# Atmospheric Waves During the Fall 2017 Season: Observations, Analysis, and Correlations Based On Rossby Wave Theory

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#### **ABSTRACT**

For several months in the fall of 2017, 500 hPa and 150-300 hPa wind measurements were taken at 50° latitude in both the North and South hemispheres. Similarly, wave data at 500 hPa in both hemispheres was gathered, including wave speed as well as minimum and maximum heights for each day at 50° latitude. Hand analysis as well as automated analysis was done for these variables with the data coming from the Global Forecast System (GFS) analyses. Understanding large scale wind and wave patterns is crucial to forecasting because small scale weather patterns are dependant on the large scale features creating them. Various variables from the collected data were plotted against each other to show characteristics that could be compared to Rossby Wave Theory. The goal of this study was to compare how well Rossby Wave Theory explains the behaviors of the large waves in our atmosphere. There were times where the results matched the expectations of the theory as well as times when they disagreed with theory. There are a number of reasons why some of these results did not fit with theory, including potential data analysis errors and unsatisfied assumptions. However, the general results did indicate that the observed atmospheric wave dynamics produce a reasonably predictable wave pattern. This information can aid forecasters' understanding of waves to allow for more detailed and accurate forecasts.

#### 1. Introduction

Understanding the evolution and patterns of large scale waves is the foundation of forecasting weather systems around the planet. Variations are common within wave patterns—meteorologists call these variations troughs and ridges. Collectively, these wave patterns typically average out in the atmosphere, as the physical system of the atmosphere seeks to restore itself to equilibrium. Troughs and ridges both induce different changes within the atmosphere, which results in different kinds of vertical motion. Ahead of a ridge, negative vorticity advection induces downward motion, resulting in a surface high pressure, which leads to fair weather and cooler temperatures. Ahead of troughs, positive vorticity advection causes low pressure to form, resulting in upward motion and low pressure to form, leading to clouds, precipitation, and warmer temperatures. These trough and ridge patterns fluctuate as the atmosphere attempts to balance itself out, resulting in changes in weather patterns over time. Other important features of these wave patterns their daily and seasonal variation. Waves vary daily in the atmosphere as either long waves and short waves. Long waves are waves with a

wavelength of greater than  $1 \times 10^3$  kilometers. A short wave is an atmospheric wave with wavelengths between  $1 \times 10^2$  and  $1 \times 10^3$  kilometers. Short waves that are associated with troughs can lead to enhanced areas of precipitation in small areas along the trough with the greatest amount of upward motion. The strength and wavenumbers change through the year usually on a seasonal basis. Waves behave differently during transitional seasons such as spring and fall, they tend to be more powerful with higher wavenumbers due to atmospheric fluctuations between the two more extreme times of the year.

Wave characteristics are studied including wave number, wave speed, and wave amplitude. The zonal wind within different levels of the atmosphere was studied as well. Their variables are important because they communicate how the wave patterns are evolving. Waves with higher amplitude will contain more energy than shallower waves. The wave number helps give insight into the trough and ridge pattern, thus displaying the general weather pattern that is unfolding. The speed of a wave is related to the zonal wind within the atmosphere—a relationship described by Rossby Wave Theory. The phase speed of a Rossby wave is given by:

$$C_x = u - [\beta/(k^2 + l^2)]$$
 (Eq. 1) (Holton and Hakim 2012),

where  $C_x$  is the speed in the x direction,  $\beta$  is the beta parameter (meridional derivative of Coriolis parameter), k is the wavenumber in the x direction, l is the wavenumber in the y direction, and u is the zonal wind velocity. Through this equation, the relationship between the different wave characteristics can be shown. The phase speed will change respectively depending on how the wave number or zonal wind evolve over time. It is this theoretical result that this paper seeks to analyze with respect to the observed data.

### 2. Data and Methods

Two procedures were utilized for our analysis: a manual procedure based on observations collected from plots of the Global Forecast System (GFS) analysis data, and an automated procedure based on raw analysis data, both described below.

a. Manual Procedure

I. Waves

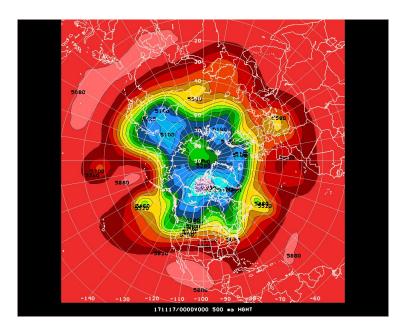


Figure 1. An example of a plot of 500 mb heights in the Northern hemisphere. It is from daily plots such as this that the waves data were collected.

