spSUR: an R package for specification, estimation and testing of spatial and spatio-temporal SUR econometric models

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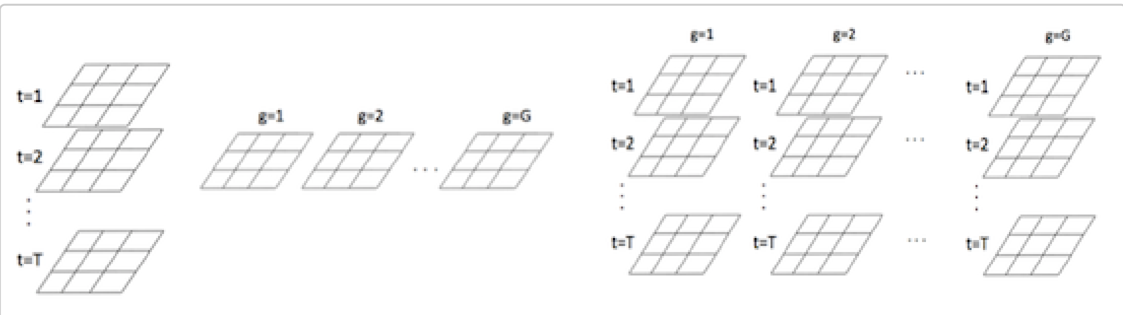
Abstract

A collection of functions to test and estimate Spatial Seemingly Unrelated Regression (SUR) models by maximum likelihood and three-stage least square for the more usual SUR specifications with spatial effects (SUR-SLX; SUR-SAR; SUR-SEM;SUR-SDM; SUR-SDEM and SUR-SARAR) and non spatial SUR model (SUR-SIM).

Motivation

1. From Zellner (1962) Seemingly Unrelated Model (SUR) is a popular model in Econometrics.
2. Spatial SUR models are a powerful multi-equational models in Spatial Econometrics.
    - Could be used to take account spatial autocorrelation and temporal correlation.
    - Correlation between residual of different equations for same cross section.
    - Even could be used in a panel data framework (López et al. 2010).

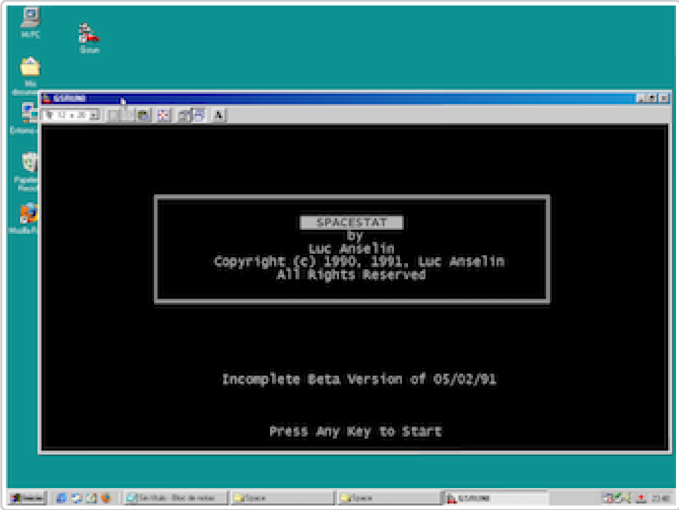


3. But surprisingly it is unpopular model in Spatial Econometrics.
4. **The question is: Why?**
    - No friendly software is available to estimate Spatial SUR

History: You know how to estimate a Spatial SUR model?

The first software was develop by Anselin: May91

- Completely unknown.
- Is an incomplete Beta version (only for Windows 98).
- Only work for estimate SUR-SAR models by Maximum-Likelihood.



Our Matlab Codes NOBODY USES IT!

- Testing for and ML estimation Spatial Sur
- Matlab is not a free-software

Phyton Codes with PySAL

- Reduced functionalities (SUR-ERR by ML & SUR-SAL by IV)
- Python has less diffusion than R among statisticians (although its community is increasing very quickly…)

We think that `spSUR` will be a welcome R-package to estimate Spatial SUR model

---

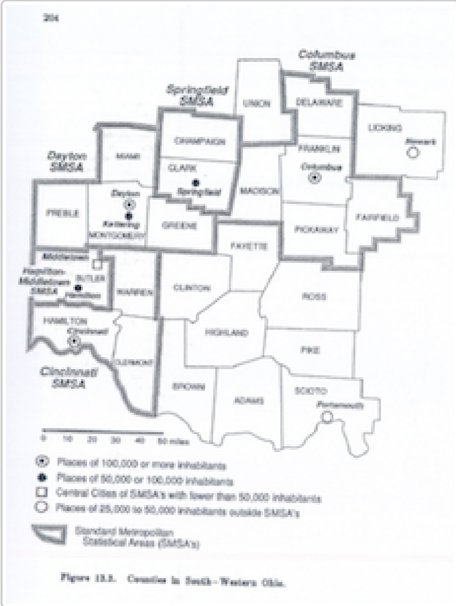
Before to present the `spSUR` package:

0.1 Data sets in `spSUR`

The `spSUR` package include several data sets:

The spc (Spatial Phillips-Curve). A classical data set from Anselin (1988, p.203)

A total of nR=25 observations and nT=2 time periods

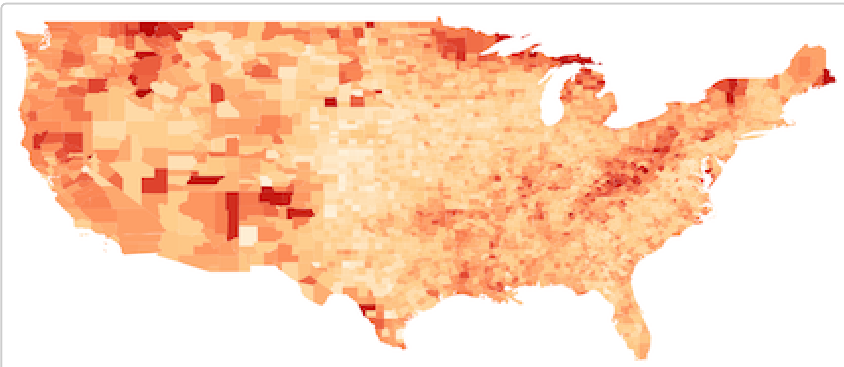


COUNTY	WAGE83	UN83	NMR83	SMSA	WAGE82	WAGE81	UN80	NMR80	WAGE80
UNION	1.003127	0.080500	-0.002217	1	1.108662	1.146178	0.130375	-0.010875	1.084886
DELAWARE	1.039972	0.122174	0.018268	1	1.071271	1.104241	0.189603	0.041886	1.110426
LICKING	1.050196	0.095821	-0.013681	1	1.058375	1.094732	0.124125	-0.004158	1.069776

2. Homicides + Socio-Economics characteristics for U.S. counties (1960-90)

from [https://geodacenter.github.io/data-and-lab/ncovr/]

Homicides and selected socio-economic characteristics for continental U.S. counties. Data for four decennial census years: 1960, 1970, 1980 and 1990.
A total of nR=3,085 US counties



NAME	STATE_NAME	STATE_FIPS	CNTY_FIPS	FIPS	STFIPS	COFIPS	FIPSNO	SOUTH
Lake of the Woods	Minnesota	27	077	27077	27	77	27077	0
Ferry	Washington	53	019	53019	53	19	53019	0
Stevens	Washington	53	065	53065	53	65	53065	0

