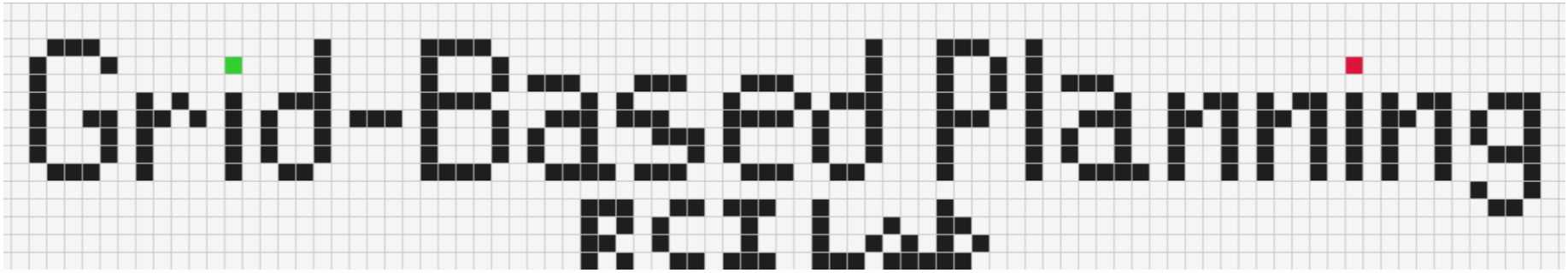
Algorithms – DFS, BFS



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KHU

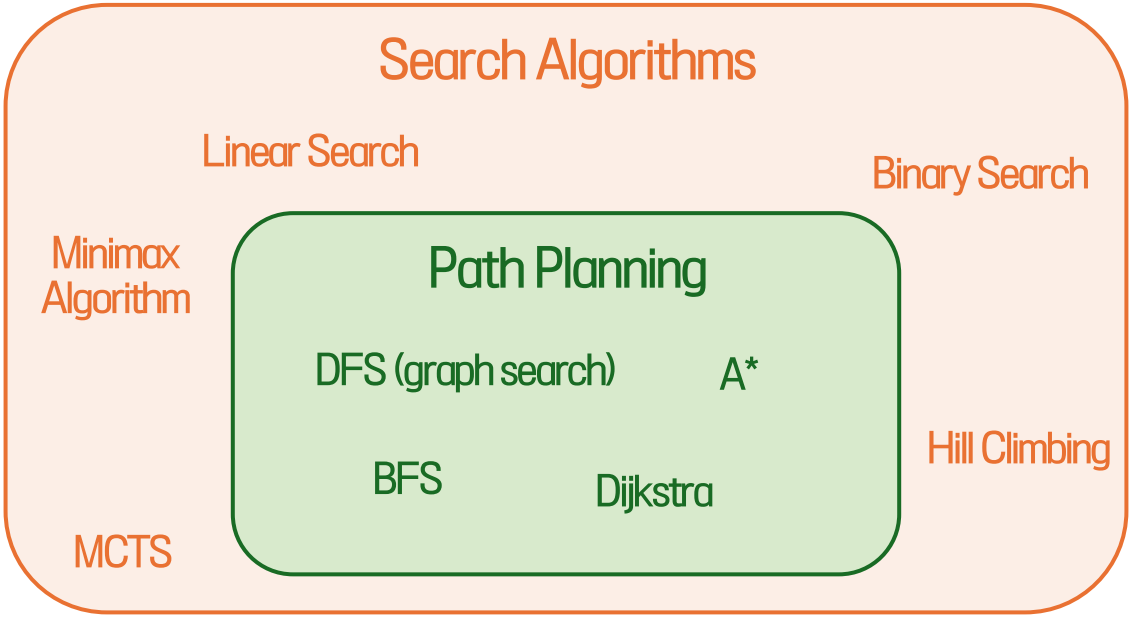
Table of Contents

01 Search Algorithm

01 Search Algorithm

Search Algorithms and Path Planning

- Search Algorithms are systematic state-space exploration to reach a goal or solution.
- Path Planning(finding) is a specialized search task focusing on a path under cost and constraints.

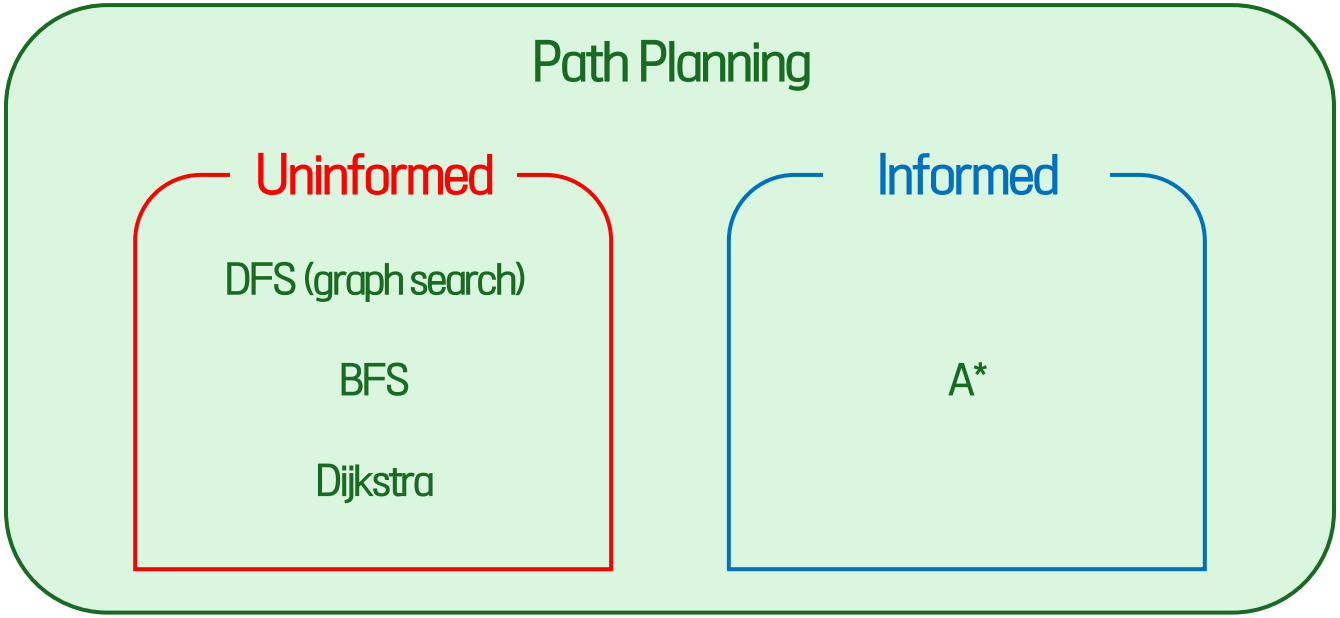


01 Search Algorithm

Uninformed vs. Informed

- Uninformed(Blind) Search :
  No information about the remaining distance to goal. (No heuristic $h(n)$)
  The expansion order is determined solely by the graph's structure.

- Informed(Heuristic) Search :
  Uses a heuristic $h(n)$ to estimate how close a node is to the goal,
  and expands more promising nodes first based on that estimate.

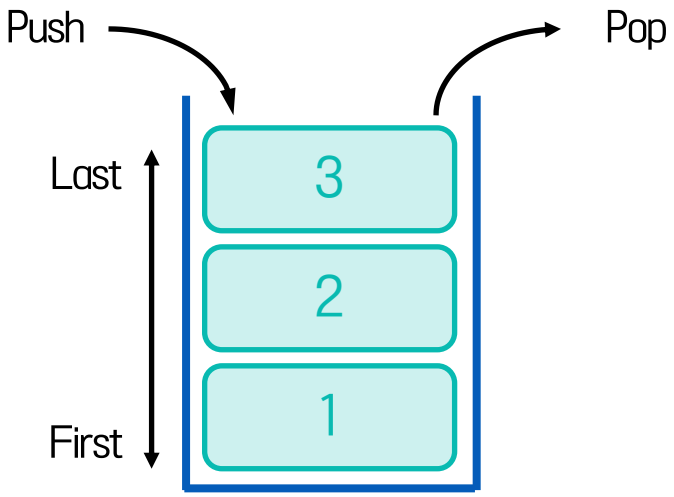


02 Stack vs. Queue

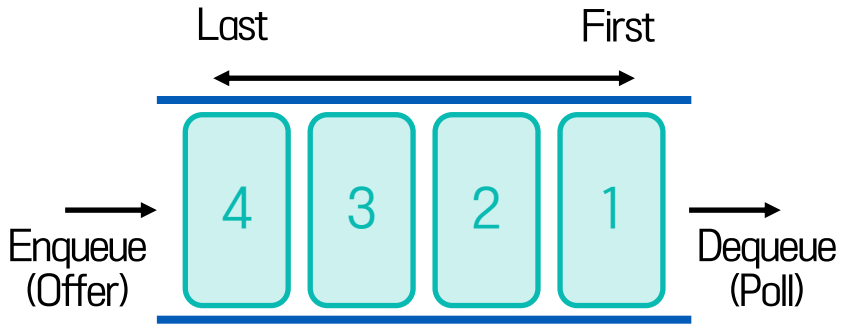
02 Stack vs. Queue

LIFO vs. FIFO

- LIFO(Last In First Out): Data structure where the object you put in later leaves first
- FIFO(First In First Out): Data structure where the object you put in first escapes first



Stack(LIFO)



Queue(FIFO)

→ Logics for path planning varies depending on the data structure that stores information about path.

↳ visited, weight

02 Stack vs. Queue

LIFO vs. FIFO

- LIFO(Last In First Out): Data structure where the object you put in later leaves first
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Representative Algorithms

Algorithm	Main Data Structure	Order Type
DFS	Stack(or recursion stack)	LIFO
BFS	Queue	FIFO

Algorithm	Pros	Cons
DFS	• Simple to implement • Uses relatively little memory	• Not complete (Can go infinitely deep in very large graphs) • Does not guarantee the shortest path
BFS	• On an unweighted graph(unit-cost), it guarantees the shortest path • Complete for finite graphs	• Needs to store many nodes at the same depth → higher memory usage than DFS

03 DFS Algorithm

03 DFS Algorithm

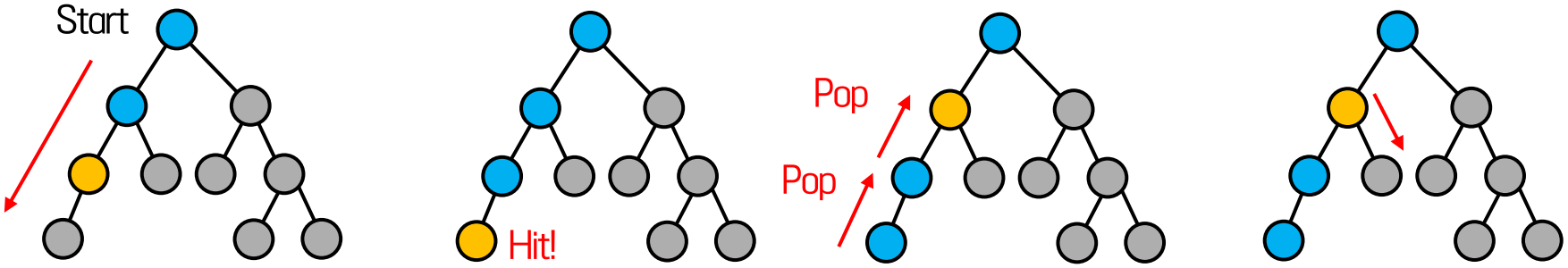
DFS (Depth First Search)

- DFS goes as deeply as possible along a branch. On dead-ends, it backtracks to try another.
- DFS is a basic graph analysis like detection of connected components, cycle, maze generation/solving, etc.

