National Tsing Hua University 國立清華大學 | 資訊工程学系
Fall 2024 | Linear Algebra | 線性代數
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Assignment 4: Image Compression

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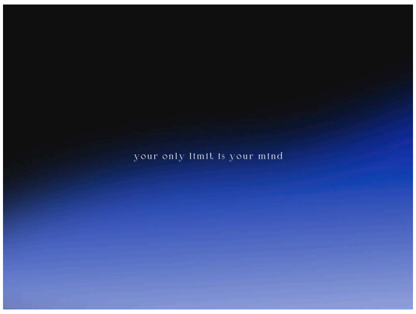
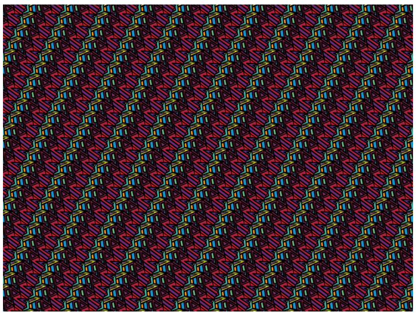
Question 1:



Attached are the original images I used and the ones that have been performed SVD compression is on the table below ▼



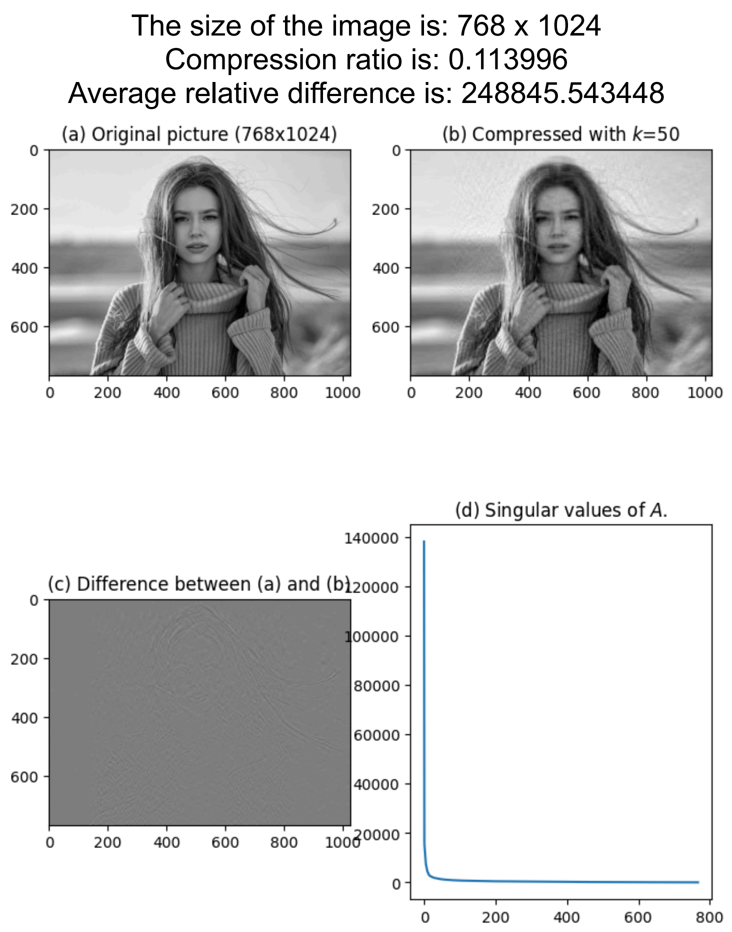
Question 2:

To clearly show the relation between compression ratio and average relative difference, in Q1, I used photos with the same dimensions but different pixel values, that is, I picked photos of different styles with different structural complexity and brightness intensity.