0.2 How to specify multiple equations: The `Formula` package

By example: two equations with different number of regressors

$$Y_1 = \beta_{10} + \beta_{11}\ X_{11} + \beta_{12} X_{12} + \epsilon_1$$
$$Y_2 = \beta_{20} + \beta_{21}\ X_{21} + \epsilon_2$$

formula <- Y_1 | Y_2 ~ X_{11} + X_{12} | X_{21}

The `spSUR` package step by step

Step 1: Testing for spatial effects

Step 2: Estimation of the Spatial SUR models

Step 3: Looking for the correct especification

Step 4: Impacts: Directs, Indirects and Total effects

Step 5: `spSUR` in a panel data framework

Step 6: Additional functionalities

Step 7: Conclusion and work to do

Step 1: Testing for: `lmtestspsur`

The function `lmtestspsur` obtain five LM statistis for testing spatial dependence in Seemingly Unrelated Regression models
(Mur J, López FA, Herrera M, 2010: Testing for spatial effect in Seemingly Unrelated Regressions. *Spatial Economic Analysis* 5(4) 399-440).

H_0 : No spatial autocorrelation
H_A : SUR-SAR or
H_A : SUR-SEM or
H_A : SUR-SARAR

- **LM-SUR-SAR**
- **LM-SUR-SEM**
- **LM-SUR-SARAR**

and two robust LM tests

- **LM*-SUR-SAR**
- **LM*-SUR-SEM**

Example 1: with Anselin's data we can test spatial effects in the SUR model:

$$
\begin{aligned}
WAGE_{83} &= \beta_{10} + \beta_{11}\, UN_{83} + \beta_{12}\, NMR_{83} + \beta_{13}\, SMSA + \epsilon_{83} \\
WAGE_{81} &= \beta_{20} + \beta_{21}\, UN_{80} + \beta_{22}\, NMR_{80} + \beta_{23}\, SMSA + \epsilon_{81} \\
Corr(\epsilon_{83}, \epsilon_{81}) &\neq 0
\end{aligned}
$$

```
library(spSUR)
data("spc")
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
LMs <- lmtestspsur(Form=Tformula,data=spc,W=Wspc)
```

```
## Computing model whitout spatial effects...
## Computing LM-SAR test...
## Computing LM-SEM test...
## Computing LM-SARAR test...
## Computing Robust LM*-SUR-SAR test...
## Computing Robust LM*-SUR-SEM test...
##
##
##                LM-Stat. DF p-value
## LM-SUR-SAR       3.5698  2   0.168
## LM-SUR-SEM       2.1396  2   0.343
## LM*-SUR-SAR      2.2161  2   0.330
## LM*-SUR-SEM      0.7859  2   0.675
## LM-SUR-SARAR     4.4157  4   0.353
```

In this example no spatial autocorrelation is identified! (25 observations).

Example 2: Homicides + Socio-Economics data (year 1980)

With **different number** of exogenous variables in each equation

$$
\begin{aligned}
HR_{80} &= \beta_{10} + \beta_{11}\, PS_{80} + \beta_{12}\, UE_{80} + \epsilon_{HR} \\
DV_{80} &= \beta_{20} + \beta_{21}\, PS_{80} + \beta_{22}\, UE_{80} + \beta_{23}\, SOUTH + \epsilon_{DV} \\
FP_{79} &= \beta_{30} + \beta_{31}\, PS_{80} + \epsilon_{FP}
\end{aligned}
$$

```
Tformula <- HR80 | DV80  | FP79 ~ PS80 + UE80 | PS80 + UE80 + SOUTH | PS80
LMs <- lmtestspsur(Form=Tformula,data=NAT,W=W)
```

```
## Computing model whitout spatial effects...
## Computing LM-SAR test...
## Computing LM-SEM test...
## Computing LM-SARAR test...
## Computing Robust LM*-SUR-SAR test...
## Computing Robust LM*-SUR-SEM test...
##
##
##                LM-Stat. DF  p-value
## LM-SUR-SAR     5543.75  2  < 2e-16 ***
## LM-SUR-SEM     5672.62  2  < 2e-16 ***
## LM*-SUR-SAR      49.23  2 2.04e-11 ***
## LM*-SUR-SEM     178.10  2  < 2e-16 ***
## LM-SUR-SARAR   5802.53  4  < 2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Step 2: Estimation of a Spatial SUR

Two alternative estimation methods are implemented:

2.1. Maximum likelihood estimation: `spsurml`

Maximum likelihood estimation for differents spatial SUR models using `spsurml`. The models are:

- **SUR-SIM:** with out spatial autocorrelation
- **SUR-SLX:** Spatial Lag of X SUR model
- **SUR-SAR:** Spatial Autorregresive SUR model
- **SUR-SEM:** Spatial Error SUR model
- **SUR-SDM:** Spatial Durbin SUR model
- **SUR-SDEM:** Spatial Durbin Error SUR model
- **SUR-SARAR:** Spatial Autorregresive with Spatial Error SUR model / (SUR-SAC)

2.1.1 Anselin data set

SUR-SIM: SUR model without spatial effects
$(y_t = X_t\beta_t + \epsilon_t; t = 1, \ldots, T)$

$$\begin{aligned} WAGE_{83} &= \beta_{10} + \beta_{11}UN_{83} + \beta_{12}NMR_{83} + \beta_{13}SMSA + \epsilon_{83} \\ WAGE_{81} &= \beta_{20} + \beta_{21}UN_{80} + \beta_{22}NMR_{80} + \beta_{23}SMSA + \epsilon_{81} \\ Corr(\epsilon_{83}, \epsilon_{81}) &\neq 0 \end{aligned}$$

```
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
spcSUR.sim <-spsurml(Form=Tformula,data=spc,type="sim",W=Wspc)
```

```
## Initial point:
## log_lik:  110.423
## Iteration:  1  log_lik:  111.201
## Iteration:  2  log_lik:  111.348
## Iteration:  3  log_lik:  111.378
## Iteration:  4  log_lik:  111.383
## Iteration:  5  log_lik:  111.385
## Iteration:  6  log_lik:  111.385
## Time to fit the model:  0.14  seconds
##
## Time to compute covariances:  0  seconds
```

```
summary(spcSUR.sim)
```

```
## Call:
## spsurml(Form = Tformula, data = spc, W = Wspc, type = "sim")
```

```
##
##
## Spatial SUR model type:  sim
##
## Equation  1
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)_1  0.9853976  0.0205012 48.0654  < 2e-16 ***
## UN83_1         0.6458915  0.2886069  2.2380  0.03101 *
## NMR83_1       -0.3545941  0.2697722 -1.3144  0.19638
## SMSA_1        -0.0078705  0.0132102 -0.5958  0.55476
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.4159
##   Equation  2
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)_2  1.1588338  0.0456535 25.3832   <2e-16 ***
## UN80_2        -0.5518074  0.4136358 -1.3340   0.1899
## NMR80_2        0.5679754  0.3793640  1.4972   0.1424
## SMSA_2         0.0098439  0.0261665  0.3762   0.7088
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.2745
##   Variance-Covariance Matrix of inter-equation residuals:
##   0.0004596374 -0.0006881683
##  -0.0006881683  0.0021204332
## Correlation Matrix of inter-equation residuals:
##   1.0000000 -0.6970672
##  -0.6970672  1.0000000
##
##  R-sq. pooled: 0.5306
##  Log-Likelihood:  111.385
##  Breusch-Pagan: 12.159  p-value: (0.000489)
```

Only change the **'type'** argument in `spsurml` function it is possible to estimate several spatial model

SUR-SAR: Spatial autorregresive model:
$(y_t = \lambda_t W y_t + X_t \beta_t + \epsilon_t;\ t = 1, \ldots, T\ )$

$$\begin{array}{l} WAGE_{83} = \lambda_{83} WWAGE_{83} + \beta_{10} + \beta_{11} UN_{83} + \beta_{12} NMR_{83} + \beta_{13} SMSA + \epsilon_{83} \\ WAGE_{81} = \lambda_{81} WWAGE_{81} + \beta_{20} + \beta_{21} UN_{80} + \beta_{22} NMR_{80} + \beta_{23} SMSA + \epsilon_{81} \\ Corr(\epsilon_{83}, \epsilon_{81}) \neq 0 \end{array}$$