Main Idea

: unvisited : visited : frontier

Dive deeply as possible › Hit a dead-end › Backtrack by popping › Try unvisited branches



03 DFS Algorithm

Main Idea

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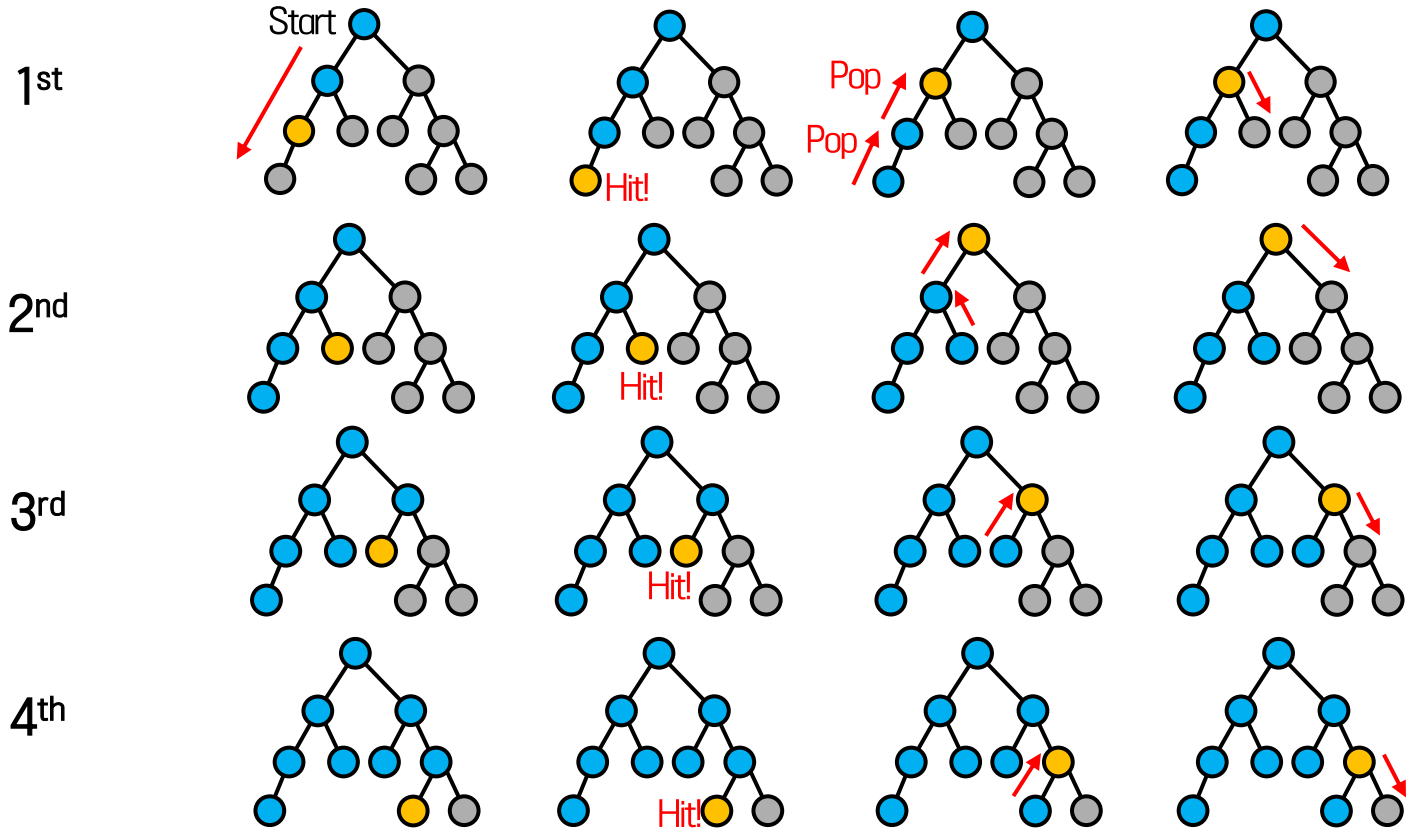
Dive deeply as possible › Hit a dead-end › Backtrack by popping › Try unvisited branches › Repeat until finding the goal



03 DFS Algorithm

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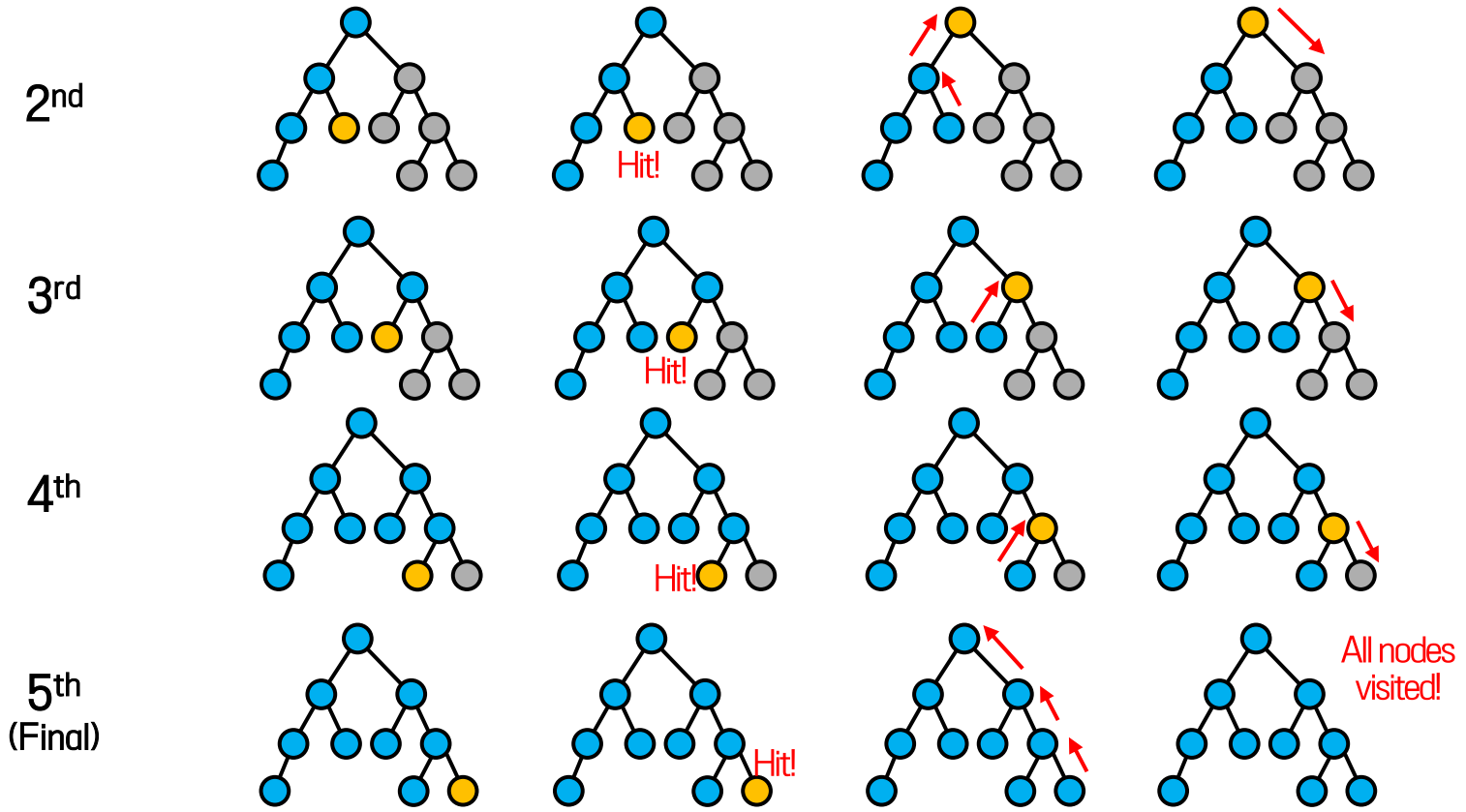
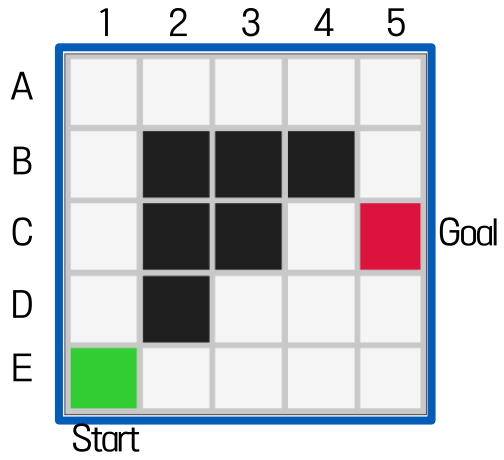


03 DFS Algorithm

Application DFS to Path Planning

- Grid as a graph:
  1. Nodes → Cells
  2. Edges → Connectivity
  3. Dead-end → Obstacle (or boundaries of map)
  4. Rule of visiting: Left first → N, E, S, W (4-connectivity)
- Obstacles block transitions.
- Note that DFS finds a path, not a shortest one.



03 DFS Algorithm

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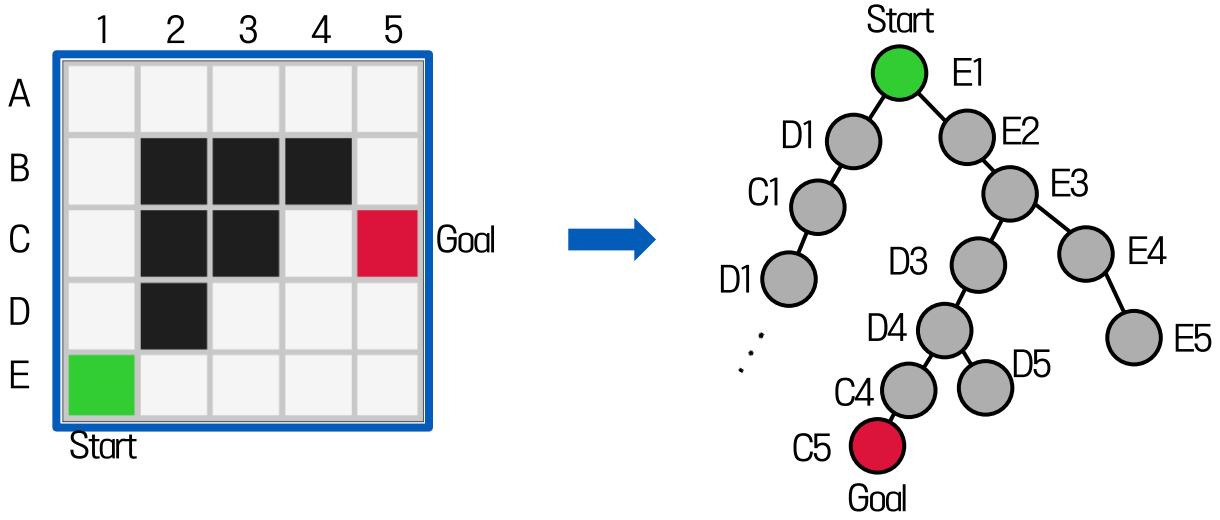


03 DFS Algorithm

Application DFS to Path Planning

Pseudocode

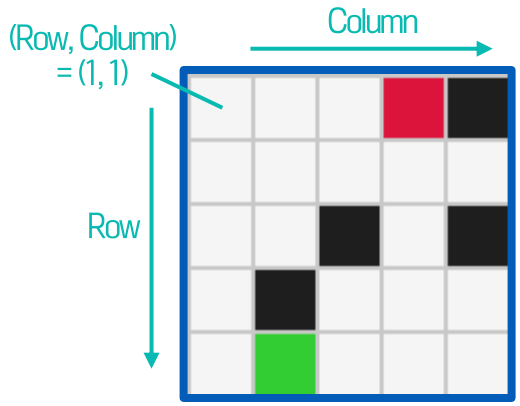
```
1)  S ← [start], parent[start] ← None                ▶ init stack & parent
2)  while S not empty:
3)          u ← pop(S)                               ▶ expand stack(LIFO)
4)          mark u as visited
5)          if u == goal:                            ▶ Check goal when popping
6)                  return reconstruct(parent, u)
7)          for v in neighbors(u) in reverse(N, E, S, W):   ▶ push in reverse of NESW
8)                  if valid(v) and v not visited:
9)                          parent[v] ← u
10)                         push(S, v)               ▶ treat v as visited
11) //                      if v == goal:            ▶ Check goal when pushing
12) //                              return reconstruct(parent, v)
13) return failure
```

In this lecture, we're going to check the goal when popping.