Data were collected visually from September 11, 2017 to November 27, 2017 from the Iowa State University (ISU) Meteorological Weather Products website. The website contains access to the current and past eight day loop of the 500 mb heights map for both the Northern and Southern Hemisphere from the Global Forecast System analysis. The data collected included wave number, wave amplitude, and wave motion. The wave number, k, was calculated by:

$$k_{north} = N_{north}/2$$
 and  $k_{south} = N_{south}/2$ , (Eq. 2)

where N is the number of times the height line, 5580m or 5520m, crossed the 50° latitude line in the Northern and Southern Hemispheres, respectively. The heights of the wave were determined visually based on a key for the heights on the map. The wave amplitude was computed based on the average minimum (trough) and average maximum (ridge) heights of each wave for each day. The average daily amplitude, A, was calculated as

$$A = (\mu_{ridge} + \mu_{trough})/2 \text{ (Eq. 3), where}$$

$$\mu_{ridge} = (a_{ridge1} + a_{ridge2} + ...)/k \text{ (Eq. 4),} \quad \text{and}$$

$$\mu_{trough} = (a_{trough1} + a_{trough2} + ...)/k \text{ (Eq. 5),}$$

where  $\mu$  is the average local amplitude of the ridge or trough, and a is the amplitude each individual trough or ridge. Wave motion was calculated by locating the position of the wave

(longitude line it was located at) on the previous day and the position on the day after the target day (Eq. 6). The positions were then averaged together. This wave speed C was calculated as

$$C = \{LON(day + 1) - LON(day - 1)\}/2$$
 (Eq. 6)

where LON is the longitude location and day is the target day for the wave motion. The daily wave velocity was found by averaging the speed of every wave per day. And so, eastward propagation is considered positive wave speed.

### II. Zonal Wind

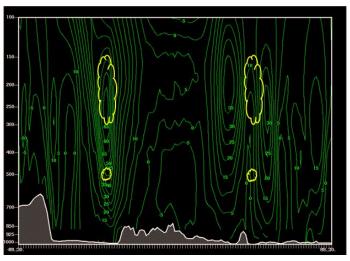


Figure 2. Cross section of zonal winds throughout the atmosphere with regions highlighted at 50 degrees latitude. The small highlighted areas in yellow displays how 500 hPa zonal wind maximums were found. The larger shaded areas above that highlight our upper level wind region in which we are again finding the maximum zonal winds in that region.

Zonal wind speed data were collected from the same time period as the wave data. The ISU Meteorological Weather Products website also provides the current and past eight day loop of global average zonal wind speeds from the Global Forecast System analysis. The data of interest were the average 500 hPa zonal wind speed and maximum average zonal wind speed for the 150-300 hPa layer for both the Northern and Southern hemispheres. These wind speeds were determined at the 50° latitude circle for both hemispheres. The 50° latitude circle was visually estimated to be about a quarter in on the zonal wind plot from each side. This latitude line was chosen because jet streams are frequently in close proximity to this latitude. These data were collected over the same period as the waves data—from September 11, 2017 to November 27, 2017. Positive values of zonal wind indicate an eastward wind, while negative values indicate a westward wind.

#### b. Automated Procedure

A similar method to the manual procedure was done using Python to create an automated comparison to our hand collected data. While the manual method was based on visual interpretations of the data, the automated procedure accessed data from the National Center for Environmental Information's NOAA Operational Model Archive Distribution System (NCEI NOMADS) and computed the same information directly on the data. These data were collected from September 1, 2017 to November 30, 2017, allowing for a full three months of data. Additionally, while the manual procedure analyzed data every 24 hours, the automated procedure analyzed data every 6 hours.

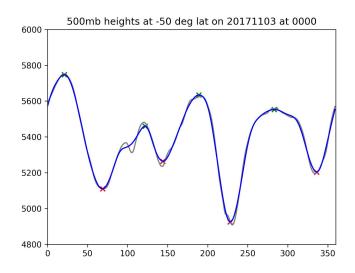


Figure 3. Plot of smoothed heights (blue line), raw heights (grey line), maxima (green ticks), and minima (red ticks), as given by the automated analysis procedure for the Southern hemisphere on November 3 at 0Z. Longitude east of 0• is given on the x-axis, with height in meters on the y-axis. The effects of smoothing can be clearly seen in the short wave and noise removal.