```
spcSUR.sar <-spsurml(Form=Tformula,data=spc,type="sar",W=Wspc)
```

```
## Initial point:   log_lik:  113.198  lambdas:  -0.472 -0.446
## Iteration:  1   log_lik:  114.088  lambdas:  -0.506 -0.482
## Iteration:  2   log_lik:  114.098  lambdas:  -0.506 -0.482
## Iteration:  3   log_lik:  114.099  lambdas:  -0.505 -0.482
## Time to fit the model:  2.58  seconds
##
## Computing marginal test...
## Time to compute covariances:  0.26  seconds
```

```r
summary(spcSUR.sar)
```

```
## Call:
## spsurml(Form = Tformula, data = spc, W = Wspc, type = "sar")
##
##
## Spatial SUR model type:  sar
##
## Equation  1
##                 Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_1  1.4947179  0.2467450  6.0577 5.244e-07 ***
## UN83_1         0.8053338  0.2558760  3.1474  0.003249 **
## NMR83_1       -0.5165301  0.2590369 -1.9940  0.053557 .
## SMSA_1        -0.0072526  0.0118566 -0.6117  0.544484
## lambda_1      -0.5048489  0.2405967 -2.0983  0.042763 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.622
##   Equation  2
##                 Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_2  1.7088172  0.2925087  5.8419 1.028e-06 ***
## UN80_2        -0.6736472  0.3870209 -1.7406   0.09007 .
## NMR80_2        0.7475734  0.3840119  1.9467   0.05919 .
## SMSA_2         0.0013487  0.0241871  0.0558   0.95583
## lambda_2      -0.4816232  0.2557338 -1.8833   0.06754 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.4743
##   Variance-Covariance Matrix of inter-equation residuals:
##   0.0003091646 -0.0003578072
##  -0.0003578072  0.0015874437
## Correlation Matrix of inter-equation residuals:
##   1.0000000 -0.5107461
##  -0.5107461  1.0000000
##
##  R-sq. pooled: 0.6601
##  Log-Likelihood:  114.099
##  Breusch-Pagan: 6.5431  p-value: (0.0105)
##  LMM: 0.50474  p-value: (0.477)
```

SUR-SEM: Spatial error model:
$(y_t = X_t\,\beta_t + u_t\,; u_t = \rho u_t + \epsilon_t\; t = 1, \ldots, T)$

$$
\begin{aligned}
&WAGE_{83} = \beta_{10} + \beta_{11}UN_{83} + \beta_{12}NMR_{83} + \beta_{13}SMSA + u_{83};\; u_{83} = \rho W\,u_{83} + \epsilon_{83}\\
&WAGE_{81} = \beta_{20} + \beta_{21}UN_{80} + \beta_{22}NMR_{80} + \beta_{23}SMSA + u_{81};\; u_{81} = \rho W\,u_{81} + \epsilon_{81}\\
&Corr(\epsilon_{83}, \epsilon_{81}) \neq 0
\end{aligned}
$$

```r
spcSUR.sem <-spsurml(Form=Tformula,data=spc,type="sem",W=Wspc)
```

```
## Initial point:   log_lik:  112.821  deltas:  -0.556 -0.477
## Iteration:  1  log_lik:  113.695  rhos:  -0.618 -0.537
## Iteration:  2  log_lik:  113.719  rhos:  -0.628 -0.548
```

```
## Iteration:  3  log_lik:  113.725  rhos:  -0.634 -0.553
## Iteration:  4  log_lik:  113.728  rhos:  -0.638 -0.557
## Iteration:  5  log_lik:  113.73  rhos:  -0.642 -0.559
## Iteration:  6  log_lik:  113.731  rhos:  -0.645 -0.561
## Iteration:  7  log_lik:  113.732  rhos:  -0.647 -0.563
## Time to fit the model:  5.37  seconds
##
## Computing marginal test...
## Time to compute covariances:  0.22  seconds
```

```
summary(spcSUR.sem)
```

```
## Call:
## spsurml(Form = Tformula, data = spc, W = Wspc, type = "sem")
##
##
## Spatial SUR model type:  sem
##
## Equation  1
##                  Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_1  0.9798052  0.0150105 65.2748 < 2.2e-16 ***
## UN83_1         0.7508713  0.2253264  3.3324  0.001963 **
## NMR83_1       -0.5084650  0.2541467 -2.0007  0.052805 .
## SMSA_1        -0.0139768  0.0098142 -1.4241  0.162788
## rho_1         -0.6468828  0.2776346 -2.3300  0.025370 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.6652
##   Equation  2
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)_2  1.1473041  0.0385173 29.7867  < 2e-16 ***
## UN80_2        -0.4391249  0.3597557 -1.2206  0.22995
## NMR80_2        0.8530412  0.4077931  2.0918  0.04337 *
## SMSA_2         0.0046617  0.0203355  0.2292  0.81995
## rho_2         -0.5627421  0.2825038 -1.9920  0.05379 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.5123
##   Variance-Covariance Matrix of inter-equation residuals:
##   0.0002908458 -0.0003037319
##  -0.0003037319  0.0015339725
## Correlation Matrix of inter-equation residuals:
##   1.0000000 -0.4547265
##  -0.4547265  1.0000000
##
##  R-sq. pooled: 0.677
##  Log-Likelihood:  113.732
##  Breusch-Pagan: 5.1422  p-value: (0.0234)
##  LMM: 1.4089  p-value: (0.235)
```

SUR-SARAR: Spatial autoregressive model with spatial autoregressive error term: $(y_t = \lambda_t W y_t + X_t \, \beta_t + u_t \, ; u_t = \rho_t u_t + \epsilon_t \; t = 1, \ldots, T)$

