CIEG 604 Prestressed Concrete Design

Final Project

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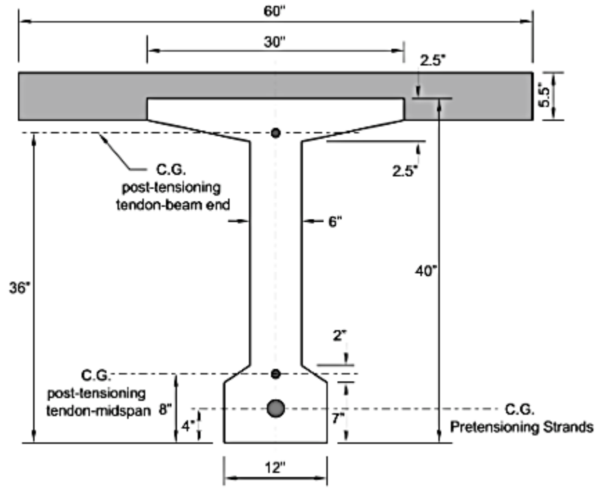
Problem Statement

Given a 100 ft. simple span composite decked bulb T beam with 12-straight 1/2in. diameter 270 ksi low relaxation steel pretensioning strands and a draped post-tensioned tendon (see Figure 1). Determine the service stresses in the beam, size of the post-tensioning tendon needed to carry the service loads, and final service level stresses from full sequence of loads. Also, check the nominal capacity of the beam and design shear reinforcement. Make sure to adhere to ACI Building Code requirements.



Section Properties

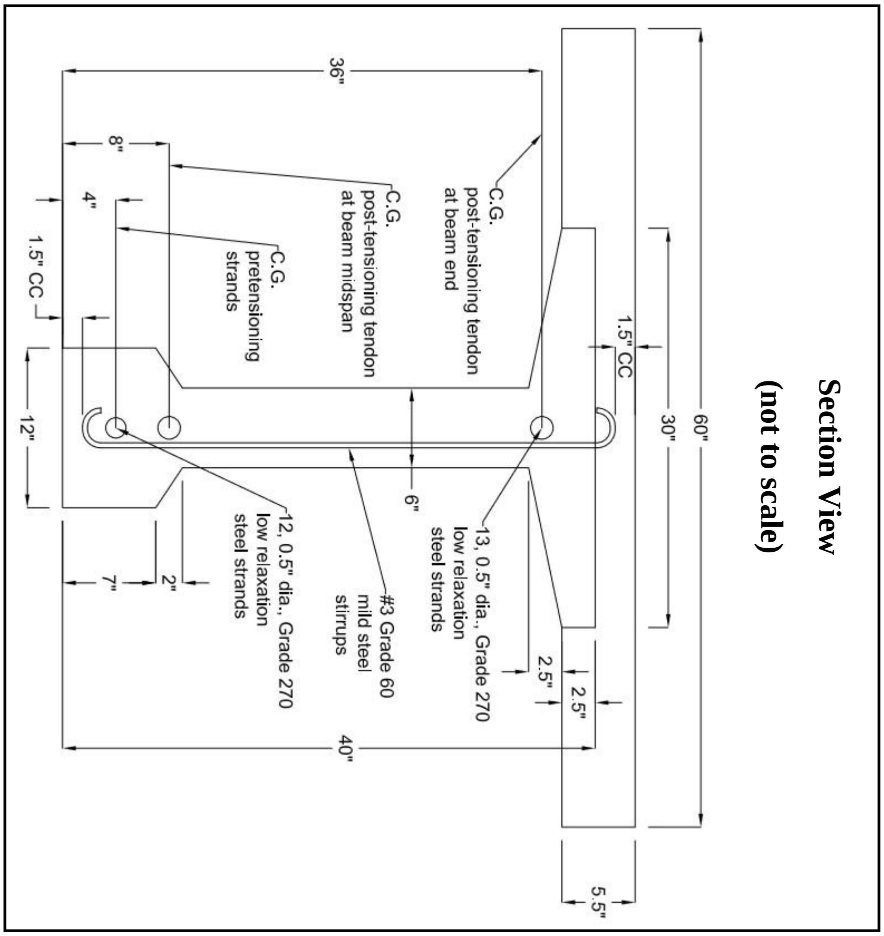
Basic Beam
A = 378 in^2, I = 72,041 in^4, y_b = 22.27", y_t = 17.73"