# I. Waves

The variables of interest were the same as the manual method: wave number, wave amplitude and wave motion. The height data were obtained from the NCEI NOMADS archive at the 50° latitude circle for both North and South hemispheres. The height data were smoothed using a Savitzky-Golay filter (Schafer 2011) to remove the smallest shortwaves and noise without removing the major features of the long waves of interest to this study. The wave number was determined by the number of maxima and minima in the data. The wave amplitude was computed similarly to the manual methods as half the difference between the average ridge and trough heights. The wave motion calculation matched the longitudinal position of the maxima and minima along the 50° latitude line from one time to the nearest maxima or minima 6 hours later and the difference in positions was calculated. During the evolution of the wave pattern, waves and their respective extrema would appear or disappear, and so, outliers of these position differences were filtered out. Then, all the differences within a

twelve hour period before and after the time period of choice were averaged to compute the wave speed at a particular time.

#### II. Zonal Wind

The zonal wind speeds for the Northern and Southern hemispheres were found for the 500 hPa level and the 150-300 hPa level along the  $50 \cdot \text{latitude}$  line. The zonal wind speeds were directly averaged around the latitude circles for the 500 hPa level and all the levels in the 150-300 hPa layer. From this 150-300 hPa layer, the maximum average zonal wind was recorded as the maximum upper level wind.

#### c. Procedural Limitations

Every computed variable calculated had limitations both in the manual and automated methods. The visual depiction of waves allows for different interpretations of the wavenumber depending on the person and procedure of choice. For example, if there were four distinct trough-ridge patterns but only two crossed the 50. latitude line, it was only counted as two waves with the manual method. However, this same day may have registered as four waves in the automated method. This would have caused the wave number to be less than what may have actually been occurring in the wave pattern. This limitation is a reason as to why a comparison to the automated method was done. The amplitude calculations also had limitations. In the manual method, if the height was in the middle between two height contours around the 50 circle, the average between the two was taken as the height for that trough or ridge. This led to multiple interpretations as there were multiple occasions where the height contour spacing and wave shapes made discerning the height at the 50. circle difficult. In contrast, the automated method was observed to have included some short waves despite the smoothing. Incorporating these short waves could have led to an overall smaller amplitude and higher wavenumber than the true long wave pattern of interest would have indicated. The limitations in the calculation of wave motion included estimating how far the wave moved. If a wave flattened out, it was hard to depict the peak to measure how many degrees it moved based on the manual method. For example, if the wave number went from five waves to three waves or three waves to five waves, it was difficult to calculate the exact motion of the waves. The automated method experienced a similar problem, which necessitated the filtering of extreme outliers of time-to-time differences. With the manual method, it was possible instead to estimate what happened to the waves and make a subjective value assessment as to what the proper wave speed should be. Zonal wind data collection, at least in the manual method, may be the most imprecise. There was no exact marker for the 50. latitude line, and so, the proper location was estimated at each time, likely resulting in error. Additionally, at times, a strong gradient was present at 500 hPa or within the 150-300 hPa in the atmosphere. This led to a difficulty in determining an accurate wind value because it is hard to read such close gradient lines with the human eye. In contrast, the automated calculations of the two zonal wind speeds were directly computed from the model output, and should therefore be more accurate than the manual method.

### 3. Results

a. Evolution of Atmospheric Wave and Wind Properties Over Time

# I. Wavenumber

The evolution of wavenumber over our sample period gives us information regarding how long identifiable wave patterns last, as well as some information on seasonal variation. In Figures 4 and 5 below, we see the trends of wavenumber over time from the data of our manual procedure for the Northern and Southern hemispheres, respectively:

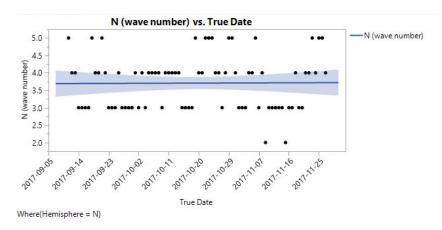


Figure 4. Wavenumber versus Date for the Northern hemisphere from the manual procedure.

