

The problem of surface reconstruction from point clouds is ill-posed if prior assumptions are not defined, as an infinite number of surfaces can pass through a given set of points. Each surface reconstruction algorithm has its assumptions on its input point cloud data, the scanned shape, and information about the device that the data was collected from. The properties of input point clouds that can have the most impact on the reconstruction results are *sampling density*, *noise*, *outliers*, *misalignment*, and *missing data*.

Sampling density is important for defining a *neighborhood*, a set of points close to a given point which describes the local geometry of the surface. 3D scans are often *non-uniform* in sampling density, which means sampling density varies spatially. To capture local variation in sampling density, k nearest neighbors are commonly used. Alternatively, a spatially varying ϵ -ball can be used [GG07]. Depending on the sampling density of our point cloud data, more sophisticated sampling density estimation approaches can be used. These methods include reconstruction error bounds [LCOL06], which finds the neighborhood size at each point by bounding the error of a moving least squares surface approximation, where the selected ϵ minimizes this error bound.

Noise. Points that are randomly distributed along the surfaces are noise. The goal of surface reconstruction is to produce a surface that passes near the points without overfitting to the noise. To handle noise, we can have robust algorithms that impose smoothness on the output [KBH06], or methods that employ robust statistics [OGG09].

Outliers. Unlike noise, outliers are points that are far from the true surface and that should not be used to infer the surface. We can handle outliers by either explicitly through detection [LCOLTE07] or implicitly through robust methods [MDGD*10].

Misalignment. The imperfect registration of range scans results in misalignment. A misaligned scan can be corrected if it fails to conform to the remaining discovered repetitive relationships between geometric primitives [LWC*11].

Missing data. Missing data are due to factors such as limited sensor range, high light absorption and occlusions in the scanning process where large portions of the shape are not sampled. Methods that deal with missing data assume that the scanned shape is watertight [CBC*01, Kaz05, KBH06, HK06, ACSTD07], where the goal is to handle the aforementioned challenges where data exist and infer geometry in parts of the surface that have not been sampled.

Surface reconstruction also depends on different types of input information associated with a point cloud. The bare minimum input requirement is a set of 3D points that sample the surface. These points alone, however, may fail to sufficiently regularize the problem of reconstruction for certain point clouds. Input information such as *surface normals*, *scanner information*, and *RGB imagery* can be beneficial in reconstruction from challenging point clouds.

Surface normals are an extremely useful input for surface reconstruction. Without surface normals, the only surface reconstruction method is to *interpolate* a point cloud from solely the *xyz* coordinates. Delaunay-based methods are commonly used in this domain. However, interpolatory reconstruction requires the sampling of the input point cloud to be sufficiently dense in order to capture the surface, and it is impractical for point clouds with significant imperfections such as noise and sparseness. A survey of these methods is in [CG06], as well as [Dey07]. A recent approach that handles moderate levels of noise is presented in [DMSL11], which uses a scale-space approach to interpolatory reconstruction.

In the case of point sets generated by laser scanners, the data is often noisy (due to the inherent uncertainty of measurements), and possibly unorganized (due to the merging of several scans). In this context, surface reconstruction can only be *approximating*, instead of *interpolating*, since data points are more of an indication of proximity to the surface than actual sample points. Approximation is when an implicit function is computed such that one of its isosurfaces best fits the data. The Poisson reconstruction method we have been using uses oriented point sets in which an implicit function f is derived from a Poisson equation $\Delta f = \nabla \cdot \vec{V}$, providing the best match between the gradient of f and the input normals n represented as the vector field \vec{V} [KBH06]. With the availability of oriented normals, this approach becomes a direct solve for a linear system, and it can scale remarkably well.

