Interpolating a fixed-step ODE solution

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Problem

Given a differential equation

$$\frac{\mathrm{d}\boldsymbol{y}(t)}{\mathrm{d}t} = f\left(t, \boldsymbol{y}\right),$$

initial condition $\boldsymbol{y}(t_0) = \boldsymbol{y}_0$, and n increasing time points $t_i > t_0$, $i = 1, \ldots, n$, solve values $\boldsymbol{x}_i = \boldsymbol{y}(t_i)$, where $i = 1, \ldots, n$, using a fixed-step ODE solver with step size h.

Algorithm

1. For $i = 1, \ldots, n$, define R_i as the smallest integer r which satisfies $t_0 + r \cdot h \geq t_i$.
2. For $i = 1, \ldots, n$, define $A_i = \frac{D_i}{h}$, where $D_i = t_0 + R_i \cdot h - t_i$.
3. Set $R = \max_i R_i$.
4. Solve $\boldsymbol{y}_j = \boldsymbol{y}(t_0 + j \cdot h)$, for $j = 0, \ldots, R$, using the fixed-step solver.
5. Use linear interpolation

   $$\boldsymbol{x}_i = (1 - A_i) \cdot \boldsymbol{y}_{R_i - 1} + A_i \cdot \boldsymbol{y}_{R_i}.$$

   for each $i = 1, \ldots, n$.