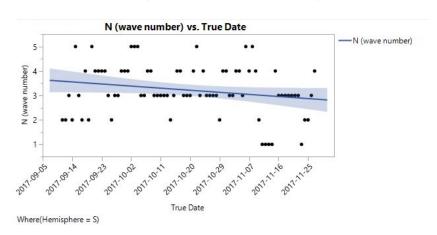


Figure 5. Wavenumber versus Date for the Southern hemisphere from the manual procedure.

Here, in general, we observe no significant linear trend over time in either hemisphere. However, more importantly than a sustained trend over time (which would point to a seasonal trend), we can observe the finer scale variations and how they tend to oscillate. Of note, we see that a particular wavenumber is sustained between 1 and 5 days in the Northern hemisphere

and between 1 and 7 days in the Southern hemisphere, both averaging out to roughly 2 or 3 days.

We can also contrast this with the trends found in the data from the automated procedure, in Figures 6 and 7 below:

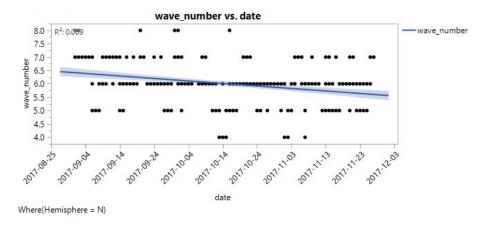
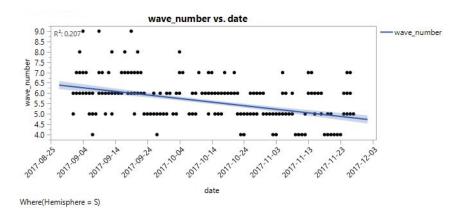


Figure 6. Wavenumber versus Date for the Northern hemisphere from the automated procedure.



*Figure 7. Wavenumber versus Date for the Southern hemisphere from the automated procedure.* 

Here, in general, we observe a statistically significant (at 95% confidence) negative linear trend over time. But, we can also pay attention to how wavenumber is sustained over time. Due to the 6-hourly data collection in this method, we can often see multiple different wavenumbers recorded each day, but also many days in a row seeing the same wavenumber occur at least once. And so, this automated method indicates a higher variability in wavenumber from time-to-time, but also a longer continuation of a similar, but not identical, wave pattern.

As an aside, contrast the wavenumbers recorded between the two methods: in the manual method, the average wavenumber is between 3 and 4 for both hemispheres, but in the automated method, the average is higher at around 6. This is likely due to two procedural differences: the automated method counted off of extrema, instead of a threshold value, as well

as including some short waves that failed to be filtered out, both of which allow more waves to appear in the automated method.

# II. Wave Amplitude

The evolution of wave amplitude over time gives us general information about how the intense or energetic the waves are, and how that varies both in the short-run and seasonally. In Figures 8 and 9 below, we see the trends of wave amplitude over time for both hemispheres from the manual procedure data:

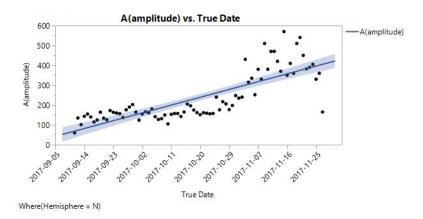


Figure 8. Wave amplitude versus Date for the Northern hemisphere from the manual procedure.

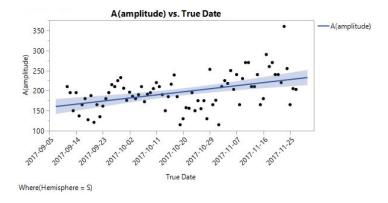


Figure 9. Wave amplitude versus Date for the Southern hemisphere from the manual procedure.

In general, we have a statistically significant positive linear trend over time for both hemispheres, meaning that the waves were generally gaining in intensity over time as we move towards the solstice. However, oscillations in the data can also be observed on a finer scale, indicating that waves can increase or decrease in amplitude reasonably quickly (up to 200 m in a day). In contrast, the waves can also be seen to maintain similar amplitudes over longer periods of time.

We can also contrast this with the wave amplitude trend observed from the automated procedure in Figures 10 and 11 below:

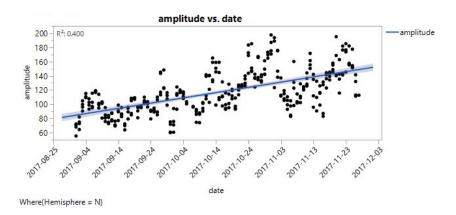


Figure 10. Wave amplitude versus Date for the Northern hemisphere from the automated procedure.