※ You may check the goal either on push or on pop. Pick one and keep it consistent. On-push ends earlier, on-pop aligns with Dijkstra / A*.

03 DFS Algorithm

Application DFS to Path Planning – Example(4-connectivity)



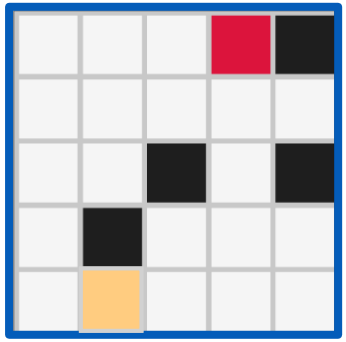
: unvisited
: visited
: obstacle
: frontier
: start
: goal

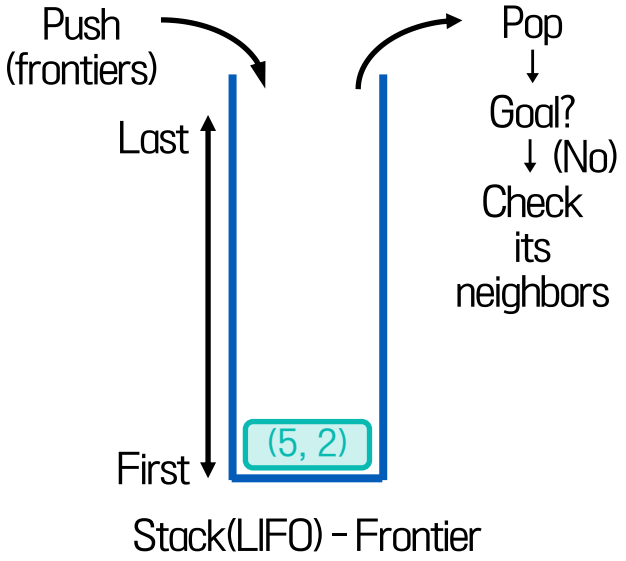
※ Rule of Transition : N → E → S → W

Step 1

- push Start(5,2)
- frontier: {(5,2)}
- parent[(5, 2)] = None

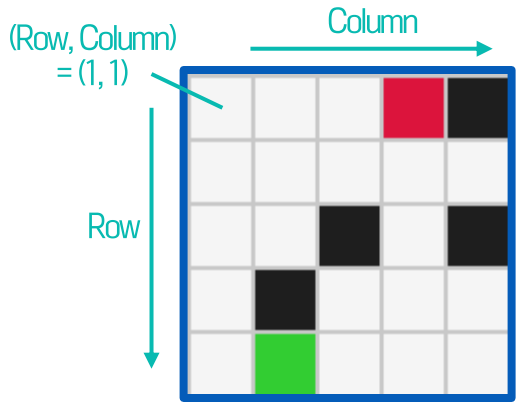
// initialize





03 DFS Algorithm

Application DFS to Path Planning – Example(4-connectivity)



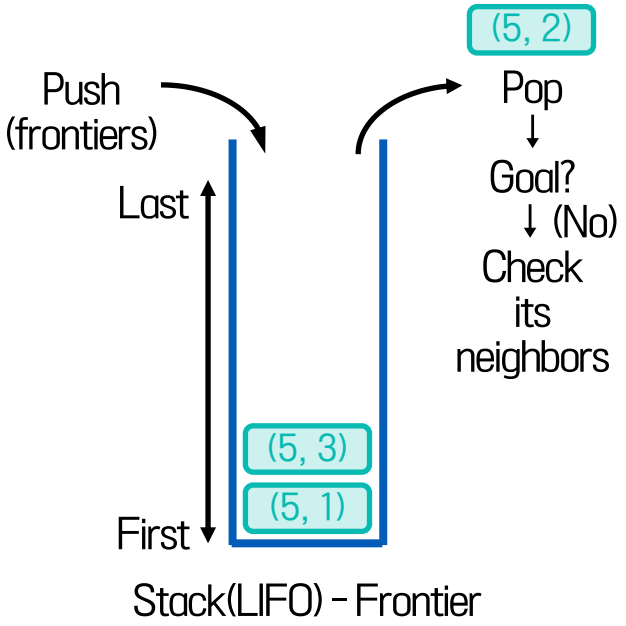
: unvisited
: visited
: obstacle
: frontier
: start
: goal

※ Rule of Transition : N → E → S → W

Step 2

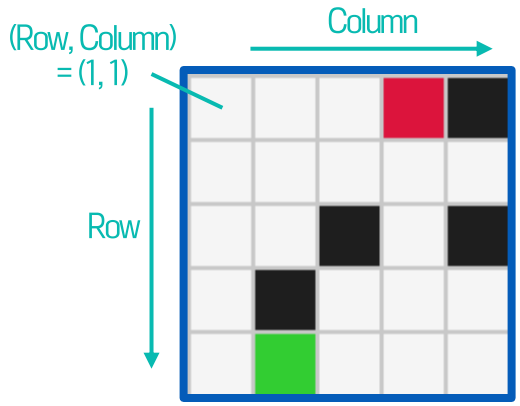
- pop (5, 2) & Check
- W(5, 1): O / E(5, 3): O / N(4, 2): X
  → push W(5, 1) → push E(5, 3)

- frontier: {(5, 3), (5, 1)}
- parent[(5, 1)] = (5, 2)
  parent[(5, 3)] = (5, 2)



03 DFS Algorithm

Application DFS to Path Planning – Example(4-connectivity)



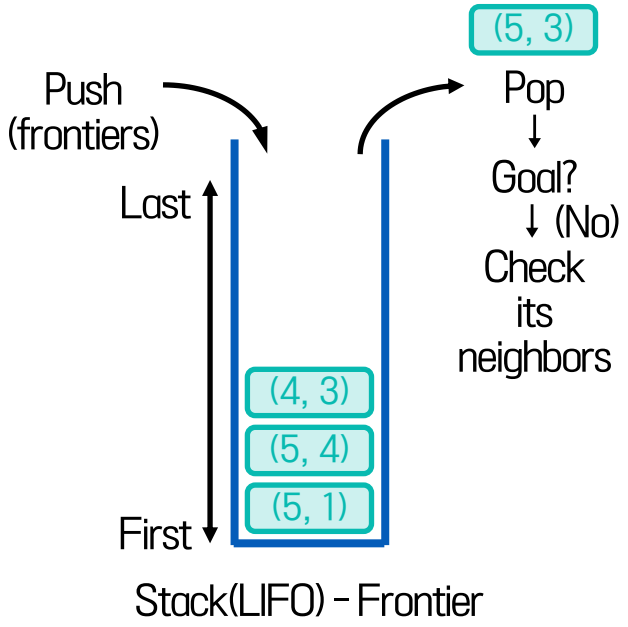
: unvisited
: visited
: obstacle
: frontier
: start
: goal

※ Rule of Transition : N → E → S → W

Step 3

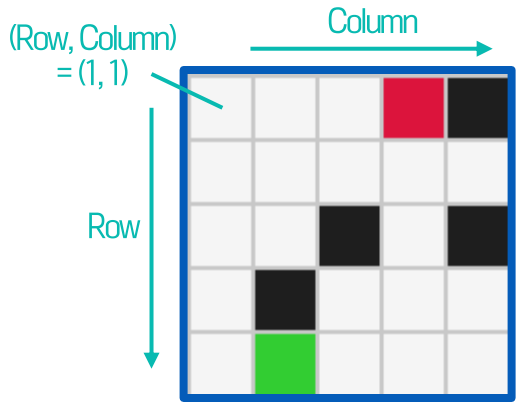
- pop (5, 3) & Check
- E(5, 4): O / N(4, 3): O
  → push E(5, 4) → push N(4, 3)

- frontier: {(4, 3), (5, 4), (5, 1)}
- parent[(5, 4)] = (5, 3)
  parent[(4, 3)] = (5, 3)



03 DFS Algorithm

Application DFS to Path Planning – Example(4-connectivity)



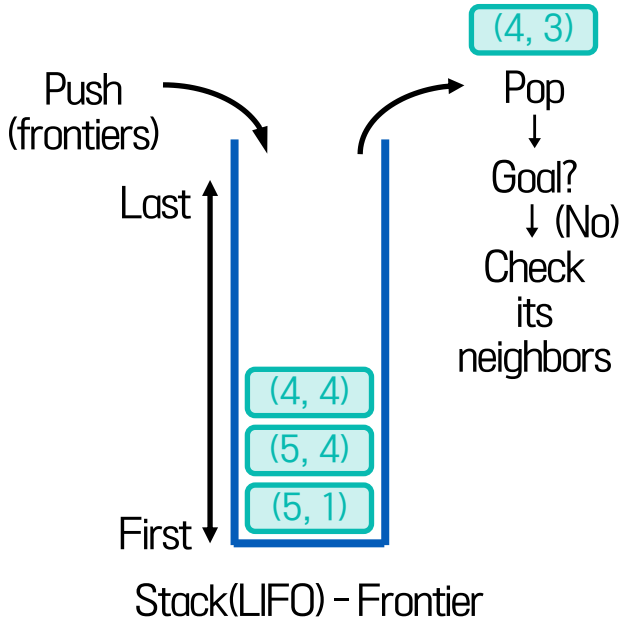
: unvisited
: visited
: obstacle
: frontier
: start
: goal

※ Rule of Transition : N → E → S → W

Step 4

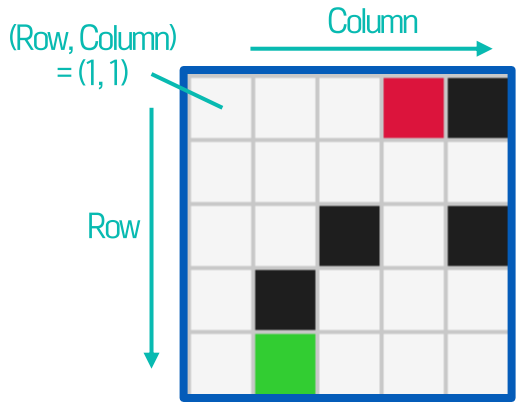
- pop (4, 3) & Check
- S(4, 2): X / E(4, 4): O / N(3, 3): X
  → push E(4, 4)

- frontier: {(4, 4), (5, 4), (5, 1)}
- parent[(4, 4)] = (4, 3)



03 DFS Algorithm

Application DFS to Path Planning – Example(4-connectivity)



