Research Notes/TODOs

JT Laune

September 1, 2021

Contents

1 capture

1.1 TODO run longer integration for $Te1/Te2 = 0.1$

1.2 TODO write why Dpomeg to 0

1.3 TODO comment about setting $Tw0 = 10$ kyr

1.4 TODO comment on loose terminology

1.5 DONES

1.5.1 DONE Solve for Tej (ej?) if Tek = infty algebraic/numerical

1.5.2 DONE Test different tolerance settings & integrators for ecc decay

- plot avg(sin(theta)) for diff tol/integrators to see if numerical issue

1.5.3 DONE Integrate a reasonable time for orbit evolution of q=0.001 for ecc decay

1.5.4 DONE solve analytical eq for e1/2d driving eccs

1.5.5 DONE Test different Te1/Te2 ratios for ecc decay

1.5.6 DONE read petigura 2019

1.5.7 DONE read ragusa 2018

2 Meetings

2.1 *[2021-08-27 Fri]* MEETING

1. fix equation 4
2. discuss plus/minus migrations, define 1/Tm = diff
3. fig 4 why not symmetric? put q inside fix y labels
4. fix figure 5 theta hat

2.2 *[2021-08-20 Fri]* MEETING

1. label all equations
2. Hhat to end

3. Include system parameters in captions
4. Fig 2 at beginning of 2.2
   - example/answer -> then explain
5. check error bars if so then show example of large e variations
6. add analytic estimate to figure 5
7. erase w/o secular line in Figure 5
8. Figure 6 plot Delta pomega -180 to 180, not absolute value
9. Fewer dots/integrations for figures 4 and 5

2.3 *[2021-08-12 Thu]* MEETING

2.4 *[2021-08-06 Fri]* MEETING

1. Mention K2-19 in intro/sec2
2. Integrate sec4 into sec3; it's not useful to fit the system exactly
3. seems like we reach a different equilibrium depending on initial conditions
4. ecc driving ansatz -> robust alignment
5. possibly high initial eccentricities ? less robust? another channel?
6. Make analytical argument with the first 2 equilibrium equations for the plot:
   - $|\Delta\varpi$

0.2 1 5 Te1/Te2

2.5 *[2021-07-30 Fri]* MEETING

1. First do natural Te e/Te e->0
2. Section 2 standard picture
3. 1st thing show secular coupling, 1 example
4. section 2 like a recap, review, show why $\Delta\varpi \to \pi$, with small correction as a function of (q, Tw0)
5. make a detailed outline
6. section 3 consider toy model e1d>0, same parameters

2.5.1 Dong's outline sketch

1. Introduction
2. Recap of "standard" picture
   - forces: e1/Te1, e2/Te2
   - cases q=2 q=1 q=1/2
   - |e1eq, e2eq

     0.2 1 5 Te1/Te2
   - $|\Delta\varpi$

     0.2 1 5 Te1/Te2
3. Toy Model, e1d>0
   - forces: (e1-e1d)/Te1, e2/Te2
   - cases q=2 q=1 q=1/2
   - |e1eq, e2eq

0.2 1 5 Te1/Te2

- $|\Delta\varpi$

0.2 1 5 Te1/Te2

4. "Fancy" Hamiltonian

2.6 *[2021-07-23 Fri]* MEETING

1. try q ~ 1 for T » Te2 to see if equilibrium is reached
2. try runs with the "story" of the capture process:
   - for alignment must have $e_{20} >$ $mu_1{}^2/3$ and $e_{10} >$ $mu_2{}^2/3$ to avoid capture into theta1/2 resonances
   - must have hat(e) within resonance capture range for hat(theta)
   - damping stops before theta1/2 equilibrium is reached

2.6.1 Plans for draft of paper

1. Introduction
   - K2-19 is puzzling in light of anti-alignment outcome
2. Summarize q first
   - example, stable case T to infty
   - secular term modification "canonical case"
   - why anti-alignment, small secular effects
3. TP case (possibly sec1 or in appendix if not relevant)

2.6.2 Plans for research talk

- K2-19 system
- Subresonances
- reproducing TP results with q=1000
  - analytic equilibrium results, not a true equilibrium

- Driving eccentricities for q~O(1) cases
  - reproducing K2-19 alignment
  - disucssion of Tei physics?

