Barcast Model

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1 Barcast Model

1.1 Data Model

$$\boldsymbol{W}_t = \boldsymbol{H}_t\boldsymbol{T}_t + \boldsymbol{B}_t + \boldsymbol{\eta}_t \quad \boldsymbol{\eta}_t \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Sigma}_t) \quad \boldsymbol{\Sigma}_t = \begin{pmatrix} \tau_I^2\boldsymbol{I}_{N_{It}} & \mathbf{0} \\ \mathbf{0} & \tau_P^2\boldsymbol{I}_{N_{Pt}} \end{pmatrix} \tag{1}$$

, $\boldsymbol{H}_t = \begin{pmatrix} \boldsymbol{H}_{It} \\ \beta_1\boldsymbol{H}_{Pt} \end{pmatrix}$, $\boldsymbol{B}_t = \begin{pmatrix} \mathbf{0}_{N_{It}} \\ \beta_0\mathbf{1}_{N_{Pt}} \end{pmatrix}$, and $\boldsymbol{\Sigma}_t = \begin{pmatrix} \tau_I^2\boldsymbol{I}_{N_{It}} & \mathbf{0} \\ \mathbf{0} & \tau_P^2\boldsymbol{I}_{N_{Pt}} \end{pmatrix}$

1.2 Process Model

$$\boldsymbol{T}_t - \mu\mathbf{1} = \alpha\left(\boldsymbol{T}_{t-1} - \mu\mathbf{1}\right) + \boldsymbol{\epsilon}_t \quad \boldsymbol{\epsilon}_t \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Sigma}_\epsilon) \quad \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}} = \sigma^2 \exp\left(-\phi\boldsymbol{D}\right) \tag{2}$$

where $\boldsymbol{D}$ is the distance matrix between observation points

1.3 Parameter Model

$$\boldsymbol{T}_0 \sim \mathrm{N}\left(\tilde{\boldsymbol{\mu}}_0, \tilde{\boldsymbol{\Sigma}}_0\right) \qquad \tilde{\boldsymbol{\mu}}_0 = \mathbf{0} \qquad \tilde{\boldsymbol{\Sigma}}_0 = \tilde{\sigma}_0^2\boldsymbol{I} \tag{3}$$

$$\alpha \sim \mathrm{U}\left(0, 1, \right) \tag{4}$$

$$\mu \sim \mathrm{N}\left(\mu_0, \sigma_0^2\right) \qquad \text{For PDSI } \mu_0 = 0 \qquad \sigma_0^2 = 1 \tag{5}$$

$$\sigma^2 \sim \mathrm{IG}\left(\alpha_{\sigma^2}, \beta_{\sigma^2}\right) \tag{6}$$

$$\phi \sim \mathrm{IG}\left(\alpha_\phi, \beta_\phi\right) \tag{7}$$

$$\tau_I \sim \mathrm{IG}\left(\alpha_I, \beta_I\right) \tag{8}$$

$$\tau_P \sim \mathrm{IG}\left(\alpha_P, \beta_P\right) \tag{9}$$

$$\beta_1 \sim \mathrm{N}\left(\mu_{\beta_1}, \sigma_{\beta_1}^2\right) \qquad \mu_{\beta_1} = \left(\frac{(1-\tau_p^2)(1-\alpha^2)}{\sigma^2}\right)^{-\frac{1}{2}} \qquad \sigma_{\beta_1}^2 = 8 \tag{10}$$

$$\beta_0 \sim \mathrm{N}\left(\mu_{\beta_0}, \sigma_{\beta_0}^2\right) \qquad \mu_{\beta_0} = -\mu\beta_1 \qquad \sigma_{\beta_1}^2 = 8 \tag{11}$$

$$\tag{12}$$