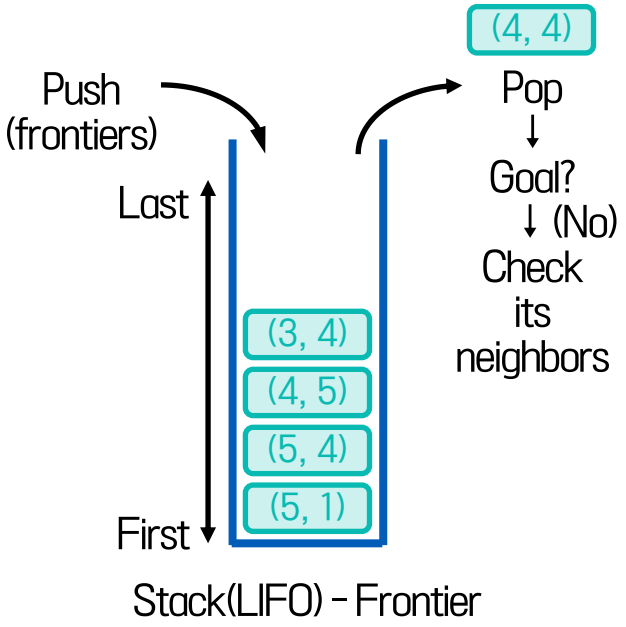
: unvisited
: visited
: obstacle
: frontier
: start
: goal

※ Rule of Transition : N → E → S → W

Step 5

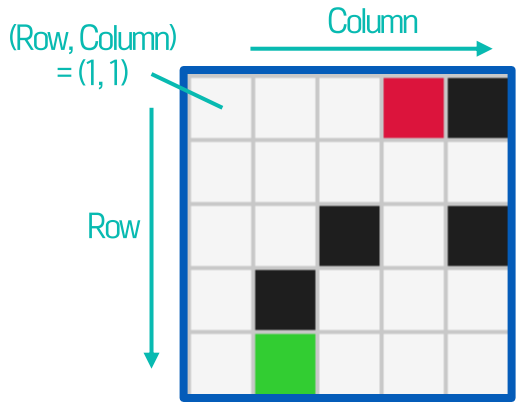
- pop (4, 4) & Check
- E(4, 5): O / N(3, 4): O
  → push E(4, 5) → push N(3, 4)

- frontier:
  {(3, 4), (4, 5), (5, 4), (5, 1)}
- parent[(3, 4)] = (4, 4)
  parent[(4, 5)] = (4, 4)



03 DFS Algorithm

Application DFS to Path Planning – Example(4-connectivity)



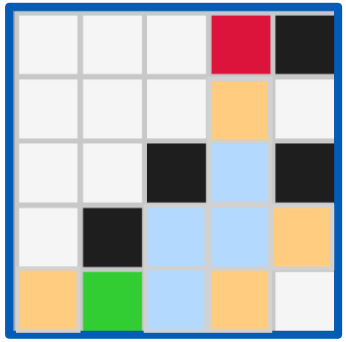
: unvisited
: visited
: obstacle
: frontier
: start
: goal

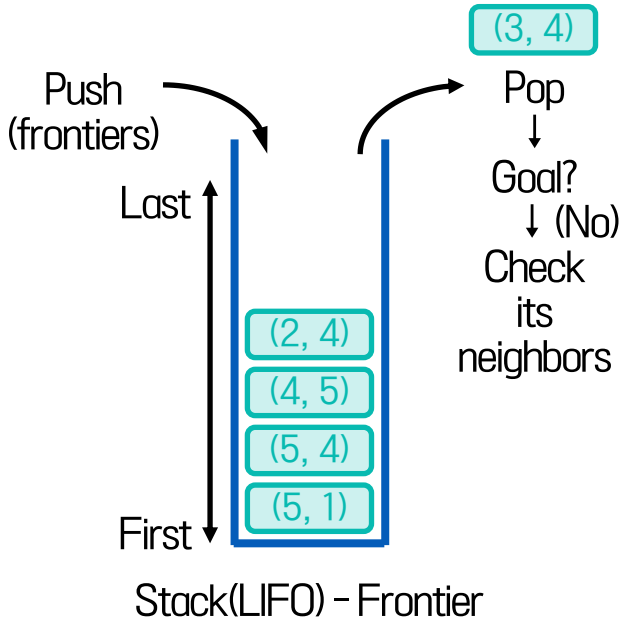
※ Rule of Transition : N → E → S → W

Step 6

- pop (3, 4) & Check
- W(3, 3): X / E(3, 5): X / N(2, 4): O
  → push N(2, 4)

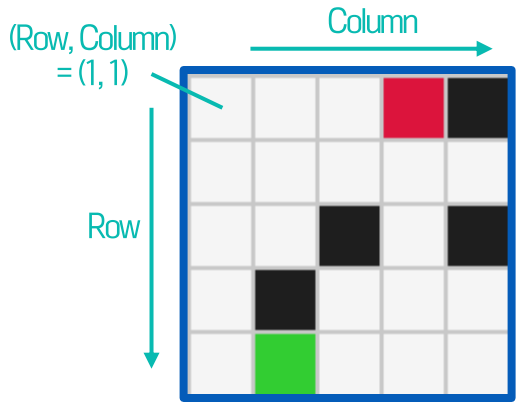
- frontier:
  {(2, 4), (4, 5), (5, 4), (5, 1)}
- parent[(2, 4)] = (3, 4)





03 DFS Algorithm

Application DFS to Path Planning – Example(4-connectivity)



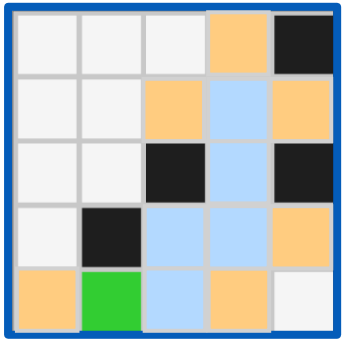
: unvisited
: visited
: obstacle
: frontier
: start
: goal

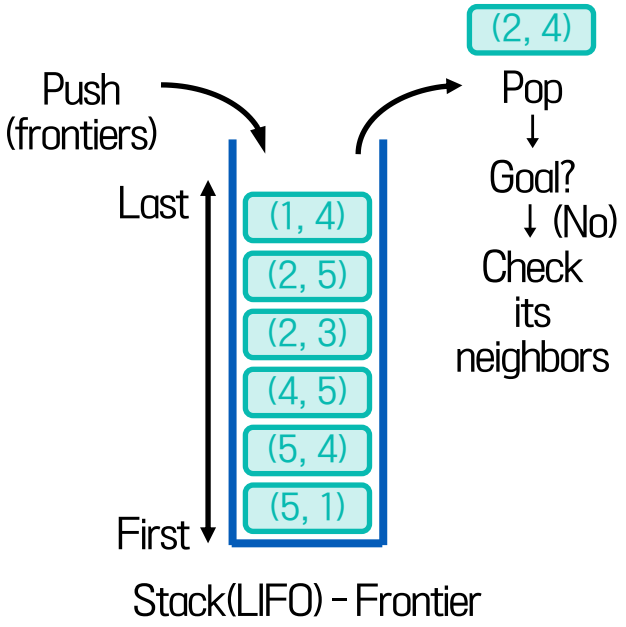
※ Rule of Transition : N → E → S → W

Step 7

- pop (2, 4) & Check
- W(2, 3): O / E(2, 5): O / N(1, 4): O
  → push W(2, 3) → push E(2, 5) → push N(1, 4)

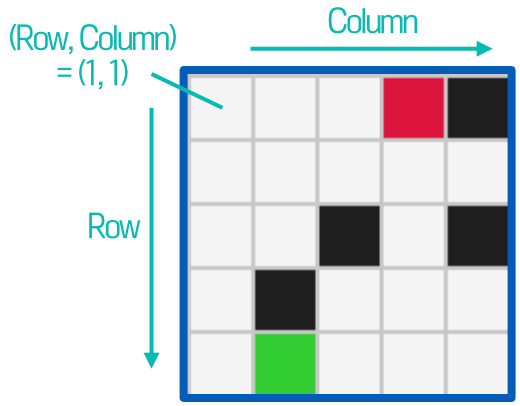
- frontier:
  {(1, 4), (2, 5), (2, 3), (4, 5), (5, 4), (5, 1)}
- parent[(1, 4)] = (2, 4)
  parent[(2, 5)] = (2, 4)
  parent[(2, 3)] = (2, 4)





03 DFS Algorithm

Application DFS to Path Planning – Example(4-connectivity)



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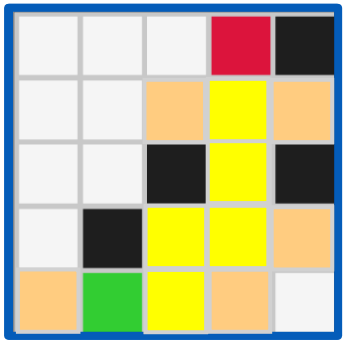
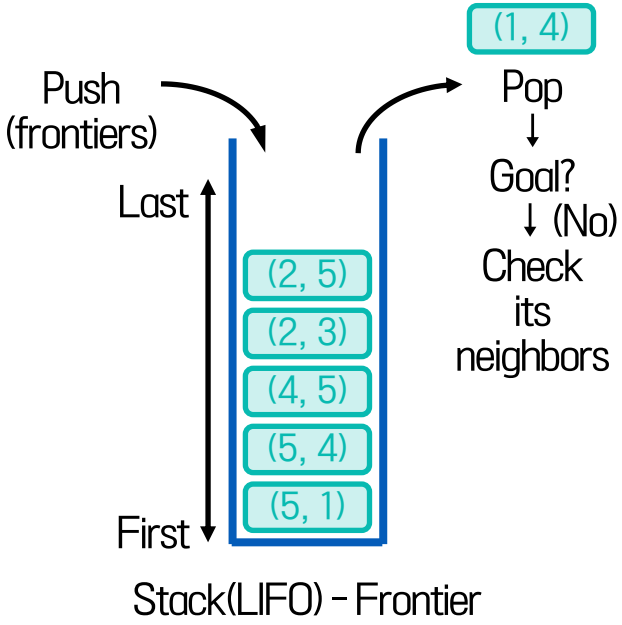
※ Rule of Transition : N → E → S → W

Step 8

- pop (1, 4) & Check ▶ (1, 4) == Goal
- Path Reconstruction from Parent:
  (※ prt == parent) // recursive
  prt[(1, 4)] → prt[(2, 4)] → prt[(3, 4)] →
  prt[(4, 4)] → prt[(4, 3)] → prt[(5, 3)] →
  prt[(5, 2)] // == None



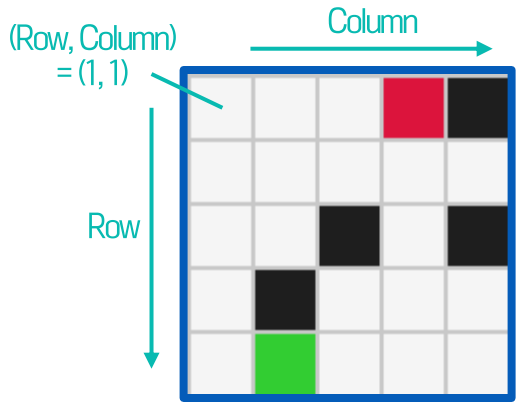
- Reverse:
  Start(5, 2) → (5, 3) → (4, 3) → (4, 4) → (3, 4)
  → (2, 4) → Goal(1, 4)



Stack(LIFO) - Frontier

03 DFS Algorithm

Application DFS to Path Planning – Example(4-connectivity)