```
spcSUR.sarar <-spsurml(Form=Tformula,data=spc,type="sarar",W=Wspc)
```

```
## Initial point:   log_lik:  113.408  lambdas:  -0.343 -0.526  rhos:  -0.28 0.113
## Iteration:  1  log_lik:  114.578  lambdas:  -0.384 -0.784  rhos:  -0.307 0.411
## Iteration:  2  log_lik:  114.828  lambdas:  -0.394 -0.905  rhos:  -0.303 0.553
## Iteration:  3  log_lik:  115.023  lambdas:  -0.396 -0.996  rhos:  -0.296 0.657
## Iteration:  4  log_lik:  115.19  lambdas:  -0.388 -1  rhos:  -0.271 0.686
## Iteration:  5  log_lik:  115.24  lambdas:  -0.381 -1  rhos:  -0.254 0.699
## Iteration:  6  log_lik:  115.264  lambdas:  -0.376 -1  rhos:  -0.243 0.707
## Iteration:  7  log_lik:  115.276  lambdas:  -0.371 -1  rhos:  -0.235 0.712
## Iteration:  8  log_lik:  115.282  lambdas:  -0.368 -1  rhos:  -0.23 0.716
## Iteration:  9  log_lik:  115.286  lambdas:  -0.366 -1  rhos:  -0.226 0.719
## Iteration:  10  log_lik:  115.288  lambdas:  -0.364 -1  rhos:  -0.224 0.721
## Iteration:  11  log_lik:  115.289  lambdas:  -0.363 -1  rhos:  -0.222 0.722
## Iteration:  12  log_lik:  115.289  lambdas:  -0.362 -1  rhos:  -0.221 0.723
## Time to fit the model:  48.59  seconds
##
## Time to compute covariances:  0.19  seconds
```

```
summary(spcSUR.sarar)
```

```
## Call:
## spsurml(Form = Tformula, data = spc, W = Wspc, type = "sarar")
##
##
## Spatial SUR model type:  sarar
##
## Equation  1
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)_1  1.354490   0.522876  2.5905 0.013880 *
## UN83_1         0.746298   0.252728  2.9530 0.005591 **
## NMR83_1       -0.438911   0.242607 -1.8091 0.079025 .
## SMSA_1        -0.011239   0.011215 -1.0021 0.323176
## lambda_1      -0.361735   0.514247 -0.7034 0.486443
## rho_1         -0.220734   0.584606 -0.3776 0.708024
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.6453
##   Equation  2
##                Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_2  2.256459   0.292920  7.7033 4.838e-09 ***
## UN80_2        -0.571501   0.326532 -1.7502 0.0888444 .
## NMR80_2        0.392295   0.274278  1.4303 0.1615033
## SMSA_2         0.022427   0.026351  0.8511 0.4005058
## lambda_2      -1.000000   0.270682 -3.6944 0.0007482 ***
## rho_2          0.722962   0.162677  4.4442 8.487e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.6888
##   Variance-Covariance Matrix of inter-equation residuals:
##   0.0003261247 -0.000399029
##  -0.0003990290  0.001215184
## Correlation Matrix of inter-equation residuals:
##   1.0000000 -0.6338573
##  -0.6338573  1.0000000
##
##  R-sq. pooled: 0.7402
```

```
##  Log-Likelihood:  115.289
##  Breusch-Pagan: 10.071  p-value: (0.00151)
```

2.1.2 Homicides + Socio-Economics (counties U.S.)

It is a **fast code**. Note the time to estimate a SUR-SAR with **3085 observations**

SUR-SAR: Spatial autorregresive model
$(y_g = \lambda_g W y_g + X_g \beta_g + \epsilon_g; g = 1, \ldots, G)$

$$
\begin{array}{l}
HR_{80} = \lambda_{HR}\, WHR_{80} + \beta_{10} + \beta_{11} PS_{80} + \beta_{12} UE_{80} + \epsilon_{HR} \\
DV_{80} = \lambda_{DV}\, WDV_{80} + \beta_{20} + \beta_{21} PS_{80} + \beta_{22} UE_{80} + \beta_{23} SOUTH + \epsilon_{DV} \\
FP_{79} = \lambda_{FP}\, WFP_{80} + \beta_{30} + \beta_{31} PS_{80} + \epsilon_{FP}
\end{array}
$$

```
data(NAT)
Tformula <- HR80 | DV80  | FP79 ~ PS80 + UE80 | PS80 + UE80 + SOUTH | PS80
NATSUR.sar <-spsurml(Form=Tformula,data=NAT,type="sar",W=W)
```

```
## Initial point:   log_lik:  -23517.27  lambdas:  0.483 0.6 0.631
## Iteration:  1   log_lik:  -22866.07  lambdas:  0.531 0.676 0.742
## Iteration:  2   log_lik:  -22857.5  lambdas:  0.535 0.681 0.754
## Iteration:  3   log_lik:  -22857.42  lambdas:  0.535 0.681 0.755
## Iteration:  4   log_lik:  -22857.42  lambdas:  0.535 0.682 0.755
## Iteration:  5   log_lik:  -22857.42  lambdas:  0.535 0.682 0.755
## Time to fit the model:  34.02  seconds
##
## Computing marginal test...
## Time to compute covariances:  71.67  seconds
```

```
summary(NATSUR.sar)
```

```
## Call:
## spsurml(Form = Tformula, data = NAT, W = W, type = "sar")
##
##
## Spatial SUR model type:  sar
##
## Equation  1
##                  Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_1 3.070929   0.259057 11.8543 < 2.2e-16 ***
## PS80_1        0.586505   0.105973  5.5345 3.207e-08 ***
## UE80_1        0.019504   0.029405  0.6633    0.5072
## lambda_1      0.535122   0.019782 27.0505 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.3073
##   Equation  2
##                   Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_2  1.1722543  0.0777684 15.0737 < 2.2e-16 ***
```

```
## PS80_2         0.2254184  0.0201623 11.1802 < 2.2e-16 ***
## UE80_2         0.0466549  0.0059494  7.8420 4.927e-15 ***
## SOUTH_2       -0.0411057  0.0364872 -1.1266    0.2599
## lambda_2       0.6815413  0.0157208 43.3528 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.4963
##   Equation  3
##                Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_3  3.074903   0.173140  17.760 < 2.2e-16 ***
## PS80_3        -1.079359   0.073941 -14.598 < 2.2e-16 ***
## lambda_3       0.754935   0.012749  59.215 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.6614
##   Variance-Covariance Matrix of inter-equation residuals:
##  33.560585  1.2156737  8.1703336
##   1.215674  1.0940386 -0.4140574
##   8.170334 -0.4140574 13.9639287
## Correlation Matrix of inter-equation residuals:
##  1.0000000  0.2006254  0.3774168
##  0.2006254  1.0000000 -0.1059352
##  0.3774168 -0.1059352  1.0000000
##
##  R-sq. pooled: 0.6038
##  Log-Likelihood:  -22857.4
##  Breusch-Pagan: 598.23  p-value: (2.44e-129)
##  LMM: 176.46  p-value: (5.12e-38)
```

---

or a **SUR-SDEM** Spatial Durbin error model:
($y_g = X_g\beta_g + WX_g\theta_g + u_t; u_t = \rho W u_t + \epsilon_g$; g=1,...,G)

```
Tformula <- HR80 | DV80  | FP79 ~ PS80 + UE80 | PS80 + UE80 + SOUTH | PS80
NATSUR.sdem <-spsurml(Form=Tformula,data=NAT,type="sdem",W=W)
```

```
## Initial point:   log_lik:  -23423.67  deltas:  0.495 0.624 0.641
## Iteration:  1  log_lik:  -22702.07  rhos:  0.549 0.713 0.763
## Iteration:  2  log_lik:  -22689.42  rhos:  0.553 0.721 0.778
## Iteration:  3  log_lik:  -22689.29  rhos:  0.554 0.721 0.779
## Iteration:  4  log_lik:  -22689.29  rhos:  0.554 0.722 0.779
## Iteration:  5  log_lik:  -22689.29  rhos:  0.554 0.722 0.779
## Time to fit the model:  30.75  seconds
##
## Computing marginal test...
## Time to compute covariances:  150.7  seconds
```

