Parallel & Concurrent Haskell 1: Basic Pure Parallelism

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Haskell’s philosophy

- We want to give you the right tool for the job
  - Even if that means having many tools
  - so unlike some languages that focus on just one parallel programming model (e.g. CSP or message-passing) in Haskell there are lots to choose from

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- But the guiding principle is
  - provide *minimal* built-in functionality
  - so that we can give a simple semantics
  - then implement nice abstraction layers on top

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- But the guiding principle is
  - provide *minimal* built-in functionality
  - so that we can give a simple semantics
  - then implement nice abstraction layers on top
- Problem: how do you, the programmer, decide which tool (API) you need?

First, we divide the landscape in two: *Parallel* and *Concurrent* applications/programming models

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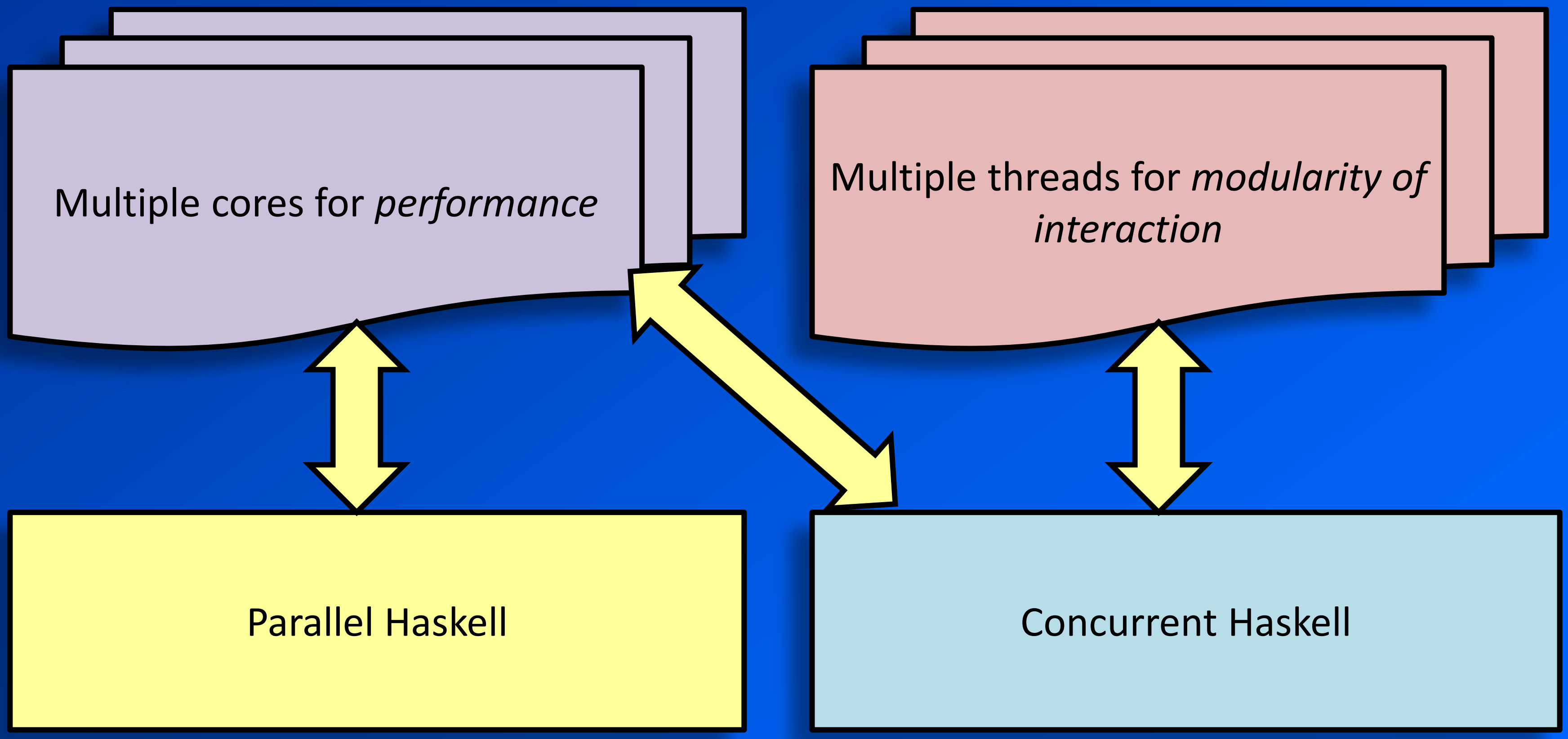
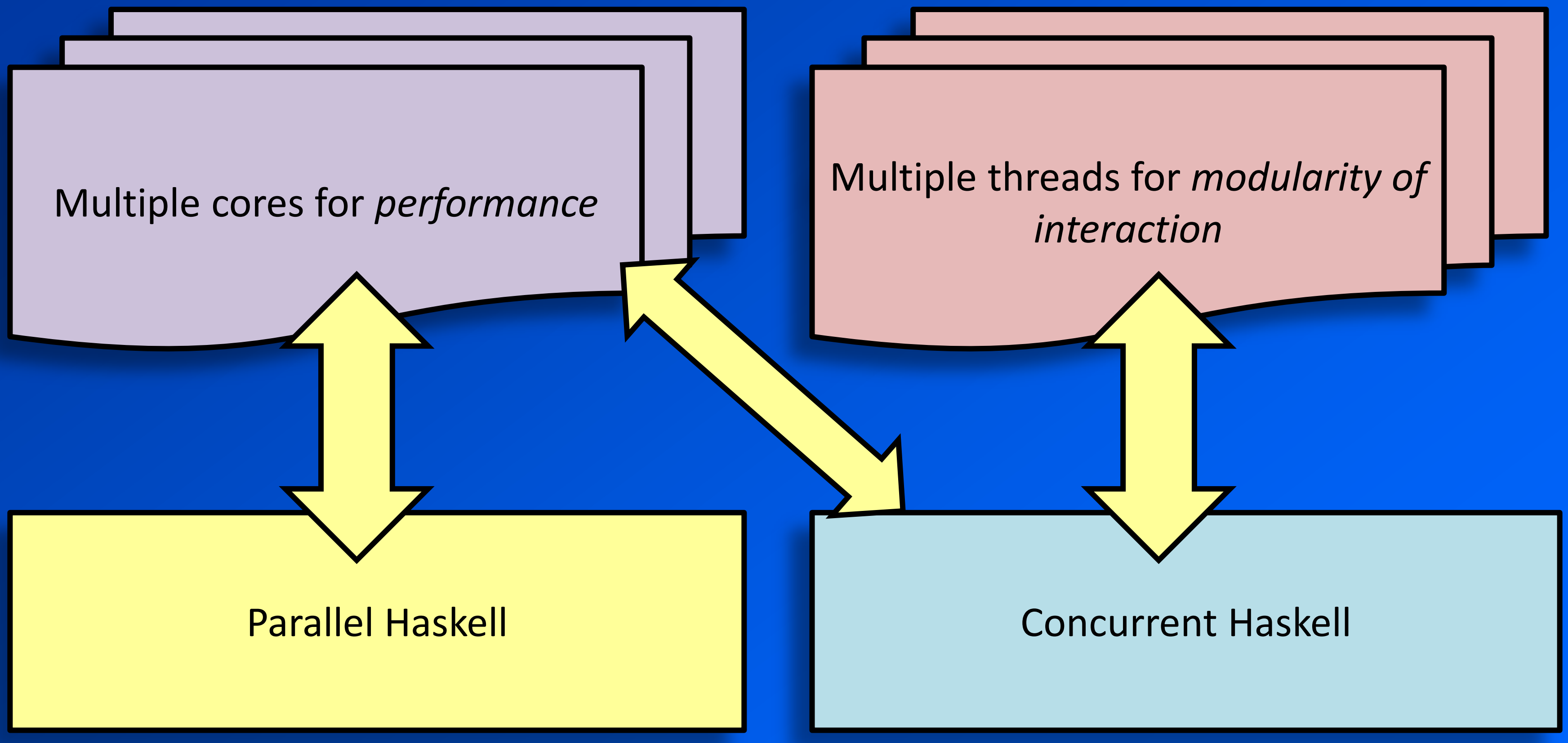
What’s the difference?

