

# The normal distribution

In this lab we'll investigate the probability distribution that is most central to statistics: the normal distribution. If we are confident that our data are nearly normal, that opens the door to many powerful statistical methods. Here we'll use the graphical tools of R to assess the normality of our data and also learn how to generate random numbers from a normal distribution.

## The Data

This week we'll be working with measurements of body dimensions. This data set contains measurements from 247 men and 260 women, most of whom were considered healthy young adults.

```
load("more/bdims.RData")
```

Let's take a quick peek at the first few rows of the data.

```
head(bdims)
```

```
##   bia.di bii.di bit.di che.de che.di elb.di wri.di kne.di ank.di sho.gi
## 1   42.9   26.0   31.5   17.7   28.0   13.1   10.4   18.8   14.1  106.2
## 2   43.7   28.5   33.5   16.9   30.8   14.0   11.8   20.6   15.1  110.5
## 3   40.1   28.2   33.3   20.9   31.7   13.9   10.9   19.7   14.1  115.1
## 4   44.3   29.9   34.0   18.4   28.2   13.9   11.2   20.9   15.0  104.5
## 5   42.5   29.9   34.0   21.5   29.4   15.2   11.6   20.7   14.9  107.5
## 6   43.3   27.0   31.5   19.6   31.3   14.0   11.5   18.8   13.9  119.8
##   che.gi wai.gi nav.gi hip.gi thi.gi bic.gi for.gi kne.gi cal.gi ank.gi
## 1   89.5   71.5   74.5   93.5   51.5   32.5   26.0   34.5   36.5   23.5
## 2   97.0   79.0   86.5   94.8   51.5   34.4   28.0   36.5   37.5   24.5
## 3   97.5   83.2   82.9   95.0   57.3   33.4   28.8   37.0   37.3   21.9
## 4   97.0   77.8   78.8   94.0   53.0   31.0   26.2   37.0   34.8   23.0
## 5   97.5   80.0   82.5   98.5   55.4   32.0   28.4   37.7   38.6   24.4
## 6   99.9   82.5   80.1   95.3   57.5   33.0   28.0   36.6   36.1   23.5
##   wri.gi age  wgt  hgt sex
## 1   16.5  21 65.6 174.0  1
## 2   17.0  23 71.8 175.3  1
## 3   16.9  28 80.7 193.5  1
## 4   16.6  23 72.6 186.5  1
## 5   18.0  22 78.8 187.2  1
## 6   16.9  21 74.8 181.5  1
```

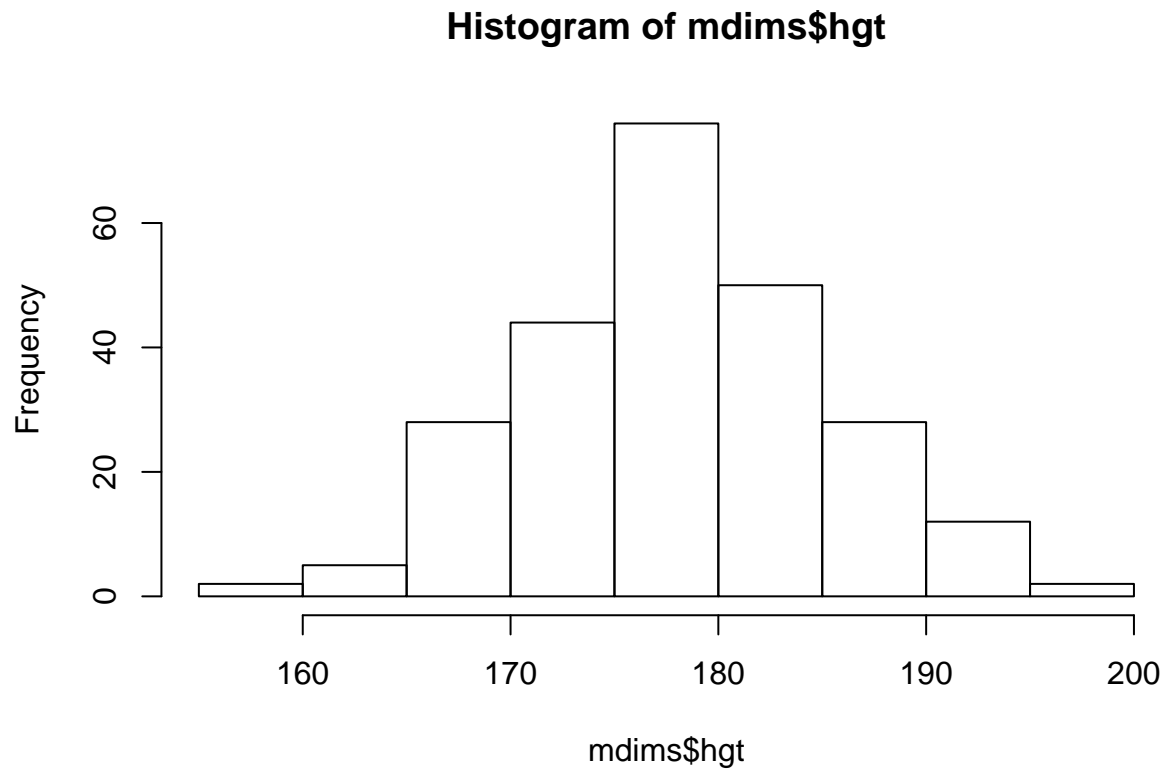
You'll see that for every observation we have 25 measurements, many of which are either diameters or girths. A key to the variable names can be found at <http://www.openintro.org/stat/data/bdims.php>, but we'll be focusing on just three columns to get started: weight in kg (`wgt`), height in cm (`hgt`), and `sex` (1 indicates male, 0 indicates female).

Since males and females tend to have different body dimensions, it will be useful to create two additional data sets: one with only men and another with only women.

