Bootstrapping in Vector Autoregressions: An Application to the Pork Sector*

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Abstract

Standard bootstrap method is used to generate confidence intervals (CIs) of impulse response functions of VAR and SVAR models in the pork sector. In the VAR model, the bootstrap method does not produce significant different results from Monte Carlo simulations. In the SVAR analysis, on the other hand, the bootstrap CIs are significantly different from Monte Carlo CIs after a six period forecast intervals. This suggests that the choice of method used to measure reliability of IRFs is not trivial. Furthermore, bootstrap CIs in SVAR model seem to be more stable than MC CIs, which tend to be wider in the longer horizons.

Keywords: Vector Autoregressions (VAR), Structural VAR, Bootstrapping, Monte Carlo Integration, Confidence Intervals.

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Introduction

Market dynamics using Vector Autoregressions (VAR) models are usually evaluated through impulse response functions which allow to trace out the time path of the various shocks on the variables contained in the VAR system. The impulse response functions (IRF) are generally obtained from estimating VAR and commonly used to analyze the response of current and future values of economic variables to a one unit increase in the current value of the VAR errors. Shocks are usually identified by either: (1) requiring coefficient restrictions on lagged coefficients (as in the standard simultaneous equation literature); (2) imposing zero restriction on contemporaneous effects, which can be recursive as in Sims (1980), or non-recursive as in Bernanke (1986); and (3) imposing restrictions on the long-run effect of the shocks, as in Blanchard and Quah (1989), King, Plosser, Stock and Watson (1991), Gali (1992).

The impulse response functions are estimates based on the VAR specification which require reliability measures. In empirical work, such measures are usually given by the confidence intervals of IRFs. Methods for constructing IRFs and their confidence intervals depend on several assumptions and conditions such as auxiliary assumptions on the order of integration of the variables, the lag length of the VAR, and the sample size.

There are three principal procedures cited in most literature to obtain the confidence intervals: asymptotic, Monte Carlo, and bootstrap. Confidence intervals based on the asymptotic normal distribution are known to fail asymptotically to some cases and their small sample properties might be bad (Killian, 1998). Nonetheless, some empirical

work gave different perspectives about these methods. Griffiths and Lutkepohl (1991), for instance, argued that none of these methods is generally superior in terms of confidence level and power. While Fachin and Bravetti (1996) concluded that asymptotic method turns out to be surprisingly robust with respect to the distribution of the errors and the bootstrap provide results superior in terms of both length of the confidence interval and coverage when highly non-linear statistics are considered.

This primary objective of this study is to apply the bootstrap method in generating confidence intervals of the impulse response functions of VAR and structural VAR (SVAR) models in the pork sector. The bootstrap method is used because it offers greater potential value in the context of prediction and provides a sound methodological basis for estimating forecast intervals (McCullough, 1994). Furthermore, the bootstrap is logically superior to Monte Carlo simulation method (Fachin and Bravetti, 1996). In so doing, we construct and estimate VAR and SVAR models of the pork sector. Further, we bootstrap the confidence intervals (CIs) of the IRFs and evaluate their performance by comparing with CIs generated by Monte Carlo integration.

Vector Autoregressions

VAR models are equivalent to a system of reduced form equations relating each endogenous variable to lagged endogenous (predetermined) and exogenous variables. The exogenous variables typically do not appear in the VARs as argued forcefully by Sims (1980). A reduced form of VAR representation can be written as

(1) $$A(L)y_t = \mathbf{e}_t$$

where A(L) is a pth order matrix polynomial in the lag operator L, such that

$$A(L) = A_0 - A_1 L - A_2 L^2 - \ldots\ldots - A_p L^p.$$ A_0 is an identity matrix. y_t is a vector containing n

economic variables and e_t is has an independent multivariate normal distribution with

zero mean. The variance covariance matrix of e_t is denoted by Σ and non-singular.

Associated with the reduced form is the structural VAR model which is written as

(2) $$B(L) y_t = u_t$$

where B(L) is a pth order matrix polynomial in the lag operator L, such that

$$B(L) = B_0 - B_1 L - B_2 L^2 - \ldots\ldots - B_p L^p.$$ B_0 is a non-singular matrix normalized to have

ones on the diagonal. It also summarizes the contemporaneous relationships between the

variables in the model contained in the vector y_t. Y_t is a vector containing n economic

variables and u_t is vector white noise. The variance of u_t is denoted by D, where D is a

diagonal matrix with the diagonal elements are the variances of structural disturbances

such that the structural disturbances are serially uncorrelated and uncorrelated with each

other (Hamilton, 1994).

