Article

Sheaf Mereology

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1. Classical Mereological Notions in the Sheaf-theoretic Setting

In this section, we provide a discussion of what classical notions of mereology look like in the sheaf-theoretic setting.

- *Cambridge fusions*. Sheaves handle Cambridge fusions correctly.
- *Mere collections*. The collection of all dogs. Is that a "whole"? Well, we could build a sheaf whose atomic regions are filled with dogs, none of which glue. Then we have a collection of dogs, but no glued object. That matches exactly the intuition: yes, we have a "collection" (we built a sheaf for it, after all), but the internals of that sheaf reveal that it's *merely* a collection, i.e., that its parts are not glued.
- *Co-habitating fusions*. Sheaves allow multiple fusions to occupy the same locale, without being glued. For instance, in the sheaf of real-valued functions over real number line, there are many functions that glue together, and occupy the same locale.
- *Non-boolean algebra*. The parts space is Heyting, not Boolean. We're not saddled with such a strong complement operation. You can pick a locale that is Boolean if you need it, but this framework doesn't require it. In fact, the positive logic of a locale is "geometric logic."
- *Reflexivity, antisymmetry, and transitivity*. These are guaranteed. Locally, of course, you may not have transitivity. But globally, it's a theorem. [Check that.]
- *Distributivity*.

  do the glued sections of a sheaf have to be distrubitive? Only inside what glues (since we glue pairwise, so every $i \vee j$ of the cover.

- *An empty element*. There is a need for a bottom element in the *algebra* of parts, but a sheaf need not contain any such thing. There is no need here to try and construct awkward mathematical structures that do algebra on parts but yet don't have a bottom element because our ontological intuitions tell us there can be no such thing. That confuses two issues: algebra and integrity. So here we separate those cleanly, and the algebra can do algebra while the sheaf can do integrity. [In a sheaf you CAN'T assign an empty element to bottom, for coherence, so the bottom element is special...need to say more about that and figure it out.]
- *Supplementation principles*. Sheaves don't constrain one way or another. [Is that really true? Maybe it's better to say that it doesn't force any supplementation principles, which might provide a reason to call into question whether supplementation is another one of those ideas that is about integrity of parts but has been confused with the algebra of parts.]
- *Ordering of parts*. Consider that "pit" and "tip" have the same parts but are different words. These differences can be handled by different sheaves over a 3-stage prefix-ordered locale as in the example of concurrent processes. Note that we retain extensionality.
- *Extensionality*. Classical mereology's notion of extensionality essentially flattens any structure and is thus overly aggressive. This is why extensionality is so controversial. The sheaf-theoretic perspective retains extensionality, but is much more nuanced. [Here too, I suspect that mereological discussions of extensionality have confused the algebra of parts and the integrity of wholes.]
- *Gunk and atoms*. You can model continuity and gunky parts if you so desire. You just need a sober space to do it.

  check that we can model continuity in the locale in this way.

can you do continuity only in the sheaf data, without an underlying continuous decompositon in the locale? I would think that if you can't infinitely decompose into smaller opens around a point in the locale, you couldn't do such a thing in the sheaf data?

- *Priority of wholes*. The framework is agnostic as to whether you take an Aristotelian-Thomistic approach

  cite Aquinas, Arlig, and that guy who wrote that recent book defending the Aristotelian view

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- *The whole is greater than its parts*. The framework is agnostic as to whether you want to be a Scotist and say that the whole is something over and above its parts (cite Cross) or an Ockhamist who says it is not

  cite Normore, Arlig

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Todo list