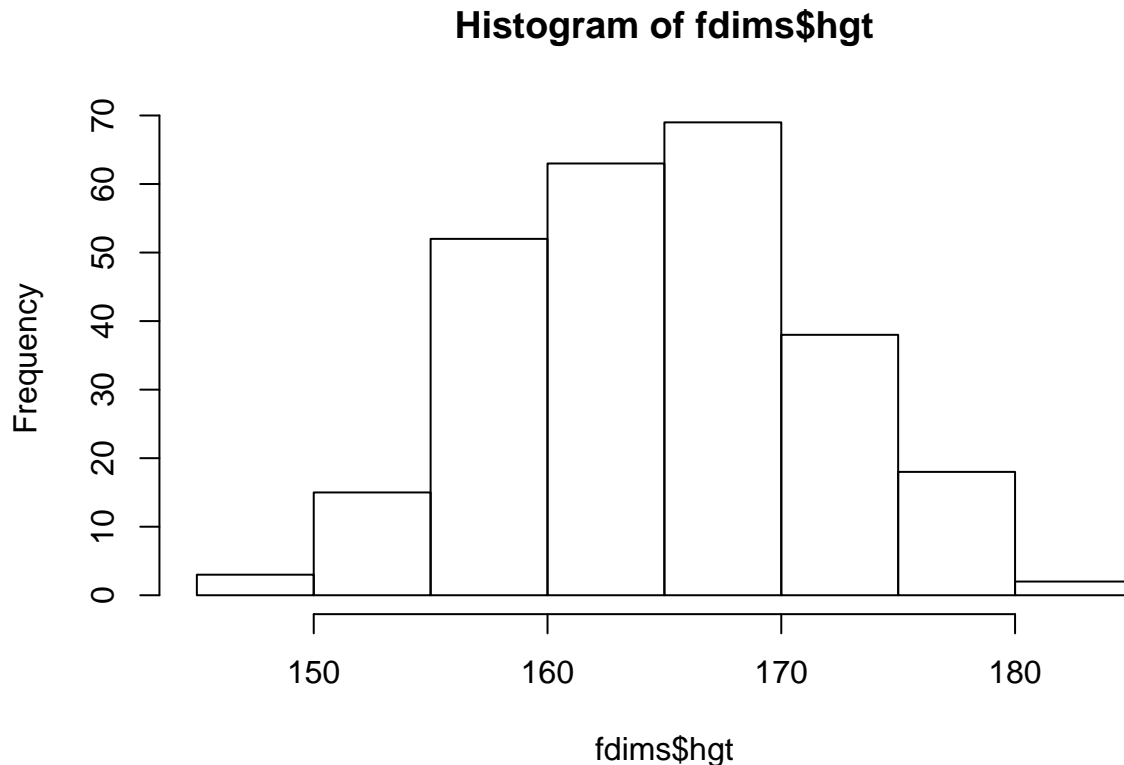
```
mdims <- subset(bdims, sex == 1)
fdims <- subset(bdims, sex == 0)
```

1. Make a histogram of men's heights and a histogram of women's heights. How would you compare the various aspects of the two distributions?

```
hist(mdims$hgt)
```



```
hist(fdims$hgt)
```



The distributions show that heights of men and women both appear to be nearly normally distributed as indicated by their nearly bell-shaped, symmetric shapes. However, the distributions also show that the heights of males are spread over a wider range (155 -> 200 = approx. 45cm) than are the heights of females (145 -> 185 = approx 40cm). Also, the mean height of males (approx. 178cm) appears to be 10 to 15 cm greater than that of females (approx 165cm).

## The normal distribution

In your description of the distributions, did you use words like *bell-shaped* or *normal*? It's tempting to say so when faced with a unimodal symmetric distribution.

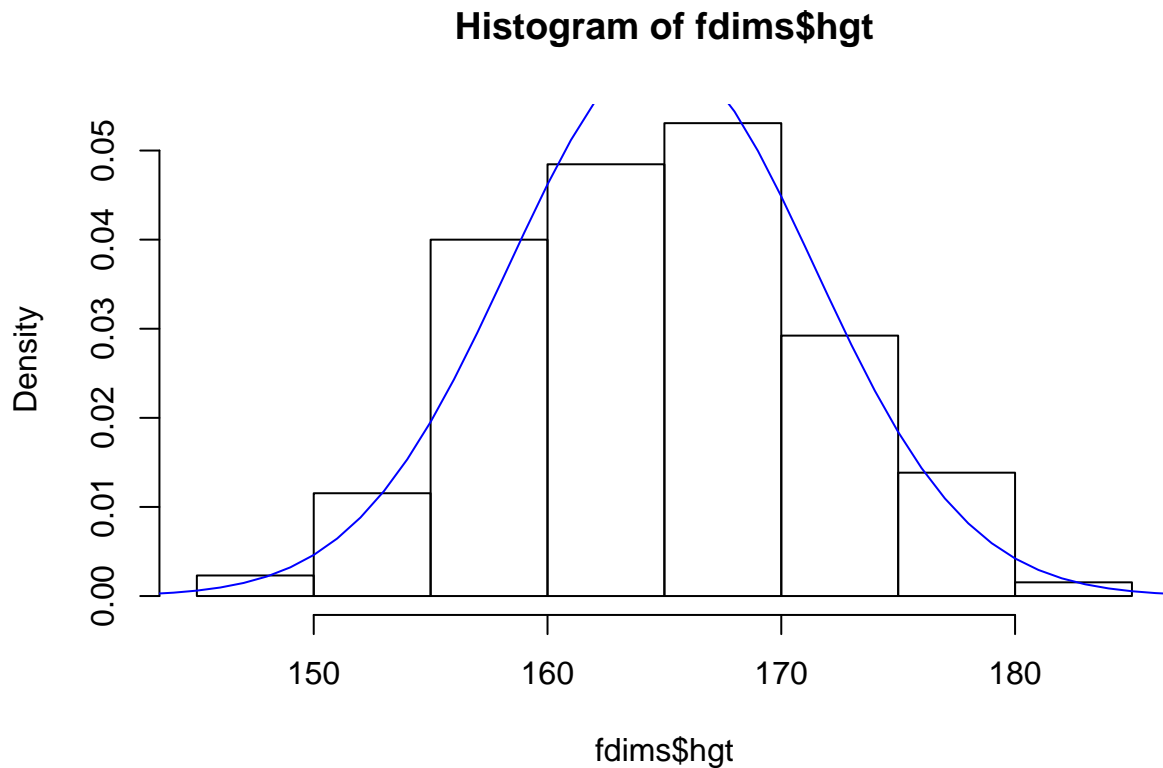
To see how accurate that description is, we can plot a normal distribution curve on top of a histogram to see how closely the data follow a normal distribution. This normal curve should have the same mean and standard deviation as the data. We'll be working with women's heights, so let's store them as a separate object and then calculate some statistics that will be referenced later.

```
fhgtmean <- mean(fdims$hgt)
fhgtstd  <- sd(fdims$hgt)
```