On the other hand, if we only have *unoriented normals*, we would need to solve for a generalized eigenvalue problem. As Poisson [KBH06] relies on oriented normals, it can only tolerate sparsely distributed normal orientation flips. Large clusters of incorrect normal orientation significantly impact

the results of this method. [ACSTD07] uses covariance matrices instead of normals to represent *unsigned* orientations. The covariance matrix of the Voronoi cell of a sample point not only provides an estimate of the normal direction, but it also provides a measure of how reliable this estimate is. The eigenvector associated to the largest eigenvalue indicates the axis along which the cell is elongated – a good approximate of the normal direction if the samples all lie on a common manifold. After normal information has been estimated through Voronoi-PCA approach, we would have a direction field of unoriented normals. We can find an implicit function f such that its gradient ∇f is everywhere best aligned to the maximum eigenvalue direction of the tensor field C , where the best alignment is weighted by the local confidence in the normal direction. The optimization problem they proposed penalizes misalignment of data points, in which an isotropic tensor (unknown normal direction) has little influence while an anisotropic tensor (high confidence in the normal direction) plays a major role. Solving this amounts to balancing smoothness of ∇f vs. alignment of the gradient – it aligns ∇f to C when the normal direction is particularly reliable, and when the tensor is isotropic (unknown normal direction) the solver favors smoothness of the gradient function instead. This global balancing act induces a consistent orientation as it flips the sign of ∇f .

- [ACSTD07] ALLIZ P., COHEN-STEINER D., TONG Y., DESBRUN M.: Voronoi-based variational reconstruction of unoriented point sets. *Computer Graphics Forum (Proceedings of the Symposium on Geometry Processing)* (2007), vol. 7, pp. 39–48.
- [CBC*01] CARR J. C., BEATSON R. K., CHERRIE J. B., MITCHELL T. J., FRIGHT W. R., MCCALLUM B. C., EVANS T. R.: Reconstruction and representation of 3d objects with radial basis functions. In *Proceedings of ACM SIGGRAPH* (2001).
- [CG06] Cazals F., Giesen J.: Delaunay triangulation based surface reconstruction.
- [Dey07] DEY T. K.: *Curve and Surface Reconstruction: Algorithms with Mathematical Analysis*. Cambridge University Press, New York, NY, USA, 2007.
- [DMSL11] DIGNE J., MOREL J.-M., SOUZANI C.-M., LARTIGUE C.: Scale space meshing of raw data point sets. *Computer Graphics Forum* 30, 6 (2011), 1630–1642.
- [GG07] GUENNEBAUD G., GROSS M.: Algebraic point set surfaces. *ACM Transactions on Graphics (Proc. SIGGRAPH)* 26, 3 (2007), 23.
- [HK06] HORNUNG A., KOBELT L.: Robust reconstruction of water-tight 3d models from non-uniformly sampled point clouds without normal information. In *Computer Graphics Forum (Proc. of the Symposium on Geometry Processing)* (2006), 41–50.
- [Kaz05] KAZHDAN M.: Reconstruction of solid models from oriented point sets. In *Proceedings of the EG/SIGGRAPH Symposium on Geometry processing* (2005).
- [KBH06] KAZHDAN M., BOLITHO M., HOPPE H.: Poisson surface reconstruction. In *Proceedings of the EG/SIGGRAPH Symposium on Geometry Processing* (2006).
- [LCOL06] LIPMAN Y., COHEN-OR D., LEVIN D.: Error bounds and optimal neighborhoods for mls approximation. *Computer Graphics Forum (Proc. of the Symposium on Geometry Processing)* (2006), 71–80.
- [LCOLTE07] LIPMAN Y., COHEN-OR D., LEVIN D., TAL-EZER H.: Parameterization-free projection for geometry reconstruction. *ACM Transactions on Graphics (Proc. SIGGRAPH)* 26, 3 (2007), 22.
- [LWC*11] LI Y., WU X., CHRYSATHOU Y., SHARF A., COHEN-OR D., MITRA N. J.: Globfit: Consistently fitting primitives by discovering global relations. *ACM Transactions on Graphics (Proc. SIGGRAPH)* 30, 4 (2011), 52.
- [MDGD*10] MULLEN P., DE GOES F., DESBRUN M., COHEN-STEINER D., ALLIEZ P.: Signing the unsigned: Robust surface reconstruction from raw pointsets. *Computer Graphics Forum (Proc. of the Symposium on Geometry Processing)* 29, 5 (2010), 1733–1741.
- [OGG09] OZTIRELI C., GUENNEBAUD G., GROSS M.: Feature preserving point set surfaces based on non-linear kernel regression. *Computer Graphics Forum* 28, 2 (2009), 493–501.