2.7 *[2021-07-16 Fri]* MEETING

1. Solve for Tej if Tek = infty algebraic/numerical
2. Only drive the larger planet's eccentricity to be nonzero
3. Look at observations of $\Delta\varpi$. How do they measure it?
   (a) are there any observed aligned cases in the literature?
   (b) If so, this is counter to the strong conclusion that the resonance is resilient to the the Te1/Te2 ratio and that in resonance the planets are always anti-aligned. **This could be the argument for your paper.**
4. Try comparable mass for e2d->0.1, maybe q=2
5. Ragusa 2018 eccentricity evolution during planet disk interaction
   (a) Long hydro simulation
6. **Problem of why q=1000 affects teh larger planet so much!!!!**
   (a) **Integrate a reasonable time for orbit evolution of q=0.001**
   (b) Compare Te1/Te2 reasonable case to crazy large case
   (c) Compare Te2 timescale to theta2 resonance timescale. Is it constant on a reasonable timescale of integration?
   (d) Run for only a few Te of the smaller planet
   (e) Try BS integrator & vary tolerance while plotting avg(sin(theta)) to see if results agree and it's not the code's fault

2.8 *[2021-07-09 Fri]* MEETING

1. It seems like the secular terms don't matter that much for the q~[0.1-1] case for comparable masses
2. For more extreme mass ratios, such as q~[0.001-0.01], dynamics may be more interesting

- in this regime Te2 is an arbitrary *parameter* because a massive planet's eccentricity damping will not be identical to a very small neighbor
- Gap opening planet, sustained eccentricity, negative Te2?

3. If we are writing a paper, what comes next?

   - brief introduction of TP case
   - Parameter study of q~[0,1e-2], Te2 parameter, how this relates to apsidal alignment/equilibrium eccentricity

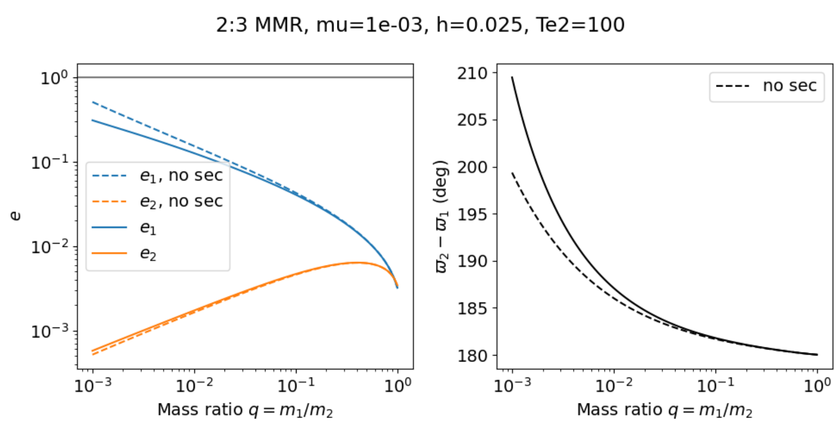
4. Papers mentioned:

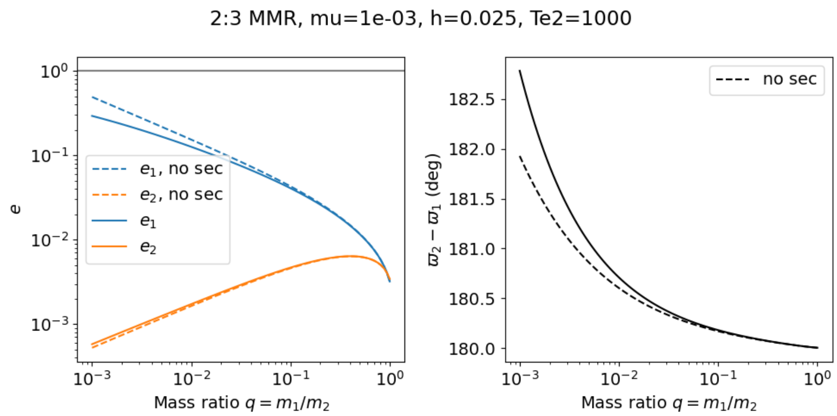
   - Chelsea Huang Warm Jupiter Neighbors, Wasp-47 system

5. **Big picture**:

   - What happens to a smaller (<~1%) mass planet when approaching a MMR with a massive planet?
   - How does this relate to apsidal alignment?