The relationships between (1) and (2) are as follows

(3 $$A(L) = B_0^{-1} B(L) = I - A_1 L - A_2 L^2 - \ldots\ldots - A_p L^p$$

and

(4) $$e_t = B_0^{-1} u_t.$$

Since (1) represents the reduced form, hence the system can be consistently estimated

with OLS equation by equation (Sims, 1980; Hamilton, 1994).). However, the matrix B_0

which represents the contemporaneous relationships and the structural disturbances in (2)

cannot be estimated. They can be recovered from the estimated reduced form coefficients

through identifying restriction. Using equations (3) and (4), the parameters in the structural form equation and those in the reduced form equation are related by

$$(5) \qquad A(L) = I - B_0^{-1}(B_1 L - B_2 L^2 - \ldots\ldots - B_p L^p$$

and

$$(6) \qquad \Sigma = B_0^{-1} \Lambda (B_0^{-1})'.$$

Maximum likelihood estimates of B_0 and Λ can be obtained only through sample estimates of Σ. The right hand side of (6) has $n(n + 1)$ free parameters to be estimated. Since Σ contains $n[(n + 1)/2]$, we need at least $n[(n + 1)/2]$ restrictions. By normalizing n diagonal elements of B_0 to ones, we need at least $n[(n - 1)/2]$ restrictions on B_0 to achieve identification. In the structural model, B_0 can be any structure as long as it has sufficient restrictions.

There are several ways of specifying the restrictions to achieve identification of the structural parameters. One procedure for determining appropriate restrictions to identify structural VAR is to use restrictions that are implied by economic model. A popular and straightforward method is to orthogonalize the reduced form errors by Choleski decomposition as originally applied by Sims (1980). The general method for imposing restrictions was suggested by Bernanke (1986), Blanchard and Watson (1986), and Sims (1986), while still giving restrictions on only contemporaneous structural parameters. This method permits non-recursive structures and the specification of restrictions based on prior theoretical and empirical about private sector behavior and policy reaction functions.

Amisano and Giannini (1997) classify SVAR into three different classes based on the identifying restrictions on instantaneous correlations, namely K-model, C-model, and

AB-model. The K-model puts restriction on the contemporaneous correlations among the elements of y_t. The C-model constructs a structural form where no instantaneous relationships among the endogenous variables are explicitly modeled. The AB-model is the most general parameterization nesting the C and K models as special cases.

Note that the relationships of (1) and (2) as described in (4) is equivalent to the K model of Amisano and Giannini's classification. This can be seen from (4) where $u_t = B_0 e_t$, given B_0 is invertible matrix. So the matrix B_0 is just the K matrix. Following Amisano and Giannini's classification, we can construct the following relationships. Letting K be a (n x n) invertible matrix, we have

$$KA(L)y_t = Ke_t$$

$$Ke_t = u_t$$

$$E(u_t) = 0 \qquad E(u_t u_t^{'}) = I_n.$$

The K matrix premultiplies the autoregressive representation and induces a transformation of the e_t disturbances by generating a vector (u_t) of orthonormalised disturbances. Hence the contemporaneous correlations among the elements of y_t are modeled through the specification of the invertible matrix K.

Impulse Response Functions

In order to assess dynamic effects, VAR models usually apply the so-called *impulse response analysis* (IRA). This method allows to trace out the time path of the various shocks on the variables contained in the VAR system. The impulse response analysis (IRA) in VAR system is a descriptive device representing the reaction of each variable to shocks in the different equations of the system (Amisano and Giannini, 1997).

IRA analysis is intended to pursue the dynamic interactions among the variables included in the VAR system (y_t), say the effects on y_i of a change occurred in y_j p periods before. In doing so, a VAR representation is usually transformed into a vector of moving average (VMA) representation. VMA allows the time path of the various shocks on the variables contained in the VAR system to be plotted.

The VAR system in equation (1) can be written in the VMA representation (Wold representation) as [1]

(7) $y_t = A(L)^{-1} e_t = \Psi(L) e_t$

or $y_t = \sum_{s=0}^{\infty} \Psi_s e_{t-s}$, where $\Psi_0 = I_K$.

The matrix $\Psi_s = \dfrac{\partial y_{t+s}}{\partial e_t^{'}}$ shows that the row i, column j element of Ψ_s identifies the consequences of a one unit increase in the jth variable's innovation at date t (e_{jt}) for the value of the ith variable at time t + s ($y_{i,t+s}$), holding all other innovations at all dates constant (Hamilton, 1994). The impulse response function (non-orthogonalized) is given by the plot of row i, column j element of Ψ_s , $\dfrac{\partial y_{i,t+s}}{\partial e_{jt}}$, as a function of s.

To obtain the orthogonalized response function, consider that for any real symmetric positive definite matrix Σ , there exists a unique lower triangular matrix A with 1s along the principle diagonal and a unique diagonal matrix D with positive entries along the principal diagonal such that $\Sigma = ADA'$. Based on this condition and using matrix A, we can obtain

(8) $u_t = A^{-1} e_t$ or $Au_t = e_t$.

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[1] For a more detail discussions on this see Hamilton (1994) : Chapter 11.

The orthogonalized impulse response function is obtained by plotting $\hat{\Psi}_s \hat{a}_j$, where

$\hat{a}_j$ denotes the jth column of matrix $\hat{A}$ [2].

