Dorazio and Royle's Multispecies Model- Mixed Notation

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February 26, 2014

1 Model as described in the text

For $j = 1, \ldots, J$ sites, $i = 1, \ldots, N$ 'actual' species (out of a total of Ω species), and for $k = 1, \ldots, K$ revisits at each site

$$
\begin{aligned}
y_{ij} &\sim \text{Binom}(K, p_i Z_{ij}) \\
Z_{ij} &\sim \text{Bern}(\Psi_i w_i) \\
W_i &\sim \text{Bern}(\lambda) \\
\text{logit}(\Psi_i) &= u_i + \alpha \\
\text{logit}(p_i) &= v_i + \beta \\
\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix} &\sim N\left(\mathbf{0}, \begin{bmatrix} \sigma_u^2 & \rho\sigma_u\sigma_v \\ \rho\sigma_u\sigma_v & \sigma_v^2 \end{bmatrix}\right)
\end{aligned}
$$

where y_{ij} is the number of times that species i is seen at site j during K visits, Z_{ij} is an indicator of presence or absence of species i at site j, and p_i is the detection probability for species i. W_i is an indicator of whether a species exists or not in the population; $\sum_{i=1}^{\Omega} W_i = N$ is the species richness; Ψ_i is the presence probability for species i; and λ is the probability of a species being real. α and β represent the average occupancy and detection probabilities, respectively, for the population; u_i and v_i are the species-specific variances around those averages.

Note: in the code, they specify the multivariate distribution slightly differently:

$$
\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix} \sim N\left(\begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \rho\sigma_u\sigma_v \\ \rho\sigma_u\sigma_v & \sigma_v^2 \end{bmatrix}\right)
$$

The priors are essentially:

$$
\begin{aligned}
\text{logit}^{-1}(\alpha) &\sim U(0,1) \\
\text{logit}^{-1}(\beta) &\sim U(0,1) \\
\lambda &\sim U(0,1) \\
\rho &\sim U(-1,1) \\
\sigma_u^2 &\sim 1/\text{Gamma}(0.1, 0.1) \\
\sigma_v^2 &\sim 1/\text{Gamma}(0.1, 0.1)
\end{aligned}
$$

2 Model as written in WinBUGS

2.1 Interlude

When they code the model, they use the marginal and conditional distributions for **u** and **v**, respectively:

$$
\begin{aligned}
[u, v] &= [v|u][u] \\
[v|u] &= N\left(\beta + \rho\left(\frac{\sigma_v}{\sigma_u}\right)(\Psi_i - \alpha), \left(1 - \rho^2\right)\sigma_v^2\right) \\
[u] &= N\left(\alpha, \sigma_u^2\right)
\end{aligned}
$$

(********** The code has Ψ_i in the equation, but it should be v_i, right?)

2.2 BUGS model

For $j = 1, \ldots, J$ sites, $i = 1, \ldots, N$ 'actual' species (out of a total of Ω species), and for $k = 1, \ldots, K$ revisits at each site:

$$
\begin{aligned}
y_{ij} &\sim \mathrm{Binom}(K, p_i Z_{ij}) \\
Z_{ij} &\sim \mathrm{Bern}(\Psi_i w_i) \\
W_i &\sim \mathrm{Bern}(\lambda) \\
\mathrm{logit}(\Psi_i) &\sim N\left(\alpha, \sigma_u^2\right) \\
\mathrm{logit}(p_i) &\sim N\left(\beta + \rho\left(\frac{\sigma_v}{\sigma_u}\right)(\Psi_i - \alpha), \left(1 - \rho^2\right)\sigma_v^2\right)
\end{aligned}
$$

Priors:

$$
\begin{aligned}
\lambda &\sim U(0, 1) \\
\text{mean.psi} &\sim U(0, 1) \\
\alpha &= \mathrm{logit}(\text{mean.psi}) \\
\text{mean.p} &\sim U(0, 1) \\
\beta &= \mathrm{logit}(\text{mean.p}) \\
\sigma_u^2 &\sim 1/\mathrm{Gamma}(0.1, 0.1) \\
\sigma_v^2 &\sim 1/\mathrm{Gamma}(0.1, 0.1) \\
\rho &\sim U(-1, 1)
\end{aligned}
$$