Next we make a density histogram to use as the backdrop and use the `lines` function to overlay a normal probability curve. The difference between a frequency histogram and a density histogram is that while in a frequency histogram the *heights* of the bars add up to the total number of observations, in a density histogram the *areas* of the bars add up to 1. The area of each bar can be calculated as simply the height *times* the width of the bar. Using a density histogram allows us to properly overlay a normal distribution curve over the histogram since the curve is a normal probability density function. Frequency and density histograms

both display the same exact shape; they only differ in their y-axis. You can verify this by comparing the frequency histogram you constructed earlier and the density histogram created by the commands below.

```
hist(fdims$hgt, probability = TRUE)
x <- 140:190
y <- dnorm(x = x, mean = fhgtmean, sd = fhgtstd)
lines(x = x, y = y, col = "blue")
```



After plotting the density histogram with the first command, we create the x- and y-coordinates for the normal curve. We chose the x range as 140 to 190 in order to span the entire range of `fheight`. To create y, we use `dnorm` to calculate the density of each of those x-values in a distribution that is normal with mean `fhgtmean` and standard deviation `fhgtstd`. The final command draws a curve on the existing plot (the density histogram) by connecting each of the points specified by x and y. The argument `col` simply sets the color for the line to be drawn. If we left it out, the line would be drawn in black.

The top of the curve is cut off because the limits of the x- and y-axes are set to best fit the histogram. To adjust the y-axis you can add a third argument to the histogram function: `ylim = c(0, 0.06)`.

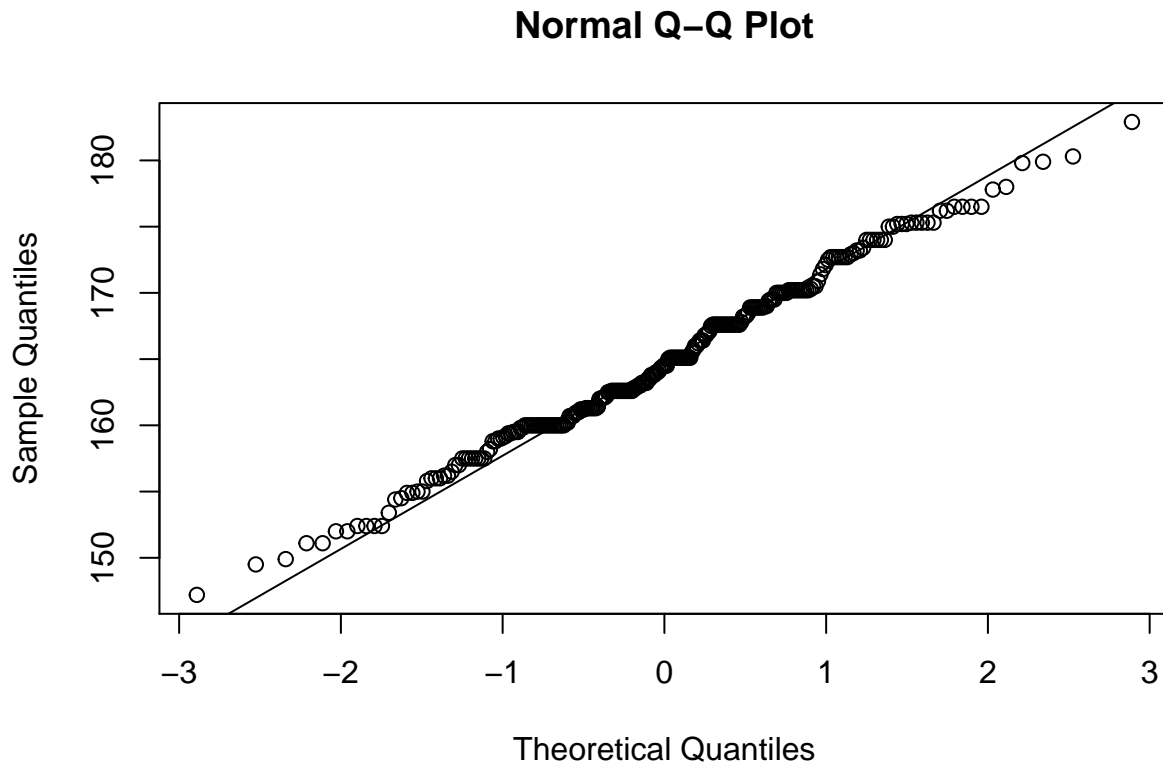
2. Based on this plot, does it appear that the data follow a nearly normal distribution?

Yes, the data do appear to follow a nearly normal distribution.

## Evaluating the normal distribution

Eyeballing the shape of the histogram is one way to determine if the data appear to be nearly normally distributed, but it can be frustrating to decide just how close the histogram is to the curve. An alternative approach involves constructing a normal probability plot, also called a normal Q-Q plot for “quantile-quantile”.

```
qqnorm(fdims$hgt)
qqline(fdims$hgt)
```



A data set that is nearly normal will result in a probability plot where the points closely follow the line. Any deviations from normality leads to deviations of these points from the line. The plot for female heights shows points that tend to follow the line but with some errant points towards the tails. We're left with the same problem that we encountered with the histogram above: how close is close enough?

A useful way to address this question is to rephrase it as: what do probability plots look like for data that I *know* came from a normal distribution? We can answer this by simulating data from a normal distribution using `rnorm`.

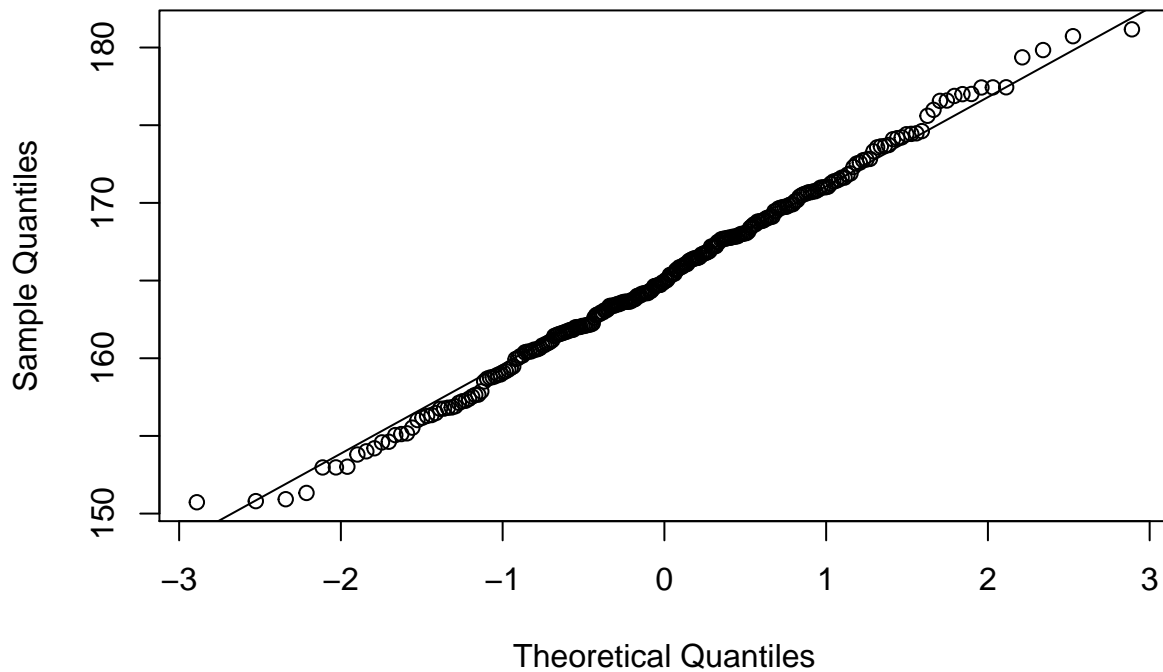
```
sim_norm <- rnorm(n = length(fdims$hgt), mean = fhgtmean, sd = fhgtstd)
```