The impulse response functions for structural VARs are obtained through moving average representation of classical VARs. The relation in (4) can be viewed as equivalent to (8) which B_0^{-1} replacing A. So we have $\frac{\partial e_t}{\partial u_t^{'}} = B_0^{-1}$. The effect on e_t of the jth structural disturbance u_{jt} is given by b_j, which is the jth column of B_0^{-1}. Hence the impulse response function is defined as

$$(9) \qquad \frac{\partial y_{t+s}}{\partial u_{jt}} = \frac{\partial \partial y_{t+s}}{\partial e_t^{'}} \frac{\partial e_t}{\partial u_{jt}} = \Psi_s b_j.$$

Bootstrapping Procedure

There are several bootstrapping methods appeared in the literature such as Efron & Tibshirani (1986) and Hall's percentile method (1992) which are outlined and used by Bentwitz., et.al. (2000). Killian (1998) also proposed bootstrap after bootstrap. An excellent discussion on bootstrapping time series is found in Berkowitz and Killian (2000). In this study, we proposed the standard bootstrap method as outlined in Benkwitz, Lutkepohl, and Neumann (2000) and also Fachin and Bravetti (1996). To simplify, consider the following model:

$$(8) \qquad y_t = \sum_{i=1}^{p} f_i y_{t-i} + e_t$$

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[2] Matrix $\hat{A}$ can be obtained from $\hat{\Sigma} = \hat{A}\hat{D}\hat{A}'$ using algorithm. $\hat{D} = \frac{1}{T}\sum_{t=1}^{T} \hat{u}_t \hat{u}_t^{'} = \hat{A}^{-1}\hat{\Sigma}(\hat{A}^{-1})'$ (See Hamilton, 1994. p.322). The Cholesky Decomposition is obtained by replacing A with P, where P is lower triangular matrix and the standard deviation of u_t along the principal diagonal.

where y_t can be multivariate. The construction of bootstrap confidence intervals in VAR analysis is based on the following steps:

(1) Estimate a VAR model of order p using OLS, in this case we obtain $\hat{f}_i$ and calculate the residuals (e.g. $\hat{e}_t = y_t - \sum \hat{f}_i y_{t-i}$.

(2) Draw with replacement random samples $\{e_t^*\}_{t=1}^n$ from the calculated residuals in point (1).

(3) Generate the series $y_t^* = \sum \hat{f}_i y_{t-i}^* + e_t^*$ and estimate $\hat{f}^*$ for each random sample.

(4) Construct an estimate of covariance matrix $\hat{\Omega}^*$ based on cov (y_t^*, y_{t-k}^*) which can be computed as

$$\text{cov}(y_t^*, y_{t-k}^*) = \frac{1}{B} \sum_{b=1}^B (y_{tb}^*, y_{t-kb}^*)$$

where y_{tb}^* is y_t^* in the b$^{\text{th}}$ bootstrap replication.

(5) The confidence intervals are constructed based on the above bootstrap statistics.

A Quarterly VAR Model of the Pork Sector

A simple model for the US pork sector consists of four variables: number of hogs and pigs (hogs inventory), quantity supplied, quantity of pork demanded, and retail price of pork. The number of hogs and pigs (HP) is defined as the number of hogs and pigs (inventory) that the farmers hold at certain periods. It is expressed in thousands of heads and extracted from National Agricultural Statistics Services (NASS), USDA. Quantity supplied (SP) is defined as the total pork production (millions pounds). The production represents both commercial pork production and other production. The quantity demanded (DP) refers to the total disappearance (millions pounds). Note that the total supply does not account for stock and import. This is because we intend to measure what would be the effect of production ("supply") shock. Similarly, quantity demanded

excluded export variables. Hence it is merely domestic consumption. Retail price (RP) is the real price of pork in cents per pound. It is the nominal price deflated by consumer price index (CPI). SP, DP, and RP are gathered from various issues of Red Meat Yearbook and Livestock and Meat Statistics. The sample period runs from 1970:1 through 2000:4.

The inclusion of the four variables in the model is because they are assumed to significantly affect the fluctuation in the pork sector. In a standard market analysis at least supply, demand, and price are the primary variables entering the model. The number of hogs and pigs (hogs inventory) is included to represent the behavior of the farm level. Previous studies such as those of Hayenga and Hacklander (1970), Arzac and Wilkinson (1979), and among others included these four variables in analyzing the pork sector. Hayenga and Hacklander, for instance, modeled intensively the hog's inventory. This is reasonable because of the fact that hogs production is subject to biological constraints. Farrowing and slaughtering are examples of decisions that are heavily dependent on such biological consideration. Farmers' decision with regard to hog inventory will considerably affect the pork production. Hence any shock in the farm level related to inventory will have significant effect on the retail level, i.e., retail price and pork supply. Economic theory advises us that an increase in the supply will force down the price level, which, in turn, affects the demand for pork. Hence, the relationships among those variables result in the so-called contemporaneous correlations.

We estimated the reduced form model of equation (3) in the level with four variables as stated above (HP, SP, RP, and DP). Dickey-Fuller and Phillip Perron unit root tests suggested that the four variables were absence of unit roots. The lag length for

the four- variable system was chosen on the grounds of statistical tests reported in Table

1. Based on the 5% significance level, we decided to use 5 lags.

Table 1. VAR Lag Length Selection Results

Lag length	LR Tests*	Significance level
3		
4	118.85	0.000
5	59.98	0.000
6	25.81	0.057
7	23.81	0.094
8	18.61	0.289

*LR test of the hypothesis that the lag length is one less than that indicated.

The structural VAR model was structured based on the identifying restrictions as

suggested by Sims (1986) and Bernanke (1986). The identification scheme is as follows.