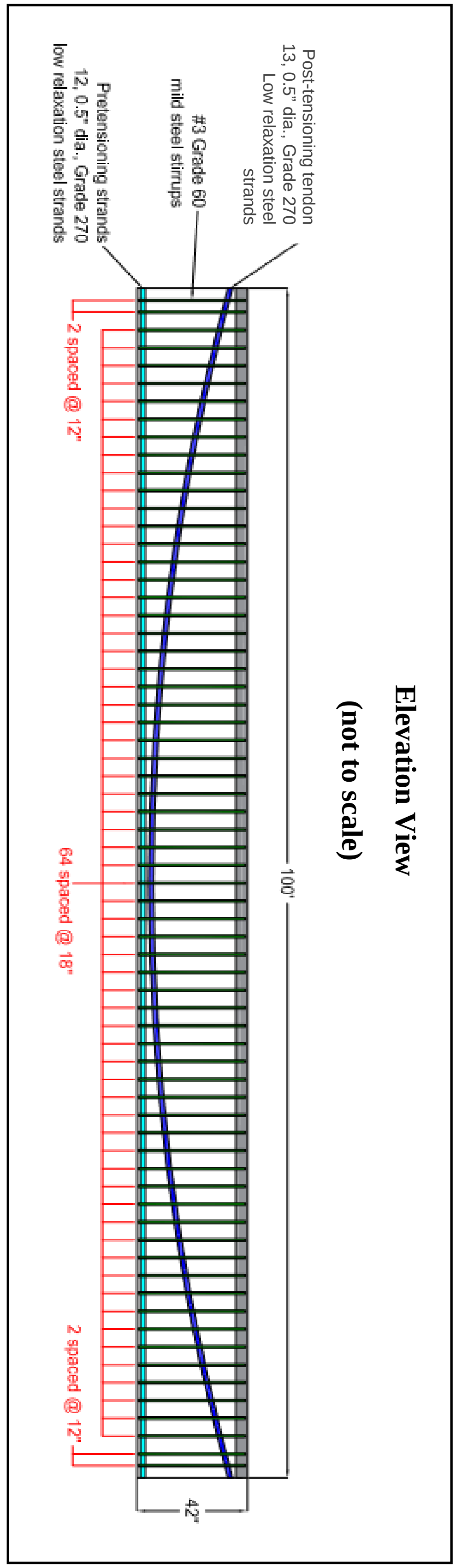
Composite Beam
A = 631.8 in^2, I = 124.243 in^4, y_b = 28.68", y_t = 13.32"

Figure 1. Section properties.