2.8.1 Laetitia's equilibrium plots:





2.8.2 DONE Read Huang paper

2.8.3 DONE Get equilibrium solving code working you idiot

- look at extreme mass ratios
- how do secular terms change the behavior?

2.8.4 DONE Find a good parameter range for q, Te2, etc

2.8.5 DONE test parameter space with time-dependent numerical runs

- in effort to answer #5 above

2.9 *[2021-07-02 Fri]* MEETING

I think I am stupid

1. Redo xu 2018 equations 16-18 but with the secular terms to see where equilibrium is, make the same plots
2. Compare numerical results of equilibrium with secular terms turned off to see the difference

3 Equations pdfs

4 dof Hamiltonian differential equations

July 20, 2021

1 Hamiltonian & Scaling

Let m_1 denote the inner planet and m_2 denote the outer planet.

$$H = -\frac{GMm_1}{2a_1} - \frac{GMm_2}{2a_2} - \frac{Gm_1m_2}{a_2}\left(f_1e_1\cos\theta_1 + f_2e_2\cos\theta_2\right) \tag{1}$$

$$\begin{aligned}
\theta &= (j+1)\lambda_2 - j\lambda_1 \\
\theta_1 &= (j+1)\lambda_2 - j\lambda_1 + \gamma_1 \\
\theta_2 &= (j+1)\lambda_2 - j\lambda_1 + \gamma_2 \\
q &= \frac{m_1}{m_2} \\
a_0 &= a_{2,\mathrm{init}} \\
H_0 &= \frac{GMm_2}{a_0} \\
\omega_0 &= n_{2,\mathrm{init}} = \sqrt{\frac{GM}{a_0^3}} \\
\Lambda_0 &= m_2\sqrt{GMa_0}
\end{aligned}$$

1

3.1 coefficients

```
sys.path.append("/home/jtlaune/multi-planet-architecture/")
from helper import *
alpha_0 = (j/(j+1))**(2./3.)
f1 = -A(alpha_0, j)
```

```
f2 = -B(alpha_0, j)
f3 = C(alpha_0)
f4 = D(alpha_0)
print([f"{fi:0.2f}" for fi in [f1, f2, f3, f4]])
```

4 Relevant Observed systems ATTACH

4.1 K2-19 b & c; Petigura et al. (2019)

- M_{star} = 0.88 Msun
- Pb = 7.9222d Pc = 11.8993d
- Mb = 32.4ME Mc = 10.8ME
- mu1 = 1.11e-4 mu2 = 3.69e-5 q = 3.00
- e_b = 0.20 e_c = 0.21
- x_b = sqrt(e_b)*cos(varpi$_b$) = 0.02 x_b = sqrt(e_b)*sin(varpi$_b$) = -0.44
- x_c = sqrt(e_c)*cos(varpi$_c$) = 0.04 x_c = sqrt(e_c)*sin(varpi$_c$) = -0.46
- Dvarpi$_{bc}$ = 2+-2 deg ~ 0.

