Consider the XXZ chain with L spins 1/2, and Hamilton operator

$$H = \sum_{\langle ij \rangle} J_{xy}(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) + J_z \sigma_i^z \sigma_j^z \tag{1}$$

The L are represented as bit patterns, such that the spin operators count/flip adjacents bits according to

pattern	$\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y$	$\sigma_i^z \sigma_j^z$
00	—	+1 × 00
01	+1 × 10	-1 × 01
10	+1 × 01	-1 × 10
11	—	+1 × 11

Because of S_z-symmetry, H preserves the number of up spins $N_\uparrow$.
$\Rightarrow$ decompose total Hilbert space of dimension 2^L into subspaces with fixed $0 \le N_\uparrow \le L$, each of which has dimension $\binom{L}{N_\uparrow}$:

	subspace size			
$N_\uparrow$	L=10	L=20	L=30	L=40
0	1	1	1	1
1	10	20	30	40
5	252	15504	142506	658008
10	1	184756	30045015	847660528
15		15504	155117520	40225345056
20		1	30045015	137846528820
30			1	847660528
40				1
total:	1024	1048576	1073741824	1099511627776

As a consequence, much fewer degenerate energy eigenvalues occur: E.g. the totally ferromagnetic state (the highest energy state for $J_{xy} = J_z > 0$) is $L+1$-fold degenerate in the full Hilbert space, but appears only once in every $N_\uparrow$-subspace.

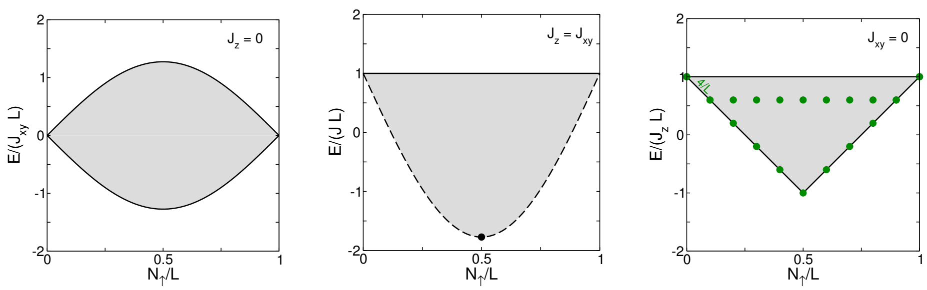

Figure 1: Minimal and maximal eigenvalues as a function of $N_\uparrow$, for $L \to \infty$. Rightmost panel: The green dots give the extremeal eigenvalues for $L = 10$, where the $4/L$ offset of the maximal eigenvalue for $1 \le N_\uparrow < L$ is clearly visible.

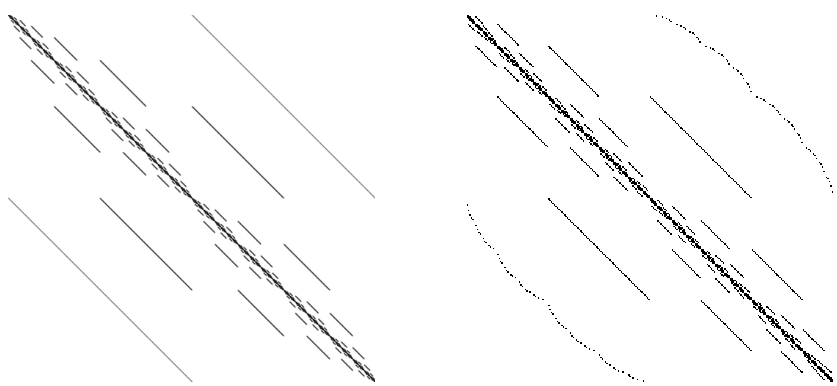

Figure 2: Sparsity pattern of the Hamilton matrix for $L = 10$ including all 2^{10} (left panel) or only the 252 states with $N_\uparrow = 5$ (right panel).