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A Modified Gurson Model: Formulation and Implementation

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A Modified Gurson Model: Formulation and Implementation

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Abstract

In this report a modified Gurson modeled is presented. It can be used to model ductile behavior up to and including material failure. The formulation incorporates the Gurson failure surface, including void nucleation, growth, and coalescence, with a J_2 yield surface with user-defined hardening behavior. Aspects of the formulation and implementation will be discussed.

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Chapter 1

Introduction

The purpose of this work is to present a modified Gurson constitutive model for use in capturing the behavior of ductile materials in the failure regime. The Gurson model has been used extensively in metal plasticity, starting with [3], and the modified Gurson model described here was recently proposed by [4], and extended to include void nucleation in [5]. In this treatment we incorporate a hyperelastic strain energy potential to define the underlying model of elasticity, and investigate a fully implicit Newton algorithm for integration of the evolution equations associated with the state variables used to define the constitutive response.

Chapter 2

Model Formulation

This section should begin with a short description about the origins of the model, including the desire to capture porous solid behavior related to void growth with an inelastic body. Then some comments about the thermodynamic motivation to place it within a hyperelastic framework. It should include sections on the flow rule and each piece of the void volume fraction evolution equation.

2.1 Background

The original Gurson model was developed by [3] based on rigorous micromechanical analysis of a characteristic volume element with spherical or cylindrical-shaped voids surrounded by rigid plastic matrix materials. This model was motivated by experimental observations of dilational plastic deformation during ductile fracture in porous metals, which generates considerable porosity due to the growth and nucleation of voids. Tvergaard and Needleman [10] later improved the original Gurson model by introducing additional parameters into the yield function and by introducing an effective void volume fraction term to account for coalescence of voids that better capture the damage growth rates. This model is known as the 'Gurson-Tvergaard-Needleman' or the GTN model.

One important limitation of the original Gurson or the GTN model is that the void growth (i.e., the damage) depends only on the mean stress. In a shear-dominated state, such as in a projectile penetration problem, the model is unable to predict damage growth if continuous void nucleation is not invoked. This limitation motivates a modification of the original void growth law to include a shear term proposed in [4]. This modification, though phenomenological in nature, has been shown to improve the prediction of the Gurson model in situations where shear stress dominates [4, 5].

The original Gurson model and the recent shear modification were both formulated either with a small deformation assumption or within a hypoelastic framework. The large deformation typically encountered in a ductile failure simulation would render the small deformation formulation inappropriate. As for the hypoelastic formulation, there are well known drawbacks such as the non-zero work done in a closed cycle of elastic deformation, which violates the most important axiom of an elastic response ([1]).

To avoid the above problems, in this section, the shear-modified Gurson model in [4, 5] will be reformulated within a large deformation hyperelastic constitutive framework. Since the elastic response is derived from a hyperelastic potential, the work done in a closed elastic deformation loop vanishes exactly. Furthermore, the hyperelastic formulation eliminates the need for incrementally objective stress update algorithms and can be easily integrated with frame-invariant formulations of anisotropic elasticity and anisotropic plastic yielding ([1]).

2.2 Preliminaries for large deformation hyperelastic formulation

2.2.1 Kinematic preliminaries

To set the stage for the hyperelastic formulation, the kinematic preliminaries for large deformation elastoplasticity are summarized in this section, along with the quantities and relations that will be used in subsequent model development.

An essential feature of this elastoplasticity framework is the multiplicative decomposition of the deformation gradient $\boldsymbol{F}$ into an elastic part $\boldsymbol{F}^e$ and a plastic part $\boldsymbol{F}^p$

$$\boldsymbol{F} = \boldsymbol{F}^e \cdot \boldsymbol{F}^p. \tag{2.1}$$

This decomposition introduces the notion of an intermediate local configuration (cf. [8] and the references therein for the motivation and micromechanical basis for such a decomposition).

Next, we introduce a set of strain measures associated with the multiplicative decomposition that will be used extensively in the model development. First is the right Cauchy-Green tensor $\boldsymbol{C}$, and its plastic counterpart $\boldsymbol{C}^p$, which are defined in the reference configuration

$$\boldsymbol{C} = \boldsymbol{F}^T \cdot \boldsymbol{F} \tag{2.2}$$

$$\boldsymbol{C}^p = \boldsymbol{F}^{pT} \cdot \boldsymbol{F}^p \tag{2.3}$$

where $\boldsymbol{F}^T$ is the transpose of $\boldsymbol{F}$.