4.2 Huang et al. (2016)

Draft version April 29, 2016
Preprint typeset using LaTeX style emulateapj v. 08/22/09

WARM JUPITERS ARE LESS LONELY THAN HOT JUPITERS: CLOSE NEIGHBOURS

Chelsea Huang[1,2], Yanqin Wu[3], Amaury H.M.J. Triaud[1,3,4]

Draft version April 29, 2016

ABSTRACT

Exploiting the *Kepler* transit data, we uncover a dramatic distinction in the prevalence of sub-Jovian companions, between systems that contain hot Jupiters (periods inward of 10 days) and those that host warm Jupiters (periods between 10 and 200 days). Hot Jupiters, with the singular exception of WASP-47b, do not have any detectable inner or outer planetary companions (with periods inward of 50 days and sizes down to $2R_{\rm Earth}$). Restricting ourselves to inner companions, our limits reach down to $1R_{\rm Earth}$. In stark contrast, half of the warm Jupiters are closely flanked by small companions. Statistically, the companion fractions for hot and warm Jupiters are mutually exclusive, particularly in regard to inner companions.

The high companion fraction of warm Jupiters also yields clues to their formation. The warm Jupiters that have close-by siblings should have low orbital eccentricities and low mutual inclinations. The orbital configurations of these systems are reminiscent of those of the low-mass, close-in planetary systems abundantly discovered by the *Kepler* mission. This, and other arguments, lead us to propose that these warm Jupiters are formed *in-situ*. There are indications that there may be a second population of warm Jupiters with different characteristics. In this picture, WASP-47b could be regarded as the extending tail of the *in-situ* warm Jupiters into the hot Jupiter region, and does not represent the generic formation route for hot Jupiters.

Subject headings:

1. FOREWORDS

The origin of hot Jupiters (HJs, period inward of ~ 10 days) has remained an unsolved issue. Although multiple scenarios have been proposed (disk migration, planet scattering, secular migration, etc.), none seem capable of satisfying all observational constraints. The recent discovery of two low-mass planetary companions (Becker et al. 2015) close to the hot Jupiter WASP-47b (Hellier et al. 2012) further obfuscates the picture. Motivated by the large population of low mass, closely-packed planets at small distances away from their host stars (Mayor et al. 2011; Howard et al. 2012; Borucki et al. 2011; Lissauer et al. 2011), and by the realization that some of them could have accumulated enough mass to undergo run-away gas accretion (Lee et al. 2014), Boley et al. (2016); Batygin et al. (2015) argue that WASP-47b, and possibly all hot Jupiters, were formed *in-situ*, instead of somehow transported inward. Only a tiny fraction of super-Earths need follow this path to be able to match the occurrence rate of hot Jupiters.

While this seems a reasonable proposal for WASP-47b, could it explain the majority of hot Jupiters? To answer this, we focus on the following issue: is WASP-47b a generic hot Jupiter in terms of co-habiting with other planets? Currently, this question is best addressed by exploiting the *Kepler* data to look for small transiting bodies in systems hosting (either confirmed or candidate) hot Jupiters. If we find that WASP-47b is truly unique among all hot Jupiters, it may suggest that the formation of hot Jupiters can have multiple pathways, with a minority being formed *in-situ*.

There is a second goal to our paper: understanding the warm Jupiters (WJs). By this term we refer specifically to those giant planets orbiting between 10 days and 200 days in period. Unlike the hot Jupiters (inward of 10 days), they are too far out to have experienced little if any tidal circularization and therefore may be difficult to migrate inward by mechanisms that invoke high-eccentricity excitation. On the other hand, they live inward of the sharp rise of giant planets outside ~ 1AU – in fact, the period range of warm Jupiters corresponds to the so-called 'period-valley', the observed dip in occupation in-between the hot Jupiters and cold Jupiters (e.g., Mayor et al. 2011; Wright et al. 2012; Santerne et al. 2016). In contrast with hot Jupiters, no theories have been proposed to explain the existence of this class of objects. So in this paper, we hope to gain some insights by studying their companion rates.