Parallelism vs. Concurrency


Multiple cores for *performance*
Multiple threads for *modularity of interaction*
Parallel Haskell
Concurrent Haskell

Parallelism vs. Concurrency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Parallelism vs. Concurrency

- Primary distinguishing feature of Parallel Haskell: determinism
  - The program always does the same thing, but may run faster when given multiple cores to run on.
  - No race conditions or deadlocks
  - add parallelism without sacrificing correctness
  - Parallelism is used to speed up pure (non-IO monad) Haskell code

Parallelism vs. Concurrency

- Primary distinguishing feature of Concurrent Haskell: threads of control
  - Concurrent programming is done in the IO monad
    - because threads have *effects*
    - effects from multiple threads are interleaved nondeterministically at runtime.
  - Concurrent programming allows programs that interact with multiple external agents to be *modular*
    - the interaction with each agent is programmed separately
    - Allows programs to be structured as a collection of interacting agents (actors)

We have a lot of ground to cover...

1. Basic pure parallelism
2. The Par Monad
3. Concurrent Haskell
4. Software Transactional Memory
5. Server applications
6. Distributed programming
7. GPU programming

Parallel

Concurrent

I. Parallel Haskell

- In this part of the course, you will learn how to:
  - Do basic parallelism:
    - compile and run a Haskell program, and measure its performance
    - parallelise a simple Haskell program (a Sudoku solver)
    - use ThreadScope to profile parallel execution
    - do dynamic partitioning
    - measure parallel speedup
      - use Amdahl's law to calculate possible speedup
  - Work with Evaluation Strategies
    - build simple Strategies

Running example: solving Sudoku

– code from the Haskell wiki (brute force search with some intelligent pruning)

– can solve all 49,000 problems in 2 mins

– input: a line of text representing a problem

```
.......2143.......6........2.15..........637...........68...4.....23........7....
.......241..8.............3...4..5..7.....1......3.......51.6....2....5..3...7...
.......24....1...........8.3.7...1..1..8..5.....2......2.4...6.5...7.3...........
```

```haskell
import Sudoku

solve :: String -> Maybe Grid
```

Solving Sudoku problems

- Sequentially:
  - divide the file into lines
  - call the solver for each line

```
main :: IO ()
main = do
    [f] <- getArgs
    grids <- fmap lines $ readFile f
    print $ length $ filter isJust $ map solve grids
```

```
solve :: String -> Maybe Grid
```

Compile the program...

```
$ ghc -O2 sudoku1.hs -rtsopts
[1 of 2] Compiling Sudoku           ( Sudoku.hs, Sudoku.o )
[2 of 2] Compiling Main             ( sudoku1.hs, sudoku1.o )
Linking sudoku1 ...
$
```

Run the program…

```
$ ./sudoku1 sudoku17.1000.txt +RTS -s
   2,392,127,440 bytes allocated in the heap
      36,829,592 bytes copied during GC
         191,168 bytes maximum residency (11 sample(s))
          82,256 bytes maximum slop
               2 MB total memory in use (0 MB lost due to fragmentation)

  Generation 0:  4570 collections,     0 parallel,  0.14s,  0.13s elapsed
  Generation 1:    11 collections,     0 parallel,  0.00s,  0.00s elapsed

...

  INIT  time    0.00s  (  0.00s elapsed)
  MUT   time    2.92s  (  2.92s elapsed)
  GC    time    0.14s  (  0.14s elapsed)
  EXIT  time    0.00s  (  0.00s elapsed)
  Total time    3.06s  (  3.06s elapsed)

...
```

Now to parallelise it...

- Doing parallel computation entails specifying coordination in some way – compute A in parallel with B
- This is a constraint on *evaluation order*
- But by design, Haskell *does not have a specified evaluation order*
- So we need to add something to the language to express constraints on evaluation order

The Eval monad

```
module Control.Parallel.Strategies (..) where

data Eval a
instance Monad Eval

runEval :: Eval a -> a

rpar :: a -> Eval a
rseq :: a -> Eval a
```

Start evaluating **a** (to WHNF) in the background

Evaluate **b** (to WHNF) and wait for the result

- Eval is pure
- Just for expressing sequencing between rpar/rseq – nothing more
- Compositional – larger Eval sequences can be built by composing smaller ones using monad combinators
- Internal workings of Eval are very simple (see Haskell Symposium 2010 paper)

- We want to do a in parallel with b.
- Which of the following is the best?

```
do
    a’ <- rpar a
    b’ <- rpar b
    return (a’,b’)
```

```
do
    a’ <- rpar a
    b’ <- rseq b
    return (a’,b’)
```

```
do
    a’ <- rpar a
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    rseq a’
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```

```
do
    a’ <- rpar a
    b’ <- rpar b
    rseq a’
    rseq b’
    return (a’,b’)
```

- We want to do a in parallel with b.
- Which of the following is the best?

```
do
    a' <- rpar a
    b' <- rpar b
    return (a',b')
```

```
do
    a' <- rpar a
    b' <- rseq b
    return (a',b')
```

```
do
    a' <- rpar a
    b' <- rseq b
    rseq a'
    return (a',b')
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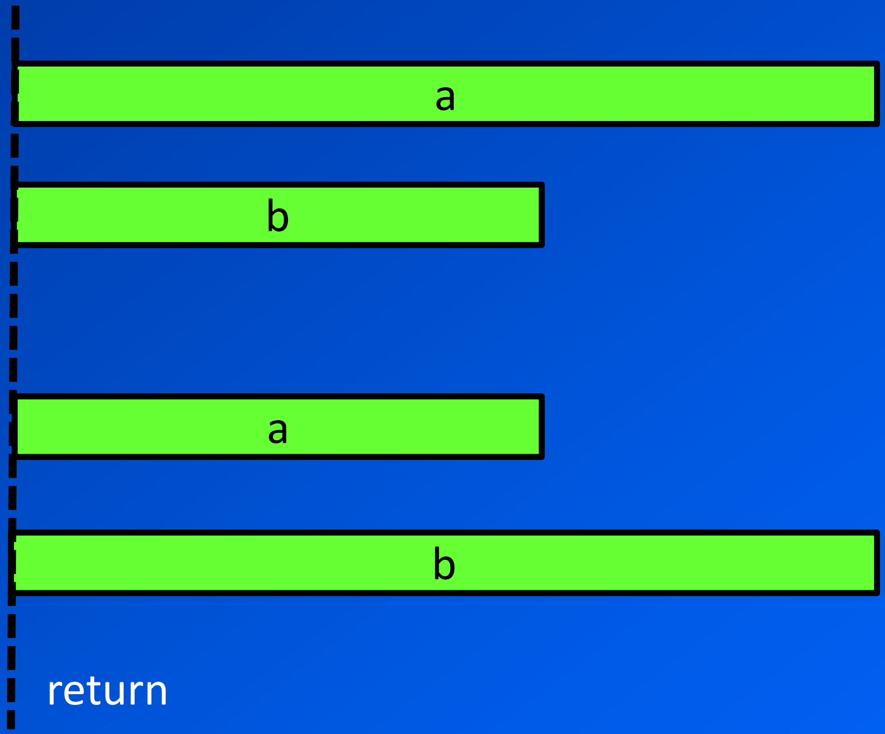
```
do
    a' <- rpar a
    b' <- rpar b
    rseq a'
    rseq b'
    return (a',b')
```