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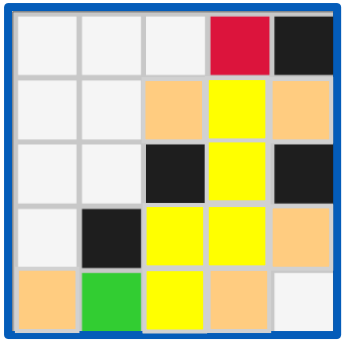
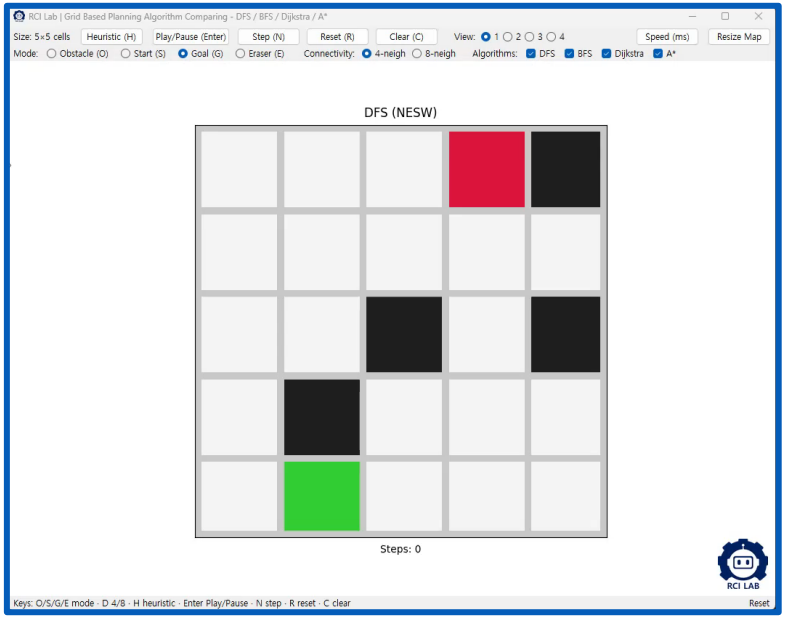
※ Rule of Transition : N → E → S → W

Demonstration in code

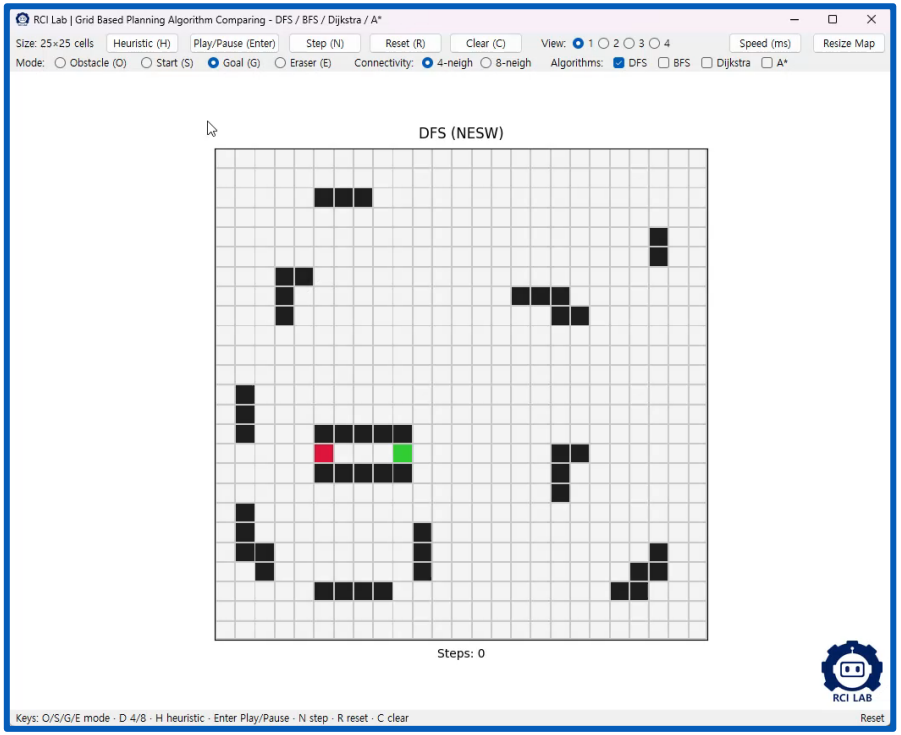




03 DFS Algorithm

Limitations of DFS Algorithm

- DFS often misses shorter routes. It can return a path, but not the shortest. → Not optimal!
- Path heavily depends on neighbor/push(LIFO) order.
- DFS ignores costs. Decisions are driven solely by neighbor/push(LIFO) order.

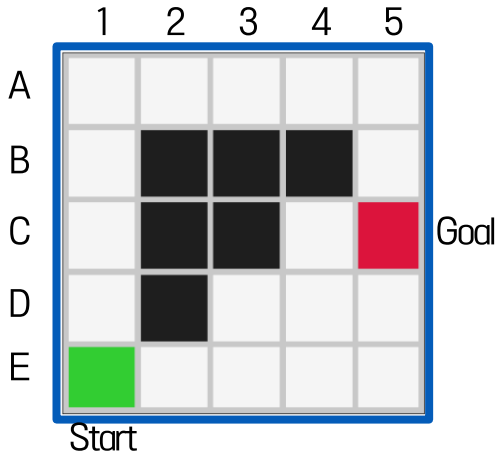


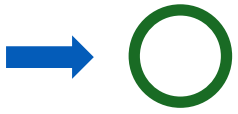
04 BFS Algorithm

04 BFS Algorithm

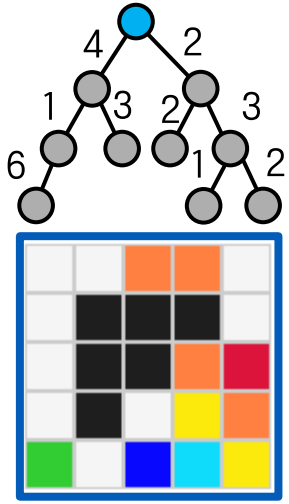
BFS (Breadth First Search)

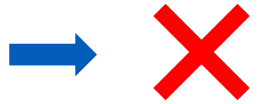
- BFS expands nodes by increasing distance(levels) from the start, using a Queue(FIFO) as the frontier.
- BFS marks visited and records parents at enqueue time.
- BFS guarantees a hop(edge)-shortest path on unit-cost graphs.





BFS can guarantee the shortest path.





BFS can't guarantee the shortest path.

Grid (= unit-cost graph)

- 4-connectivity
- 8-connectivity (Chebyshev dist)

Weighted-edge graph

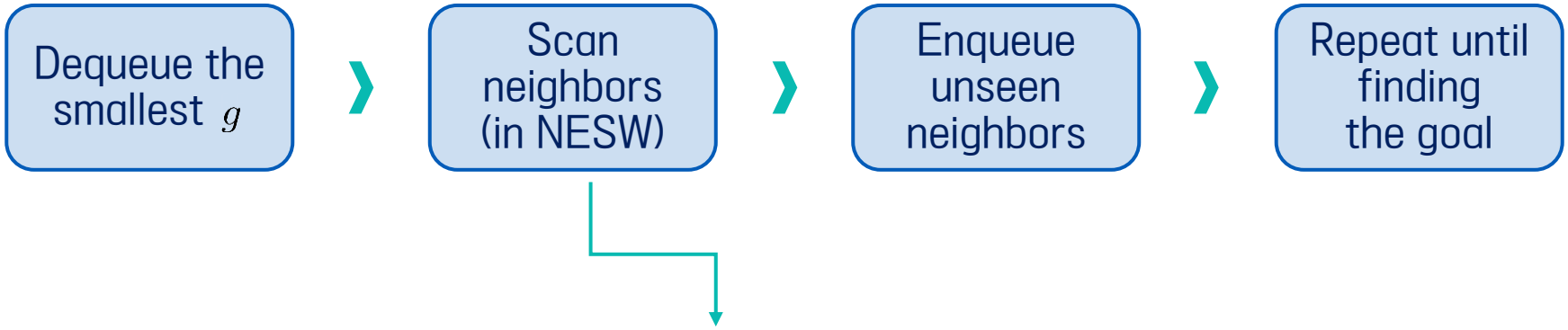
- 8-connectivity (Euclidean dist)
- Grid which is applied non-uniform costmap

04 BFS Algorithm

Main Idea

BFS grows the frontier in non-decreasing path cost(g), recording parents to reconstruct the path.

- Frontier(container of candidates) expands smallest g first. → Key Idea
- Record parent[v] = u on first valid discovery.
- Stop when the goal is selected for expansion. Then, reconstruct via parents



- When several neighbors have same cost, check/enqueue them in NESW order.
- This only decides which one goes first, it does not change shortest-path guarantees(unit-cost) or optimality(Dijkstra)
- A fixed order makes the run deterministic (stable animation, easy debugging, reproducible results).

04 BFS Algorithm

Application BFS to Path Planning

Pseudocode

```
1)  Q ← [start], parent[start] ← None
2)  while Q not empty:
3)          u ← dequeue(S)
4)  //      if u == goal:
5)  //              return reconstruct(parent, goal)
6)          for v in neighbors(u) in (N, E, S, W):
7)                  if valid(v) and v not visited:
8)                          mark v as visited
9)                          parent[v] ← u
10)                         enqueue(Q, v)
11)                         if v == goal:
12)                                 return reconstruct(parent, v)
13) return failure
```

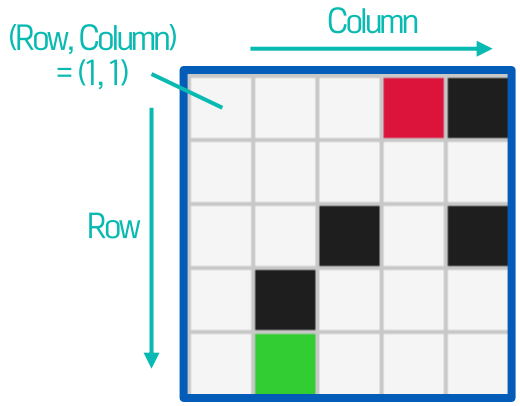
- ▶ init queue & parent
- ▶ expand stack(LIFO)
- ▶ check goal when dequeuing
- ▶ fixed order NESW
- ▶ treat v as visited
- ▶ check goal when enqueuing

In this lecture, we're going to check the goal when enqueuing.

※ Stopping on dequeuing can delay to find the goal.