Let the endogenous vector $y_t = (HP_t, SP_t, RP_t, DP_t)$; and the vector of structural shocks u_t

$= (u_{hp}, u_{sp}, u_{dp}, u_{rp})$ where u_{hp} is a hogs inventory shock, u_{sp} is a production shock, u_{dp} is a

demand shock, and u_{rp} is a price shock. Letting $e_{hp}, e_{sp}, e_{rp}, and \, e_{dp}$ be the unrestricted

VAR residuals, we propose the following contemporaneous relationships among the

variables:

$$
\begin{aligned}
e_{hp} &= u_{hp} \\
e_{sp} &= g_{21}e_{hp} + u_{sp} \\
e_{rp} &= g_{31}e_{hp} + g_{32}e_{sp} + u_{rp} \\
e_{dp} &= g_{43}e_{rp} + u_{dp}
\end{aligned}
$$

(8)

The first equation of (8) postulates that the innovation to hogs inventory within a

quarter is a structural disturbance. It is not correlated with any other structural

disturbances. However, this assumption does not say that HP is uncorrelated with the

other observable variables: SP, RP, and DP. The second equation shows that current pork

production will respond to current hog's inventory, but not the current price, i.e., current

price does not have contemporaneous correlation with pork production. Retail price of pork is assumed to react to the current pork production as well as current hog inventory, as shown in the third equation of (8). Any structural shocks in either pork production or hogs inventory is expected to have impact on the retail price. The last equation of (8) indicates that any structural shocks on the retail price will affect the quantity demanded.

Based on equation (8) we can construct a matrix that shows contemporaneous correlations among the endogenous variables included in the model. Letting K be such matrix, we can obtain

(9)
$$K = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -g_{21} & 1 & 0 & 0 \\ -g_{31} & -g_{32} & 1 & 0 \\ 0 & 0 & -g_{43} & 1 \end{bmatrix}$$

Clearly, matrix K is over identified. The model was estimated using Rats with cvmodel procedure. The estimated coefficients of matrix K are given in Table 2.

Table 2
Maximum Likelihood Estimates of Contemporaneous Pork Model Parameters

Parameters	Coefficients	Standard error	Significance level
g_{21}	-0.0418	0.097	0.668
g_{31}	0.1547	0.0867	0.074
g_{32}	-0.7135	0.0813	0.000
g_{43}	-0.5341	0.0636	0.000

The $c_{(2)}^{2}$ for the LR test is 138.16, indicating the rejection of the null hypothesis.
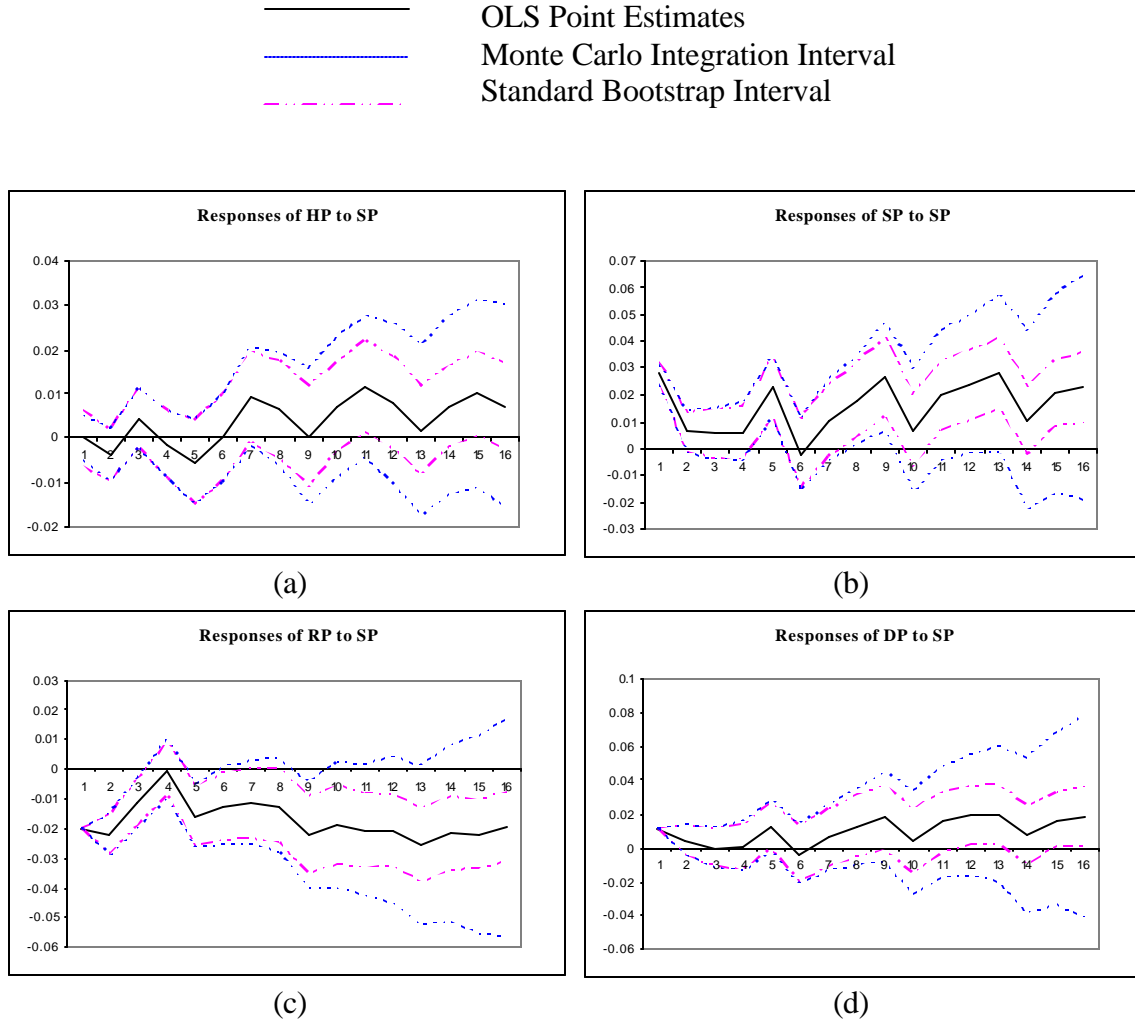
Table 2 shows that all the estimated coefficients of the K matrix are significant (at least at 10% level), except for g_{21}. The $c_{(2)}^{2}$ for the likelihood ratio test is also significant, suggesting the rejection of the null hypothesis.