The first argument indicates how many numbers you'd like to generate, which we specify to be the same number of heights in the `fdims` data set using the `length` function. The last two arguments determine the mean and standard deviation of the normal distribution from which the simulated sample will be generated. We can take a look at the shape of our simulated data set, `sim_norm`, as well as its normal probability plot.

3. Make a normal probability plot of `sim_norm`. Do all of the points fall on the line? How does this plot compare to the probability plot for the real data?

```
qqnorm(sim_norm)
qqline(sim_norm)
```

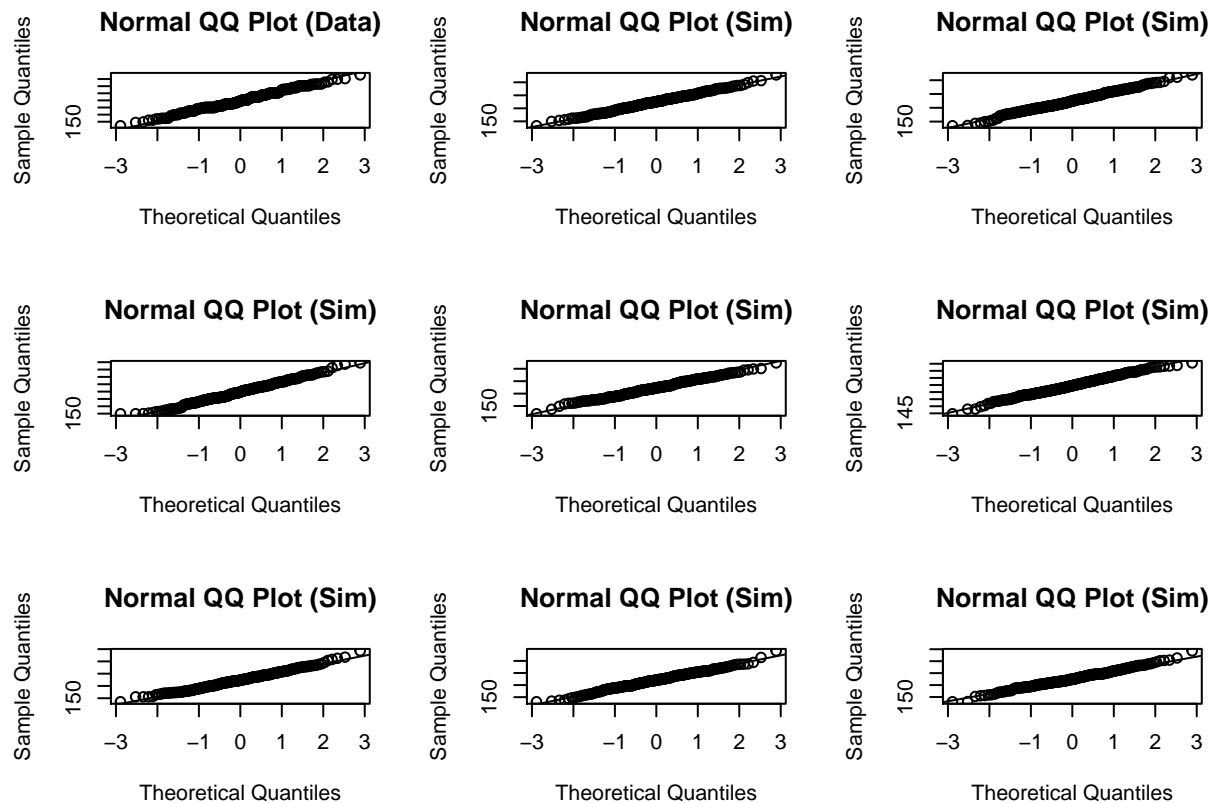
## Normal Q-Q Plot



All of the points do not fall on the line. The plot varies from that of the actual data in that the actual data appear to be somewhat “stepped”, likely due to the binning of samples into specific quantiles.

Even better than comparing the original plot to a single plot generated from a normal distribution is to compare it to many more plots using the following function. It may be helpful to click the zoom button in the plot window.

```
qqnormsim(fdims$hgt)
```



4. Does the normal probability plot for `fdims$ht` look similar to the plots created for the simulated data? That is, do plots provide evidence that the female heights are nearly normal?