04 BFS Algorithm

Application BFS to Path Planning – Example(4-connectivity)

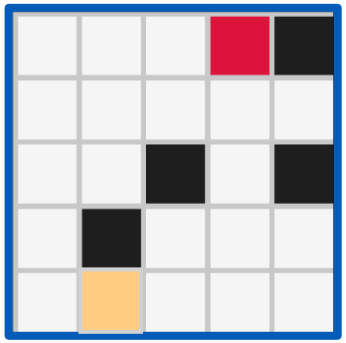


: unvisited
: visited
: obstacle
: frontier
: start
: goal

※ Rule of Transition : N → E → S → W

Step 1

- dequeue X
- enqueue (5, 2) // initialize
- frontier: {(5, 2)}
- parent[(5, 2)] = None

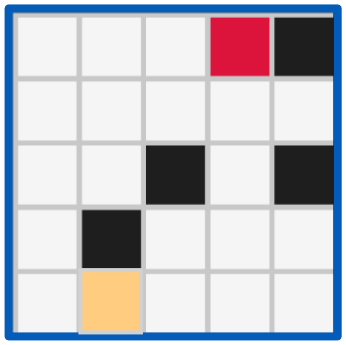


04 BFS Algorithm

Application BFS to Path Planning – Example(4-connectivity)

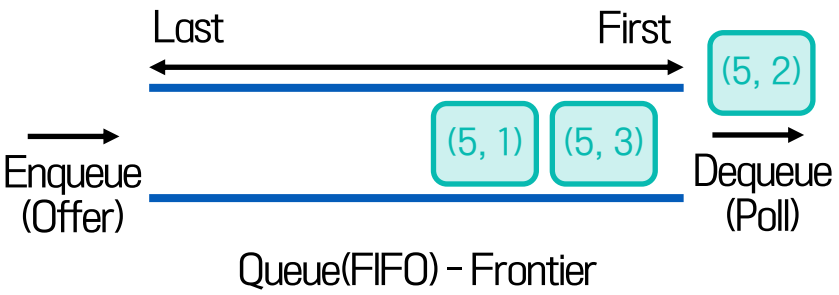
Step 1

- dequeue X
- enqueue (5, 2) // initialize
- frontier: {(5, 2)}
- parent[(5, 2)] = None



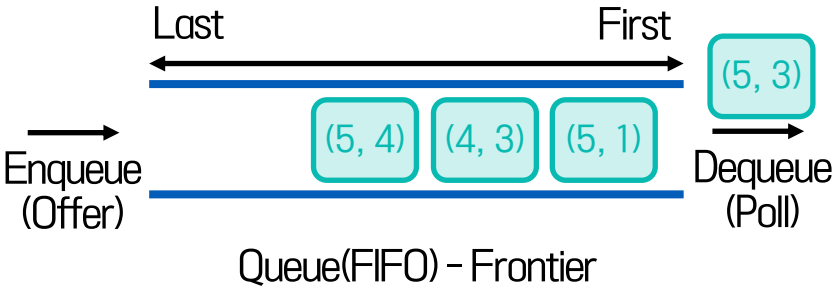
Step 2

- dequeue (5, 2)
- enqueue (5, 3), (5, 1) // NESW
- frontier: {(5, 3), (5, 1)}
- parent[(5, 3)] = (5, 2)
- parent[(5, 1)] = (5, 2)



Step 3

- dequeue (5, 3)
- enqueue (4, 3), (5, 4) // NESW
- frontier: {(5, 1), (4, 3), (5, 4)}
- parent[(4, 3)] = (5, 3)
- parent[(5, 4)] = (5, 3)

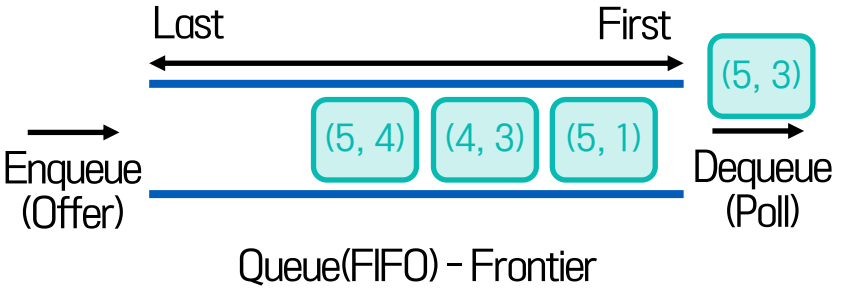


04 BFS Algorithm

Application BFS to Path Planning – Example(4-connectivity)

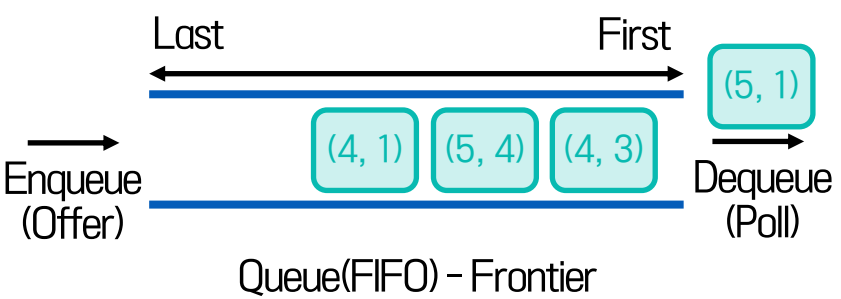
Step 3

- dequeue (5, 3)
- enqueue (4, 3), (5, 4) // NESW
- frontier: {(5, 1), (4, 3), (5, 4)}
- parent[(4, 3)] = (5, 3)
- parent[(5, 4)] = (5, 3)



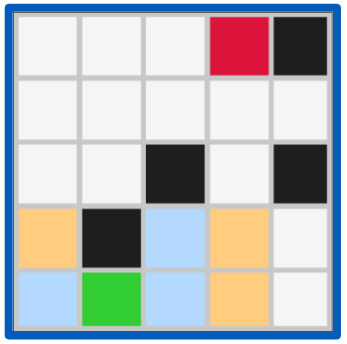
Step 4

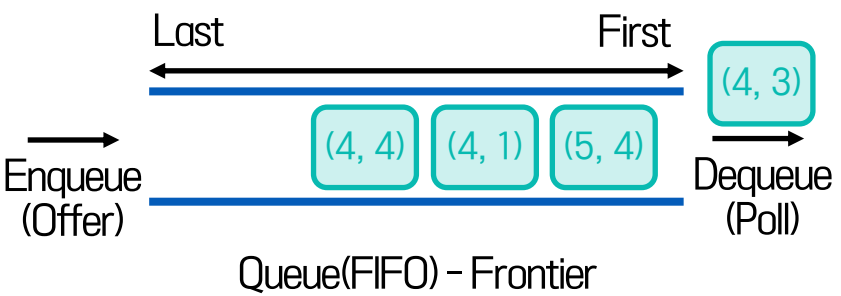
- dequeue (5, 1)
- enqueue (4, 1) // NESW
- frontier: {(4, 3), (5, 4), (4, 1)}
- parent[(4, 1)] = (5, 1)



Step 5

- dequeue (4, 3)
- enqueue (4, 4) // NESW
- frontier: {(5, 4), (4, 1), (4, 4)}
- parent[(4, 4)] = (4, 3)



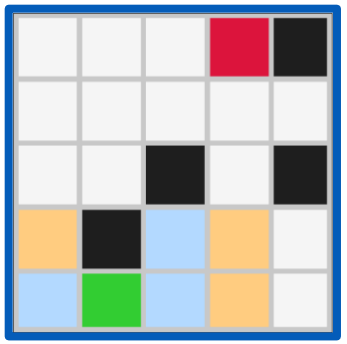


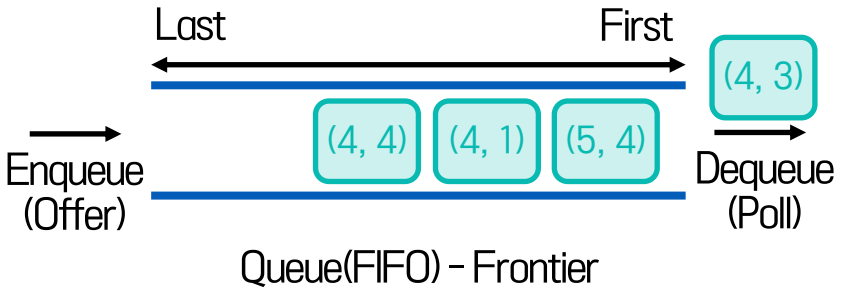
04 BFS Algorithm

Application BFS to Path Planning – Example(4-connectivity)

Step 5

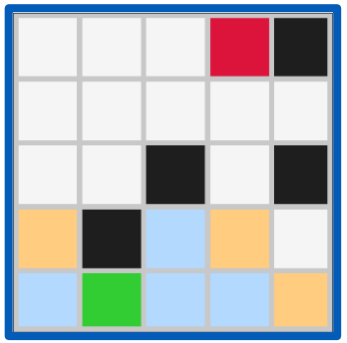
- dequeue (4, 3)
- enqueue (4, 4) // NESW
- frontier: {(5, 4), (4, 1), (4, 4)}
- parent[(4, 4)] = (4, 3)

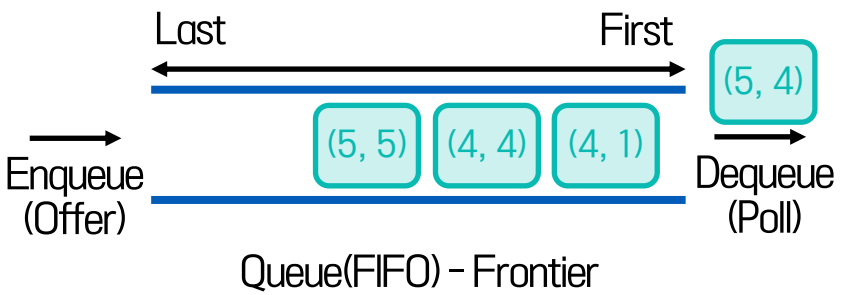




Step 6

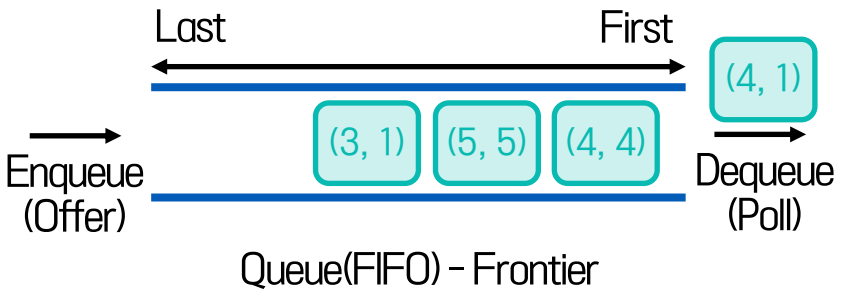
- dequeue (5, 4)
- enqueue (5, 5) // NESW
- frontier: {(4, 1), (4, 4), (5, 5)}
- parent[(5, 5)] = (5, 4)





Step 7

- dequeue (4, 1)
- enqueue (3, 1) // NESW
- frontier: {(4, 4), (5, 5), (3, 1)}
- parent[(3, 1)] = (4, 1)

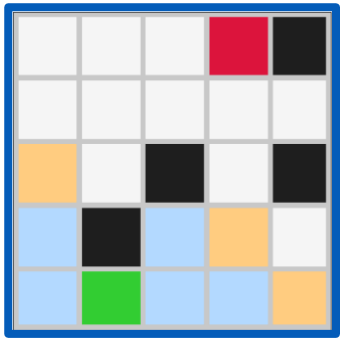


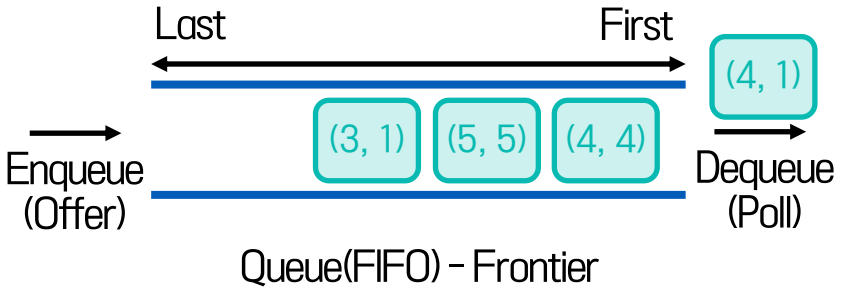
04 BFS Algorithm

Application BFS to Path Planning – Example(4-connectivity)

Step 7

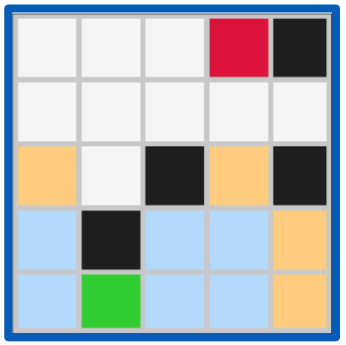
- dequeue (4, 1)
- enqueue (3, 1) // NESW
- frontier: {(4, 4), (5, 5), (3, 1)}
- parent[(3, 1)] = (4, 1)