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    rseq b’
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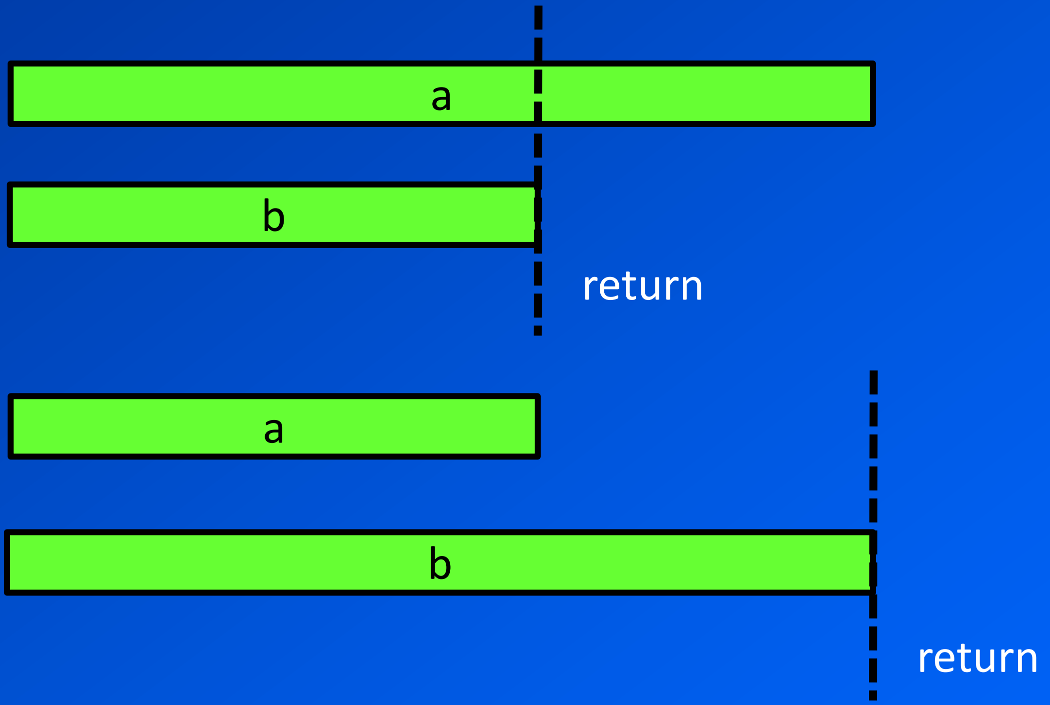

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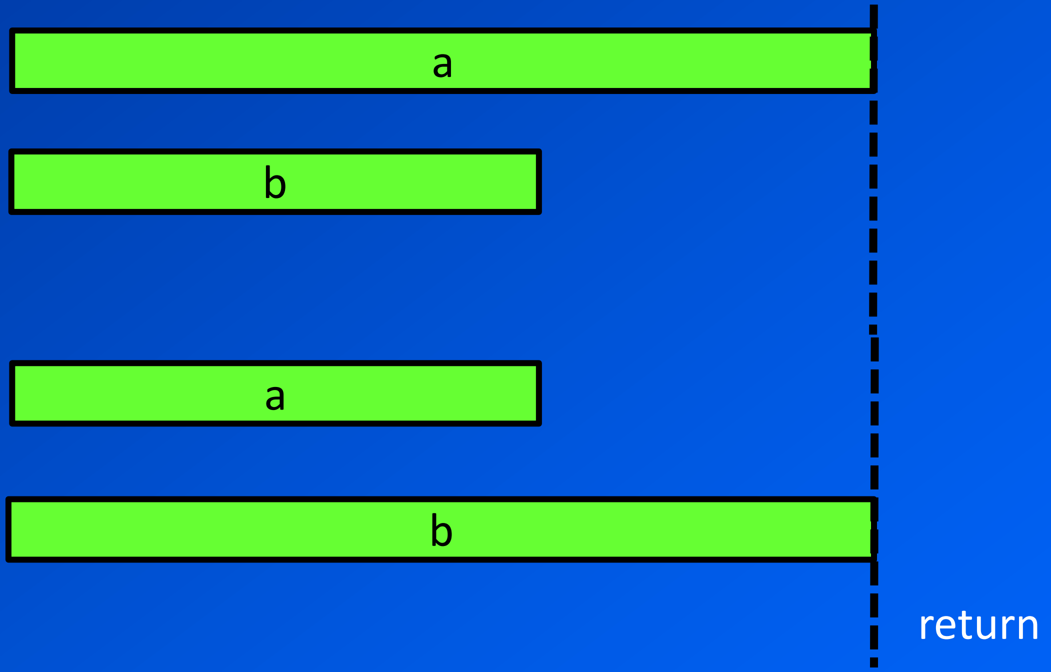
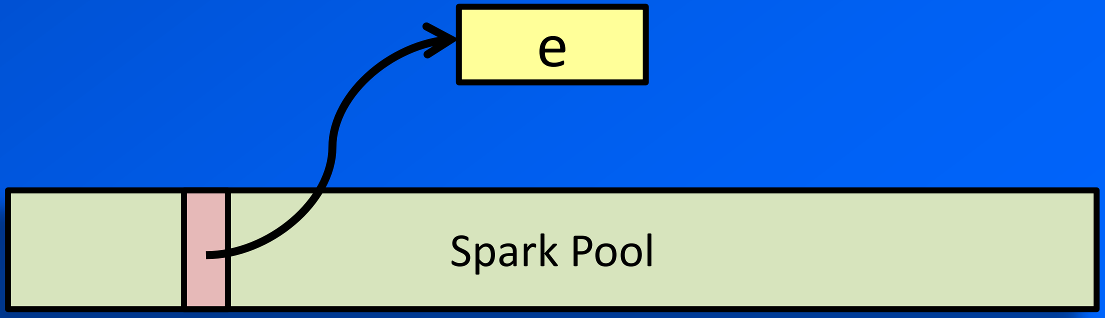


What does rpar *actually do?*

```
x <- rpar e
```

- rpar creates a *spark* by writing an entry in the *spark pool*
  - rpar is very cheap! (not a thread)
- the spark pool is a circular buffer
- when a processor has nothing to do, it tries to remove an entry from its own spark pool, or steal an entry from another spark pool (*work stealing)*
- when a spark is found, it is evaluated
- The spark pool can be full – new sparks are discarded when the pool is full. Watch out!



Parallelising Sudoku

- Let’s divide the work in two, so we can solve each half in parallel:

```
 let (as,bs) = splitAt (length grids `div` 2) grids
```

- Now we need something like

```
runEval $ do
   as’ <- rpar (map solve as)
   bs’ <- rpar (map solve bs)
   rseq as’
   rseq bs’
   return (as’ ++ bs’)
```

But this won’t work...

```
runEval $ do
   as’ <- rpar (map solve as)
   bs’ <- rpar (map solve bs)
   rseq as’
   rseq bs’
   return (as’ ++ bs’)
```

- rpar evaluates its argument to Weak Head Normal Form (WHNF)
- what is WHNF?
  - evaluates as far as the *first constructor*
  - e.g. for a list, we get either [] or (x:xs)
  - e.g. WHNF of “map solve (a:as)” would be “solve a : map solve as”
- But we want to evaluate the whole list, and the elements

We need to go deeper

```haskell
module Control.DeepSeq ( .. ) where

class NFData a where
  rnf :: a -> ()

deepseq :: NFData a => a -> b -> b
deepseq a b = rnf a `seq` b

force :: NFData a => a -> a
force a = deepseq a a
```

We need this

- provided by the ‘deepseq’ package
- force fully evaluates a nested data structure and returns it
  - e.g. a list: the list is fully evaluated, including the elements
- uses overloading: the argument must be an instance of NFData
  - instances for most common types are provided by the library

Ok, adding force

```
runEval $ do
   as’ <- rpar (force (map solve as))
   bs’ <- rpar (force (map solve bs))
   rseq as’
   rseq bs’
   return (as’ ++ bs’)
```

- Now we just need to evaluate this at the top level in ‘main’:

```
print $ length $ filter isJust $ runEval $ do
       as’ <- rpar (force (map solve as))
       ...
```

Let’s try it...

```
$ ghc --make -O2 sudoku2.hs -rtsopts -threaded
[1 of 2] Compiling Sudoku           ( Sudoku.hs, Sudoku.o )
[2 of 2] Compiling Main             ( sudoku2.hs, sudoku2.o )
Linking sudoku2 ...
$
```

Run it on one processor first

```
$ ./sudoku2 sudoku17.1000.txt +RTS -s
./sudoku2 sudoku17.1000.txt +RTS -s
1000
   2,400,398,952 bytes allocated in the heap
      48,900,472 bytes copied during GC
       3,280,616 bytes maximum residency (7 sample(s))
         379,624 bytes maximum slop
              11 MB total memory in use (0 MB lost due to fragmentation)

  …

  INIT  time    0.00s  (  0.00s elapsed)
  MUT   time    2.91s  (  2.91s elapsed)
  GC    time    0.19s  (  0.19s elapsed)
  EXIT  time    0.00s  (  0.00s elapsed)
  Total time    3.09s  (  3.09s elapsed)
 …
```

A tiny bit slower (was 3.06 before). Splitting and reconstructing the list has some overhead.