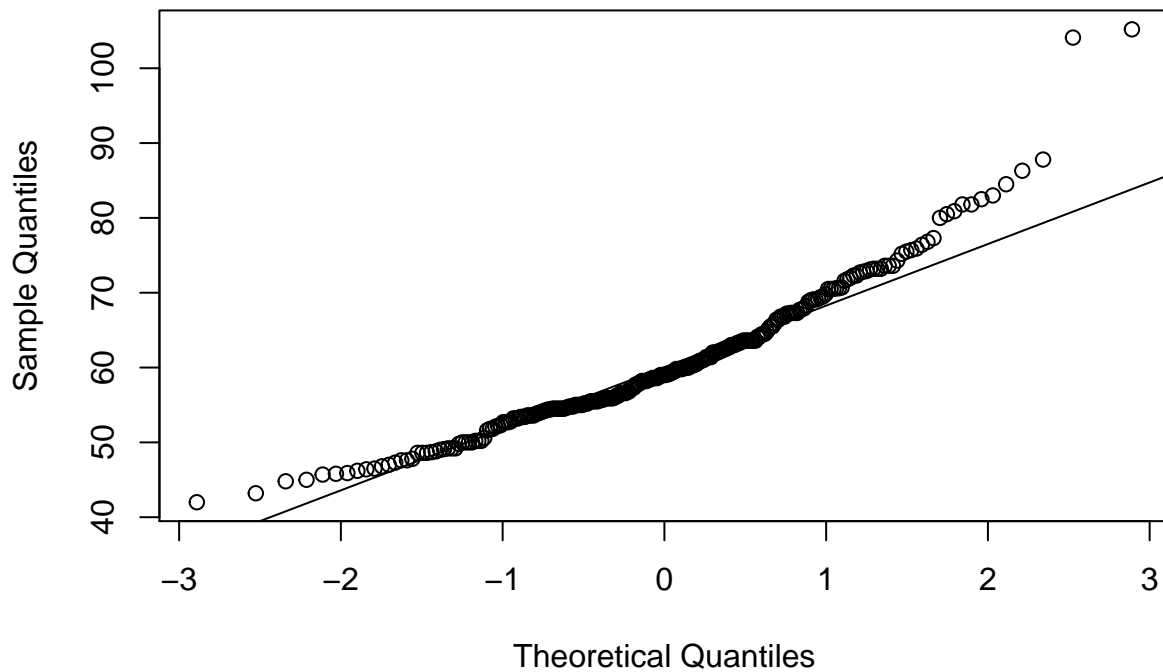
Yes, the actual data for female heights does appear to approximate a normal distribution.

5. Using the same technique, determine whether or not female weights appear to come from a normal distribution.

We'll start by creating a normal probability plot for female weights:

```
qqnorm(fdims$wgt)
qqline(fdims$wgt)
```

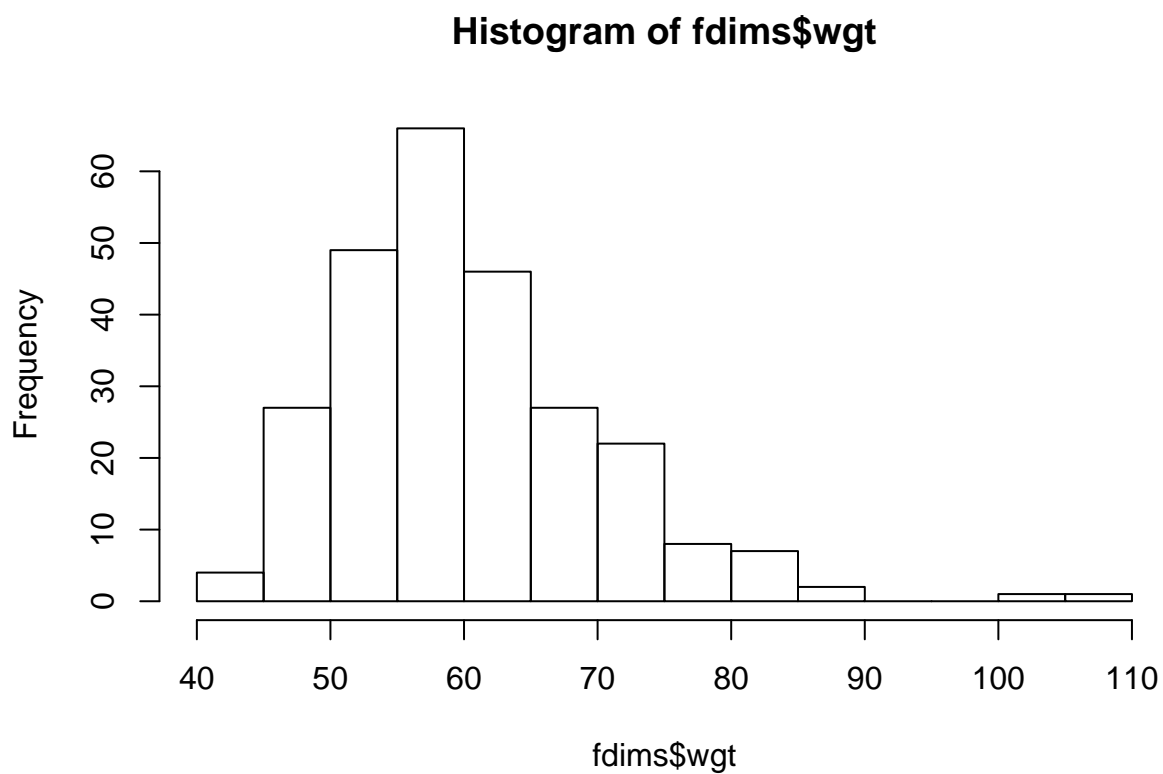
## Normal Q-Q Plot



From this plot it appears as though the female weight data do not come from a normal distribution. For example, weights above the “+1” quantile skew quite drastically away from the line. This is typically an indication that a distribution is right-skewed. We can examine whether this is in fact the case by plotting a histogram of female weights:

```
hist(fdims$wgt)
```