```
summary(NATSUR.sdem)
```

```
## Call:
## spsurml(Form = Tformula, data = NAT, W = W, type = "sdem")
##
##
## Spatial SUR model type:  sdem
```

```
##
## Equation  1
##                 Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_1  7.584547   0.536368 14.1406 < 2.2e-16 ***
## PS80_1         1.001711   0.149782  6.6878 2.397e-11 ***
## UE80_1         0.226484   0.043837  5.1665 2.436e-07 ***
## W_PS80_1      -0.123111   0.261546 -0.4707    0.6379
## W_UE80_1      -0.327565   0.073016 -4.4862 7.339e-06 ***
## rho_1          0.554004   0.019701 28.1205 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.3347
##   Equation  2
##                 Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_2  4.228925   0.157365 26.8733 < 2.2e-16 ***
## PS80_2         0.500468   0.026934 18.5811 < 2.2e-16 ***
## UE80_2         0.100294   0.008472 11.8382 < 2.2e-16 ***
## SOUTH_2        0.111463   0.179136  0.6222   0.53381
## W_PS80_2      -0.240875   0.056776 -4.2426 2.231e-05 ***
## W_UE80_2      -0.031839   0.017131 -1.8586   0.06312 .
## W_SOUTH_2     -0.226163   0.209832 -1.0778   0.28114
## rho_2          0.721527   0.015585 46.2966 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.5425
##   Equation  3
##                 Estimate Std. Error  t value Pr(>|t|)
## (Intercept)_3 12.479519   0.298334  41.8307  < 2e-16 ***
## PS80_3        -1.869599   0.100406 -18.6204  < 2e-16 ***
##               -0.468096   0.226073  -2.0706  0.03843 *
## rho_3          0.779293   0.013175  59.1508  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.6779
##   Variance-Covariance Matrix of inter-equation residuals:
##  32.39737  0.9822900  7.7446204
##   0.98229  1.0013733 -0.3468253
##   7.74462 -0.3468253 13.3569909
## Correlation Matrix of inter-equation residuals:
##  1.0000000  0.17245939  0.37229819
##  0.1724594  1.00000000 -0.09483276
##  0.3722982 -0.09483276  1.00000000
##
##  R-sq. pooled: 0.6199
##  Log-Likelihood:  -22689.3
##  Breusch-Pagan:  547.1  p-value: (2.96e-118)
##  LMM: 536.73  p-value: (5.23e-116)
```

2.2. Three-Stage Least Square estimation (3SLS): `spsur3sls`

The function `spsur3sls` estimate by IV the models

- **SUR-SAR:** Spatial autorregresive model
- **SUR-SDM:** Spatial Durbin model

By example:

$$
\begin{aligned}
&HR_{80} = \lambda_{HR}\, WHR_{80} + \beta_{10} + \beta_{11} PS_{80} + \beta_{12} UE_{80} + \epsilon_{HR}\\
&DV_{80} = \lambda_{DV}\, WDV_{80} + \beta_{20} + \beta_{21} PS_{80} + \beta_{22} UE_{80} + \beta_{23} SOUTH + \epsilon_{DV}\\
&FP_{79} = \lambda_{FP}\, WFP_{80} + \beta_{30} + \beta_{31} PS_{80} + \epsilon_{FP}
\end{aligned}
$$

```
data(NAT)
Tformula <- HR80 | DV80  | FP79 ~ PS80 + UE80 | PS80 + UE80 + SOUTH | PS80
NATSUR.sar.3sls <- spsur3sls(Form=Tformula,data=NAT,type="sar",W=W,maxlagW=2)
```

```
## Time to fit the model:  0.17  seconds
```

```
summary(NATSUR.sar.3sls)
```

```
## Call:
## spsur3sls(Form = Tformula, data = NAT, W = W, type = "sar", maxlagW = 2)
##
##
## Spatial SUR model type:  sar
##
## Equation  1
##                  Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_1  8.877617   1.307821  6.7881 1.206e-11 ***
## PS80_1         0.892351   0.138898  6.4245 1.388e-10 ***
## UE80_1        -0.060245   0.033200 -1.8146   0.06962 .
## lambda_1      -0.221472   0.191153 -1.1586   0.24664
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.1405
##   Equation  2
##                  Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_2 2.9935206  0.3798303  7.8812  3.61e-15 ***
## PS80_2        0.2533626  0.0262643  9.6467 < 2.2e-16 ***
## UE80_2        0.0914432  0.0095959  9.5294 < 2.2e-16 ***
## SOUTH_2       0.0346575  0.0471028  0.7358    0.4619
## lambda_2      0.2124110  0.0900048  2.3600    0.0183 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.291
##   Equation  3
##                  Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_3  7.670117   1.095699  7.0002 2.734e-12 ***
## PS80_3        -1.585681   0.160403 -9.8856 < 2.2e-16 ***
## lambda_3       0.386346   0.087474  4.4167 1.014e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.5611
##   Variance-Covariance Matrix of inter-equation residuals:
##  45.377766  1.867308 18.961586
##   1.867308  1.938112 -1.427426
##  18.961586 -1.427426 34.820491
## Correlation Matrix of inter-equation residuals:
##  1.0000000  0.1991154  0.4770190
```

```
##  0.1991154  1.0000000 -0.1737588
##  0.4770190 -0.1737588  1.0000000
##
##  R-sq. pooled: 0.3595
```

Step 3: Testing for misspecification in spatial SUR

3.1 Testing for the diagonality of Σ

The Breush-Pagan test of diagonality of Σ

$$H_0 : \Sigma = \sigma^2 I_R$$
$$H_A : \Sigma \neq \sigma^2 I_R$$

```
summary(NATSUR.sar)
```

```
## Call:
## spsurml(Form = Tformula, data = NAT, W = W, type = "sar")
##
##
## Spatial SUR model type:  sar
##
## Equation  1
##               Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_1 3.070929   0.259057 11.8543 < 2.2e-16 ***
## PS80_1        0.586505   0.105973  5.5345 3.207e-08 ***
## UE80_1        0.019504   0.029405  0.6633    0.5072
## lambda_1      0.535122   0.019782 27.0505 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.3073
##   Equation  2
##                 Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_2  1.1722543  0.0777684 15.0737 < 2.2e-16 ***
## PS80_2         0.2254184  0.0201623 11.1802 < 2.2e-16 ***
## UE80_2         0.0466549  0.0059494  7.8420 4.927e-15 ***
## SOUTH_2       -0.0411057  0.0364872 -1.1266    0.2599
## lambda_2       0.6815413  0.0157208 43.3528 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.4963
##   Equation  3
##                Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_3  3.074903   0.173140  17.760 < 2.2e-16 ***
## PS80_3        -1.079359   0.073941 -14.598 < 2.2e-16 ***
## lambda_3       0.754935   0.012749  59.215 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
## R-squared: 0.6614
##   Variance-Covariance Matrix of inter-equation residuals:
##  33.560585  1.2156737  8.1703336
##   1.215674  1.0940386 -0.4140574
##   8.170334 -0.4140574 13.9639287
## Correlation Matrix of inter-equation residuals:
##  1.0000000  0.2006254  0.3774168
##  0.2006254  1.0000000 -0.1059352
##  0.3774168 -0.1059352  1.0000000
##
##  R-sq. pooled: 0.6038
##  Log-Likelihood:  -22857.4
##  Breusch-Pagan: 598.23  p-value: (2.44e-129)
##  LMM: 176.46  p-value: (5.12e-38)
```

3.2 Marginal tests: $LM(\rho|\lambda)$ & $LM(\lambda|\rho)$

The Marginal Multiplier tests (LMM) are used to test for no correlation in one part of the model allowing for spatial correlation in the other. (Mur J, López FA and Herrera M, 2010)

- The $LM(\rho|\lambda)$ is the test for spatial error correlation in a model with subtantive spatial correlation (SUR-SAR; SUR-SDM).

$$H_0 : SUR - SAR$$
$$H_A : SUR - SARAR$$

