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A Modified Gurson Model: Formulation and Implementation

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A Modified Gurson Model: Formulation and Implementation

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Abstract

In this report a modified Gurson modeled is presented. It can be used to model ductile behavior up to and including material failure. The formulation incorporates the Gurson failure surface, including void nucleation, growth, and coalescence, with a J_2 yield surface with user-defined hardening behavior. Aspects of the formulation and implementation will be discussed.

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Chapter 1

Introduction

The purpose of this work is to present a modified Gurson constitutive model for use in capturing the behavior of ductile materials in the failure regime. The Gurson model has been used extensively in metal plasticity, starting with [2], and the modified Gurson model described here was recently proposed by [3], and extended to include void nucleation in [4]. In this treatment we incorporate a hyperelastic strain energy potential to define the underlying model of elasticity, and investigate a fully implicit Newton algorithm for integration of the evolution equations associated with the state variables used to define the constitutive response.

Chapter 2

Model Formulation

This section should begin with a short description about the origins of the model, including the desire to capture porous solid behavior related to void growth with an inelastic body. Then some comments about the thermodynamic motivation to place it within a hyperelastic framework. It should include sections on the flow rule and each piece of the void volume fraction evolution equation.

TODO: Short description of the model and motivations behind

TODO: add some comments on the thermodynamic motivation for hyperelastic framework

In this section, the hyperelastic formulation of the shear-modified Gurson model is presented. The framework extends the small-deformation version presented in [3, 4] to a hyperelastic finite deformation formulation.

2.1 Preliminaries for hyperelastic finite deformation formulation

2.1.1 Kinematic preliminaries

To set the stage for the hyperelastic constitutive relation, the kinematic preliminaries for finite deformation elastoplasticity are summarized in this section, along with the quantities and relations used in subsequent model development.

An essential feature of this elastoplasticity framework is the multiplicative decomposition of deformation gradient $\boldsymbol{F}$ into an elastic part $\boldsymbol{F}^e$ and a plastic part $\boldsymbol{F}^p$

$$\boldsymbol{F} = \boldsymbol{F}^e \boldsymbol{F}^p. \tag{2.1}$$

This decomposition introduces the notion of an intermediate local configuration (cf. [6]

and the references therein for the motivation and micromechanical basis for such a decomposition).

Next, we introduce a set of strain measures associated with the multiplicative decomposition that will be used extensively in the model development. First is the right Cauchy-Green tensor $\boldsymbol{C}$, and its plastic counterpart $\boldsymbol{C}^p$, which are defined in the reference configuration

$$\boldsymbol{C} = \boldsymbol{F}^T \boldsymbol{F}, \tag{2.2}$$
$$\boldsymbol{C}^p = \boldsymbol{F}^{pT} \boldsymbol{F}^p. \tag{2.3}$$

In the current configuration we consider the left Cauchy-Green tensor $\boldsymbol{b}$, and its elastic counterpart $\boldsymbol{b}^e$

$$\boldsymbol{b} = \boldsymbol{F}\boldsymbol{F}^T, \tag{2.4}$$
$$\boldsymbol{b}^e = \boldsymbol{F}^e \boldsymbol{F}^{eT}. \tag{2.5}$$
$$\tag{2.6}$$

The above fundamental strain measures are related via pull-back and push-forward operations

$$\boldsymbol{C}^{-p} = \boldsymbol{F}^{-1} \boldsymbol{b}^e \boldsymbol{F}^{-T}, \quad \boldsymbol{b}^e = \boldsymbol{F} \boldsymbol{C}^{-p} \boldsymbol{F}^T. \tag{2.7}$$

In metal plasticity, a standard assumption is that plastic flow is isochoric (volume-preserving), i.e. $\det(\boldsymbol{F}^p) = 1$, which implies

$$J = \det(\boldsymbol{F}) = \det(\boldsymbol{F}^e). \tag{2.8}$$

Defining $J^e = \det(\boldsymbol{F}^e)$, we have the volume preserving part of the elastic left Cauchy-Green tensor $\bar{\boldsymbol{b}}^e$

$$\bar{\boldsymbol{b}}^e = J^{e^{-2/3}} \boldsymbol{b}^e = J^{-2/3} \boldsymbol{b}^e. \tag{2.9}$$

2.1.2 Hyperelastic constitutive relation

The starting point of the hyperelastic constitutive relation is the assumption of the existence of the following strain-energy function

$$\Psi = \Psi^{\text{vol}}[J^e] + \Psi^{\text{iso}}[\bar{\boldsymbol{b}}^e] \tag{2.10}$$

