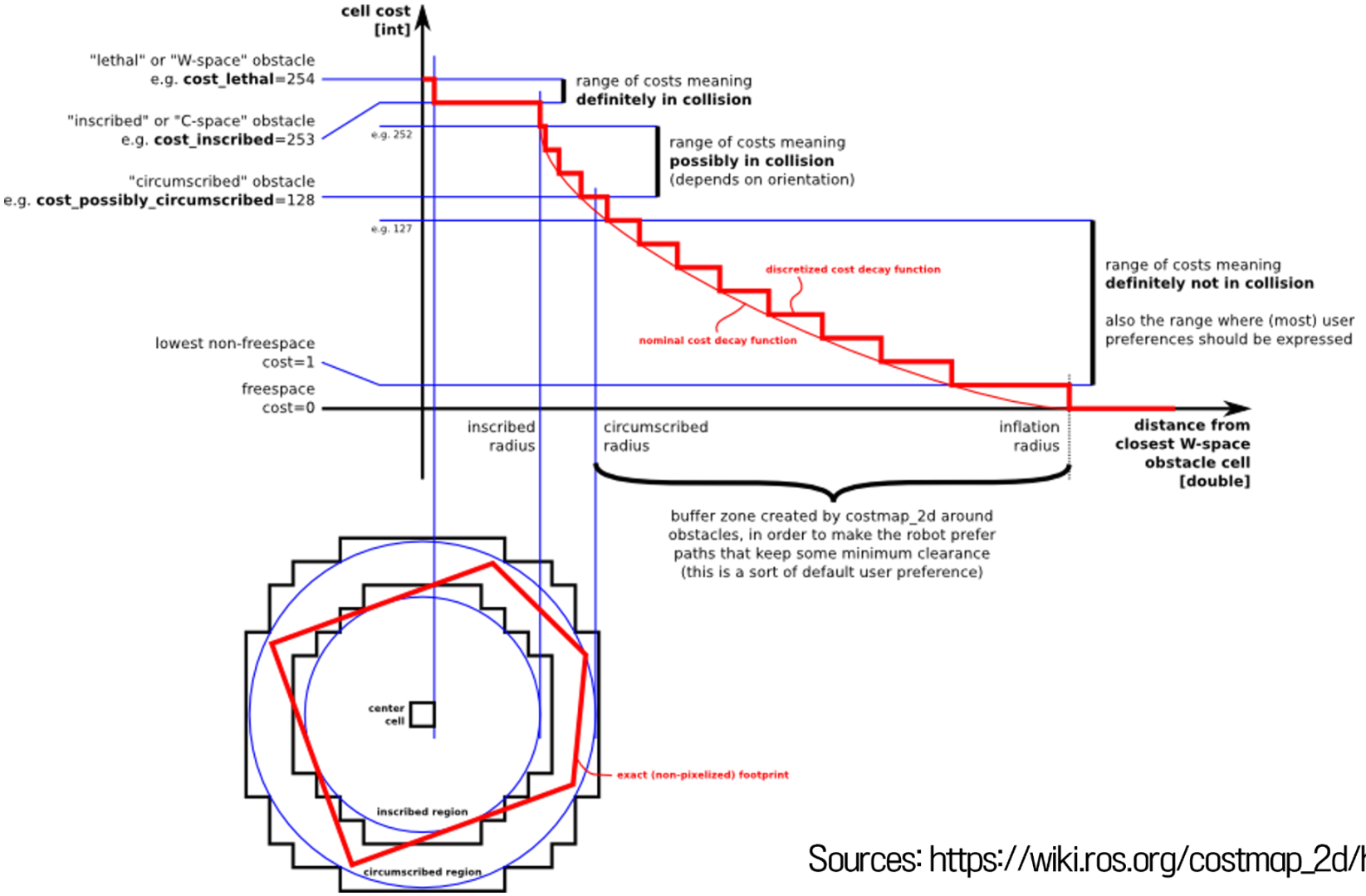
$C_{\max}(= 252)$

$$C_{\text{infl}}(d) = \begin{cases} \text{Obstacle core} & d = 0 \\ C_{\max} e^{-\alpha d} & 0 < d \leq R \\ 0 & d > R \end{cases}$$

< Exponential Decay >

04 Inflation

- Application of Inflation in a Real ROS/Nav2 Environment



Sources: https://wiki.ros.org/costmap_2d/hydro/inflation

05 Connectivity

05 Connectivity

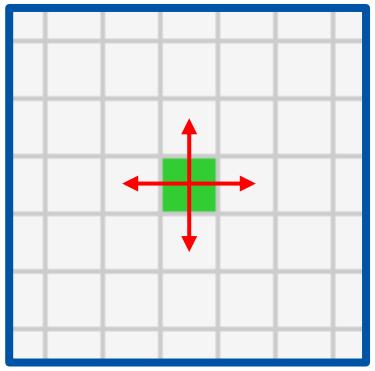
What does connectivity mean in path planning?