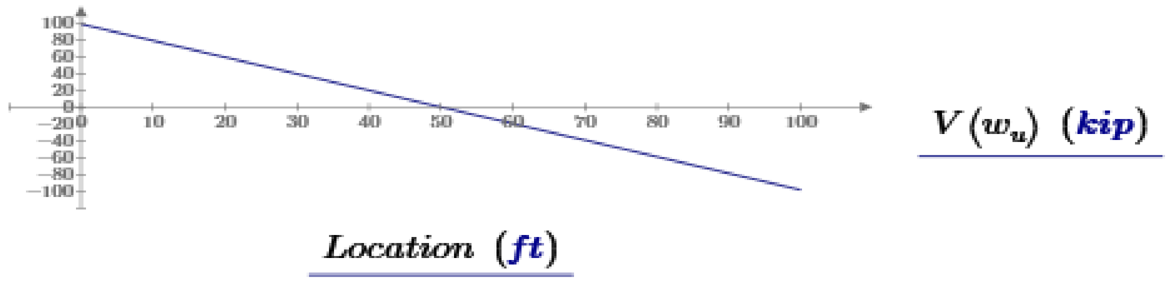
Design Methodology

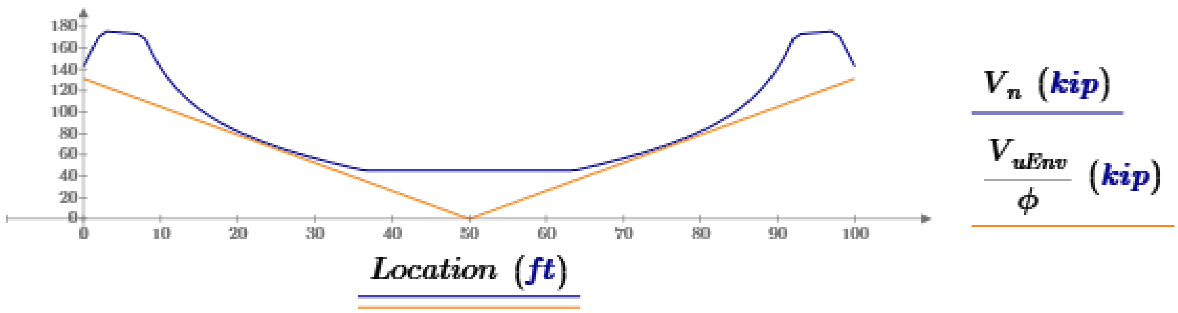
Geometry, section, and material properties as well as loading of the beam were determined (see pages 5-7). The stresses in the concrete due to the pretensioned strands in Load Stage 1 were determined and checked with the ACI Building code requirements and assuming allowable stresses of $\pm 0.8f'_{ci}$ at the ends of the member (see page 11). The final service stresses in the concrete were then calculated considering the full sequence of loads (see pages 12-14) and the post-tensioned tendon was sized with 4, 0.5 diameter strands to ensure the member satisfied the minimum stresses to classify it as Class U according to ACI 24.5.2.1. Next, the flexural strength of the section was calculated and it was determined that the moment capacity was not sufficient to support the factored moment. The post-tensioned tendon was then resized to 13, 0.5 diameter strands in order to ensure moment capacity was sufficient (see page 20). After recalculating the final service stresses, the member was still classified as Class U (see page 11). Finally, the shear strength of the member was determined and compared with the factored shear (see pages 21-23). Shear reinforcement was sized with #3 stirrups and spaced to ensure the member's shear capacity was sufficient and that minimum reinforcement and spacing requirements of ACI were met (see pages 24-28). Section and elevation drawings of the final member are shown on page 3. Factored shear and moment as well as final shear and moment capacity diagrams are shown on page 4.


Section View
(not to scale)
60"
30"
1.5" CC
36"
8"
4"
1.5" CC
12"
6"
7"
2"
2.5"
2.5"
40"
5.5"
C.G.
post-tensioning tendon
at beam end
C.G.
post-tensioning tendon
at beam midspan
C.G.
pretensioning
strands
13, 0.5" dia., Grade 270
low relaxation
steel strands
#3 Grade 60
mild steel
stirrups
12, 0.5" dia., Grade 270
low relaxation
steel strands

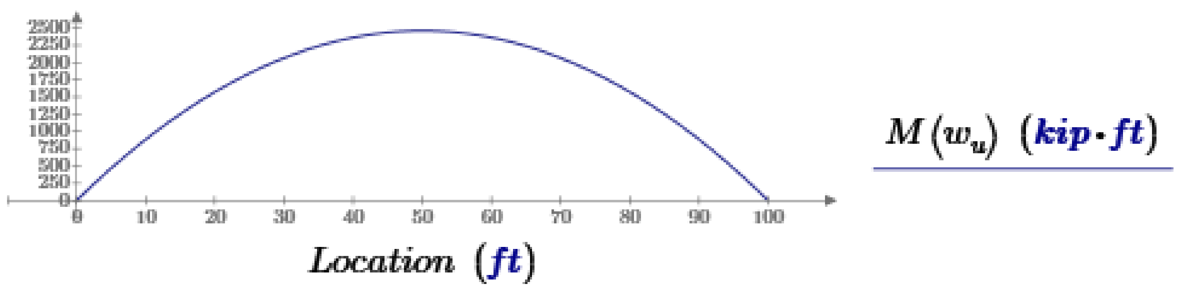

Elevation View
(not to scale)
Post-tensioning tendon
13, 0.5" dia., Grade 270
Low relaxation steel
strands
#3 Grade 60
mild steel stirrups
Pretensioning strands
12, 0.5" dia., Grade 270
low relaxation steel strands
100'
42"
2 spaced @ 12"
64 spaced @ 18"
2 spaced @ 12"