Here the strain-energy function Ψ is a decoupled function of the isochoric part (i.e., $\bar{\boldsymbol{b}}^e$) and the volumetric part (i.e., $J^e = \det \boldsymbol{F}^e$) of the elastic deformation. $\boldsymbol{b}^e$ is the elastic left Cauchy-Green tensor defined in (2.5). The volumetric and the isochoric parts of the strain-energy function are given as

$$\Psi^{\text{vol}}[J^e] = \frac{1}{2}\kappa(\ln J^e)^2, \quad \Psi^{\text{iso}}[\bar{\boldsymbol{b}}^e] = \frac{1}{4}\mu \ln \bar{\boldsymbol{b}}^e : \ln \bar{\boldsymbol{b}}^e \tag{2.11}$$

where κ and μ are the bulk and shear modulus. Following the argument in [7], elastic logarithmic Henchky strains $\ln \boldsymbol{b}^e$ is used as the strain measure, and the elastic constitutive law and the Kirchhoff stresses are given as

$$\boldsymbol{\tau} = \kappa \ln J^e \boldsymbol{g}^{-1} + \mu \ln \bar{\boldsymbol{b}}^e = \kappa(\ln \boldsymbol{b}^e : \boldsymbol{g})\boldsymbol{g}^{-1} + \mu \operatorname{dev} \ln \boldsymbol{b}^e \tag{2.12}$$

where $\boldsymbol{g}$ is the metric tensor. The Kirchhoff pressure p and deviatoric stress tensor $\boldsymbol{s}$ can be obtained

$$p = \frac{1}{3}\operatorname{tr}(\boldsymbol{\tau}) = \frac{1}{2}\kappa \operatorname{tr} \ln \boldsymbol{b}^e = \frac{1}{2}\kappa \ln \det \boldsymbol{b}^e \tag{2.13}$$

$$\boldsymbol{s} = \operatorname{dev}(\boldsymbol{\tau}) = \mu \operatorname{dev} \ln \boldsymbol{b}^e \tag{2.14}$$

2.2 Constitutive relations of shear-modified Gurson model

2.2.1 Yield function and hardening law

Within the previously described hyperelastic constitutive framework, the yield function Φ of the Gurson model can be written as

$$\Phi = \|\boldsymbol{s}\| - \sqrt{\frac{2}{3}}\operatorname{sign}(\psi)\sqrt{|\psi|}Y \tag{2.15}$$

where $\boldsymbol{s}$ is the deviatoric Kirchhoff stress defined in Eq.(2.14). It should be noted that Eq.(2.15) is a linear form (in terms of $\|\boldsymbol{s}\|$) of the Gurson yield function . This particular form of Gurson yield function is implemented to facilitate comparisons with existing J2-like models in Albany analysis code [CITE], where the yield functions are also written as linear function of $\|\boldsymbol{s}\|$.

The function ψ contains contributions from the damage in the material, which directly relates to the void volume fraction f of the porous solid:

$$\psi = 1 + f^2 - 2f\cosh(v), \quad v = \frac{3p}{2Y} \tag{2.16}$$

where p is the Kirchhoff pressure defined in Eq.(2.13), and f is the void volume fraction of the porous solid. The Kirchhoff yield stress Y describes the hardening of the matrix material. One example of a nonlinear hardening law proposed by [6] for metal is written as

$$Y = Y_0 + Y_\infty\left[1 - \exp(-\delta\varepsilon_q)\right] + K\varepsilon_q \tag{2.17}$$

where ε_q is a strain-like internal variable (e.g., the equivalent plastic strain), Y_0 is the initial yield strength, Y_∞ is the residual flow stress, K is the hardening coefficient, and δ is the saturation exponent. Other forms of hardening law can also be used depending on the observed material behavior.

2.2.2 Flow rule

Following the standard procedure of the principle of maximum dissipation, [5] proposed a general form of associate flow rule, which is adopted in the current formulation and is given as

$$-\frac{1}{2}L_v(\boldsymbol{b}^e)\cdot\boldsymbol{b}^{e^{-1}} = \gamma\frac{\partial\Phi}{\partial\boldsymbol{\tau}} = \gamma\boldsymbol{n} + \frac{1}{3}t\mathbf{1} \tag{2.18}$$

where $L_v(\boldsymbol{b}_e)$ is the Lie derivative of the elastic left Cauchy-Green tensor $\boldsymbol{b}^e$, and $\boldsymbol{n}$ and t are computed given the yield function (2.15) as

$$\boldsymbol{n} = \frac{\boldsymbol{s}}{\|\boldsymbol{s}\|} \tag{2.19}$$

$$t = \mathrm{tr}\left(\gamma\frac{\partial\Phi}{\partial\boldsymbol{\tau}}\right) = \sqrt{\frac{3}{2}}\gamma f\sinh(\frac{3p}{2Y})\frac{1}{\sqrt{|\psi|}}\mathrm{sign}(\psi) \tag{2.20}$$