Main Results and Conclusions

As our interest in to obtain the confidence intervals of the IRFs, our next step is to bootstrap confidence intervals using standard bootstrap as previously outlined for both VAR and SVAR models. For the purpose of comparison, we also conducted Monte Carlo simulations (MC) for generating confidence intervals. Bootstrap and Monte Carlo simulations were conducted using two different sample drawings: 500 and 1500. Because the full set of results is rather large, with no loss of information, we will restrict our discussion to two given shocks: pork production (SVAR model) and retail prices (VAR model). Our investigation shows that there does not seem to be much different between the outcomes from different shocks and responses. The graphical representations of the confidence intervals of the impulse response functions of these two shocks are given in figure 1 though figure 4.

Figure 1 shows responses of the four variables to a shock in pork production with N=500 for SVAR model. The four panels of figure 1 suggested that in the first six periods, both bootstrap and MC deliver approximately equivalent results in the sense that the CIs generated by the two methods coincide. As the lag length increases, MC yields larger CIs than the bootstrap does. Panel (a) of figure 1 shows that the response of Hog inventory (HP) to a shock in pork production (SP) is around the zero point in the first 6 periods. There is a tendency the response to move into positive direction afterwards; but the CIs of the two methods indicated insignificant movement. Panel (d) exhibits responses of demand for pork to a shock in pork production. There seem to be positive responses at all horizons, except in the sixth period. Evaluating using the CIs, the positive responses are not significant.

Figure 1: 95% Confidence Interval of Impulse Response Function
To A Shock in Pork Production – SVAR Model (N=500)

————————— OLS Point Estimates

····················· Monte Carlo Integration Interval

·–··–··–··–· Standard Bootstrap Interval

(a)

(b)

(c)

(d)

A closer investigation should be given to panels (b) and (c). Although panel (b) may not be of interest since it describes responses of SP to its own shock, we should give a closer look to the CIs and their possible inferences. In fact this panel shows that there are some periods (period 10 and beyond) in which the bootstrap CIs show significant positive responses but not the MC CIs. Panel (c) shows negative responses of retail price (RP) to a shock in SP, which is expected. The two methods provided evidence of significant negative responses in the first three periods. Between period 3 and period 8,

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the zero point seems to be covered in both CIs. After period 8, however, there is a clear discrepancy of CIs generated by the two methods. More precisely, the bootstrap method provided a significant evidence of negative responses of RP to a shock in SP, which suggested significant permanent reduction in retail price.

Adding the sample drawings does not seem to change the patterns significantly as shown in figure 2. All four panels of this figure follow the same patterns as those of figure 1. However, if we investigate further and compare the standard errors of both methods, we found that the standard errors generated using 1500 drawings are slightly smaller than those from 500 drawings, as shown in figure 5. This evidence should be taken with great care, however, since we did not conduct any statistical inferences with respect to these differences.

The bootstrap and MC CIs for VAR model are presented in figure 3 and 4. Unlike in the SVAR model, the CIs in the VAR model generated by the two methods deliver similar results in the whole horizons. There is no significant indication that the two methods give different economic interpretations. All panels of figure 3 (N=500) and figure 4 (N=1500) display that the two methods provide very close estimates of CIs, although in some cases the bootstrap CIs are smaller than MC CIs (see figure 5). In lower right panel of these figures, for instance, a shock in retail price had a significant decrease in the demand for pork at the first two periods. Since we did not intend to compare between VAR and SVAR models, we did not elaborate such differences. The hint, however, can be traced by the fact that SVAR model takes into account contemporaneous correlation as shown in matrix K.