```
summary(spcSUR.sar)
```

```
## Call:
## spsurml(Form = Tformula, data = spc, W = Wspc, type = "sar")
##
##
## Spatial SUR model type:  sar
##
## Equation  1
##                  Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_1  1.4947179  0.2467450  6.0577 5.244e-07 ***
## UN83_1         0.8053338  0.2558760  3.1474  0.003249 **
## NMR83_1       -0.5165301  0.2590369 -1.9940  0.053557 .
## SMSA_1        -0.0072526  0.0118566 -0.6117  0.544484
## lambda_1      -0.5048489  0.2405967 -2.0983  0.042763 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.622
##   Equation  2
##                  Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_2  1.7088172  0.2925087  5.8419 1.028e-06 ***
## UN80_2        -0.6736472  0.3870209 -1.7406   0.09007 .
## NMR80_2        0.7475734  0.3840119  1.9467   0.05919 .
## SMSA_2         0.0013487  0.0241871  0.0558   0.95583
## lambda_2      -0.4816232  0.2557338 -1.8833   0.06754 .
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.4743
##   Variance-Covariance Matrix of inter-equation residuals:
##   0.0003091646 -0.0003578072
##  -0.0003578072  0.0015874437
## Correlation Matrix of inter-equation residuals:
##   1.0000000 -0.5107461
##  -0.5107461  1.0000000
##
##  R-sq. pooled: 0.6601
##  Log-Likelihood:  114.099
##  Breusch-Pagan: 6.5431  p-value: (0.0105)
##  LMM: 0.50474  p-value: (0.477)
```

- The $LM(\lambda|\rho)$ is the test for subtantive spatial autocorrelation in a model with spatial autocorrelation in error term (SUR-SEM; SUR-SDEM).

$$H_0 : SUR - SEM$$
$$H_A : SUR - SARAR$$

```
summary(spcSUR.sem)
```

```
## Call:
## spsurml(Form = Tformula, data = spc, W = Wspc, type = "sem")
##
##
## Spatial SUR model type:  sem
##
## Equation  1
##                 Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_1  0.9798052  0.0150105 65.2748 < 2.2e-16 ***
## UN83_1         0.7508713  0.2253264  3.3324  0.001963 **
## NMR83_1       -0.5084650  0.2541467 -2.0007  0.052805 .
## SMSA_1        -0.0139768  0.0098142 -1.4241  0.162788
## rho_1         -0.6468828  0.2776346 -2.3300  0.025370 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.6652
##   Equation  2
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)_2  1.1473041  0.0385173 29.7867  < 2e-16 ***
## UN80_2        -0.4391249  0.3597557 -1.2206  0.22995
## NMR80_2        0.8530412  0.4077931  2.0918  0.04337 *
## SMSA_2         0.0046617  0.0203355  0.2292  0.81995
## rho_2         -0.5627421  0.2825038 -1.9920  0.05379 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.5123
##   Variance-Covariance Matrix of inter-equation residuals:
##   0.0002908458 -0.0003037319
##  -0.0003037319  0.0015339725
```

```
## Correlation Matrix of inter-equation residuals:
##   1.0000000 -0.4547265
##  -0.4547265  1.0000000
##
##  R-sq. pooled: 0.677
##  Log-Likelihood:  113.732
##  Breusch-Pagan: 5.1422  p-value: (0.0234)
##  LMM: 1.4089  p-value: (0.235)
```

3.3 Coefficient stability/homogeneity

3.3.1 Wald tests for beta coefficients: `wald_betas`

In a **SUR-SAR** the model:

$$\begin{aligned} WAGE_{83} &= \lambda_{83} W\ WAGE_{83} + \beta_{10} + \beta_{11} UN_{83} + \beta_{12} NMR_{83} + \boldsymbol{\beta_{13}} SMSA + \epsilon_{83} \\ WAGE_{81} &= \lambda_{81} W\ WAGE_{81} + \beta_{20} + \beta_{21} UN_{80} + \beta_{22} NMR_{80} + \boldsymbol{\beta_{23}} SMSA + \epsilon_{81} \end{aligned}$$

It's possible to test equality between SMSA coefficients in both equations:

$$\begin{aligned} H_0 &: \beta_{13} = \beta_{23} \\ H_A &: \beta_{13} \neq \beta_{23} \end{aligned}$$

```
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
spcSUR.sar <-spsurml(Form=Tformula,data=spc,type="sar",W=Wspc)
```

```
## Initial point:   log_lik:  113.198  lambdas:  -0.472 -0.446
## Iteration:  1   log_lik:  114.088  lambdas:  -0.506 -0.482
## Iteration:  2   log_lik:  114.098  lambdas:  -0.506 -0.482
## Iteration:  3   log_lik:  114.099  lambdas:  -0.505 -0.482
## Time to fit the model:  2.86  seconds
##
## Computing marginal test...
## Time to compute covariances:  0.3  seconds
```

```
R1 <- matrix(c(0,0,0,1,0,0,0,-1),nrow=1)
r1 <- matrix(0,ncol=1)
print(R1)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
## [1,]    0    0    0    1    0    0    0   -1
```

```
Wald_beta <- wald_betas(results=spcSUR.sar,R=R1,r=r1)
```

```
##
##  Wald stat.:  0.076  p-value ( 0.7824697 )
```

More complex hypothesis about β coefficients could be tested using R1 vector

$$H_0 : \beta_{13} = \beta_{23} \text{ and } \beta_{12} = \beta_{22}$$
$$H_A : \beta_{13} \neq \beta_{23} \text{ or } \beta_{12} \neq \beta_{22}$$

```
# Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
# spcSUR.sar <-spsurml(Form=Tformula,data=spc,type="sar",W=Wspc,trace=F)
R1 <- t(matrix(c(0,0,0,1,0,0,0,-1,0,0,1,0,0,0,-1,0),ncol=2))
r1 <- matrix(0,ncol=2)
print(R1)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
## [1,]    0    0    0    1    0    0    0   -1
## [2,]    0    0    1    0    0    0   -1    0
```

```
Wald_beta <- wald_betas(results=spcSUR.sar,R=R1,r=r1)
```

```
##
##  Wald stat.:  6.128  p-value ( 0.04670461 )
```

Estimate the restricted model

In case don't reject the null, it's possible to estimate the model with equal coefficient in both equations:

$$\begin{aligned} WAGE_{83} &= \lambda_{83} W\ WAGE_{83} + \beta_{10} + \beta_{11} UN_{83} + \beta_{12} NMR_{83} + \boldsymbol{\beta_{13}} SMSA + \epsilon_{83} \\ WAGE_{81} &= \lambda_{81} W\ WAGE_{81} + \beta_{20} + \beta_{21} UN_{80} + \beta_{22} NMR_{80} + \boldsymbol{\beta_{13}} SMSA + \epsilon_{81} \end{aligned}$$