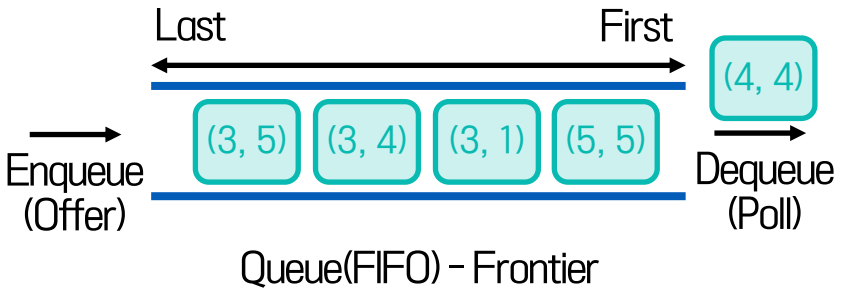




Step 8

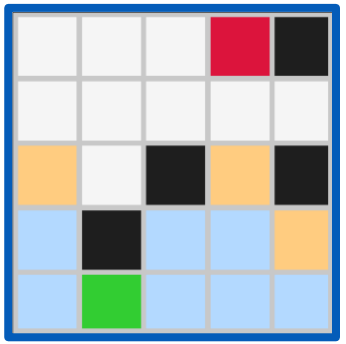
- dequeue (4, 4)
- enqueue (3, 4), (4, 5) // NESW
- frontier: {(5, 5), (3, 1), (3, 4), (3, 5)}
- parent[(3, 4)] = (4, 4)
- parent[(4, 5)] = (4, 4)

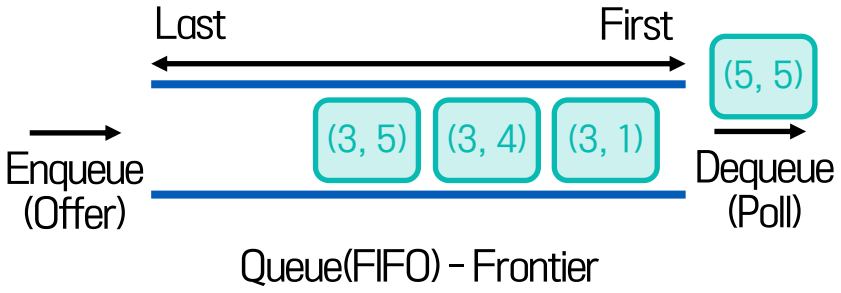




Step 9

- dequeue (5, 5)
- enqueue X // there isn't non-visited.
- frontier: {(3, 1), (3, 4), (3, 5)}



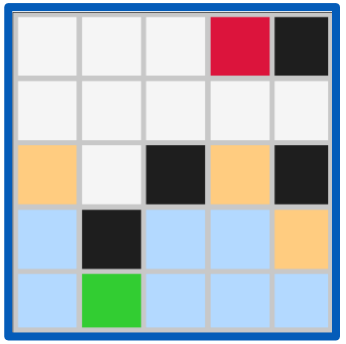


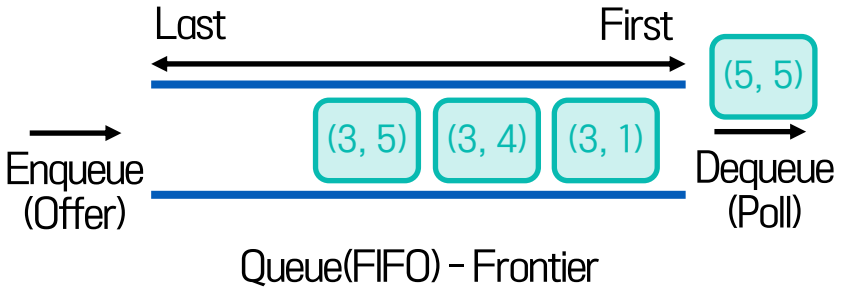
04 BFS Algorithm

Application BFS to Path Planning – Example(4-connectivity)

Step 9

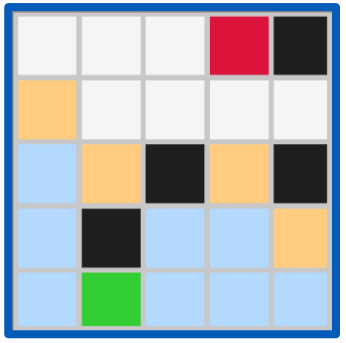
- dequeue (5, 5)
- enqueue X // there isn't non-visited.
- frontier: {(3, 1), (3, 4), (3, 5)}

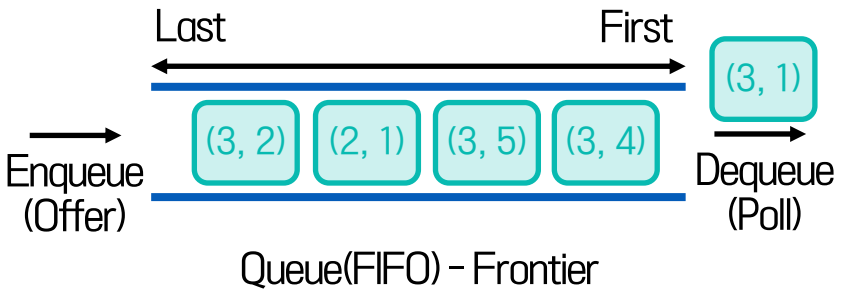




Step 10

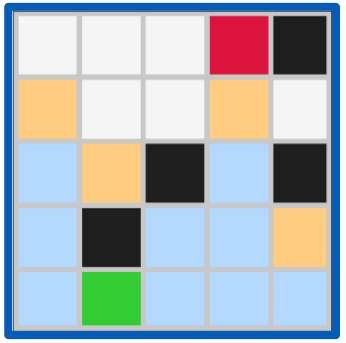
- dequeue (3, 1)
- enqueue (2, 1), (3, 2) // NESW
- frontier: {(3, 4), (3, 5), (2, 1), (3, 2)}
- parent[(2, 1)] = (3, 1)
- parent[(3, 2)] = (3, 1)

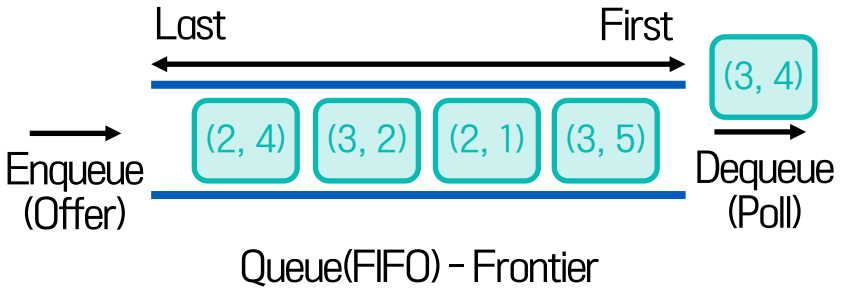




Step 11

- dequeue (3, 4)
- enqueue (2, 4) // NESW
- frontier: {(3, 5), (2, 1), (3, 2), (2, 4)}
- parent[(2, 4)] = (3, 4)



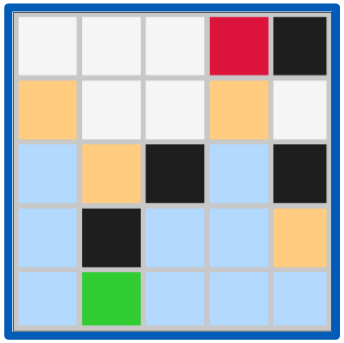


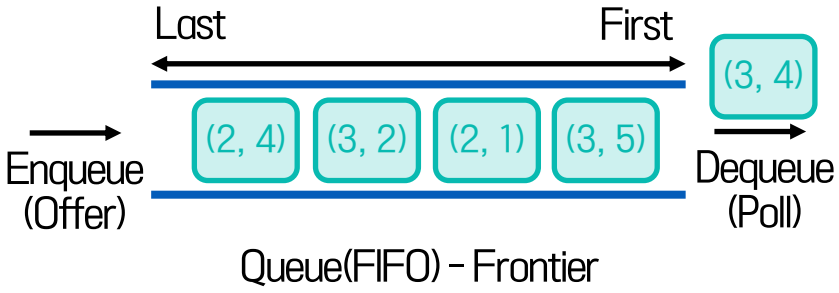
04 BFS Algorithm

Application BFS to Path Planning – Example(4-connectivity)

Step 11

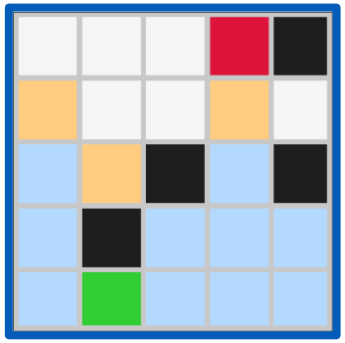
- dequeue (3, 4)
- enqueue (2, 4) // NESW
- frontier: {(3, 5), (2, 1), (3, 2), (2, 4)}
- parent[(2, 4)] = (3, 4)

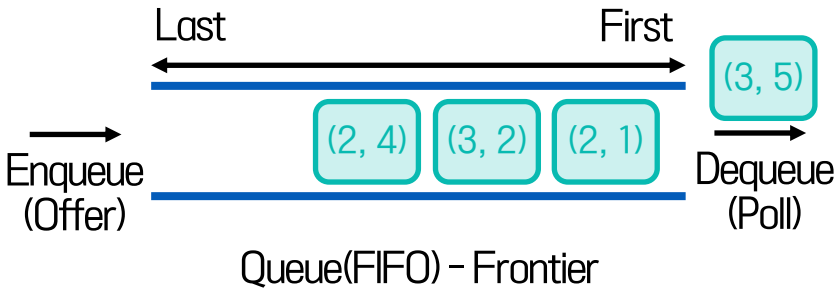




Step 12

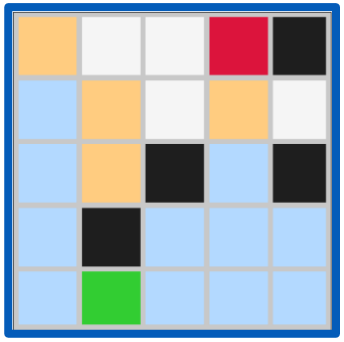
- dequeue (3, 5)
- enqueue X // there isn't non-visited.
- frontier: {(2, 1), (3, 2), (2, 4)}

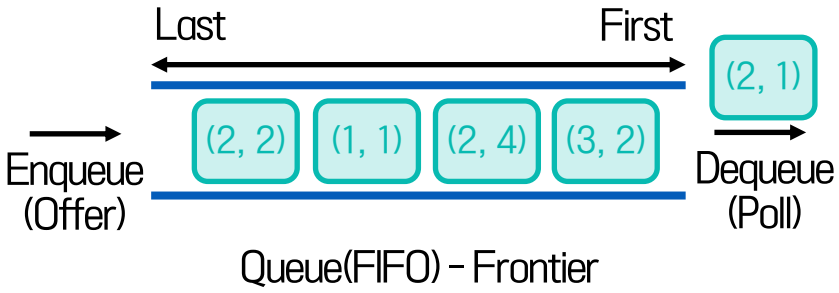




Step 13

- dequeue (2, 1)
- enqueue (1, 1), (2, 2) // NESW
- frontier: {(3, 2), (2, 4), (1, 1), (2, 2)}
- parent[(1, 1)] = (2, 1)
- parent[(2, 2)] = (2, 1)