Figure 2: 95% Confidence Interval of Impulse Response Function
To A Shock in Pork Production – SVAR Model (N=1500)

OLS Point Estimates
Monte Carlo Integration Interval
Standard Bootstrap Interval
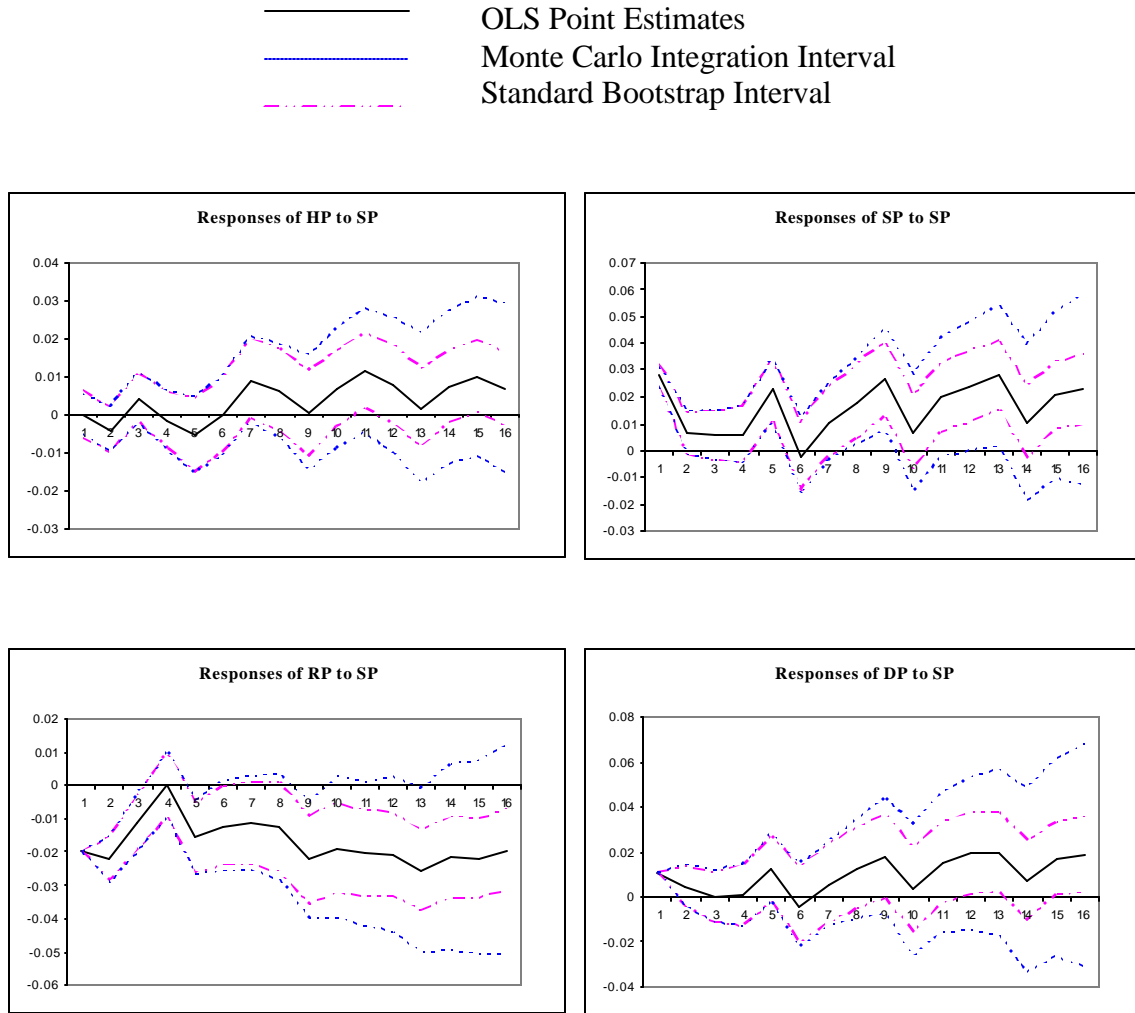
Although our experiment has been necessarily limited, we are nevertheless able to
draw some conclusions. First, in the VAR model, the bootstrap and MC methods do not
display significant differences in generating CIs of IRFs. In the SVAR model, we found
significant differences in CIs generated by the two methods, especially after the sixth
period. In fact, to some degree, this experiment illustrates that using bootstrap method in
SVAR analysis can change the way we interpret economic data, especially in further

Figure 3:   95% Confidence Interval of Impulse Response Function
To A Shock in Retail Price of Pork – VAR Model (N=500)