```
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
R1 <- matrix(c(0,0,0,1,0,0,0,-1),nrow=1)
r1 <- matrix(0,ncol=1)
spcSUR.sar.restring <- spsurml(Form=Tformula, data=spc, type="sar", W=Wspc,R=R1,r=r1)
```

```
## Initial point:   log_lik:  113.163  lambdas:  -0.428 -0.422
## Iteration:  1   log_lik:  114.012  lambdas:  -0.482 -0.465
## Iteration:  2   log_lik:  114.051  lambdas:  -0.495 -0.476
## Iteration:  3   log_lik:  114.061  lambdas:  -0.501 -0.481
## Iteration:  4   log_lik:  114.065  lambdas:  -0.504 -0.484
## Iteration:  5   log_lik:  114.066  lambdas:  -0.506 -0.486
## Iteration:  6   log_lik:  114.066  lambdas:  -0.507 -0.486
## Time to fit the model:  4  seconds
##
## Computing marginal test...
## Time to compute covariances:  0.19  seconds
```

```
summary(spcSUR.sar.restring)
```

```
## Call:
## spsurml(Form = Tformula, data = spc, R = R1, r = r1, W = Wspc,
##     type = "sar")
##
##
## Spatial SUR model type:  sar
##
## Equation  1
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)_1  1.4990577  0.2438736  6.1469 3.58e-07 ***
## UN83_1         0.7621012  0.2283530  3.3374 0.001901 **
## NMR83_1       -0.4822134  0.2557282 -1.8856 0.067004 .
## SMSA_1        -0.0045365  0.0082392 -0.5506 0.585133
## lambda_1      -0.5070051  0.2386772 -2.1242 0.040217 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.6151
##   Equation  2
##               Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_2  1.71095    0.28908  5.9187 7.361e-07 ***
## UN80_2        -0.61585    0.32533 -1.8930   0.06599 .
## NMR80_2        0.70981    0.37559  1.8899   0.06642 .
## NA                  NA         NA      NA        NA
## lambda_2      -0.48645    0.25027 -1.9437   0.05936 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.4702
##   Variance-Covariance Matrix of inter-equation residuals:
##   0.0003161332 -0.0003795889
##  -0.0003795889  0.0016045055
## Correlation Matrix of inter-equation residuals:
##   1.0000000 -0.5329763
##  -0.5329763  1.0000000
##
##  R-sq. pooled: 0.656
##  Log-Likelihood:  114.066
##  Breusch-Pagan: 7.0904  p-value: (0.00775)
##  LMM: 0.38563  p-value: (0.535)
```

3.3.2 Wald test for 'spatial' coefficients homogeneity: `wald_deltas`

In same way a test for equal spatial autocorrelation coefficients can be obtain with `wald_deltas` function:
In the model:

$$\begin{aligned} WAGE_{83} &= \boldsymbol{\lambda_{83}} W\ WAGE_{83} + \beta_{10} + \beta_{11}UN_{83} + \beta_{12}NMR_{83} + \beta_{13}SMSA + \epsilon_{83} \\ WAGE_{81} &= \boldsymbol{\lambda_{81}} W\ WAGE_{81} + \beta_{20} + \beta_{21}UN_{80} + \beta_{22}NMR_{80} + \beta_{23}SMSA + \epsilon_{81} \end{aligned}$$

In this case the null is:

$$\begin{aligned} &H_0 : \lambda_{83} = \lambda_{81} \\ &H_A : \lambda_{83} \neq \lambda_{81} \end{aligned}$$

```
# Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
# spcSUR.sar <-spsurml(Form=Tformula,data=spc,type="sar",W=Wspc,trace=F)
```

```
R1 <- matrix(c(1,-1),nrow=1)
r1 <- matrix(0,ncol=1)
res1 <- wald_deltas(results=spcSUR.sar,R=R1,r=r1)
```

```
##
##  R:      lambda_1 lambda_2
## [1,]        1       -1
##
##  r:      [,1]
## [1,]    0
##
##  statistical discrepancies:             [,1]
## [1,] -0.02322566
##
##  Wald stat.:  0.006  p-value ( 0.9402822 )
```

3.3.3 Likekihood ratio tests `lr_betas_spsur`

Alternatively to wald test, the Likelihoo Ration (LR) tests can be obtain using the `lr_betas_spsur` function.

```
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
R <- matrix(c(0,0,0,1,0,0,0,-1),nrow=1)
r <- matrix(0,ncol=1)
LR_SMSA <-  lr_betas_spsur(Form=Tformula,data=spc,W=Wspc,type="sar",R=R,r=r,trace=F,printmodels=F)
```

```
##
##  Fitting unrestricted model ...
##
##  Time to fit unrestricted model:  2.62  seconds
##
##  Fitting restricted model ...Time to fit restricted model:  4.28  seconds
##
##  LR-Test
##
##  Log-likelihood unrestricted model:  114.0988
##  Log-likelihood restricted model:  114.0664
##  LR statistic:  0.065  degrees of freedom:  1  p-value: ( 0.7989591 )
```

4. Step 4: Marginal Effects: `impacts`

The marginal effects `impacts` of spatial autoregressive models (SUR-SAR; SUR-SDM; SUR-SARAR) has been calculated following the propose of LeSage and Pace (2009).

```
eff.spcSUR.sar <-impacts(spcSUR.sar,nsim=299)
```

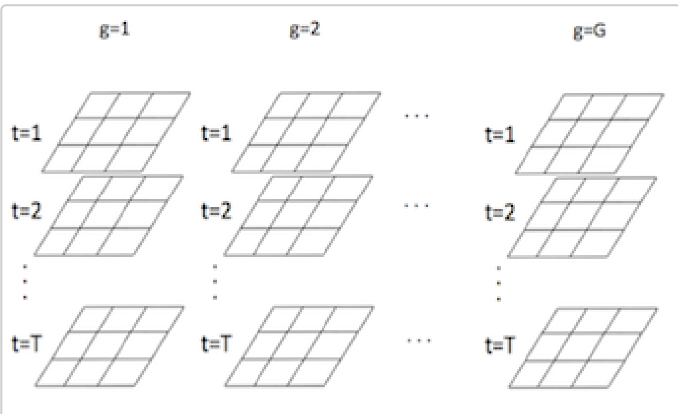
```
##
##
## Spatial SUR model type:  sar
##
##
##  Direct effects
##
```

```
##                mean         sd  t-stat    p-val   
## UN83_1   0.5420928  0.2016249  2.6886 0.007175 **
## NMR83_1 -0.3332948  0.1892673 -1.7610 0.078243 . 
## SMSA_1  -0.0044030  0.0083124 -0.5297 0.596329   
## UN80_2  -0.4813234  0.2922734 -1.6468 0.099594 . 
## NMR80_2  0.5060073  0.3009373  1.6814 0.092678 . 
## SMSA_2   0.0013184  0.0194590  0.0678 0.945981   
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## 
##  Indirect effects 
## 
##                 mean          sd  t-stat   p-val  
## UN83_1  -0.32109050  0.19076804 -1.6831 0.09235 .
## NMR83_1  0.18596093  0.12800270  1.4528 0.14628  
## SMSA_1   0.00173167  0.00478248  0.3621 0.71729  
## UN80_2   0.26052177  0.20698670  1.2586 0.20816  
## NMR80_2 -0.26588364  0.20653858 -1.2873 0.19798  
## SMSA_2   0.00024799  0.01078069  0.0230 0.98165  
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## 
##  Total effects 
## 
##                mean         sd  t-stat    p-val   
## UN83_1   0.5420928  0.2016249  2.6886 0.007175 **
## NMR83_1 -0.3332948  0.1892673 -1.7610 0.078243 . 
## SMSA_1  -0.0044030  0.0083124 -0.5297 0.596329   
## UN80_2  -0.4813234  0.2922734 -1.6468 0.099594 . 
## NMR80_2  0.5060073  0.3009373  1.6814 0.092678 . 
## SMSA_2   0.0013184  0.0194590  0.0678 0.945981   
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