Runtime results...

```
$ ./sudoku2 sudoku17.1000.txt +RTS -N2 -s
   2,400,125,664 bytes allocated in the heap
      48,845,008 bytes copied during GC
       2,617,120 bytes maximum residency (7 sample(s))
         313,496 bytes maximum slop
               9 MB total memory in use (0 MB lost due to fragmentation)

  Generation 0:  2975 collections,  2974 parallel,  1.04s,  0.15s elapsed
  Generation 1:     7 collections,     7 parallel,  0.05s,  0.02s elapsed

  Parallel GC work balance: 1.52 (6087267 / 3999565, ideal 2)

  SPARKS: 2 (1 converted, 0 pruned)

  INIT  time    0.00s  (  0.00s elapsed)
  MUT   time    2.21s  (  1.80s elapsed)
  GC    time    1.08s  (  0.17s elapsed)
  EXIT  time    0.00s  (  0.00s elapsed)
  Total time    3.29s  (  1.97s elapsed)
```

Calculating Speedup

- Calculating speedup with 2 processors:
  - Elapsed time (1 proc) / Elapsed Time (2 procs)
  - NB. not CPU time (2 procs) / Elapsed (2 procs)!
  - NB. compare against sequential program, not parallel program running on 1 proc
    - why? introducing parallelism may add some overhead compared to the sequential version

- Speedup for sudoku2: 3.06/1.97 = 1.55
  - not great...

Why not 2?

- there are two reasons for lack of parallel speedup:
  - less than 100% utilisation (some processors idle for part of the time)
  - extra overhead in the parallel version
- Each of these has many possible causes...

A menu of ways to go wrong

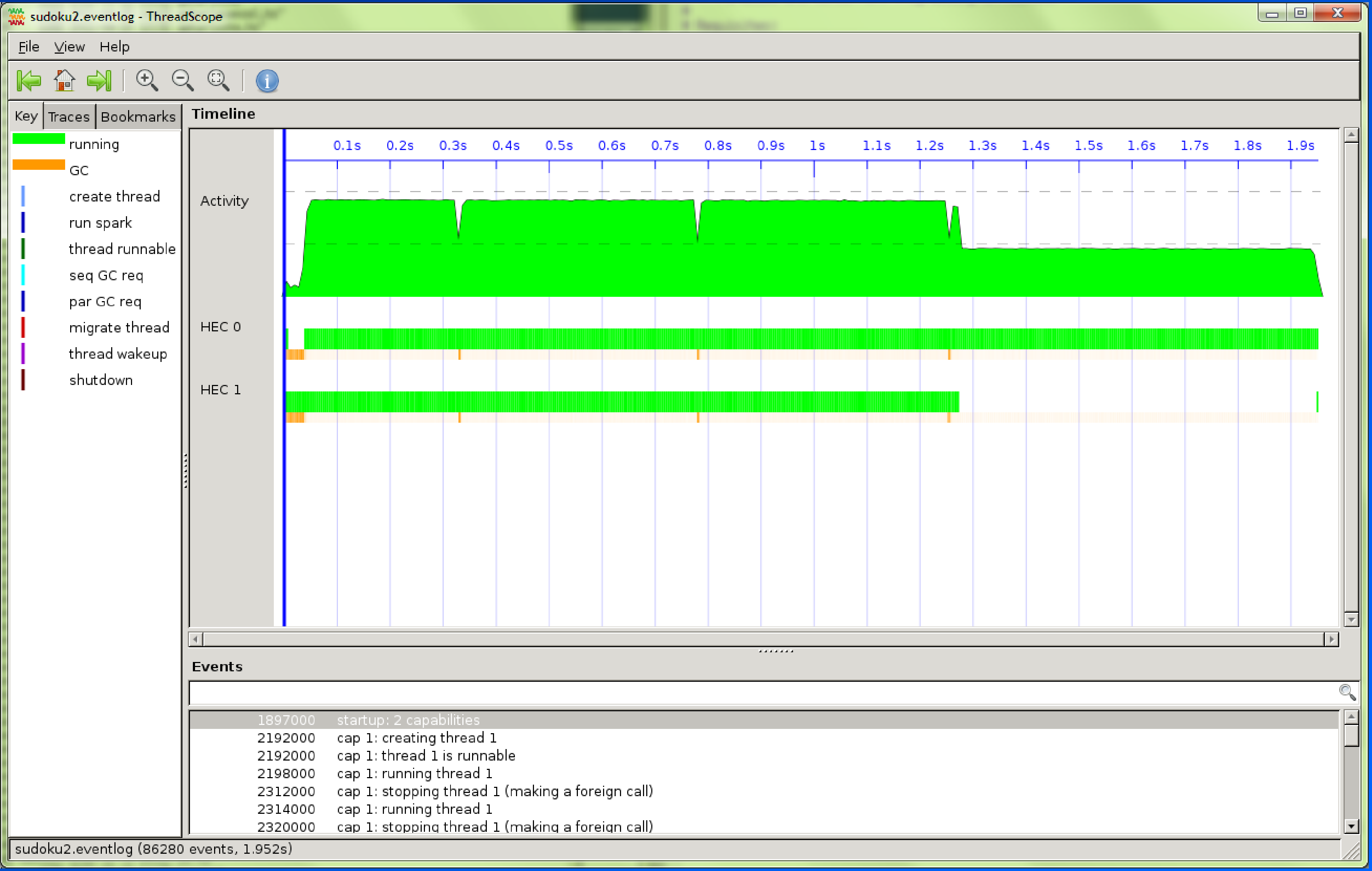
- less than 100% utilisation
  - parallelism was not created, or was discarded
  - algorithm not fully parallelised – residual sequential computation
  - uneven work loads
- extra overhead in the parallel version
  - overheads from rpar, work-stealing, force, ...
  - larger memory requirements leads to GC overhead
- low-level issues that are Simon’s problem:
  - poor scheduling
  - communication latency
  - GC synchronisation
  - duplicating work
  - poor locality, cache effects

So we need *tools*

- to tell us why the program isn't performing as well as it could be
- For Parallel Haskell we have ThreadScope

```
$ rm sudoku2; ghc -O2 sudoku2.hs -threaded -rtsopts -eventlog
$ ./sudoku2 sudoku17.1000.txt +RTS -N2 -ls
$ threadscope sudoku2.eventlog
```

- -eventlog has very little effect on runtime
  – important for profiling parallelism



sudoku2.eventlog - ThreadScope

🟩	running
🟧	GC
|	create thread
|	run spark
|	thread runnable
|	seq GC req
|	par GC req
|	migrate thread
|	thread wakeup
|	shutdown

```
1897000   startup: 2 capabilities
2192000   cap 1: creating thread 1
2192000   cap 1: thread 1 is runnable
2198000   cap 1: running thread 1
2312000   cap 1: stopping thread 1 (making a foreign call)
2314000   cap 1: running thread 1
2320000   cap 1: stopping thread 1 (making a foreign call)
```

sudoku2.eventlog (86280 events, 1.952s)

Uneven workloads...

- So one of the tasks took longer than the other, leading to less than 100% utilisation

```
let (as,bs) = splitAt (length grids `div` 2) grids
```

- One of these lists contains more work than the other, even though they have the same length
  - sudoku solving is not a constant-time task: it is a searching problem, so it depends on how quickly the search finds the solution

Partitioning

```
let (as,bs) = splitAt (length grids `div` 2) grids
```

- Dividing up the work along fixed pre-defined boundaries, as we did here, is called *static partitioning*
  - static partitioning is simple, but can lead to under-utilisation if the tasks can vary in size
  - static partitioning does not adapt to varying availability of processors – our solution here can use only 2 processors

Dynamic Partitioning

- Dynamic partitioning involves
  - dividing the work into smaller units
  - assigning work units to processors dynamically at runtime using a *scheduler*
  - good for irregular problems and varying number of procoessors
- GHC’s runtime system provides spark pools to track the work units, and a work-stealing scheduler to assign them to processors
- So all we need to do is use smaller tasks and more sparks, and we get dynamic partitioning

Revisiting Sudoku...