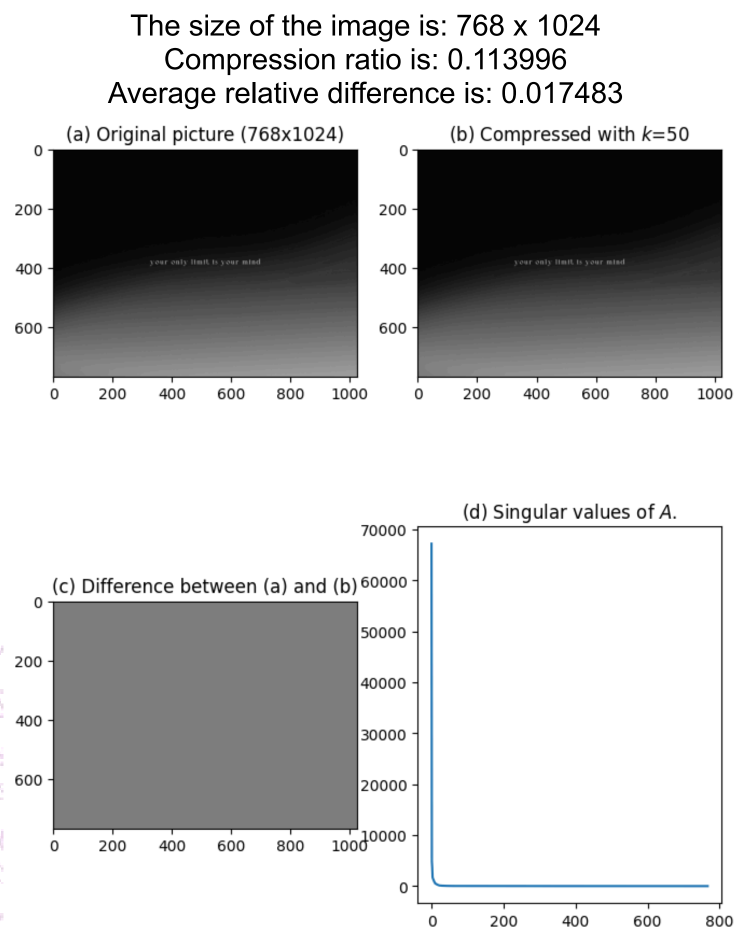
After that, I tried to run the code by trying different ranks and then plot my result in a graph, as shown in the table below whose figures are photos that have been performed SVD compression.

From all the graphs shown below, despite the variations due to the structural complexity and brightness intensity, we can still draw a conclusion that for an image, a lower compression ratio (ρ) means that more data is discarded during compression (greater reduction in size), which leads to a higher average relative difference (δ) because the image loses more information, resulting in more distortion or loss in quality. However, this relationship also depends on the content and complexity of the image, which is shown in the table below.

Also, It is also worth mentioning that the average relative difference (δ) stabilizes at a specific value for each photo because SVD reaches its limit of approximation. At high ranks, all meaningful singular values have been included, and further increases do not improve reconstruction. This represents the best possible compression quality for each image, and this can be shown in the graph that at a certain point, no matter how high the compression ratio is, the average relative difference will stay at that point.

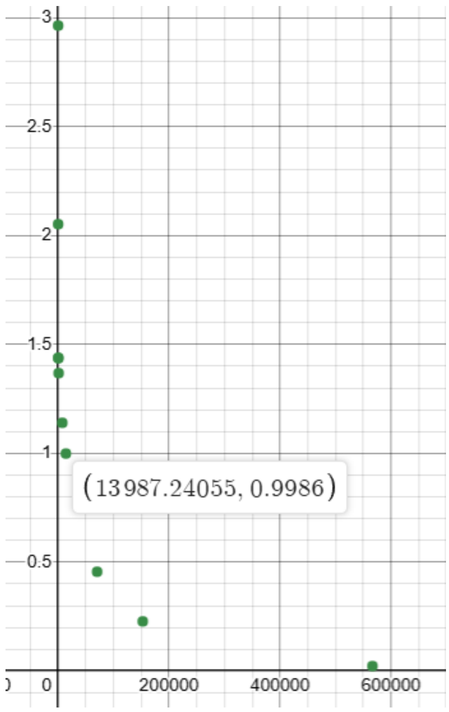


▲ figure 1: Human Portrait



▲ figure 2: Gradient Wallpaper with Text

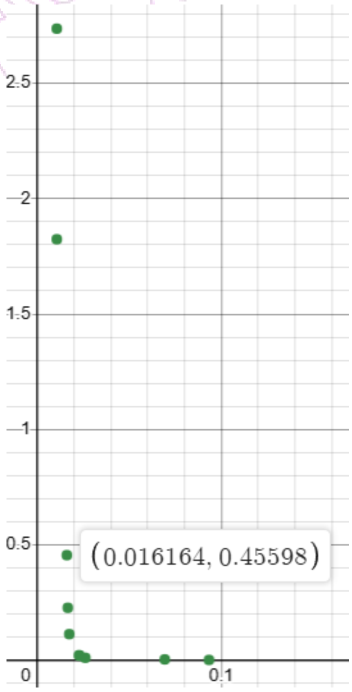
Q2. The relation of compression ratio ρ and the average relative difference δ:



y-axis: compression ratio
x-axis: average relative difference

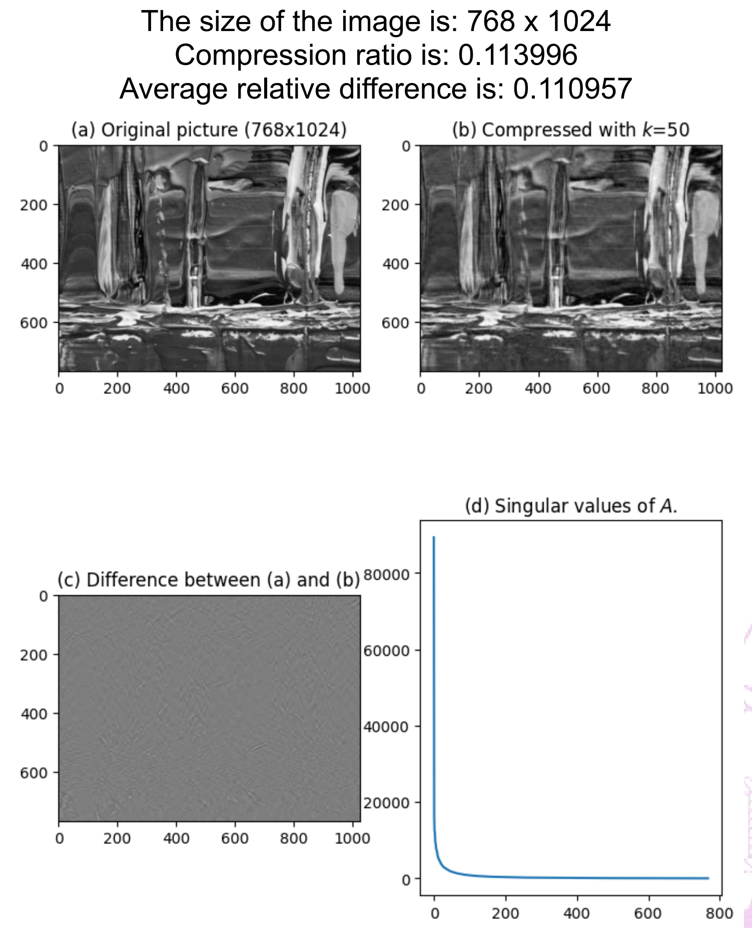
Moderate δ values imply that portraits preserve key features but may lose more detailed features and textures. It is quite balanced due to the blur part of the image.

Q2. The relation of compression ratio ρ and the average relative difference δ:

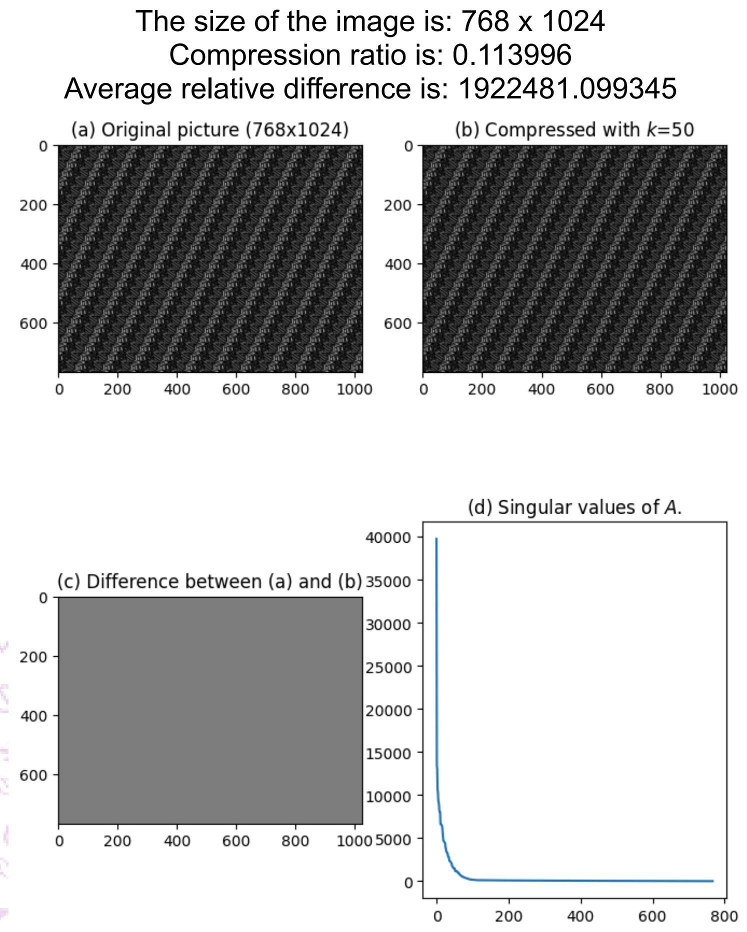


y-axis: compression ratio
x-axis: average relative difference

The smooth gradient and low pixel variation result in a highly compressible image with minimal loss. The graph highlights a steady compression performance at low δ. It would even be even easier to compress with more minimal loss if there were no text in the middle of the image.