In the current configuration we consider the left Cauchy-Green tensor $\boldsymbol{b}$, and its elastic counterpart $\boldsymbol{b}^e$

$$\boldsymbol{b} = \boldsymbol{F} \cdot \boldsymbol{F}^T \tag{2.4}$$
$$\boldsymbol{b}^e = \boldsymbol{F}^e \cdot \boldsymbol{F}^{eT} \tag{2.5}$$

The above fundamental strain measures are related via pull-back and push-forward operations

$$\boldsymbol{C}^{-p} = \boldsymbol{F}^{-1} \cdot \boldsymbol{b}^e \cdot \boldsymbol{F}^{-T} \tag{2.6}$$
$$\boldsymbol{b}^e = \boldsymbol{F} \cdot \boldsymbol{C}^{-p} \cdot \boldsymbol{F}^T \tag{2.7}$$

In metal plasticity, a standard assumption is that plastic flow is isochoric (volume-preserving), i.e. $\det(\boldsymbol{F}^p) = 1$, which implies

$$J = \det(\boldsymbol{F}) = \det(\boldsymbol{F}^e) \tag{2.8}$$

Defining $J^e = \det(\boldsymbol{F}^e)$, we have the volume preserving part of the elastic left Cauchy-Green tensor $\bar{\boldsymbol{b}}^e$

$$\bar{\boldsymbol{b}}^e = J^{e^{-2/3}} \boldsymbol{b}^e = J^{-2/3} \boldsymbol{b}^e. \tag{2.9}$$

2.2.2 Hyperelastic constitutive relation

The starting point of the hyperelastic constitutive formulation is the assumption of the existence of a strain-energy function, which is proposed to have the following form

$$\Psi = \Psi^{\mathrm{vol}}[J^e] + \Psi^{\mathrm{iso}}[\bar{\boldsymbol{b}}^e] \tag{2.10}$$

Here the strain-energy function Ψ is a decoupled function of the volumetric part (i.e., $J^e = \det \boldsymbol{F}^e$) and the isochoric part (i.e., $\bar{\boldsymbol{b}}^e$) of the elastic deformation. The volumetric and the isochoric parts of the strain-energy function are given as

$$\Psi^{\mathrm{vol}}[J^e] = \frac{1}{2}\kappa(\ln J^e)^2 \tag{2.11}$$

$$\Psi^{\mathrm{iso}}[\bar{\boldsymbol{b}}^e] = \mu(\frac{1}{2}\ln \bar{\boldsymbol{b}}^e) : (\frac{1}{2}\ln \bar{\boldsymbol{b}}^e) \tag{2.12}$$

where κ and μ are the bulk and shear modulus, and the elastic logarithmic Hencky strains $\frac{1}{2}\ln \boldsymbol{b}^e$ is used as the strain measure ([9]). The elastic constitutive law and the Kirchhoff stresses are given as

$$\boldsymbol{\tau} = \kappa \ln J^e \boldsymbol{g}^{-1} + \mu \ln \bar{\boldsymbol{b}}^e = \kappa(\ln \boldsymbol{b}^e : \boldsymbol{g})\boldsymbol{g}^{-1} + \mu \operatorname{dev} \ln \boldsymbol{b}^e \tag{2.13}$$

where $\boldsymbol{g}$ is the metric tensor. The Kirchhoff pressure p and the deviatoric stress tensor $\boldsymbol{s}$ are related to the elastic strain measure as

$$p = \frac{1}{3}\operatorname{tr}(\boldsymbol{\tau}) = \frac{1}{2}\kappa \operatorname{tr} \ln \boldsymbol{b}^e = \frac{1}{2}\kappa \ln \det \boldsymbol{b}^e \tag{2.14}$$

$$\boldsymbol{s} = \operatorname{dev}(\boldsymbol{\tau}) = \mu \operatorname{dev} \ln \boldsymbol{b}^e \tag{2.15}$$

2.3 Constitutive relations of the Gurson model

Within the previously described large deformation hyperelastic framework, key components of the constitutive relations of the shear-modified Gurson model are presented in this section, including the yield function, the hardening law, the flow rule and the evolution law for the shear-modified void growth.