- So previously we had this:

```
runEval $ do
          a <- rpar (force (map solve as))
          b <- rpar (force (map solve bs))
          ...
```

- We want to push rpar down into the map
  - so each call to solve will be a separate spark

A parallel map

```
parMap :: (a -> b) -> [a] -> Eval [b]
parMap f [] = return []
parMap f (a:as) = do
   b <- rpar (f a)
   bs <- parMap f as
   return (b:bs)
```

Create a spark to evaluate (f a) for each element a

Return the new list

- Provided by Control.Parallel.Strategies
- Also:

```
parMap f xs = mapM (rpar . f) xs
```

Putting it together...

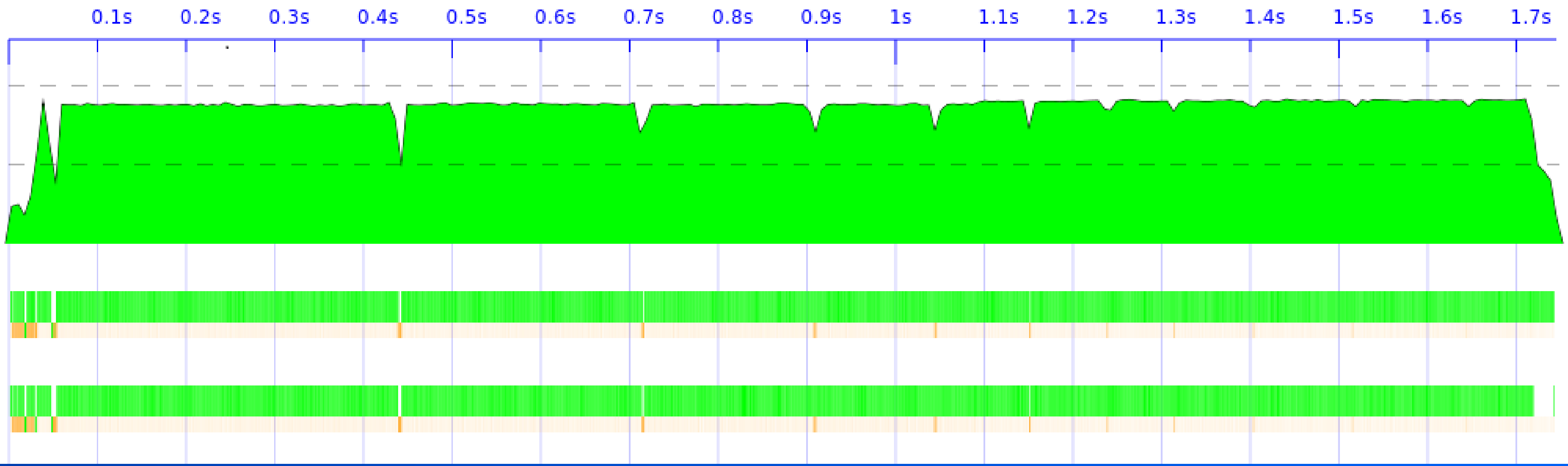
```
runEval $ parMap solve grids
```

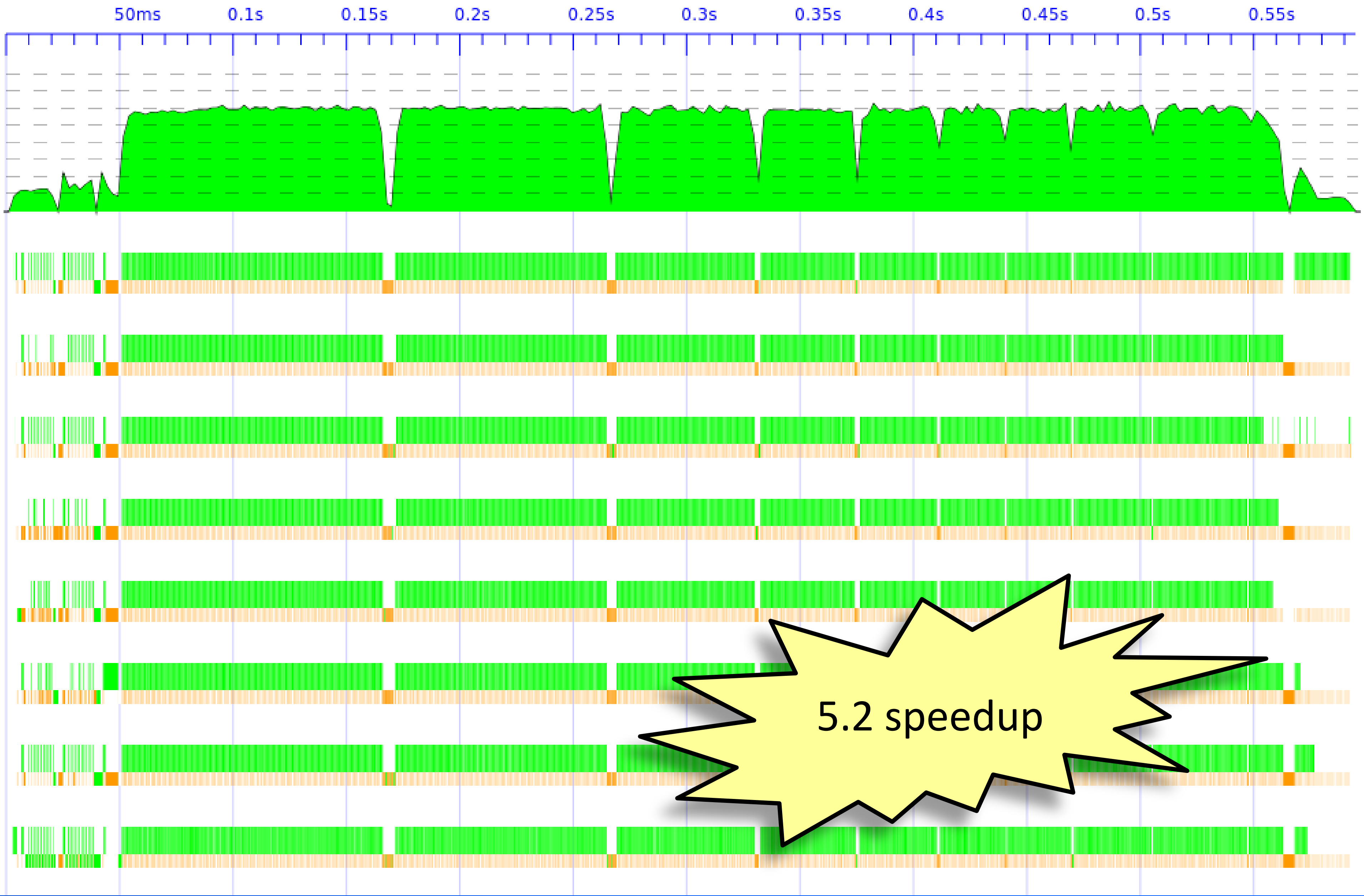
- Code is simpler than the static partitioning version!