OLS Point Estimates
Monte Carlo Integration Interval
Standard Bootstrap Interval
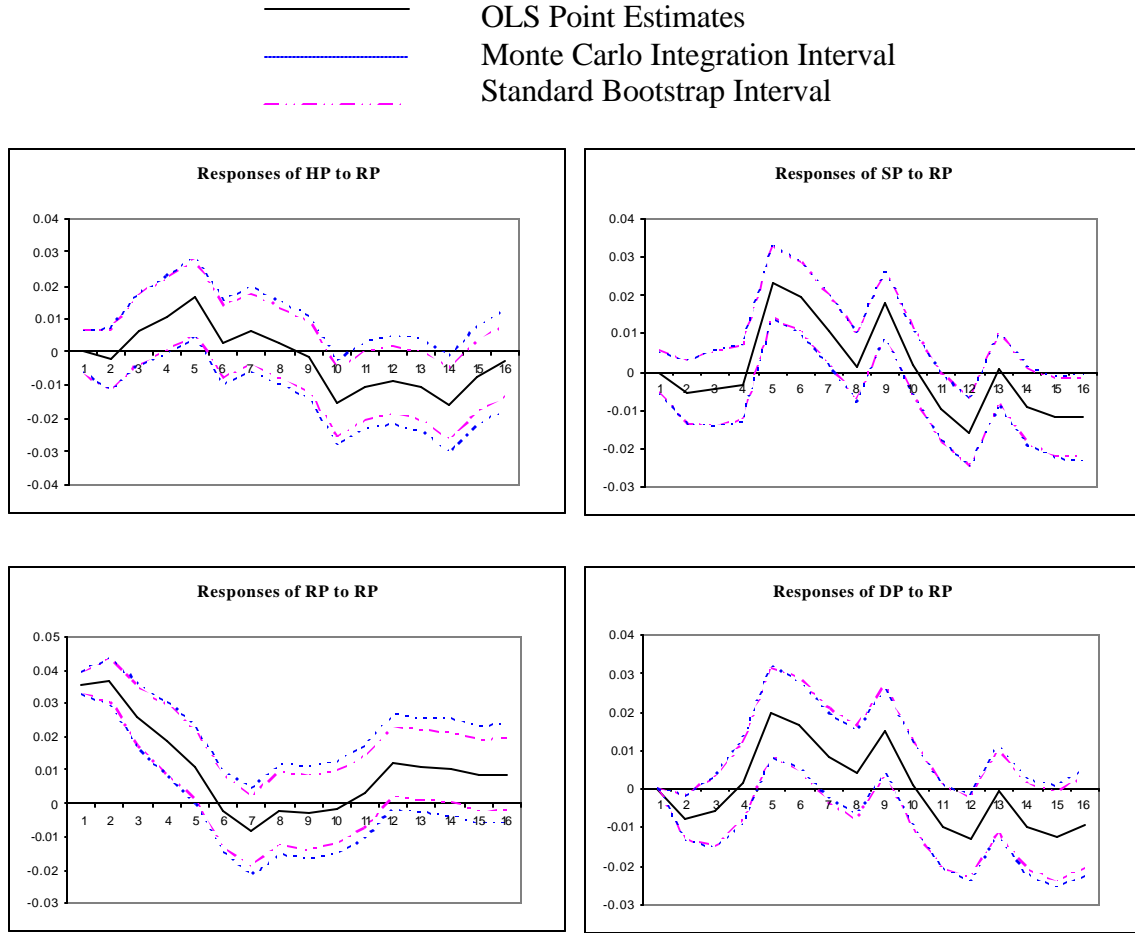
horizons. If we rely on the common usage of checking if zero is included in the

confidence interval in order to evaluate the significance of the effect, the bootstrap

method provided some evidence of clear different inferences in some responses to a

shock. Second, the bootstrap seems superior to the MC procedure in the case of the

variance of the forecast errors. There is a slight difference in the standard errors of the

bootstrap and standard errors of MC method in VAR model; still there is an indication

that the bootstrap gives lower standard errors than the MC procedure in the forecast

horizons. In the SVAR model, we found significant differences where the MC CIs tend

Figure 4: 95% Confidence Interval of Impulse Response Function
To A Shock in Retail Price of Pork – VAR Model (N=1500)

OLS Point Estimates
Monte Carlo Integration Interval
Standard Bootstrap Interval
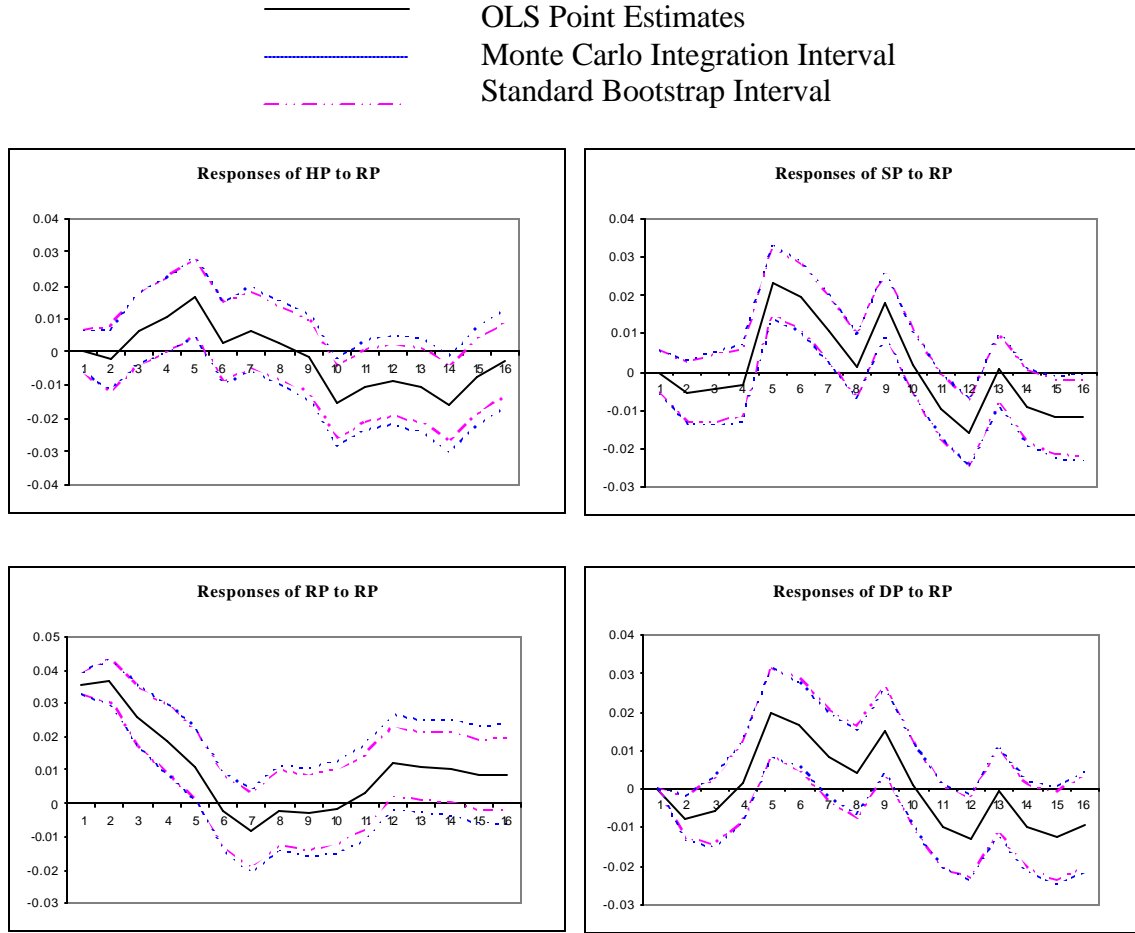
to widen and explode after certain periods; suggesting that the bootstrap CIs are more stable than the MC CIs (see figure 5 for standard errors comparison).