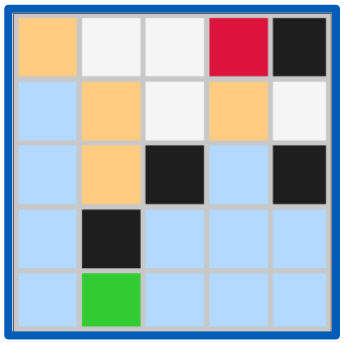


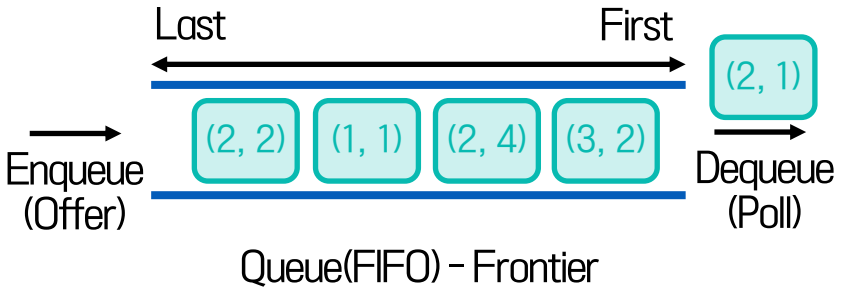
04 BFS Algorithm

Application BFS to Path Planning – Example(4-connectivity)

Step 13

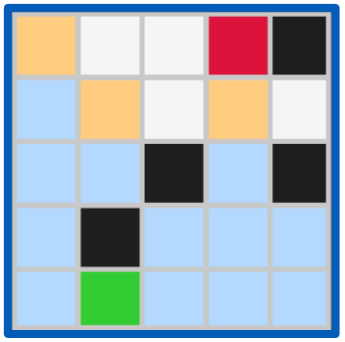
- dequeue (2, 1)
- enqueue (1, 1), (2, 2) // NESW
- frontier: {(3, 2), (2, 4), (1, 1), (2, 2)}
- parent[(1, 1)] = (2, 1)
- parent[(2, 2)] = (2, 1)

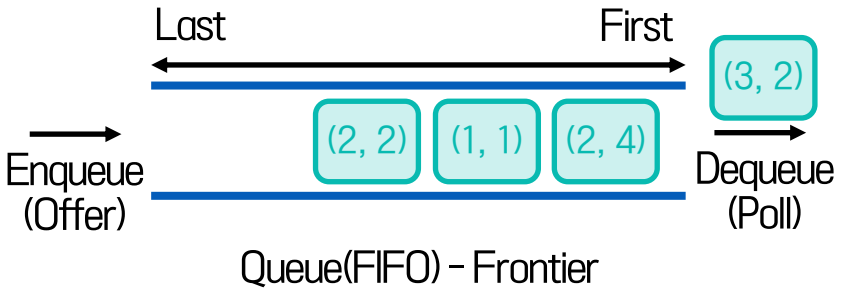




Step 14

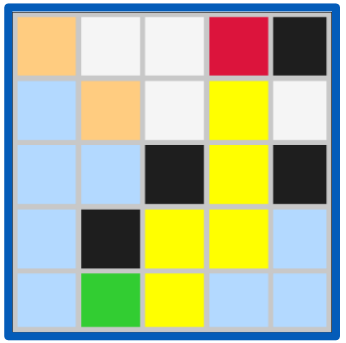
- dequeue (3, 2)
- enqueue X // there isn't non-visited.
- frontier: {(2, 4), (1, 1), (2, 2)}

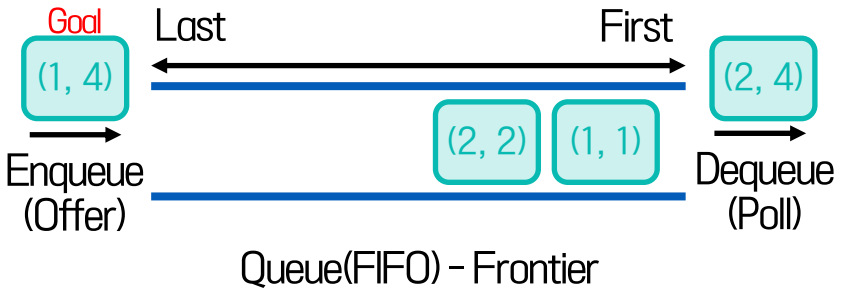




Step 15 - Final

- dequeue (2, 4)
- enqueue (1, 4) // North First & Check Goal
- frontier: {(1, 1), (2, 2), (1, 4)}
- (1, 4) == goal
  → Reconstruct Path from Parent

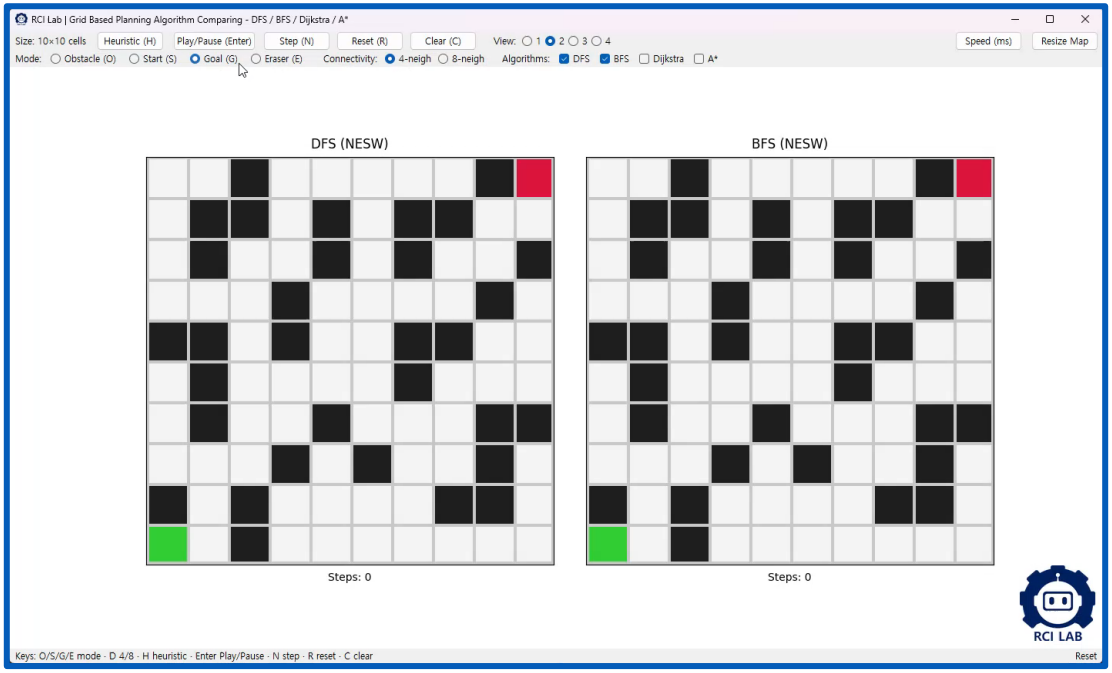




04 BFS Algorithm

DFS vs. BFS

Algorithm	Pros	Cons
DFS	• Simple to implement • Uses relatively little memory	• Not complete (Can go infinitely deep in very large graphs) • Does not guarantee the shortest path
BFS	• On an unweighted graph(unit-cost), it guarantees the shortest path • Complete for finite graphs	• Needs to store many nodes at the same depth → higher memory usage than DFS



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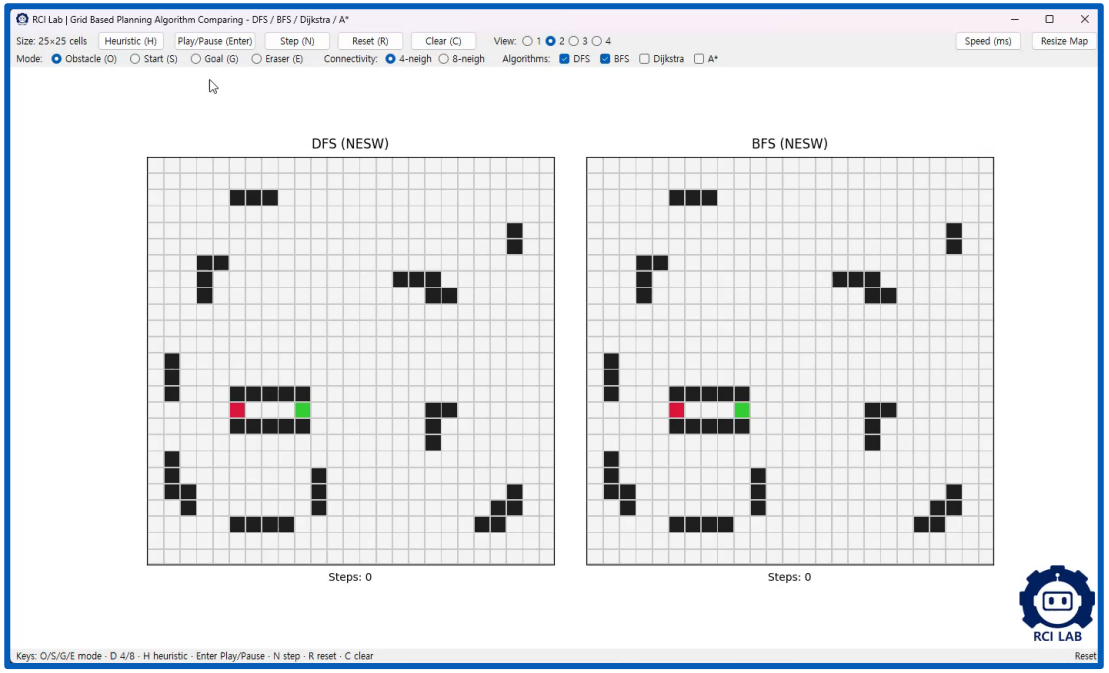
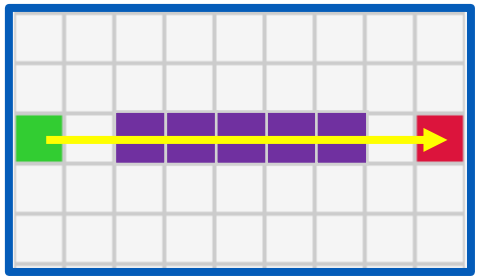


04 BFS Algorithm

Limitations of BFS Algorithm

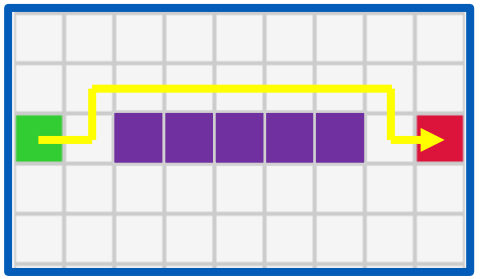
- With $\sqrt{2}$ diagonals or costmaps, fewer steps can still cost more. → Not cost-optimal!
- Level expansion can create very large frontiers in open spaces. → Memory blow-up
- BFS must traverse all shallower(low-level) layers first. → Inefficient for distant goals

: free (cost = 1) : puddle (cost = 5)



BFS

total cost: 28



Dijkstra (4-connectivity)

total cost: 10

↓

cost-optimal

04 BFS Algorithm

Uniform cost vs. Non-uniform cost

Item	BFS (Unit-Cost)	Dijkstra / Non-Unit
Container	Queue (FIFO)	Priority Queue (min g)
Visit Finalization	Mark visited on enqueue	Mark visited on dequeue
Goal Test	Stop on enqueue (on discovery) ※ Stopping on dequeue is also valid, but it can delay to stop.	Stop on dequeue (when selected)
Guarantee	Hop(edge)-shortest	Cost-optimal
Wavefront pattern	Concentric levels	Circular/Octagonal cost contours
NESW role	Order within a level	Order among equal-g candidates

Thank you