Results

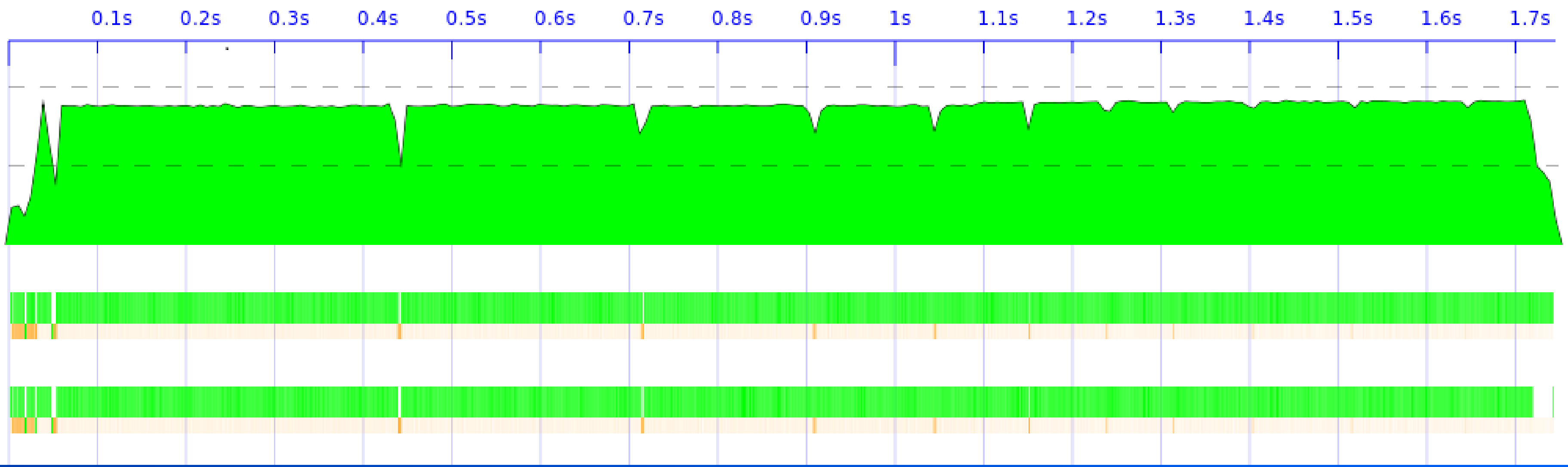
```
./sudoku3 sudoku17.1000.txt +RTS -s -N2 -ls
   2,401,880,544 bytes allocated in the heap
      49,256,128 bytes copied during GC
       2,144,728 bytes maximum residency (13 sample(s))
         198,944 bytes maximum slop
               7 MB total memory in use (0 MB lost due to fragmentation)

  Generation 0:  2495 collections,  2494 parallel,  1.21s,  0.17s elapsed
  Generation 1:    13 collections,    13 parallel,  0.06s,  0.02s elapsed

  Parallel GC work balance: 1.64 (6139564 / 3750823, ideal 2)

  SPARKS: 1000 (1000 converted, 0 pruned)

  INIT  time    0.00s  (  0.00s elapsed)
  MUT   time    2.19s  (  1.55s elapsed)
  GC    time    1.27s  (  0.19s elapsed)
  EXIT  time    0.00s  (  0.00s elapsed)
  Total time    3.46s  (  1.74s elapsed)
```

Now 1.7 speedup


0.1s
0.2s
0.3s
0.4s
0.5s
0.6s
0.7s
0.8s
0.9s
1s
1.1s
1.2s
1.3s
1.4s
1.5s
1.6s
1.7s


50ms
0.1s
0.15s
0.2s
0.25s
0.3s
0.35s
0.4s
0.45s
0.5s
0.55s
5.2 speedup

break...


0.1s
0.2s
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0.4s
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0.6s
0.7s
0.8s
0.9s
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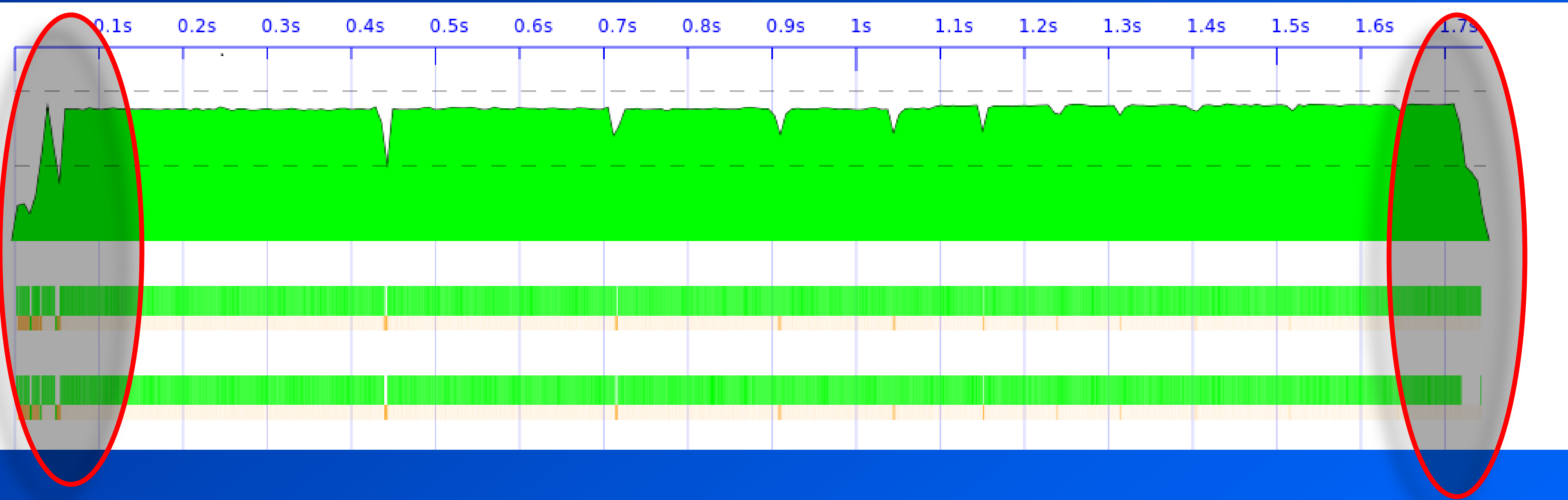
1.1s
1.2s
1.3s
1.4s
1.5s
1.6s
1.7s


0.1s
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0.3s
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0.6s
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1s
1.1s
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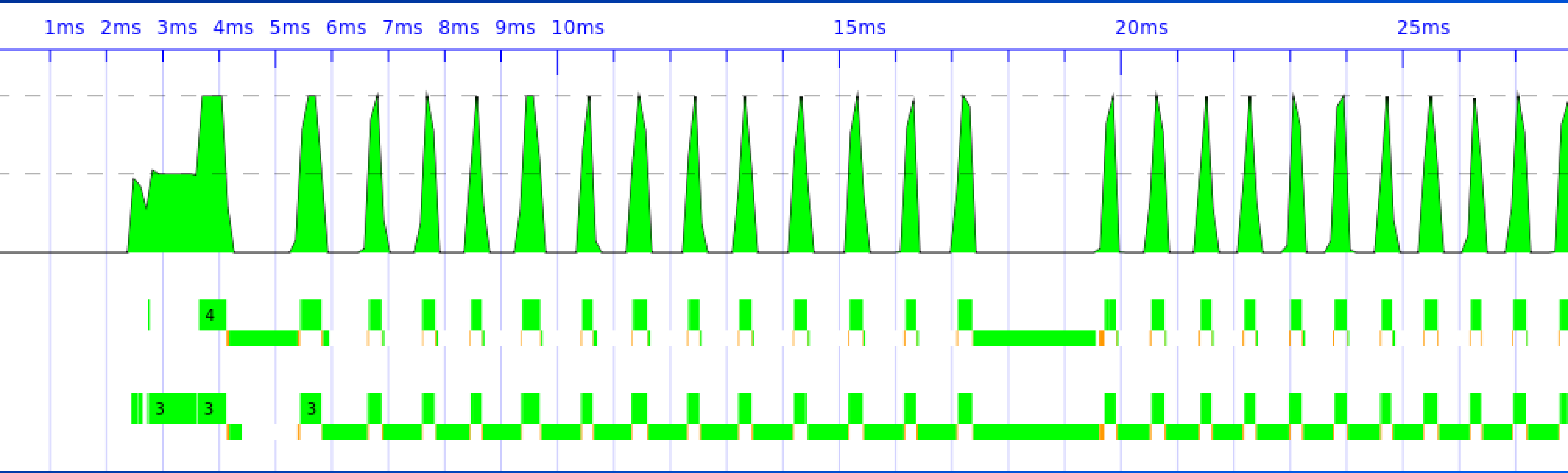


- Lots of GC
- One core doing all the GC work
  - indicates one core generating lots of data

```haskell
main :: IO ()
main = do
    [f] <- getArgs
    grids <- fmap lines $ readFile f
    print $ length $ filter isJust $
        runEval $ parMap solve grids
```