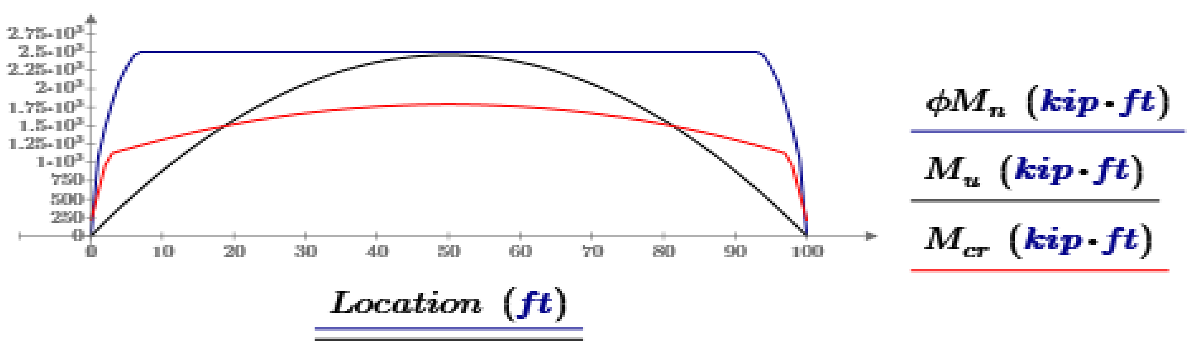
Shear Diagrams





Moment Diagrams





Geometry and Section Properties

$L:=100\ ft$ Span length

$b_{bb}:=30\ in$ Width of flange of basic beam

$b_{cb}:=60\ in$ Width of flange of composite beam

$b_w:=6\ in$ Width of web of precast section

$\gamma_c:=150\ pcf$ Unit weight of concrete

Basic Beam

$A_{bb}:=378\ in^2$ Area of basic beam

$I_{bb}:=72041\ in^4$ Moment of inertia of basic beam

$y_b:=22.27\ in$ Distance from N.A. to bottom fiber of basic beam

$S_b:=\dfrac{I_{bb}}{y_b}$ $S_b=3235\ in^3$

$y_t:=17.73\ in$ Distance from N.A. to top fiber of basic beam

$S_t:=\dfrac{I_{bb}}{y_t}$ $S_t=4063\ in^3$

$h_{bb}:=y_b+y_t$

Composite Beam

$A_{cb}:=631.8\ in^2$ Area of composite beam

$I_{cb}:=124243\ in^4$ Moment of inertia of composite beam

$y_{bp}:=28.68\ in$ Distance from N.A. to bottom fiber of composite beam

$S_{bp}:=\dfrac{I_{cb}}{y_{bp}}$ $S_{bp}=4332\ in^3$

$y_{tc}:=13.32\ in$ Distance from N.A. to top fiber of composite beam

$S_{tc}:=\dfrac{I_{cb}}{y_{tc}}$ $S_{tc}=9328\ in^3$

$h_{cb}:=y_{bp}+y_{tc}$ $h_{cb}=42\ in$

$t_c:=5.5\ in$ Thickness of topping

$y_{tp}:=h_{bb}-y_{bp}$ $y_{tp}=11.32\ in$

$S_{tp}:=\dfrac{I_{cb}}{y_{tp}}$ $S_{tp}=10976\ in^3$

$y_{bc} := h_{cb} - t_c - y_{bp}$ $y_{bc} = 7.82\ in$

$S_{bc} := \frac{I_{cb}}{y_{bc}}$ $S_{bc} = 15888\ in^3$

Pretensioned Reinforcement Data

$A_{ps} := 1.836\ in^2$ Area of pretensioned reinforcement (12, 0.5 in. dia. straight low relaxation steel strands)

$d_{ps} := 0.5\ in$ Diameter of pretensioned straight low relaxation steel strands

$e_{ps\_bb} := y_b - 4 \cdot in$ Distance from cgc to cgs of strands of basic beam

$e_{ps\_cb} := y_{bp} - 4 \cdot in$ Distance from cgc to cgs of strands of composite beam

$L_{db\_ps} := 0\ in$ Debond length

Post-Tensioned Reinforcement Data

$A_{pt} := 1.989\ in^2$ Area of post-tensioned reinforcement (13, 0.5 in. dia. low relaxation steel strands)