2.3.1 Yield function

The yield function Φ of the Gurson model can be written in terms of the previously defined Kirchhoff mean stress p and deviatoric stress tensor $\boldsymbol{s}$ as

$$\Phi = \|\boldsymbol{s}\| - \sqrt{\frac{2}{3}}\operatorname{sign}(\psi)\sqrt{|\psi|}Y \tag{2.16}$$

where ψ contains contributions from the damage in the material and Y is the Kirchhoff yield stress. This yield function (2.16) is a linear form, in terms of $\|\boldsymbol{s}\|$, of the Gurson yield criteria. This particular form is implemented to facilitate comparisons with existing J_2-like models in the Albany analysis code [6], where the yield functions are mostly written as linear functions of $\|\boldsymbol{s}\|$.

The function ψ directly relates to the void volume fraction of the porous solid and is given as

$$\psi = 1 + f^2 - 2f\cosh(v), \quad v = \frac{3p}{2Y} \tag{2.17}$$

where p is the Kirchhoff pressure defined in (2.14), and f is the void volume fraction of the porous solid. The Kirchhoff yield stress Y describes the hardening of the undamaged matrix material.

2.3.2 Hardening law

The hardening law relates the yield strength Y to some measure of plastic deformation. One example of a nonlinear hardening law proposed by [8] for metal is written as

$$Y = Y_0 + Y_\infty \left[1 - \exp(-\delta\varepsilon_q)\right] + K\varepsilon_q \tag{2.18}$$

where ε_q is the equivalent plastic strain, Y_0 is the initial yield strength, Y_∞ is the residual flow stress, K is the hardening coefficient, and δ is the saturation exponent. Other forms of hardening law can also be used depending on the observed material behavior.

The plastic work increment in the matrix material is equal to the macroscopic plastic work increment, which can be used to derive the evolution equation for the equivalent plastic strain ε_q as

$$\dot{\varepsilon}_q Y(1-f) = \boldsymbol{\tau} : \left(\gamma \frac{\partial \Phi}{\partial \boldsymbol{\tau}}\right) \tag{2.19}$$

Substituting yield function Φ into (2.19) to obtain the expression to compute $\dot{\varepsilon}_q$

$$\dot{\varepsilon}_q = \frac{1}{1-f}\left(\gamma\sqrt{\frac{2|\psi|}{3}}\operatorname{sign}(\psi) + \frac{pt}{Y}\right) \tag{2.20}$$

2.3.3 Flow rule

Following the standard procedure of the principle of maximum dissipation, Simo and Miehe [7] proposed a general form of associate flow rule, which is adopted in the current formulation and is given as

$$-\frac{1}{2}L_v(\boldsymbol{b}^e)\cdot\boldsymbol{b}^{e^{-1}} = \gamma\frac{\partial\Phi}{\partial\boldsymbol{\tau}} = \gamma\boldsymbol{n} + \frac{1}{3}t\mathbf{1} \tag{2.21}$$

where $L_v(\boldsymbol{b}^e) = \boldsymbol{F} \cdot \dot{\boldsymbol{C}}^{p^{-1}} \cdot \boldsymbol{F}^T$ is the Lie derivative, and $\boldsymbol{n}$ and t are the deviatoric and volumetric component of the gradient term, respectively. Substituting the yield function (2.16) leads to the following

$$\boldsymbol{n} = \frac{\boldsymbol{s}}{\|\boldsymbol{s}\|} \tag{2.22}$$

$$t = \mathrm{tr}\left(\gamma \frac{\partial \Phi}{\partial \boldsymbol{\tau}}\right) = \sqrt{\frac{3}{2}} \gamma f \sinh\left(\frac{3p}{2Y}\right) \frac{1}{\sqrt{|\psi|}} \mathrm{sign}(\psi) \tag{2.23}$$