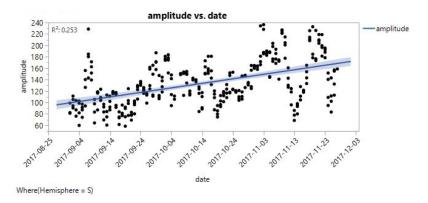


Figure 11. Wave amplitude versus Date for the Southern hemisphere from the automated procedure.

Rather similar results to the manual case can be seen from the automated data. Both hemispheres saw a statistically significant positive linear trend in wave amplitude over time, indicating a seasonal intensification of the waves. Variability on smaller scales can also be observed, but here in greater detail due to the enhanced temporal resolution of the data.

# III. Wave Speed/Motion

The wave speed simply gives us information about how rapidly the wave patterns move, which is nevertheless critical from a forecasting perspective. First, we can see the evolution of wave speed over time in both hemispheres from the manual data in Figures 12 and 13 below:

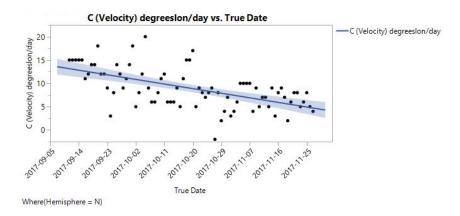


Figure 12. Wave speed versus Date for the Northern hemisphere from the manual procedure.

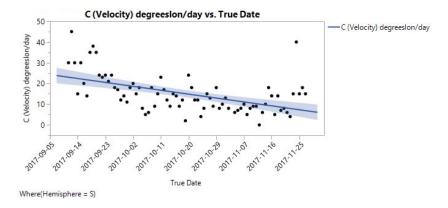


Figure 13. Wave speed versus Date for the Southern hemisphere from the manual procedure.

In general, we have a statistically significant negative linear trend of wave speed over time for both hemispheres, suggesting a seasonal slowing of the waves over this fall period. Additionally, we can note that the wave speed in the Northern hemisphere averages around 10 degrees per day, meaning that a wave would take about 36 days to circle the globe, whereas the wave speed in the Southern hemisphere averages around 15 degrees per day, meaning that a wave would take about 24 days to circle the globe.

We can also contrast these findings with those from the automated procedure's data in Figures 14 and 15 below:

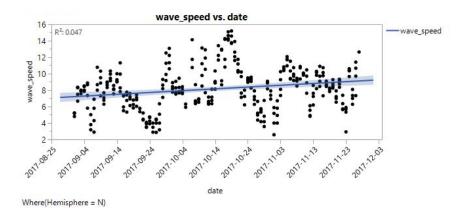


Figure 14. Wave speed versus Date for the Northern hemisphere from the automated procedure.

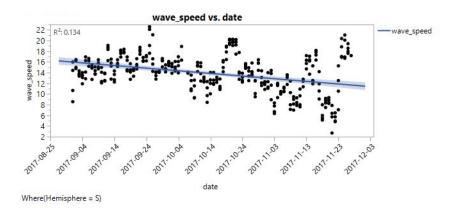


Figure 15. Wave speed versus Date for the Southern hemisphere from the automated procedure.

While, in general, we observe here a statistically significant positive linear trend of wave speed over time in the Northern hemisphere and negative in the Southern hemisphere, it is also clear that that the short- and mid-range variability dominates the pattern, which stands in contrast with the long-term trend seen predominantly in the manual data. Again, we can compute rough averages and see that in the Northern hemisphere, according to these data, the average wave speed is roughly 8 degrees per day, meaning that a wave would take roughly 45 days to circle the globe, and in the Southern hemisphere, the average wave speed is roughly (again) 15 degrees per day, meaning that a wave would take roughly 24 days to circle the globe. One area of consistency between these procedures is that the wave speed is generally higher in the Southern hemisphere than it is in the Northern hemisphere. While, in most cases, the effects of surface friction are neglected for upper level analysis, this is one way in which friction may still manifest itself in upper levels: the waves move faster over the Southern hemisphere where there is significantly less land area.

