Contents

1 Introduction

The typical outcome of planet-disk migration is thought to be capture into mean motion resonance, after which torques from the gas damp the system's angular momentum deficit (AMD) into an equilibrium. The MMR equilibrium is governed by various conditions which are well studied in the literature (e.g. [Henrard and Lemaitre, 1983, Deck and Batygin, 2015, Goldreich and Schlichting, 2014, Xu et al., 2018, Henrard and Lemaitre, 1983])

2 Methods

2.1 Hamiltonian

The Hamiltonian of a system with two planets near a first order $j : j+1$ MMR is

$$\begin{aligned} H = &-\frac{GMm_1}{2a_1} - \frac{GMm_2}{2a_2} \\ &- \frac{Gm_1m_2}{a_2}(f_1e_1\cos\theta_1 + f_2e_2\cos\theta_2 \\ &+ f_3(e_1^2+e_2^2) + f_4e_1e_2\cos(\varpi_2-\varpi_1) \end{aligned} \tag{1}$$

The first two terms in parentheses are resonant terms of order $\mathcal{O}(e_i^1)$, while the last two terms are secular effects of order $\mathcal{O}(e_i^2)$. The two resonant angles are given by

$$\theta_i = (j+1)\lambda_2 - \lambda_1 - \varpi_i.$$

Following [Wisdom, 1986, Henrard et al., 1986], through a series of canonical transformations (as outlined in the appendix), we may turn (1) into an integrable system with resonant argument $\hat\theta$ given by the equation

$$\tan\hat\theta = \frac{e_1\sin(\theta_1) + (f_1/f_2)e_2\sin(\theta_2)}{e_1\cos(\theta_1) + (f_1/f_2)e_2\cos(\theta_2)} \tag{2}$$

3 Results

3.1 Test particle results

3.2 Case $q \gg 1$

4 Discussion

5 Conclusion

References

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