2.3.4 Evolution of void volume fraction

The void volume fraction f is the internal variable that characterizes the material damage. The rate of change in total void volume fraction, $\dot{f}$, is typically given by the sum of contributions due to the void growth, $\dot{f}_g$, and the nucleation of new voids, $\dot{f}_n$.

$$\dot{f} = \dot{f}_g + \dot{f}_n \tag{2.24}$$

In the original Gurson model [3], the void growth part $\dot{f}_g$ was related to the plastic volume change as

$$\dot{f}_g = (1 - f)\, \mathrm{tr}\left(\gamma \frac{\partial \Phi}{\partial \boldsymbol{\tau}}\right) \tag{2.25}$$

To account for the void growth under shear-dominated stress state, the void growth law (2.25) was extended in [4] by adding a term that depends on the third stress invariant. This shear-modified void growth equation is written as

$$\dot{f}_g = (1 - f)\, \mathrm{tr}\left(\gamma \frac{\partial \Phi}{\partial \boldsymbol{\tau}}\right) + k_\omega f \frac{\omega(\boldsymbol{\tau})}{\tau_e} \boldsymbol{s} : \left(\gamma \frac{\partial \Phi}{\partial \boldsymbol{\tau}}\right) \tag{2.26}$$

where $\tau_e = \sqrt{3/2}\|\boldsymbol{s}\|$ is the effective deviatoric Kirchhoff stress, k_ω is a material constant that sets the magnitude of the damage growth rate in pure shear states [4]. The function $\omega(\boldsymbol{\tau})$ includes the effect of the third stress invariant on void growth and is given as

$$\omega(\boldsymbol{\tau}) = 1 - \left(\frac{27 J_3}{2\tau_e^3}\right)^2 \tag{2.27}$$

where $J_3 = \det(\boldsymbol{s})$ is the third invariant of deviatoric Kirchhoff stress tensor.

Substituting the yield function (2.16) and the expression for $\omega(\boldsymbol{\tau})$ (2.27) into the void growth law (2.26) yields

$$\dot{f}_g = (1-f)t + \sqrt{\frac{2}{3}}\gamma k_\omega f \omega(\boldsymbol{\tau}) \tag{2.28}$$

The effective increase in damage due to plastic strain controlled nucleation is given by [2] as

$$\dot{f}_{nu} = A\dot{\varepsilon}_q \tag{2.29}$$

where the parameter A is defined as a function of the matrix equivalent plastic strain ε_q

$$A = \begin{cases} \frac{f_N}{s_N\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\varepsilon_q - \epsilon_N}{s_N}\right)^2\right], & p \geq 0 \\ 0, & p < 0 \end{cases} \tag{2.30}$$

where the nucleation strain follows a normal distribution with a mean value ϵ_N and a standard deviation s_N with the volume fraction of the nucleated voids given by f_N.

Chapter 3

Implementation

This section should discuss implementation aspects of the large deformation hyperelastic model and the implicit integration scheme. I based the Sierra implementation off of the Albany version, so that would be a good place to start. In both places we are using Sacado to compute derivative for the local consistent tangent, so we should talk about what that buys us.

The details of numerical implementation of the large deformation hyperelastic Gurson model are discussed in this section. A fully implicit integration scheme ([8, 9]) is implemented to integrate stress response over a finite time step $\Delta t = t_{n+1} - t_n$, for given state represented by $\varepsilon_{q(n)}, f_n, \boldsymbol{b}_{e(n)}$ and deformation gradient $\boldsymbol{F}_{n+1}$. The integration scheme consists of an elastic trial state followed by plastic corrections.

3.1 Discrete form of the rate equations

For the discrete form of the rate equations, the flow rule (2.21) is first written in the material (reference) configuration through a pull-back operation

$$-\frac{1}{2}\dot{\boldsymbol{C}}_p^{-1} \cdot \boldsymbol{C}_p = \gamma \boldsymbol{F}^{-1} \frac{\partial \Phi}{\partial \boldsymbol{\tau}} \boldsymbol{F} \tag{3.1}$$

Then, the application of the exponential mapping to (3.1) yields an incremental objective integration algorithm

$$\boldsymbol{C}_{p(n+1)}^{-1} = \boldsymbol{F}_{n+1}^{-1} \exp\left(-2\Delta\gamma \frac{\partial \Phi_{n+1}}{\partial \boldsymbol{\tau}}\right) \boldsymbol{F}_{n+1} \boldsymbol{C}_{p(n)}^{-1} \tag{3.2}$$