Connectivity defines which neighbor cells are allowed to move to from a given cell.

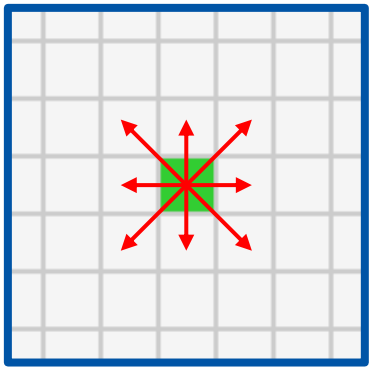
Standard Choices of Connectivity

- 4 – connected (Von Neumann) : Up, Down, Left, Right → Manhattan
- 8 – connected (Moore) : 4-connected + 4 diagonals → Octile

※ With 8-connected moves, paths can be shorter and straighter. However, it can be more expensive of more branching.



< 4 - connected >



< 8 - connected >

$$C_{\text{step}} = \begin{cases} 1 & \text{cardinal moves} \\ \sqrt{2} & \text{diagonals (if } 8-\text{connected)} \end{cases}$$

< per-step cost >

05 Connectivity

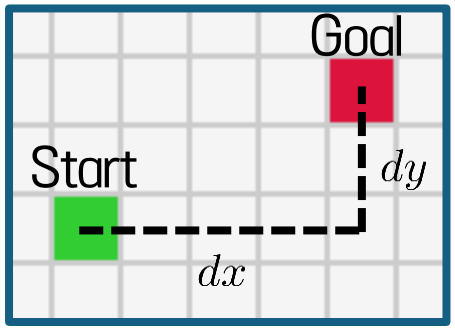
Heuristic Alignment (A* Preview)

- 4-connected → Manhattan Distance (admissible/consistent)

$$h = |dx| + |dy|$$

- 8-connected → Octile Distance ($\text{with straight} = 1,\ \text{diagonal} = \sqrt{2}$)

$$h = (|dx| + |dy|) + (\sqrt{2} - 2) \cdot \min(|dx|, |dy|)$$



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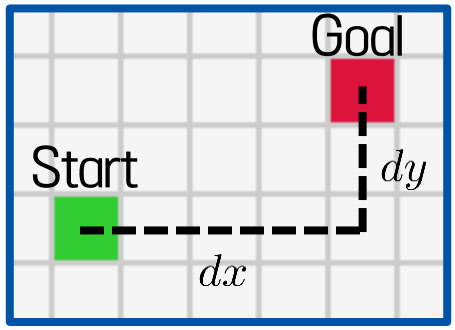

$$dx = 4$$
$$dy = 2$$

< 4-connected >

$$\begin{aligned} h &= |dx| + |dy| \\ &= |4| + |2| \\ &= 6 \end{aligned}$$

05 Connectivity

Heuristic Alignment (A* Preview)

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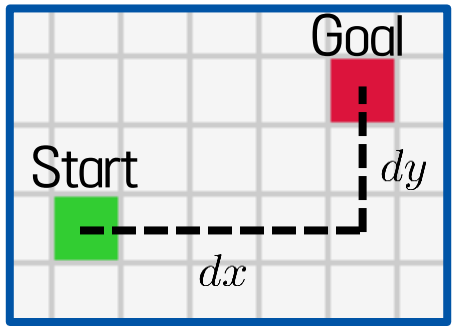
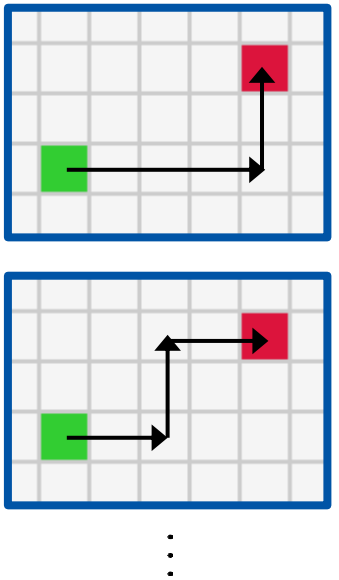
- 8-connected → Octile Distance (with straight $= 1$, diagonal $= \sqrt{2}$)