### IV. Zonal Wind

Our final variable in which we consider the evolution over time is zonal average wind speed at 500 mb and the maximum of the 150-300 mb layer. The trends from the manual method can be seen in Figure 16 below, and the automated method in Figure 17:

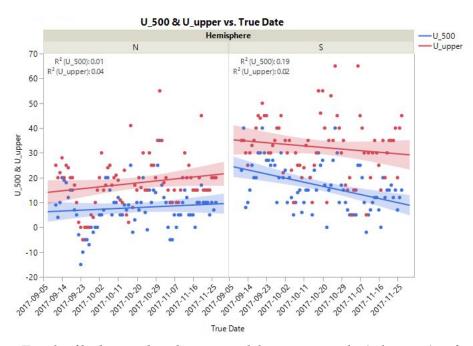


Figure 16. Trends of both 500 mb and 150-300 mb layer max zonal winds over time from both hemispheres from the manual data collection method.

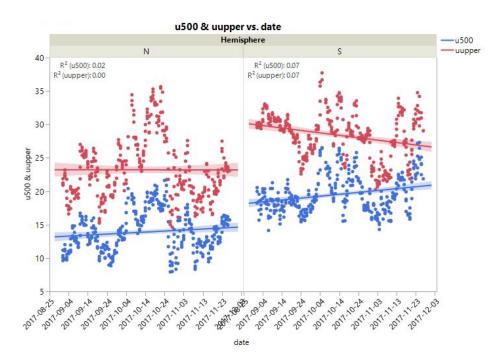


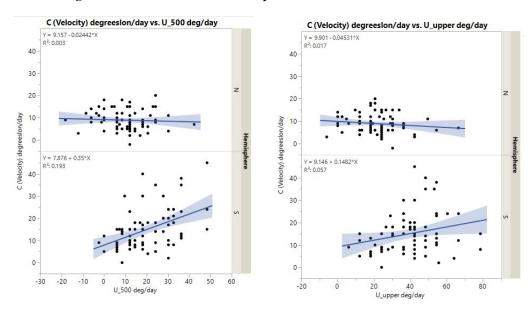
Figure 17. Trends of both 500 mb and 150-300 mb layer max zonal winds over time from both hemispheres from the automated data collection method.

Several features are of note from this data: first, from both methods and both hemispheres, we see that the upper level winds are almost always greater than those at 500 mb. This corresponds with what we would expect from the theory behind thermal wind: in the presence of typical thermal gradients, we expect a general increase in geostrophic wind speed with height. While more noticeable in the automated data, we can also see a coupling of the two wind speeds in the short-term variations: when the 500 mb winds are stronger, the upper level winds tend to be stronger as well, so that they follow generally the same oscillatory pattern. Similar to what we observed with wave speed, we also see that the speeds are higher in the Southern hemisphere, suggesting possible increased frictional effects in the Northern hemisphere where there is greater land area.

b. Correlations and Comparisons with Rossby Wave Theory

# I. Wave Speed vs. Zonal Wind

Now, we consider the correlations between our variables in accordance with the expectations of Rossby Wave Theory—first of which is between wave speed and zonal wind. In Figure 18, we see the correlations between wave speed and 500 mb wind for the manual method, and in Figure 19, that between wave speed and 150-300 mb maximum wind.

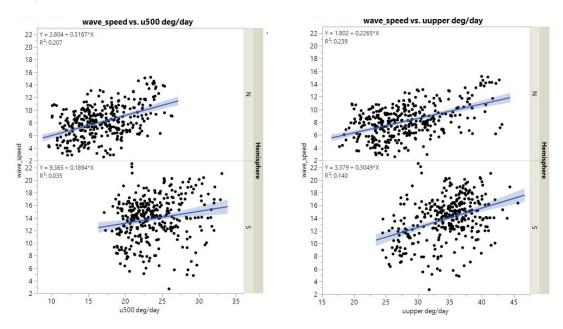


Figures 18 and 19. Associations between wave speed and 500 mb wind (left) and between wave speed and 150-300 mb maximum wind (right) for the manual method.

Based on our theoretical relationship in equation 1, we would expect the wave speed and zonal wind speed to be in direct one-to-one proportion, with variation from the wave number. From our manual data, we instead observe no significant trend in the Northern

hemisphere, and while a significant positive trend in the southern hemisphere, not close enough a slope to one to fit Rossby Wave Theory.

In contrast, we see the same associations, but instead with our automated data in Figures 20 and 21, likewise:



Figures 18 and 19. Associations between wave speed and 500 mb wind (left) and between wave speed and 150-300 mb maximum wind (right) for the automated method.