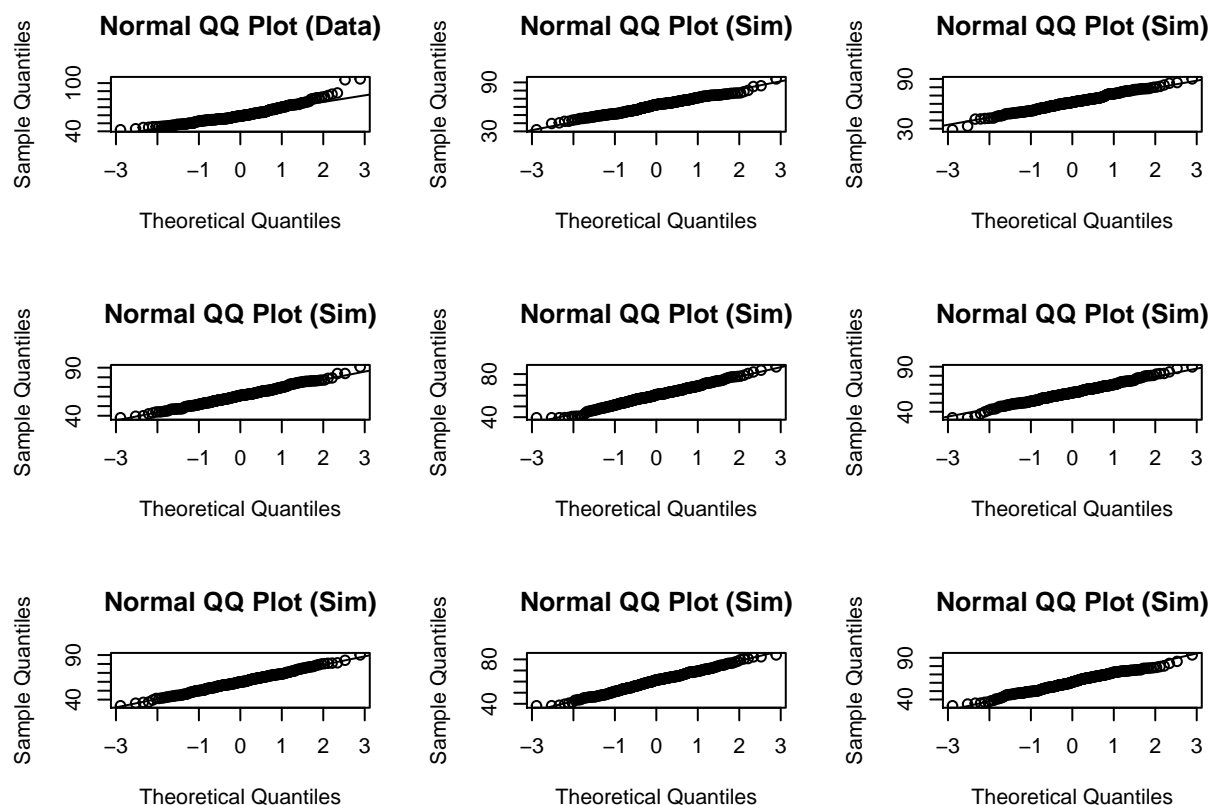




The histogram shows clear evidence of right skew in the distribution of female weights.

Finally, we can compare our normal probability plot for female weights against several simulated normal probability plots based on the mean and standard deviation of the female weight data:

```
qqnormsim(fdims$wgt)
```



The simulated plots hew much more closely to the normal line than does the actual female weight data. As such, it appears that our female weight data does not come from a normal distribution.

## Normal probabilities

Okay, so now you have a slew of tools to judge whether or not a variable is normally distributed. Why should we care?

It turns out that statisticians know a lot about the normal distribution. Once we decide that a random variable is approximately normal, we can answer all sorts of questions about that variable related to probability. Take, for example, the question of, “What is the probability that a randomly chosen young adult female is taller than 6 feet (about 182 cm)?” (The study that published this data set is clear to point out that the sample was not random and therefore inference to a general population is not suggested. We do so here only as an exercise.)

If we assume that female heights are normally distributed (a very close approximation is also okay), we can find this probability by calculating a Z score and consulting a Z table (also called a normal probability table). In R, this is done in one step with the function `pnorm`.

```
1 - pnorm(q = 182, mean = fhgtmean, sd = fhgtsd)
```

Note that the function `pnorm` gives the area under the normal curve below a given value, `q`, with a given mean and standard deviation. Since we’re interested in the probability that someone is taller than 182 cm, we have to take one minus that probability.

Assuming a normal distribution has allowed us to calculate a theoretical probability. If we want to calculate the probability empirically, we simply need to determine how many observations fall above 182 then divide

this number by the total sample size.

```
sum(fdims$hgt > 182) / length(fdims$hgt)
```

Although the probabilities are not exactly the same, they are reasonably close. The closer that your distribution is to being normal, the more accurate the theoretical probabilities will be.

6. Write out two probability questions that you would like to answer; one regarding female heights and one regarding female weights. Calculate the those probabilities using both the theoretical normal distribution as well as the empirical distribution (four probabilities in all). Which variable, height or weight, had a closer agreement between the two methods?

The two questions are as follows:

- What is the probability that a female's height is less than 66 inches (i.e., 5 feet, 6 inches) ?
- What is the probability that a female's weight is greater than 150 pounds?

Since both of these questions makes use of metrics that do not conform to the data (i.e., inches and pounds vs. centimeters and kilograms in the actual data), we'll need to be sure to convert the data from our questions to the metrics provided in the actual data.

We'll start with the height question:

```
# Answer using theoretical method:  
# Convert inches to centimeters: 2.54 cm per inch  
qfHgt <- 66 * 2.54  
  
# get the theoretical probability  
theo_prob <- pnorm(q = qfHgt, mean = fhgtmean, sd = fhgtstd)  
theo_prob
```

```
## [1] 0.6638148
```

Using R's **pnorm** function we get a theoretical probability of 0.66 for the likelihood that a female is less than 5 foot 6 inches tall.

```
# Answer using empirical method:  
# get the empirical probability  
emp_prob <- sum(fdims$hgt < qfHgt) / length(fdims$hgt)  
emp_prob
```

```
## [1] 0.6807692
```

Empirically, we find that the probability of a female being less than 5 foot 6 inches tall is 0.68.

Subtracting the theoretical probability from our empirical probability we find a difference between the two of 0.017:

```
abs(emp_prob - theo_prob)
```

```
## [1] 0.01695445
```

Turning to the weight question:

```
# convert pounds to kilos: .453 kilos per pound
qfWgt <- 150 * .453

# find mean + sd for female weights
fwgtmean <- mean(fdims$wgt)
fwgtsd    <- sd(fdims$wgt)

# get the theoretical probability of > 150 pounds
theo_wprob <- 1 - pnorm(q = qfWgt, mean = fwgtmean, sd = fwgtsd)
theo_wprob
```

```
## [1] 0.2223338
```

Using R's **pnorm** function we get a theoretical probability of 0.22 for the likelihood that a female's weight is greater than 150 pounds.

```
# Answer using empirical method:
# get the empirical probability
emp_wprob <- sum(fdims$wgt > qfWgt) / length(fdims$wgt)
emp_wprob
```

```
## [1] 0.1923077
```

Empirically, we find that the probability of a female's weight being more than 150 pounds as 0.19.

We then find a difference between the two of 0.03:

```
abs(theo_wprob - emp_wprob)
```

```
## [1] 0.03002611
```

As we can see, the weight data (.03) varies more from a purely normal distribution than does the height data (.017), so the height data has closer agreement between the theoretical and empirical methods.

---

## On Your Own

- Now let's consider some of the other variables in the body dimensions data set. Using the figures at the end of the exercises, match the histogram to its normal probability plot. All of the variables have been standardized (first subtract the mean, then divide by the standard deviation), so the units won't be of any help. If you are uncertain based on these figures, generate the plots in R to check.
  - The histogram for female biliac (pelvic) diameter (**bii.di**) belongs to normal probability plot letter **B**.
  - The histogram for female elbow diameter (**elb.di**) belongs to normal probability plot letter **C**.
  - The histogram for general age (**age**) belongs to normal probability plot letter **D**.
  - The histogram for female chest depth (**che.de**) belongs to normal probability plot letter **A**.