There have been multiple past claims that hot Jupiters lack sub-Jovian (and Jovian) close companions, by using the Radial Velocity data (Wright et al. 2009), by inferring from (the lack of) transit timing variations in these objects (Steffen & Agol 2005; Gibson et al. 2009; Latham et al. 2011; Steffen et al. 2012), and by searching for other transiting companions in the same systems (Steffen et al. 2012). The last study, in particular, is the closest to our work in spirit. Using preliminary candidates resulting from the first four months of the *Kepler* Mission (63 HJs and 31 WJs, defined differently from here), Steffen et al. (2012) found a difference between the two populations: while none of the HJs have any transit-

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:ID: 9ac2be99-7caa-47bf-b897-7babb34634a7

/home/jtlaune/multi-planet-architecture/notes/2021-07-14_14-34-29_screenshot.png

4.2.1 Kepler-30 q~0.019, q~26

Panichi et al. (2017) `https://arxiv.org/pdf/1707.04962.pdf` b,c near 2:1 first order, q~0.019 all transiting *from exoplanet catalog:* b 11.3 Me 0.18au 29.3 days e=0.04 c 2.01 Mj 0.3au 60.3 days e=0.01 d 23.1 Me 0.5au 143.3 days e=0.02

4.2.2 Wasp-47

b 1.1 Mj c 1.6 Mj d 13 Me e 6.8 Me

4.2.3 Kepler-46

b 6 Mj c 0.38 Mj d 3.3 Me

4.2.4 Kepler-302

b 16 Me c Unknown WJ

4.2.5 Kepler-419

b 2.5 Mj c 7.3 Mj

4.2.6 Kepler-289

b 7.3 Me c 0.42 Mj d 4 Me

4.2.7 Kepler-418

b 1.1 Mj

4.2.8 Kepler-117

b 30 Me c 1.8 Mj

5 validating w/ REBOUND [8/8]

5.1 DONE plot gammadot components to compare

5.2 DONE calculate ring potential

- involves elliptic integral, "'sp.special.ellipkinc"'
- research journal *[2021-02-24 Wed]*

5.3 DONE test J_2 external forcing term for perihelion precession rates

/home/jtlaune/mmr/nbody/testsuite/test-omext/mup1.00e-04/om1.00e-03/e0.00e+00.png

5.4 DONE calculate external forcing term in terms of J_2

- research journal *[2021-02-11 Thu]*

5.5 DONE try to use REBOUNDx to implement om_{eff}

- reboundx will not install on my system

5.6 DONE investigate REBOUNDx

- implemented lots of extra forces already
- `https://reboundx.readthedocs.io/en/latest/effects.html`

- going to try to use a negative J_2 value with

#+BEGIN$_{\mathrm{SRC}}$ python gh = rebx.load$_{\mathrm{force}}$("gravitational$_{\mathrm{harmonics}}$") #+END$_{\mathrm{SRC}}$ python

5.7 DONE check units on om$_{\mathrm{eff}}$ in migforce

- current results show little change in behavior, contradict semianalytical
- this cannot be right. I stupidly set the cartesian coordinates of the particle equal to the cartesian phase space coordinates: #+BEGIN$_{\mathrm{SOURCE}}$ python

if self.omext: tpart.ax += -(self.omext**2)*tpart.x tpart.ay += -(self.omext**2)*tpart.y #+END$_{\mathrm{SOURCE}}$ python

5.8 DONE compare semianalytical ext-perturber results with REBOUND [2/2]

5.8.1 DONE run bottomright test (nonchaotic for edisk = 0.01, ep = 0.1)

finally s ecc excitation, but gammas have contradicting signs and thetas arculating. i'm thinking its some kind of issue in signs for om$_{\mathrm{exuld}}$ explain both)

/home/jtlaune/mmr/nbody/testsuite/collect/precess-eq1.00e-02-ep1.00e-01-mu1.00e-03.png

/home/jtlaune/mmr/ext-per

5.8.2 DONE compare gamma derivatives

/home/jtlaune/mmr/ext-perturber/varyon

similar behavior, but the first term is circulating for nbody

/home/jtlaune/mmr/nbody/testsuite/collect/precess-gammacomps-eq1.00e-02-ep1.00e-01-om1

6 summary

6.1 characteristics

1. chaos (only when om_{ext} large)

2. internal apsidal alignment

   - $\text{om}_{\text{eff}} = 0$
     - unknown res????<—- figure this out
     - kind of all over the place if im being honest. maybe don't include? maybe leave out just migfail runs? not sure what to do here