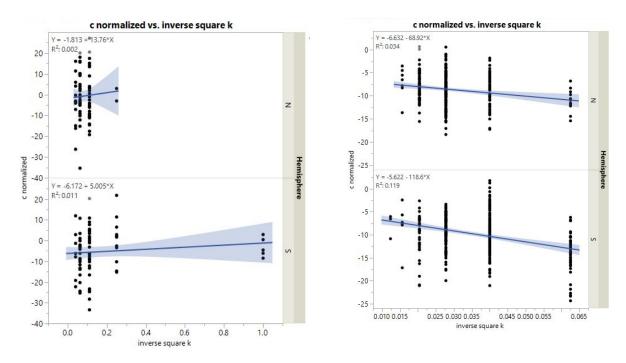
Here, we instead observe significant positive trends in all hemispheres and with both wind levels, but again, none of them have a slope close enough to one to fit Rossby Wave Theory exactly.

# II. Wave Speed vs. Wavenumber

If we take and adjust our equation 1 into a linear relationship between a normalized wave speed and linearized wave number, we obtain

$$c_x - u = (-\beta) (1/k^2)$$
 (Eq. 7).

And so, plotting this comparison between wave speed and wavenumber for both our procedures in both hemispheres, we find:

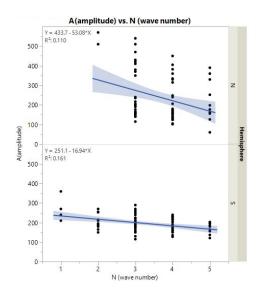


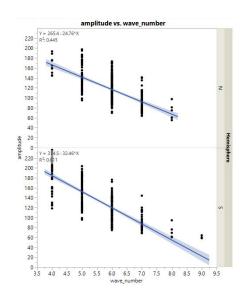
Figures 20 and 21. Associations between normalized wave speed and linearized wave number for the manual data (left) and automated data (right).

By looking at the regression statistics for both of these data sources, we find no significant trends for the manual data (countering theory), and significant negative trends for the automated method in both hemispheres (in accord with theory).

# III. Wave Amplitude vs. Wavenumber

Finally, it is a generally expected property of waves that as wavenumber increase the amplitude will decrease. As show in both Figures 22 and 23 below, we observed this negative association to a statistically significant degree in both hemispheres with both methods.





Figures 22 and 23. Associations between wave amplitude and wave number for the manual data (left) and automated data (right).

#### 4. Conclusions

Although there were times where the observations and data disagreed with Rossby Wave Theory, we also saw results that were very supportive of the theory. Some of the reasons that our results did not completely agree are because Rossby Wave Theory has multiple assumptions that are not realistic in the natural atmosphere. Foremost among these is the assumption of a barotropic atmosphere. Instead, the real atmosphere in the mid-latitudes is often rather baroclinic, as we ourselves saw when we noted that the upper level winds are almost always stronger than the 500 mb winds. This means if we take data from the natural atmosphere, it is not surprising that some if the results don't match up because all of the dynamics unique to a baroclinic atmosphere (such as thermal wind) are missing from the theoretical model. Another point of error comes from taking hand analysis. Hand analysis is subjective and will not be consistent between two people. This throws large variations in the results and makings them difficult to take too seriously. For this reason, the automated program that takes data and does analysis was put in place. This takes away the bias of hand analysis and gives us a more consistent idea of what the waves are doing throughout the atmosphere. However, this automated method still has drawbacks: first, it could not sufficiently filter out all short waves, so that the results may have skewed towards higher wavenumbers and lower amplitudes. Another drawback to the automated program is that it looks at wave maximum and minimums to find waves without being based on a given threshold (like the hand analysis was), and so, the results are only indirectly comparable. Finally, a major drawback to this study as a whole was its duration. Data were only taken for a few months and was also taken during a transition season. More consistent and complete results would likely be displayed if multiple years' worth of daily data were taken.

Even though we cannot conclusively verify or dismiss Rossby Wave Theory, it is still apparent that there is a pattern in which these waves behave. This means that the waves can be studied and predicted. If we understand these waves and know how they work, then we can use these understandings to forecast the weather. In general, our results did a decent job in displaying Rossby Wave Theory, but could still be improved by more consistent data over a longer duration in time in order to capture the full extent to which this simplified theory explains reality and the extent to which it fails.

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