The plots for each of these were generated using the following R code:

```

# a. female biiliac (pelvic) diameter bii.di plot match
qqnorm(fdims$bii.di)
qqline(fdims$bii.di)
# matches plot B

# b. female elbow diameter eld.di plot match
qqnorm(fdims$elb.di)
qqline(fdims$elb.di)
# matches plot C

# c. age plot match (gender neutral)
qqnorm(bdims$age)
qqline(bdims$age)
# matches plot D

# d. female chest depth che.de plot match
qqnorm(fdims$che.de)
qqline(fdims$che.de)
# matches plot A

```

- Note that normal probability plots C and D have a slight stepwise pattern. Why do you think this is the case?

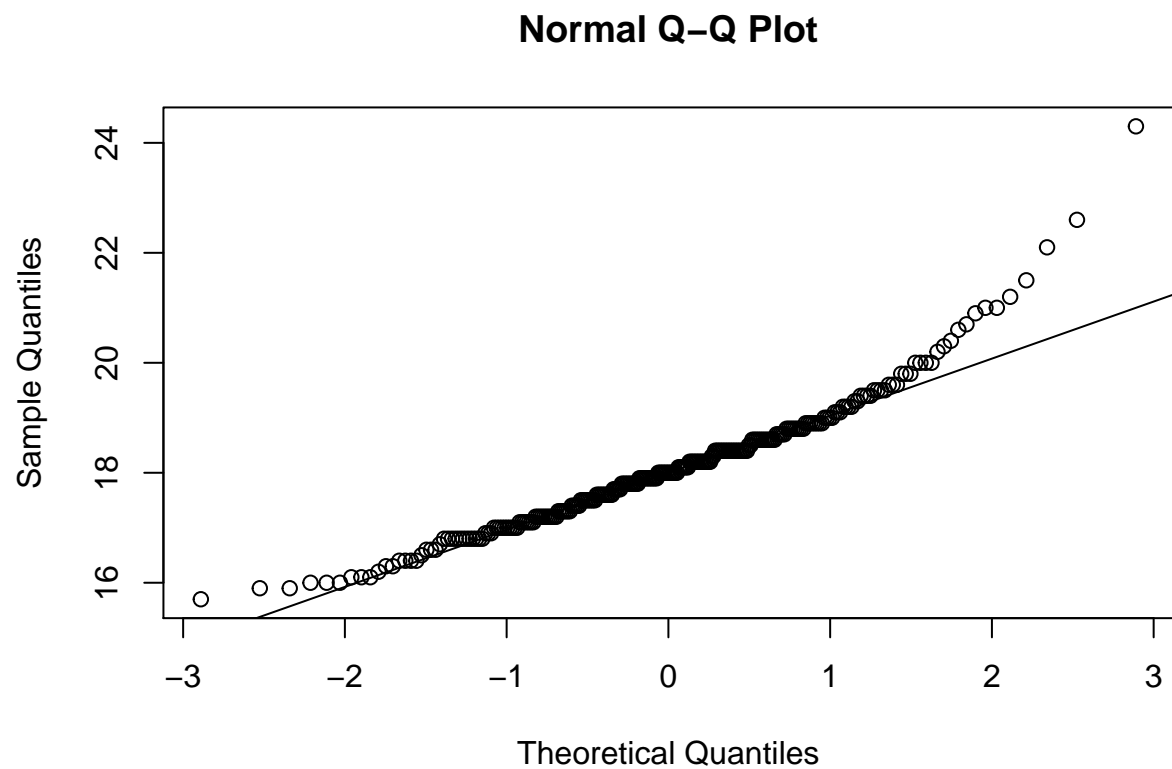
The step pattern is a result of the “binning” of samples into specific quantiles.

- As you can see, normal probability plots can be used both to assess normality and visualize skewness. Make a normal probability plot for female knee diameter (**kne.di**). Based on this normal probability plot, is this variable left skewed, symmetric, or right skewed? Use a histogram to confirm your findings.

```

#3. Make a normal probability plot for female knee diameter (kne.di).
qqnorm(fdims$kne.di)
qqline(fdims$kne.di)

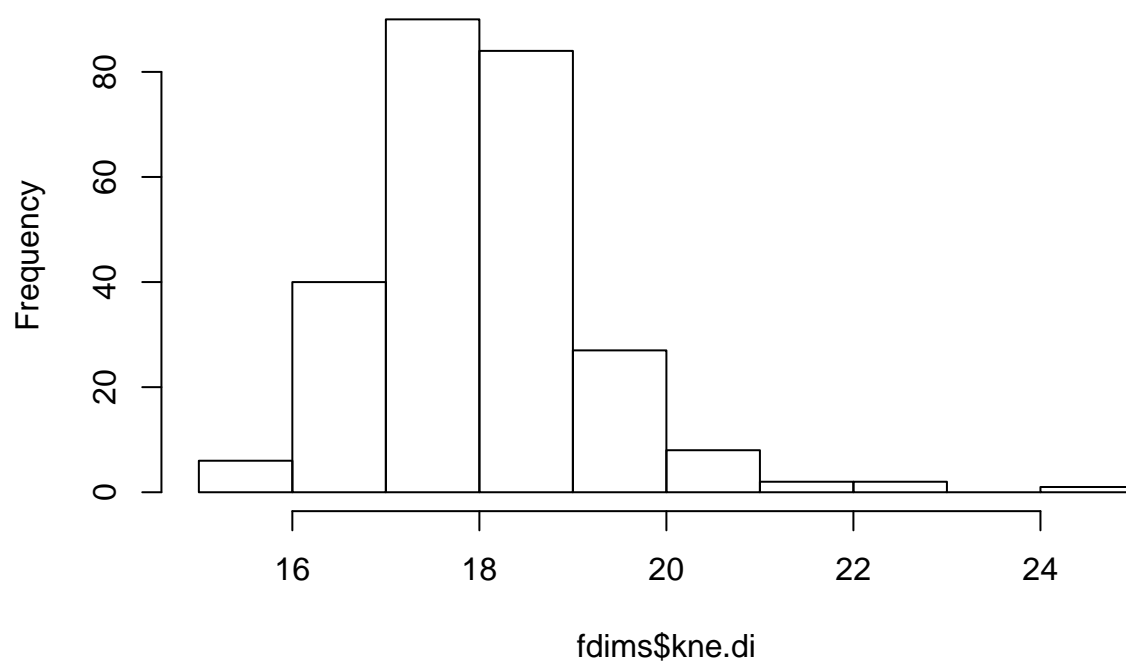
```