3. external apsidal alignment

   - $\text{om}_{\text{eff}} = 0$
     - gamma -> 0
     - ep vs edisk grid
     - EoM analytical analysis
     - plots of gamma-components summary
   - $\text{om}_{\text{eff}} > 0$
     - gamma -> pi
     - heuristic description of EoM summary
     - plot e1 eq numerical value vs om_{eff} w/ behaviors
     - **figure** gamma component term plots (from above file bottom page 2)
     - gamma component plots

4. equilibrium eccentricity

   - no om_{eff} ~ disk properties
   - large enough om_{eff} ~ 1/gammadot from above

7 results summary table

	internal		external		
	$om_{ext} = 0$	$om_{ext} = 0$	$om_{ext} <$ res width	om_{ext} ~ res width	$om_{ext} >$ res wid
$e_{disk} < e_p$	**disaster zone**	**aligned**			
e_{disk} ~ e_p	**aligned**				**chaotic**
$e_{disk} > e_p$					

7.1 DONE fill in om_{ext} columns for external

- in paper draft

7.2 DONE think about internal? is it important to include?

yes, should include internal. explain away the bad parts by saying our model fails

8 semianalytical test cases [1/1]

`test-cases.py`

8.0.1 DONE test cases [5/5]

- ☒ inner migrating out, 4 mup stability cases (no cap, cap unstable, cap librate, cap stable)
- ☒ internal equilibrium e
- ☒ outer migrating in, 2 mup capture cases, (no cap, cap)
- ☒ external equilibrium e
- ☒ stability cases w/ ep = 0.01 small

9 handwritten research journals

Feb 2020-

10 Long term objectives

10.1 DONE list of figures and outline [3/3]

10.1.1 DONE apsidal alignment [2/3]

- ☒ combine internal & external plots
- ☐ plot heuristic contours from EoM
  - important term is $\cos\theta/e$
  - g-alignment
    - ep > ed => $\theta \neq \overline{\theta}$ => θ circ => 1/e term avgs out => $\dot{\gamma} \to 0$
  - g-circulation
    - ep < ed => $\theta \approx \overline{\theta}$ => $\theta \to 0, \pi$ => 1/e term dominates => $\dot{\gamma} > 0$
- ☒ highlight example runs with red border

10.1.2 DONE example runs

- blurred scatter plots
- pick 0.01,0.1 and 0.1,0.01

10.1.3 DONE phase diagrams

10.2 Waiting on

10.2.1 WAIT write up comparison of theta1/2 resonant timescales and Te1/2 timescales

10.2.2 WAIT phase diagrams [1/3]

- ☒ semianalytical
- ☐ n body
- ☐ describe resonance splitting

10.2.3 WAIT finish summary [2/4]

- ☐ need to include N-body runs for ext-perturber, non-confirmation or confirmation
- ☒ clarify chaotic nature of e1 excitation for omext >~ dn runs
  - ep and edisk similar magnitude => chaotic based on a0
- ☒ summary table of runs, cross table, # runs, etc
- ☐ relate eeq to disk properties

10.2.4 WAIT test omext in H integrator

10.2.5 WAIT fix & shorten reference-pdf

10.2.6 WAIT sympy confirmation of sidebyside summary EoMs

10.2.7 WAIT organize [4/4]

1. **DONE** org research notes
2. **DONE** goodnotes research notes
3. **DONE** meeting notes
4. **DONE** calculation notes

10.2.8 WAIT REBOUND

1. **WAIT** matter ring potential [0/3]
   - ☐ implement force in rebound
   - ☐ test implementation
   - ☐ compare to semianalytical
2. **WAIT** add interrupt conditions

10.2.9 WAIT fix rebound mmr Tm signs. simplify

10.2.10 WAIT figure out unknown res situation to be able to include internal runs in summary

10.3 Done

10.3.1 DONE Change e1, e2 calc in `file:///home/jtlaune/multi-planet-architecture/run.py` to proper delaunay variables

10.3.2 DONE comparable mass Hamiltonian [3/3]

1. **DONE** make git commit w/ test particle test suite
2. **DONE** clean up, organize files
3. **DONE** write & test comparable mass H code