Applying the push-forward operation to (3.2) renders an update algorithm for the elastic left Cauchy-Green tensor

$$\boldsymbol{b}_{e(n+1)} = \exp\left(-2\Delta\gamma\frac{\partial\Phi_{n+1}}{\partial\boldsymbol{\tau}}\right)\cdot\boldsymbol{b}_e^{\mathrm{tr}} \tag{3.3}$$

where the trial elastic left Cauchy-Green tensor is given by

$$\boldsymbol{b}_e^{\mathrm{tr}} = \boldsymbol{F}_{n+1}\cdot\boldsymbol{C}_{p(n)}^{-1}\cdot\boldsymbol{F}_{n+1}^T \tag{3.4}$$

From elastic and plastic isotropy, $\boldsymbol{b}_{e(n+1)}, \boldsymbol{b}_e^{\mathrm{tr}}$ and $\boldsymbol{\tau}$ have identical principal axes. Then, the logarithmic Henchy strains follow as

$$\ln\boldsymbol{b}_{e(n+1)} = \ln\boldsymbol{b}_e^{\mathrm{tr}} - 2\Delta\gamma\frac{\partial\Phi_{n+1}}{\partial\boldsymbol{\tau}} \tag{3.5}$$

From the elastic constitutive relations derived in Eqs.(2.14) and (2.15), the Kirchhoff pressure and deviatoric stress tensor can be obtained as

$$p_{n+1} = p^{\mathrm{tr}} - \kappa t \tag{3.6}$$
$$\boldsymbol{s}_{n+1} = \boldsymbol{s}^{\mathrm{tr}} - 2\mu\Delta\gamma\boldsymbol{n} \tag{3.7}$$

where $\boldsymbol{n}$ and t are given by Eqs.(2.22), (2.23) and evaluated at time t_{n+1}. The trial state are

$$p^{\mathrm{tr}} = \kappa\ln J_e^{\mathrm{tr}}, \quad J_e^{\mathrm{tr}} = \det(\boldsymbol{b}_e^{\mathrm{tr}})^{1/2} \tag{3.8}$$
$$\boldsymbol{s}^{\mathrm{tr}} = \mu\,\mathrm{dev}\ln\boldsymbol{b}_e^{\mathrm{tr}} \tag{3.9}$$

The discrete form of evolution equations for internal variables ε_q and f are obtained by apply backward Euler to (2.24) and (2.20).

$$f_{n+1} = f_n + (1-f_{n+1})t + \sqrt{\frac{2}{3}}\Delta\gamma k_\omega f_{n+1}\omega(\boldsymbol{\tau}) + A_{n+1}(\varepsilon_{q(n+1)} - \varepsilon_{q(n)}) \tag{3.10}$$
$$\varepsilon_{q(n+1)} = \varepsilon_{q(n)} + \frac{1}{1-f_{n+1}}\left(\Delta\gamma\sqrt{\frac{2|\psi|}{3}}\mathrm{sign}(\psi) + \frac{p_{n+1}t}{Y}\right) \tag{3.11}$$

3.2 Implicit integration algorithm

The discrete form of the rate equations (3.3), (3.6), (3.7), (3.10) and (3.11) include four unknowns, i.e., the unknown vector

$$\boldsymbol{X} = \{p, f, \varepsilon_q, \Delta\gamma\} \tag{3.12}$$

which will be obtained from solving the following nonlinear system of equations. For simplicity, in the following section, we will omit the index $n+1$ referring to the current time step. The resulting nonlinear system of equations follow

$$R_1(\boldsymbol{X}) = \|\boldsymbol{s}^{\mathrm{tr}}\| - 2\mu\Delta\gamma - \sqrt{\frac{2}{3}}\mathrm{sign}(\psi)\sqrt{|\psi|}Y \tag{3.13}$$

$$R_2(\boldsymbol{X}) = p - p^{\mathrm{tr}} + \kappa t \tag{3.14}$$

$$R_3(\boldsymbol{X}) = f - f_n - (1-f)t - \sqrt{\frac{2}{3}}\Delta\gamma k_\omega f\omega(\boldsymbol{\tau}) - A(\varepsilon_q - \varepsilon_{q(n)}) \tag{3.15}$$

$$R_4(\boldsymbol{X}) = \varepsilon_q - \varepsilon_{q(n)} - \frac{1}{1-f}\left(\Delta\gamma\sqrt{\frac{2|\psi|}{3}}\mathrm{sign}(\psi) + \frac{pt}{Y}\right) \tag{3.16}$$

