Assignment 3: Quantum Computing, Computability and Complexity

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June 7, 2017

Question One

a) The D-Wave quantum processor is cooled so that it is at optimal performance, due to its high load in processing calculations, which increases in temperature as a result. Thus, the lower the temperature, the better the performance.

b) The latest generation D-Wave 2X system has an operating temperature of about 15mK (millikelvin), which at this temperature, is approximately 180 times colder than interstellar space.

c) The D-Wave computer stores qubits, which encodes information as 0s, 1s, or both at the same time. This superposition of statesalong with the other quantum mechanical phenomena of entanglement and tunnelingenables quantum computers to manipulate enormous combinations of states at once, which also mitigates the majority of wrong answers, with the use of probability.

d) A probabilistic machine is more like an NFA, as it acts more randomly, rather than returning a set of static answers to a problem. Like an NFA we would see that, this machine would give multiple answers, but wouldn't matter as long as one state, or trace returns the correct answer, when computing a problem.

Question Two

a) A language L is Turing-recognizable if there is a Turing machine M such that $L = L(M)$. As for an undecidable language, there is no Turing machine M such that $L = L(M)$, where on input w, it cannot decide whether it accepts or rejects a string.

b) L is Turing-recognizable as there exists two binary strings. We may use a machine M' to check whether or not they are accepted, running M on input w and w'. If there exists two distinct binary strings, accept, else reject or loop forever. Thus L(M) is recognized.

c) Given L(M), we may use a machine M' to guess two distinct strings w and w' and run M on each of these. Let M' accept iff both w and w' are accepted. Thus M' accepts iff there exist two distinct strings in L(M).

d) Here, we shall prove via contradiction. Suppose L be a decidable language, where there is a halting Turing machine B that recognizes M, such that M is a Turing machine that accepts at least two binary strings, satisfying L. Using B we construct a Turning machine A that accepts the language, <M,w> such that M is a Turing machine that accepts at least two binary strings w.

Let A be a Turing machine that satisfies B, since B is non-trivial there exists a machine that it accepts. Now A operates as follows:

1. On input <M,w>, where M is a Turing machine and w is a string:

    1. On input x, let the Turing machine M run on the string w until it accepts, otherwise run forever.

    2. Run B on x. Accept iff B does.

If M accepts w the Turing machine <M,w> has the property of B, otherwise it doesn't.

2. Feed the description of <M,w> to B, where it accepts at least two binary strings. If B accepts, accept the input $< M, w >$, else if B rejects, reject.

Question Three

a) A is a language that is mapping reducible to the language B, written $A \leq_M B$, such that if there is a computable function i.e. on-to relation, where for every w, $w \in A \Leftrightarrow f(w) \in B$. The function f is called a reduction from A to B.

b) Let $A \leq_M B$, $A \leq_M \overline{B}$, and B is Turing-recognizable, then A is decidable. We know that if $A \leq_M B$ and B is decidable, then A is also decidable, proved in theorem 5.22, and since B is decidable, it means that it is also Turing-recognizable because all decidable languages is also recognizable. Now let $X = A \leq_M B$ and $Y = A \leq_M \overline{B}$, we can say that they are equivalent as, B and $\overline{B}$ are equivalent, since B and the compliment of B are both Turing decidable, with B being recognizable and $\overline{B}$ being co-recognizable.