$$h = (|dx| + |dy|) + (\sqrt{2} - 2) \cdot \min(|dx|, |dy|)$$



< 4-connected >

$$h = |dx| + |dy|$$
$$= |4| + |2|$$
$$= 6$$



05 Connectivity

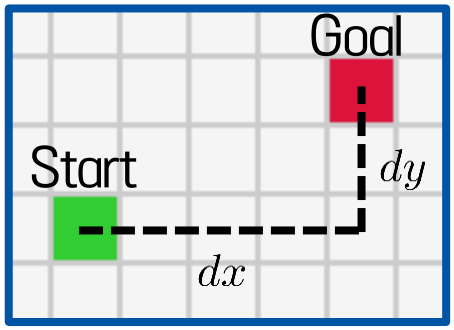
Heuristic Alignment (A* Preview)

- 4-connected → Manhattan Distance (admissible/consistent)

$$h = |dx| + |dy|$$

- 8-connected → Octile Distance ($\text{with straight} = 1,\ \text{diagonal} = \sqrt{2}$)

$$h = (|dx| + |dy|) + (\sqrt{2} - 2) \cdot \min(|dx|, |dy|)$$



$$dx = 4$$
$$dy = 2$$

< 8-connected >

$$\begin{aligned} h &= (|dx| + |dy|) + (\sqrt{2} - 2) \cdot \min(|dx|,\ |dy|) \\ &= (|4| + |2|) + (\sqrt{2} - 2) \cdot |2| \\ &= 2 + 2\sqrt{2} \end{aligned}$$

05 Connectivity

Heuristic Alignment (A* Preview)

- 4-connected → Manhattan Distance (admissible/consistent)

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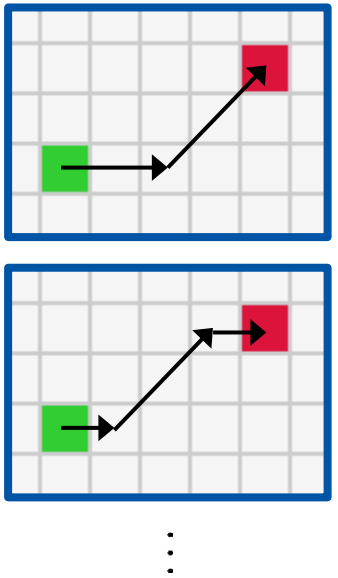
< 8-connected >

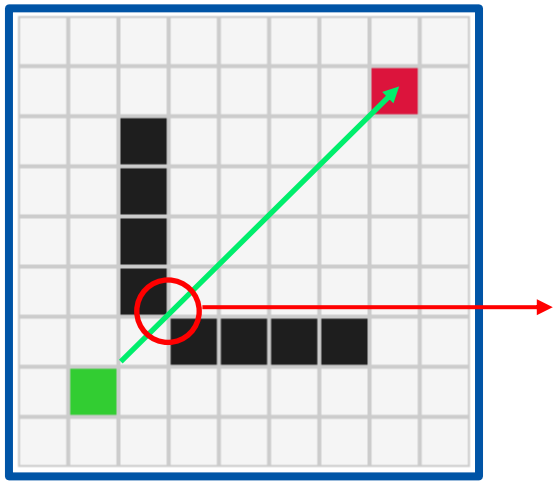
$$\begin{aligned} h &= (|dx| + |dy|) + (\sqrt{2} - 2) \cdot \min(|dx|, |dy|) \\ &= (|4| + |2|) + (\sqrt{2} - 2) \cdot |2| \\ &= 2 + 2\sqrt{2} \end{aligned}$$



05 Connectivity

Diagonal Safety Rule (Anti Corner-Cutting)

Issue: With 8-connected grids, a path may squeeze between two obstacle corners via a diagonal.



There is no problem theoretically,
but in real world, it is wrong path.

Solution(Rule) :

→ Allow a diagonal $(i, j) \to (i \pm 1, j \pm 1)$ only if both adjacent cardinal cells $(i, j \pm 1), (i \pm 1, j)$ are free.

Effects :

- It robustly prevents corner-cutting.
- It works best together with inflation.

05 Connectivity

Comparison

- 4-connected vs. 8-connected

	4-connected	8-connected
Branching	Lower (often faster)	Higher (more list expansions, computation ↑)
Path Length & Straightness	Can be longer	Often shorter & straighter
Corner-cut Risk	Low	High (mitigate with safety rule)
Heuristic	Manhattan	Octile
Implementation	Simple	Moderate (add safety rule)