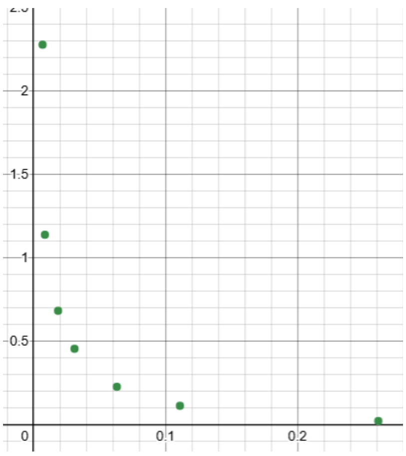


▲ figure 3: Abstract Painting



▲ figure 4: Repeated Patterns

Q2. The relation of compression ratio ρ and the average relative difference δ:

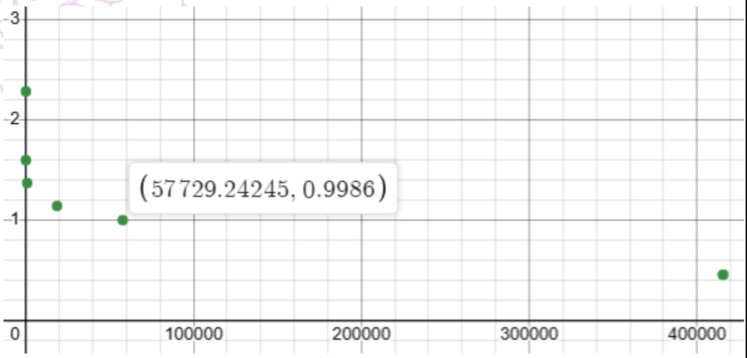


y-axis: compression ratio

x-axis: average relative difference

Moderate variations in δ suggest that abstract images maintain some detail but may lose some textures

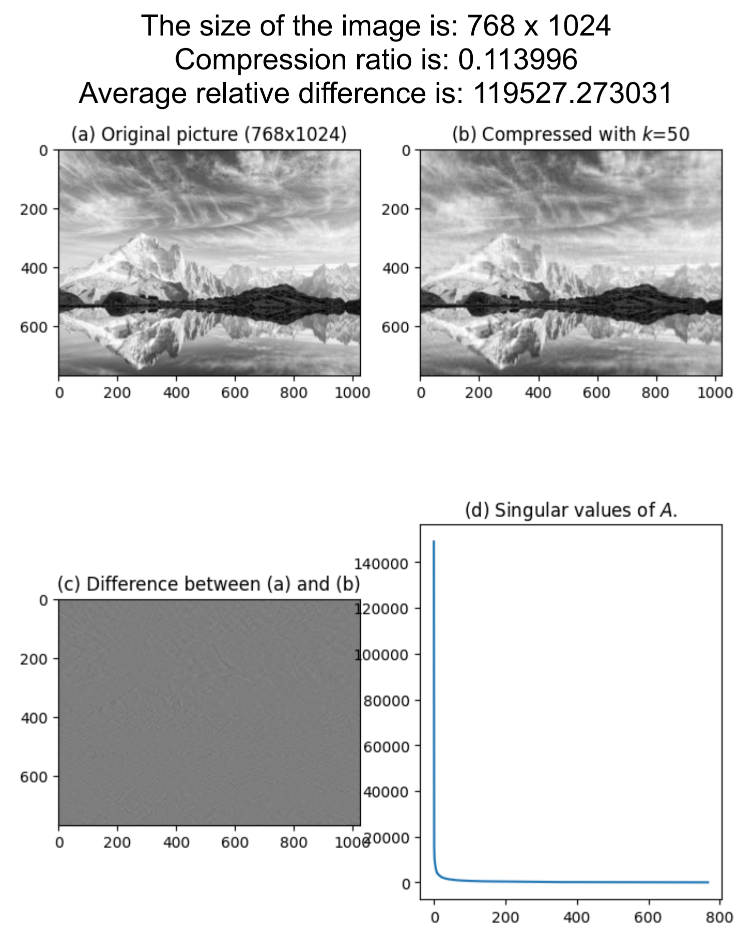
Q2. The relation of compression ratio ρ and the average relative difference δ:



y-axis: compression ratio

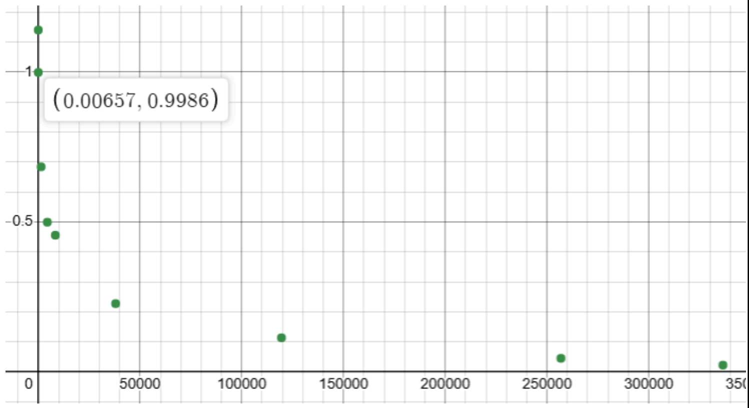
x-axis: average relative difference

Repeated patterns have lots of details which need many singular values to capture accurately. Using fewer singular values distorts those patterns, causing a big difference from the original. It can be seen that δ is growing very quickly even with small compression, meaning repeated patterns are sensitive to compression and lose quality fast.



▲ figure 5: Mountain Scenery with Reflection

Q2. The relation of compression ratio ρ and the average relative difference δ:



y-axis: compression ratio

x-axis: average relative difference

Smooth areas like the sky and water reflections are easier to compress since they don't have much detail.However, detailed parts like the mountain edges and reflections still lose some sharpness when compressed. As a result, it produced a balanced compression outcome, with δ remaining relatively low (although not that low) despite the scenic complexity.

Question 3:

(a) What kind of pictures can be compressed easily by SVD (smaller relative difference)? And why?

We know that SVD works by breaking down an image into singular values, so pictures that require fewer singular values to be approximated accurately can be compressed easily by SVD..

One of the examples of an image that can be compressed easily is shown in figure 2 (Gradient Wallpaper with Text), and it will be even easier to compress if there is no text in the middle. It can be clearly seen from the graph that δ remains very low across all compression ratios, confirming that smooth gradients and simple designs compress well without significant loss.

Pictures that can be compressed easily by SVD are those with smooth gradients, uniform color regions (plain backgrounds), or simple structures, such as gradient wallpapers or images with large areas of consistent tones. These images have fewer complex features and are dominated by low-frequency components, meaning their information is concentrated in a few singular values. Therefore, most of the image's structure can be captured using only a few components.

(b) Which means they have low ρ and δ? Try to find its relation with the distribution of singular values of the original pictures.

Images with low ρ and low δ are easily compressible, meaning they maintain high visual quality even with significant compression.

Refer to the (d) part of figure 2. Notice that a significant portion of the image's information is concentrated in the largest singular value (steep initial drop), then the smaller singular values decay very slowly (flatter tail), indicating there is still some excess information spread across them. The steep initial drop shows that the image can be approximated with a small number of singular values, which is why the compressed image in (b) looks visually similar to the original in (a). However, the flat tail in (d) means that some fine details (most likely the text in the middle for our case) or subtle gradients are distributed across smaller singular values, which might cause minor differences. This can be subtly seen in (c), where the difference is minimal but not completely zero, especially for smooth gradient transitions.

So, when the singular value plot (d) has a steep initial drop followed by a flat tail, the image can be compressed effectively because the most critical information is captured by the largest singular values. This leads to a low δ, as the flat tail contributes only minor details that are less noticeable in the compressed image. This is also evident in the graph I plotted where even at low compression ratio, the difference is already very minimal (below 0.1)

(c) Which part of the pictures have the largest difference of the compressed images? And why?

From the table above, it can be seen that with the same compression ratio (about 11% of the storage) used across all photos, figure 4 (Repeated Patterns) suffer the most significant distortion due to the compression. This can be seen from the large average relative difference (δ = 1922481.099345), which reflects the inability of SVD to preserve the precision of detailed textures when using fewer singular values, and how it needed more than 100% of its storage to maintain minimal image quality loss. The compressed image cannot accurately represent the regular and complex patterns without using a high number of singular values, leading to noticeable differences. These parts have higher storage demands to maintain minimal quality loss

Areas with high complexity or frequent patterns demand greater storage and suffer the largest differences in compressed images due to the limitation of singular value truncation in representing such details.

Coding Assignment:

For the coding assignment, I used the following resources:

- https://www.accel.ai/anthology/2022/8/17/svd-algorithm-tutorial-in-python
- ChatGPT
- Gemini AI
- PDF Attached