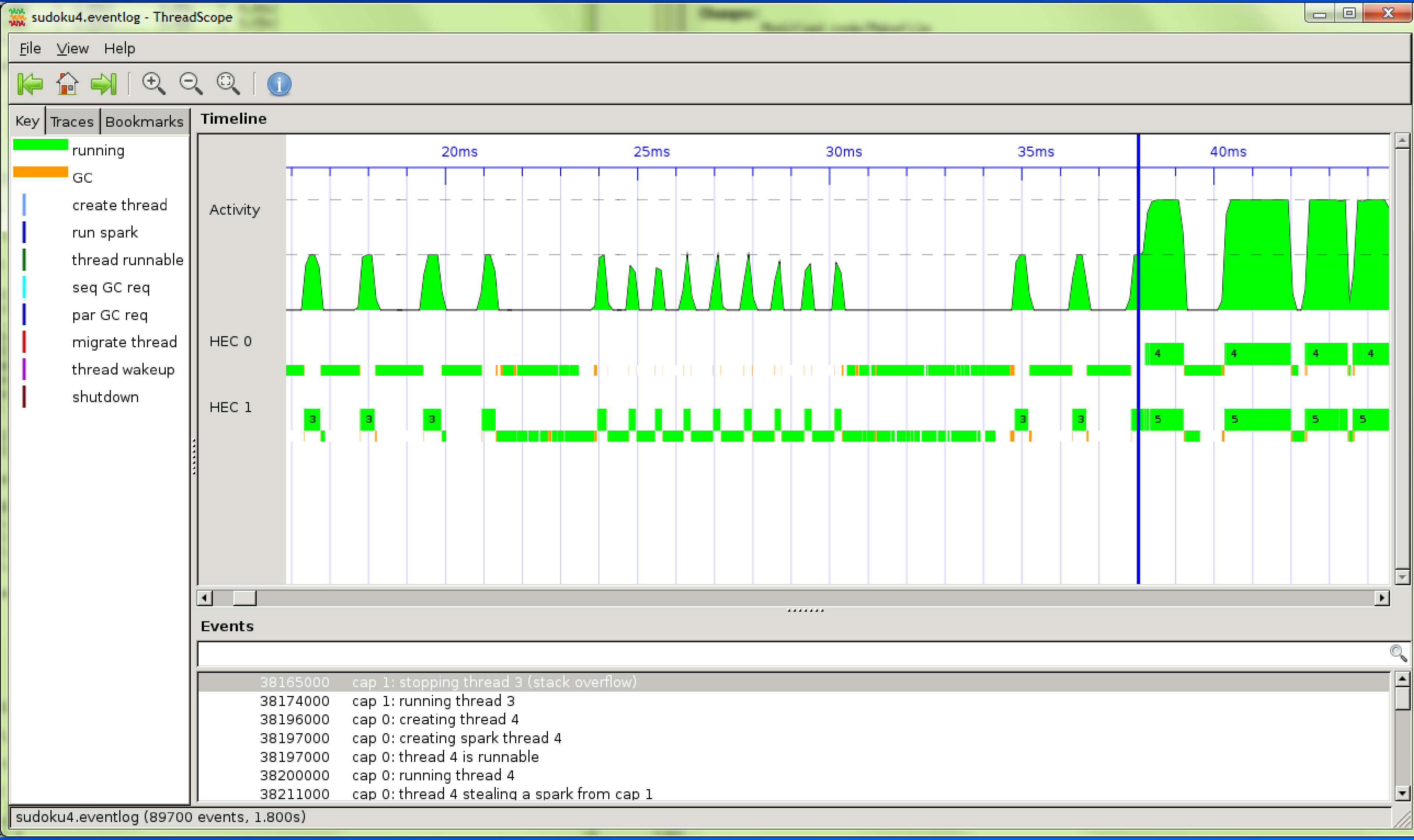
- Are there any sequential parts of this program?

```
main :: IO ()
main = do
    [f] <- getArgs
    grids <- fmap lines $ readFile f
    print $ length $ filter isJust $
        runEval $ parMap solve grids
```

- Are there any sequential parts of this program?
- readFile and lines are not parallelised

- Suppose we force the sequential parts to happen first...

```haskell
main :: IO ()
main = do
    [f] <- getArgs
    grids <- fmap lines $ readFile f
    evaluate (length grids)
    print $ length $ filter isJust $
        runEval $ parMap solve grids
```



🟩	running
🟧	GC
|	create thread
|	run spark
|	thread runnable
|	seq GC req
|	par GC req
|	migrate thread
|	thread wakeup
|	shutdown

```
38165000   cap 1: stopping thread 3 (stack overflow)
38174000   cap 1: running thread 3
38196000   cap 0: creating thread 4
38197000   cap 0: creating spark thread 4
38197000   cap 0: thread 4 is runnable
38200000   cap 0: running thread 4
38211000   cap 0: thread 4 stealing a spark from cap 1
```

sudoku4.eventlog (89700 events, 1.800s)

Calculating possible speedup

- When part of the program is sequential, Amdahl's law tells us what the maximum speedup is.

$$\frac{1}{(1-P)+\frac{P}{N}}$$

- P = parallel portion of runtime
- N = number of processors

Applying Amdahl’s law

- In our case:

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  - runtime = 3.06s (NB. sequential runtime!)

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  - non-parallel portion = 0.038s (P = 0.9876)

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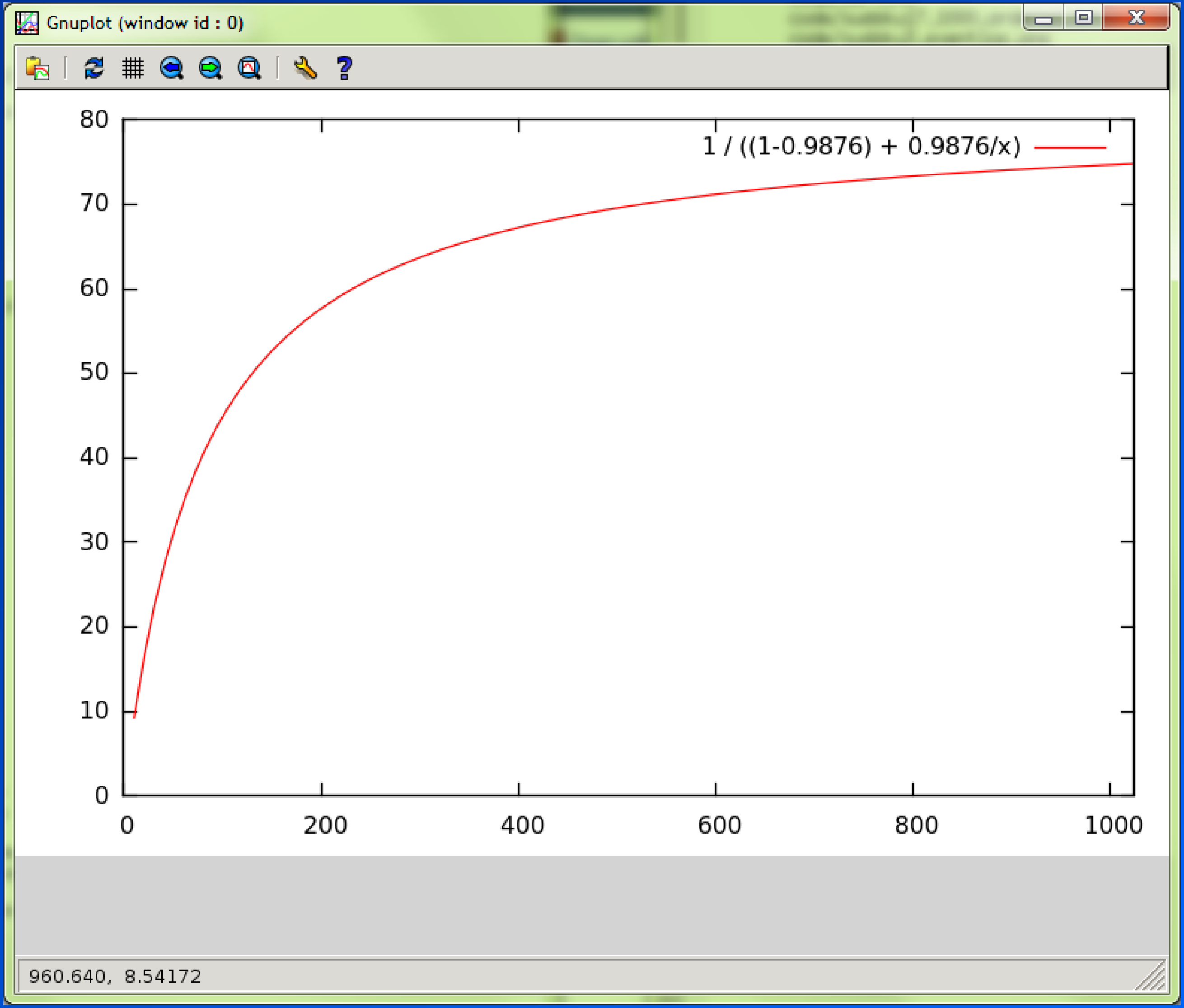
- In our case:
  - runtime = 3.06s (NB. sequential runtime!)
  - non-parallel portion = 0.038s (P = 0.9876)
  - N = 2, max speedup = 1 / ((1 – 0.9876) + 0.9876/2)
    - =~ 1.98
    - on 2 processors, maximum speedup is not affected much by this sequential portion