The above system of equations can be solved through iterative solution procedures like the Newton's method, which requires consistent linearisations.

The integration algorithm is summarized in the following box

Box 1. Integration algorithm for shear-modified finite deformation Gurson model

GIVEN: $\varepsilon_{q(n)}, f_n, \boldsymbol{b}_{e(n)}$ and $\boldsymbol{F}$
FIND: $\boldsymbol{\tau}, \varepsilon_q, f, \boldsymbol{b}_e(\boldsymbol{F}_p)$
STEP 1. Compute trial elastic left Cauchy-Green tensor $\boldsymbol{b}_e^{\mathrm{tr}}$ (3.4)
STEP 2. Compute trial Kirchhoff pressure and deviatoric tensor $p^{\mathrm{tr}}, \boldsymbol{s}^{\mathrm{tr}}$ (3.8), (3.9)
STEP 3. Check yielding (2.16): $\Phi^{\mathrm{tr}}(p^{\mathrm{tr}}, \boldsymbol{s}^{\mathrm{tr}}, \varepsilon_{q(n)}, f_n) > 0$?
No, set $p = p^{\mathrm{tr}}, \boldsymbol{s} = \boldsymbol{s}^{\mathrm{tr}}, \boldsymbol{b}_e = \boldsymbol{b}_e^{\mathrm{tr}}, \varepsilon_q = \varepsilon_{q(n)}, f = f_n$ and exit
STEP 4. Yes, local Newton loop
4.1 Initialize $\boldsymbol{X}^k$ (3.12) and iteration count $k = 0$
4.2 Assemble residual $\boldsymbol{R}(\boldsymbol{X}^k)$ (3.13) - (3.16)
4.3 Check convergence: $\| \boldsymbol{R} \| < tol$?
Yes, converged and go to STEP 5
4.4 No, compute local Jacobian matrix $\boldsymbol{J} = \partial\boldsymbol{R}/\partial\boldsymbol{X}$
4.5 Solve system of equations $\boldsymbol{J}\cdot\delta\boldsymbol{X} = \boldsymbol{R}$ for $\delta\boldsymbol{X}$
4.6 Update $\boldsymbol{X}^{k+1} = \boldsymbol{X}^k - \delta\boldsymbol{X}$, $k \to k+1$ and go to 4.2
STEP 5. Update $\boldsymbol{\tau} = \boldsymbol{s} + p\boldsymbol{g}$, and $\varepsilon_q, f, \boldsymbol{F}_p^*$

*The plastic deformation gradient, which is used in (3.1) and (3.4) to compute trial state, is updated using

$$\boldsymbol{F}_p = \exp\left(\frac{\partial \Phi}{\partial \boldsymbol{\tau}}\right) \cdot \boldsymbol{F}_{b(n)} \tag{3.17}$$

The linearization of the system of equations requires evaluating the local Jacobian matrix $\boldsymbol{J} = \partial \boldsymbol{R} / \partial \boldsymbol{X}$. In this work, we used an numerical exact method called Fast Automatic Differentiation (FAD) to evaluating all the Jacobian. FAD provides an efficient and convenient way to evaluate derivatives. A detailed description of FAD can be found in [CITE].

3.3 FAD: a numerical exact way of computing consistent tangent

Chapter 4

Results

This section could be renamed to Numerical Examples, and should include any and all verification/validation work. To be clear, for the purpose of the SAND report, we don't require any validation. I will have to include a description of the Sierra input parameters for use in their user's manual, but obviously I'll worry about that.

Chapter 5

Conclusions

This should be a summary and short discussion about the strengths and weaknesses of the model. Nothing too fancy, just practice for the journal article.

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