5. Step 5: The `spSUR` in a panel data framework

5.1 The `spSUR` with G equation and T periods

Case of T temporal cross-sections and G equations



By example with NAT data set:

- T = 4 (Four temporal periods)
- G = 2 (Two equations with different numbers of independent variables)
- R = 3085 (Spatial observations)

A **SUR-SAR-PANEL** model

$y_{gt} = \lambda_g W y_{gt} + X_{gt}\beta_g + \epsilon_{gt};$

$g = 1, \ldots, G;\ t = 1, \ldots, T$

$Corr(\epsilon_{gt}, \epsilon_{g't}) = Corr(\epsilon_g, \epsilon_{g'}) \neq 0$ for $(\forall t)$

```
data(NATPanel)
Tformula <- HR | HC ~ UE + RD | UE
spSUR.sar.panel <- spsurml(Form = Tformula, data = NATpanel, W = W,
                           type = "sar", nR = 3085, nG = 2, nT = 4)
```

```
## Initial point:   log_lik:  -102689.6  lambdas:  0.415 0.148
## Iteration:  1   log_lik:  -102604  lambdas:  0.435 0.149
## Iteration:  2   log_lik:  -102604  lambdas:  0.435 0.149
## Iteration:  3   log_lik:  -102604  lambdas:  0.435 0.149
## Time to fit the model:  8.4  seconds
##
## Computing marginal test...
## Time to compute covariances:  41.72  seconds
```

```
summary(spSUR.sar.panel)
```

```
## Call:
## spsurml(Form = Tformula, data = NATpanel, W = W, nG = 2, nR = 3085,
##     nT = 4, type = "sar")
##
##
## Spatial SUR model type:  sar
##
## Equation  1
##               Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_1 2.993127   0.124471 24.0468 < 2.2e-16 ***
## UE_1          0.066676   0.016614  4.0132 6.007e-05 ***
## RD_1          2.153448   0.057369 37.5366 < 2.2e-16 ***
## lambda_1      0.434979   0.010755 40.4434 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.3721
##   Equation  2
##               Estimate Std. Error t value  Pr(>|t|)
## (Intercept)_2 2.861119   0.879270  3.2540  0.001140 **
## UE_2          0.361676   0.134899  2.6811  0.007343 **
## lambda_2      0.148712   0.014051 10.5839 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.02017
##   Variance-Covariance Matrix of inter-equation residuals:
##  28.35597   46.6441
##  46.64410 2011.7583
## Correlation Matrix of inter-equation residuals:
```

```
##  1.000000 0.195293
##  0.195293 1.000000
##
##  R-sq. pooled: 0.02685
##  Log-Likelihood:  -102604
##  Breusch-Pagan: 470.64  p-value: (2.33e-104)
##  LMM: 1330.3  p-value: (2.9e-291)
```

5.1.1 Unobserved effects: The `demeaning` option

`spsur` package offers the possibility to transform the original data in order to remove potential unobserved effects.
The most popular transformation is demeaning the data: To subtract the sampling averages of each individual, in every equation, from the corresponding observation.

```
Tformula <- HR | HC ~ UE + RD | UE
spSUR.sar.panel <- spsurml(Form = Tformula, data = NATpanel, W = W,
                           type = "sar", nR = 3085, nG = 2, nT = 4,
                           demean = TRUE)
```

```
## Initial point:   log_lik:  -88996.94  lambdas:  0.223 0.181
## Iteration:  1   log_lik:  -88991.67  lambdas:  0.226 0.182
## Iteration:  2   log_lik:  -88991.67  lambdas:  0.226 0.182
## Time to fit the model:  6.19  seconds
##
## Computing marginal test...
## Time to compute covariances:  41.66  seconds
```

```
summary(spSUR.sar.panel)
```

```
## Call:
## spsurml(Form = Tformula, data = NATpanel, W = W, nG = 2, nR = 3085,
##     nT = 4, demean = TRUE, type = "sar")
##
##
## Spatial SUR model type:  sar
##
## Equation  1
##          Estimate Std. Error t value Pr(>|t|)
## UE_1     0.066187   0.021224  3.1185  0.00182 **
## RD_1     1.020898   0.120140  8.4975  < 2e-16 ***
## lambda_1 0.226039   0.013521 16.7174  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.04865
##   Equation  2
##          Estimate Std. Error t value  Pr(>|t|)
## UE_2     0.396048   0.097954  4.0432 5.288e-05 ***
## lambda_2 0.182391   0.013886 13.1352 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## R-squared: 0.02959
##   Variance-Covariance Matrix of inter-equation residuals:
##  17.00875  10.99058
```

```
##  10.99058 371.66373
## Correlation Matrix of inter-equation residuals:
##  1.0000000 0.1382323
##  0.1382323 1.0000000
##
##  R-sq. pooled: 0.03044
##  Log-Likelihood:  -88991.7
##  Breusch-Pagan: 235.79  p-value: (3.25e-53)
##  LMM: 1523.9  p-value: (   0)
```

6. Aditional functionalities: `dgp_spSUR`

A Data Generating Process of a spatial SUR models is avalible using the function `dgp_spSUR`

```
nT <- 1 # Number of time periods
nG <- 3 # Number of equations
nR <- 500 # Number of spatial elements
p <- 3 # Number of independent variables
Sigma <- matrix(0.3,ncol=nG,nrow=nG)
diag(Sigma)<-1
Coeff <- c(2,3)
rho <- 0.5 # level of spatial dependence
lambda <- 0.0 # spatial autocorrelation error term = 0
# ramdom coordinates
# co <- cbind(runif(nR,0,1),runif(nR,0,1))
# W <- spdep::nb2mat(spdep::knn2nb(spdep::knearneigh(co,k=5,longlat=F)))
# DGP <- dgp_spSUR(Sigma=Sigma,Coeff=Coeff,rho=rho,lambda=lambda,nT=nT,nG=nG,nR=nR,p=p,W=W)
```

7. Conclusion & work to do

- `spSUR` is a powerful R-package to test, estimate and looking for the correct specification
- More functionalities and estimation algorithm coming soon
    - GMM estimation
    - ML estimation with equal level of spatial dependence (λ/ρ=constant)
    - Orthogonal demeaning for space-time SUR models
    - …..
- The spSUR is avaliable in GitHub [https://github.com/rominsal/spSUR/]

References

- López, F.A., P. J. Martínez-Ortiz, and J.G. Cegarra-Navarro (2017). Spatial spillovers in public expenditure on a municipal level in spain. *The Annals of Regional Science* 58 (1), 39–65.
- López, F.A., J. Mur, and A. Angulo (2014). Spatial model selection strategies in a sur framework. the case of regional productivity in eu. *The Annals of Regional Science* 53 (1), 197–220.
- Mur, J., F. López, and M. Herrera (2010). Testing for spatial effects in seemingly unrelated regressions. *Spatial Economic Analysis* 5 (4), 399–440.