Applying Amdahl's law

- In our case:
  - runtime = 3.06s (NB. sequential runtime!)
  - non-parallel portion = 0.038s (P = 0.9876)
  - N = 2, max speedup = 1 / ((1 – 0.9876) + 0.9876/2)
    - =~ 1.98
    - on 2 processors, maximum speedup is not affected much by this sequential portion
  - N = 64, max speedup = 35.93
    - on 64 processors, 38ms of sequential execution has a dramatic effect on speedup



- diminishing returns...
- See *“Amdahl's Law in the Multicore Era”*, Mark Hill & Michael R. Marty

- Amdahl’s law paints a bleak picture
  - speedup gets increasingly hard to achieve as we add more cores
  - returns diminish quickly when more cores are added
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- See also Gustafson's law – the situation might not be as bleak as Amdahl's law suggests:
  - with more processors, you can solve a bigger problem
  - the sequential portion is often fixed or grows slowly with problem size
- Note: in Haskell it is hard to identify the sequential parts anyway, due to lazy evaluation

Evaluation Strategies

- So far we have used Eval/rpar/rseq
  - these are quite low-level tools
  - but it's important to understand how the underlying mechanisms work
- Now, we will raise the level of abstraction
- Goal: encapsulate parallel idioms as re-usable components that can be composed together.

The Strategy type

```
type Strategy a = a -> Eval a
```

- Strategy a is a function that:
  - when applied to a value a,
  - evaluates a to some degree
  - (possibly sparking evaluation of sub-components of a in parallel),
  - and returns an equivalent a in the Eval monad
- NB. the return value should be observably equivalent to the original
  - (why not the same? we'll come back to that...)

Example...

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parList :: Strategy [a]
```

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- So we parameterise parList over the Strategy to apply to the elements:

```
parList :: Strategy a -> Strategy [a]
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```

- This is what we mean by "composable":
  - given a Strategy on the list elements,
  - parList gives us a Strategy on lists of those elements
- We have some simple Strategies already:

```
type Strategy a = a -> Eval a

rpar :: a -> Eval a  -- same as Strategy a
rseq :: a -> Eval a  -- ditto
```

here's a couple more:

```
r0 :: Strategy a
rdeepseq :: NFData a -> Strategy a
```

so here's a simple composition:

```
parList rdeepseq :: Strategy [a]
```

Strategies are easy to define

```haskell
type Strategy a = a -> Eval a
parList :: Strategy a -> Strategy [a]
```

- We have the building blocks:

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rpar :: a -> Eval a
```

Strategies are easy to define

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type Strategy a = a -> Eval a
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- We have the building blocks:

```
rpar :: a -> Eval a
```

```
parList :: (a -> Eval a) -> [a] -> Eval [a]
        -- same as Strategy a -> Strategy [a]

parList s []     = return []
parList s (x:xs) = do
  x'  <- rpar (runEval (s x))
  xs' <- parList s xs
  return (x':xs')
```

Let’s generalise...

- parList has rpar built-in, but we might not want that. Let’s make a version without the rpar:

```
evalList :: (a -> Eval a) -> [a] -> Eval [a]
evalList f []     = return ()
evalList f (x:xs) = do
  x’  <- f x
  xs’ <- evalList f xs
  return (x’:xs’)
```

now we can define parList in terms of evalList:

```
parList f = evalList (rparWith f)
```

```
rparWith :: Strategy a -> Strategy a
rparWith s a = rpar (runEval (s a))
```

Let’s make a general Strategy on pairs:

```
evalTuple2 :: Strategy a -> Strategy b -> Strategy (a,b)
evalTuple2 sa sb (a,b) = do
  a’ <- sa a
  b’ <- sb b
  return (a’,b’)
```

Example: a Strategy on a pair that evaluates the first component and sparks the second:

```
evalTuple2 rseq rpar :: Strategy (a,b)
```

(left-then-right ordering is built into evalTuple2, if you want the other ordering you have to define a different evalTuple2)

So far:

```
rpar, rseq, r0 :: Strategy a
rdeepseq :: NFData a => Strategy a

rparWith :: Strategy a -> Strategy a

evalList   :: Strategy a -> Strategy [a]
evalTuple2 :: Strategy a -> Strategy b -> Strategy (a,b)
```

Here are some example Strategies. What do they do?

```
evalList (evalTuple2 rpar r0) :: Strategy [(a,b)]

evalList (rparWith (evalTuple2 rseq rseq))
  :: Strategy [(a,b)]

evalList (evalTuple2 rdeepseq rpar) :: Strategy [(a,b)]

evalList (evalList rpar) :: Strategy [[a]]
evalList (rparWith (evalList rseq)) :: Strategy [[a]]
evalList (rparWith (evalList rpar)) :: Strategy [[a]]
```

How do we *use* a Strategy?

```haskell
type Strategy a = a -> Eval a
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- e.g.

```
myList `using` parList rdeepseq
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How do we *use* a Strategy?

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- e.g.

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myList `using` parList rdeepseq
```

- Why better? Because we have a “law”:
  - x `using` s ≈ x
  - We can insert or delete “`using` s” without changing the semantics of the program

Is that really true?

1. It relies on a Strategy returning “the same value” (*identity-safety)*
   - Strategies from the library obey this property
   - Be careful when writing your own Strategies
2. x `using` s might do more evaluation than just x.
   - So the program with x `using` s might be _|_, but the program with just x might have a value

- if identity-safety holds, adding using cannot make the program produce a different result (other than _|_)

But we wanted parMap

- Earlier we used parMap to parallelise Sudoku
- But parMap is a combination of two concepts:
  - The *algorithm*, ‘map’
  - The *parallelism*, ‘parList’

```
parMap f x = map f xs `using` parList rseq
```

- With Strategies, the algorithm can be separated from the parallelism.
  - The algorithm produces a (lazy) result
  - A Strategy filters the result, but does not do any computation – it returns the same result.

- So a nicer way to write the Sudoku example is:

```haskell
main :: IO ()
main = do
    [f] <- getArgs
    grids <- fmap lines $ readFile f
    evaluate (length grids)
    evaluate $ force $
       map solve grids `using` parList rseq
```

- Here the parallelism is modular

Recap...

- Eval monad, for expressing evaluation order and sparking:

```
data Eval a -- instance Monad Eval

runEval :: Eval a -> a

rpar :: a -> Eval a
rseq :: a -> Eval a
```

- Strategies,
  - built using Eval, rpar, rseq
  - express compositional parallelism over data structures
  - modular: parallelism separate from algorithm

```
type Strategy a = a -> Eval a
`using` :: a -> Strategy a -> a
```

extra stuff

Generalise further...

- In fact, evalList already exists for arbitrary data types in the form of ‘traverse’.

```
evalTraversable
  :: Traversable t => Strategy a -> Strategy (t a)

evalTraversable = traverse

evalList = evalTraversable
```

- So, building Strategies for arbitrary data structures is easy, given an instance of Traversable.
- (not necessary to understand Traversable here, just be aware that many Strategies are just generic traversals in the Eval monad).

By why *do* Strategies return a value?

```
parList :: Strategy a -> Strategy [a]
parList s []     = return ()
parList s (x:xs) = do
  x'  <- rpar (runEval (s x))
  xs' <- parList s xs
  return (x':xs')
```

- Spark pool points to (runEval (s x))
- If nothing else points to this expression, the runtime will discard the spark, on the grounds that it is not required
- *Always keep hold of the return value of* rpar
- (see the notes for more details on this)