The general conclusion is that the bootstrap method seems to be able to deliver superior performances compared to Monte Carlo integration method. Furthermore, the bootstrap method could stimulate changing of economic interpretation of the data. All the conclusions are, however, subject to possible bias that may occur in the estimation, on which researchers are usually concerned. Our suggestion and our intention for future work is to apply a method as proposed by Killian (1998) in order to reduce possible bias.

Figure 5: Monte Carlo and Bootstrap Standard Errors of VAR and SVAR Models
(N=500 and N=1500)
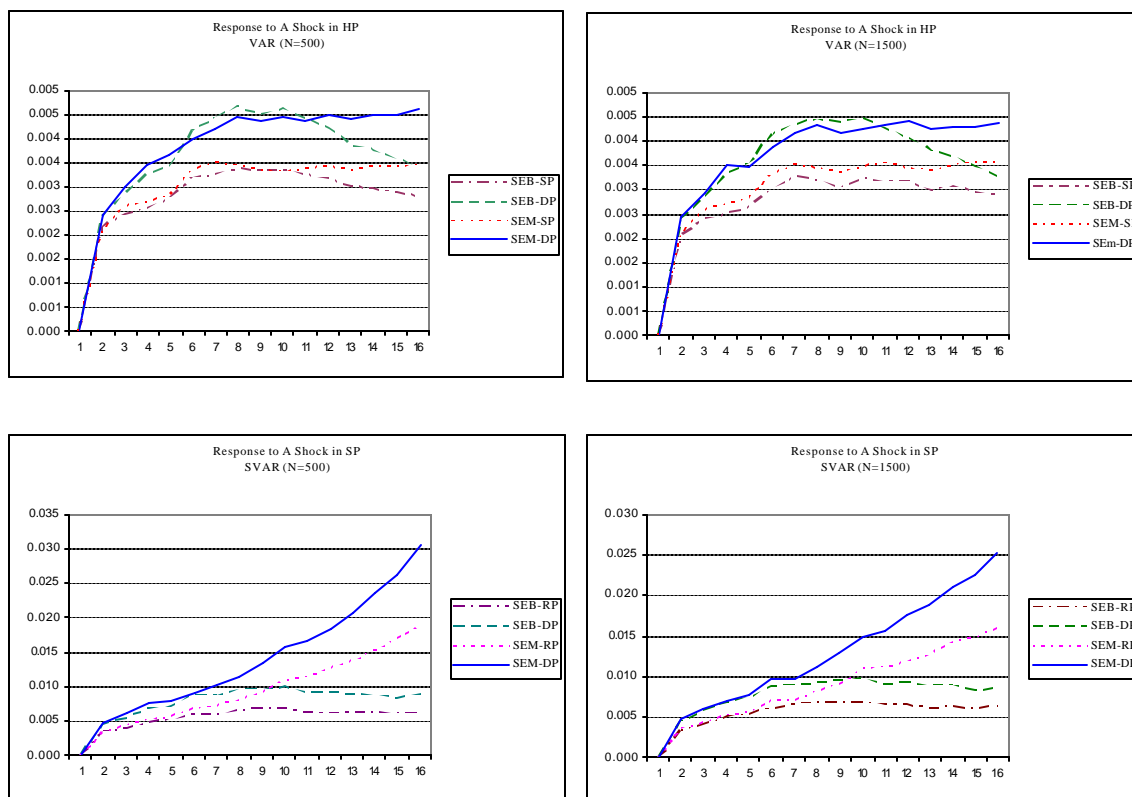
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