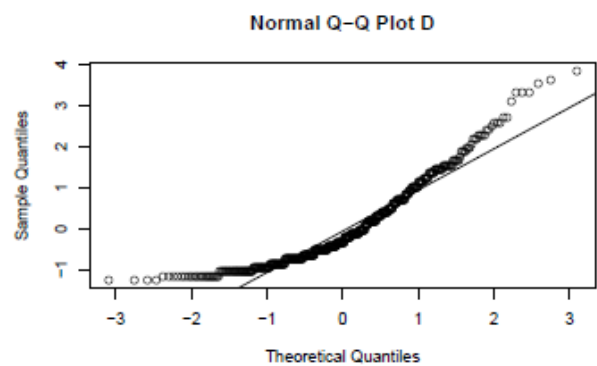
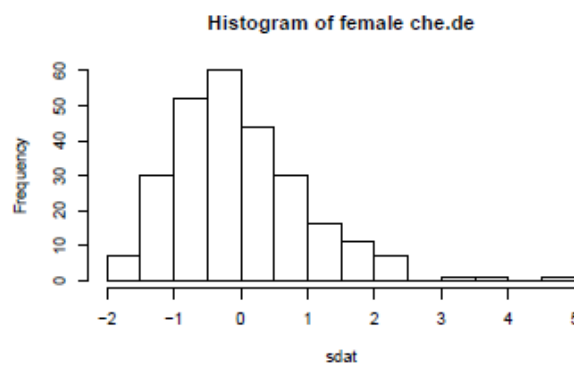
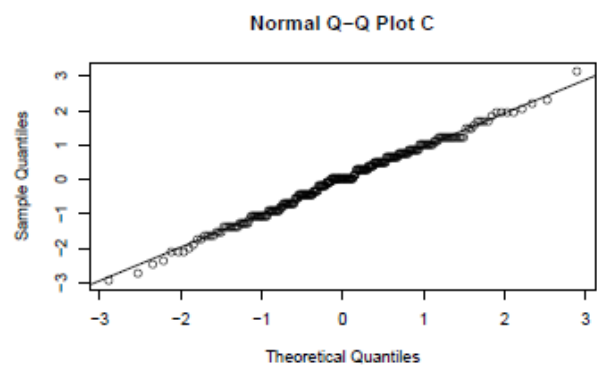
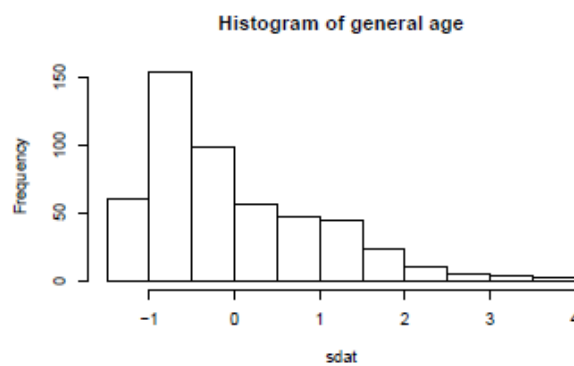
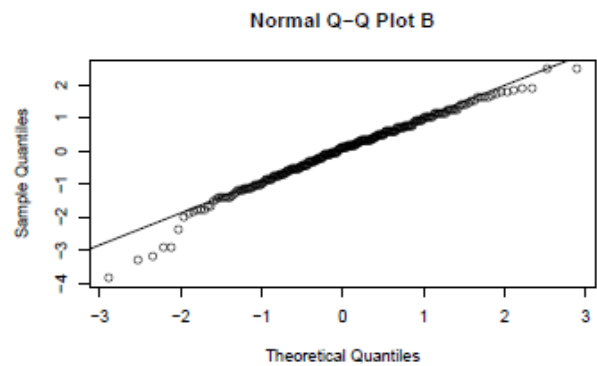
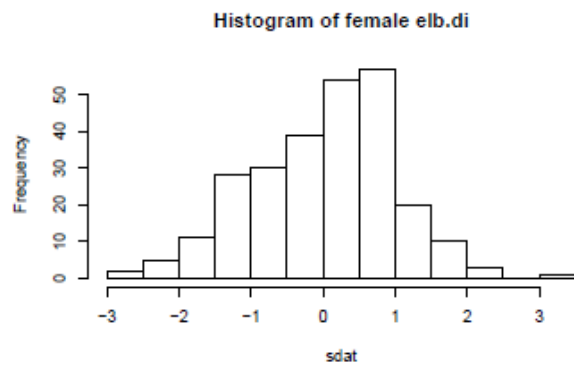
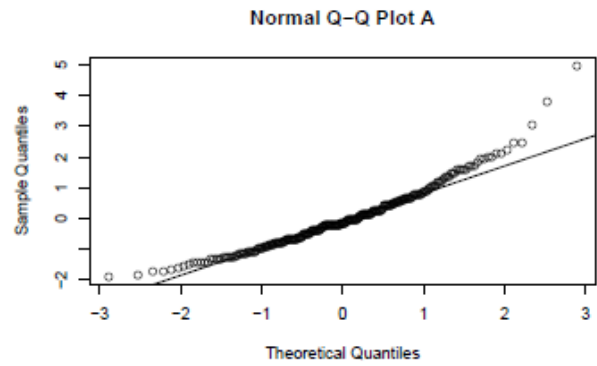
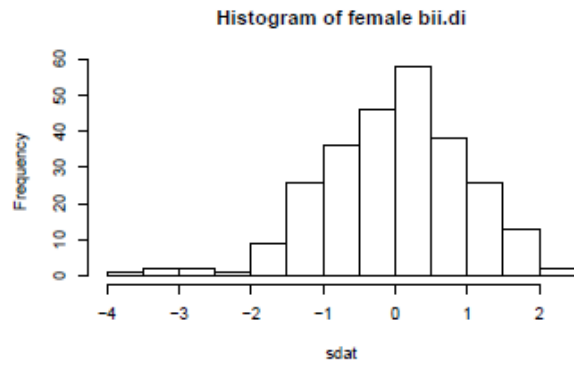


The normal probability plot appears to indicate that knee diameter is right skewed. A histogram plot confirms this:

```
hist(fdims$kne.di)
```

**Histogram of fdims\$kne.di**





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