2.2.3 Evolution of void volume fraction

The internal variable f is the void volume fraction and represents damage of the porous solid. Its evolution typically consists of a void growth part, f_g, and a void nucleation part, f_n.

$$\dot{f} = \dot{f}_g + \dot{f}_n \tag{2.21}$$

For the void growth part $\dot{f}_g$, the shear-modified Gurson model by [3] extends the original void growth law by adding the dependence on the third stress invariant. The evolution equation is given by

$$\begin{aligned} \dot{f}_g &= (1-f)\,\mathrm{tr}\left(\gamma \frac{\partial \Phi}{\partial \boldsymbol{\tau}}\right) + k_\omega f \frac{\omega(\boldsymbol{\tau})}{\tau_e} \boldsymbol{s} : \left(\gamma \frac{\partial \Phi}{\partial \boldsymbol{\tau}}\right) && (2.22) \\ &= (1-f)t + \gamma k_\omega f \frac{\omega(\boldsymbol{\tau})}{\tau_e} \|\boldsymbol{s}\| && (2.23) \\ &= (1-f)t + \sqrt{\frac{2}{3}} \gamma k_\omega f \omega(\boldsymbol{\tau}) && (2.24) \end{aligned}$$

where $\tau_e := \sqrt{3/2}\|\boldsymbol{s}\|$ is the effective deviatoric Kirchhoff stress, k_ω is a material constant that sets the magnitude of the damage growth rate in pure shear states [3]. The function $\omega(\boldsymbol{\tau})$ includes the effect of the third stress invariant on void growth and is given as

$$\omega(\boldsymbol{\tau}) = 1 - \left(\frac{27 J_3}{2\tau_e^3}\right)^2 \tag{2.25}$$

where $J_3 := \det(\boldsymbol{s})$ is the third invariant of deviatoric Kirchhoff stress tensor.

The increase in void volume fraction due to plastic strain controlled nucleation can be written as [1]

$$\dot{f}_{nu} = A \dot{\varepsilon}_q \tag{2.26}$$

A is proposed to have the following form

$$A(\varepsilon_q) = \begin{cases} \frac{f_N}{s_N\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\varepsilon_q - \epsilon_N}{s_N}\right)^2\right], & p \geq 0 \\ 0, & p < 0 \end{cases} \tag{2.27}$$

where the nucleation strain follows a normal distribution with a mean value ϵ_N and a standard deviation s_N with the volume fraction of the nucleated voids given by f_N.

2.2.4 Evolution equation for equivalent plastic strain

The evolution equation for equivalent plastic strain (Equation (2.26)) is derived from the equivalence of plastic work increment in the matrix material and the macroscopic plastic work increment as

$$\dot{\varepsilon}_q Y(1-f) = \boldsymbol{\tau} : \left(\gamma \frac{\partial \Phi}{\partial \boldsymbol{\tau}}\right) \tag{2.28}$$

$$\Rightarrow \qquad \dot{\varepsilon}_q = \frac{1}{1-f}\left(\gamma \sqrt{\frac{2|\psi|}{3}} \text{sign}(\psi) + \frac{pt}{Y}\right) \tag{2.29}$$

2.2.5 Summary of material parameters

The material parameters for the modified Gurson model include

$$\boldsymbol{P} = \{E, \nu, K, Y_0, Y_\infty, \delta, f_0, k_\omega, e_N, s_N, f_N\} \tag{2.30}$$

Chapter 3

Implementation

This section should discuss implementation aspects of the large deformation hyperelastic model and the implicit integration scheme. I based the Sierra implementation off of the Albany version, so that would be a good place to start. In both places we are using Sacado to compute derivative for the local consistent tangent, so we should talk about what that buys us.

Chapter 4

Results

This section could be renamed to Numerical Examples, and should include any and all verification/validation work. To be clear, for the purpose of the SAND report, we don't require any validation. I will have to include a description of the Sierra input parameters for use in their user's manual, but obviously I'll worry about that.

Chapter 5

Conclusions

This should be a summary and short discussion about the strengths and weaknesses of the model. Nothing too fancy, just practice for the journal article.

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