$d_{pt} := 0.5\ in$ Diameter of post-tensioned low relaxation steel strands

$y_h := 8\ in$ Distance from bottom of beam to cgs of post-tensioned strands at low point of drape

$e_{pt\_e\_cb} := y_{bp} - 36 \cdot in$ Distance from cgc to cgs of strands at end of composite beam

$e_{pt\_c\_cb} := y_{bp} - 8 \cdot in$ Distance from cgc to cgs of strands at midspan of composite beam

$L_{db\_pt} := 0\ in$ Debond length

Material Properties

$f'_c := 6500\ psi$ Specified 28-day concrete compressive strength

$E_c := 57000 \cdot \sqrt{f'_c \cdot psi}$ $E_c = 4595\ ksi$

$f'_{ci} := 4500\ psi$ Specified concrete compressive strength at prestress transfer

$E_{ci} := 57000 \cdot \sqrt{f'_{ci} \cdot psi}$ $E_{ci} = 3824\ ksi$

$f'_{ct} := 3500\ psi$ Assumed 28-day concrete compressive strength of topping

$E_{ct} := 57000 \cdot \sqrt{f'_{ct} \cdot psi}$ $E_{ct} = 3372\ ksi$

$f_y := 60\ ksi$ Specified yield strength of mild steel reinforcement

$E_s := 29000\ ksi$ Young's modulus of mild steel reinforcement

$f_{pu\_ps} := 270\ ksi$ Specified ultimate tensile strength of pretensioned strands

$f_{pi\_ps} := 0.9 \cdot 0.75 \cdot f_{pu\_ps}$ $f_{pi\_ps} = 182\ ksi$ Initial prestress in pretensioned strands (after elastic loses)

$f_{se\_ps} := 0.75 \cdot f_{pu\_ps} - 30 \cdot ksi$ $f_{se\_ps} = 173\ ksi$ Effective prestress in pretensioned strands (after all loses)

$f_{pu\_pt} := 270\ ksi$ Specified ultimate tensile strength of post-tensioned strands

$f_{pi\_pt} := 0.9 \cdot 0.8 \cdot f_{pu\_pt}$ $f_{pi\_pt} = 194\ ksi$ Initial prestress in post-tensioned strands (after elastic loses)

$f_{se\_pt} := 0.6 \cdot f_{pu\_pt}$ $f_{se\_pt} = 162\ ksi$ Effective prestress in post-tensioned strands (after all loses)

$\lambda := 1.0$ Normal weight concrete

$n_c := \frac{E_{ct}}{E_c}$ $n_c = 0.734$

Loads

$w_{0\_bb} := A_{bb} \cdot \gamma_c$ $w_{0\_bb} = 394\ plf$ Self weight of basic beam

$w_{0\_cb} := A_{cb} \cdot \gamma_c$ $w_{0\_cb} = 658\ plf$ Self weight of composite beam

$w_t := w_{0\_cb} - w_{0\_bb}$ $w_t = 264\ plf$ Self weight of deck topping

$w_{DL} := 250 \cdot plf$ Superimposed dead load

$w_{LL} := 550 \cdot plf$ Live load

Solution

1) Write equations for tendon profile and prestress over transfer length

$\mathbf{ORIGIN} := 0$

$Segments := 100$ Number of segments used to discretize beam

$j := 0..Segments$

$$Location := \left\| \begin{array}{l} Loc_0 \leftarrow 0 \cdot ft \\ Loc_{Segments} \leftarrow L \\ SegLength \leftarrow \dfrac{L}{Segments} \\ \text{for } i \in 0..(Segments-2) \\ \quad \| \; Loc_{i+1} \leftarrow Loc_i + SegLength \\ Loc \end{array} \right.$$

Equation for harped or parabolic tendon eccentricity

$$profile(e_e, e_c, L_h, L_d, L, drape) := \left\| \begin{array}{l} \text{if } drape = \text{“harped”} \\ \quad \| \text{“harped tendon”} \\ \quad \| \text{for } i \in 0..Segments \\ \quad \| \quad \| \text{if } Location_i < L_h \\ \quad \| \quad \| \quad \| \; ecc_i \leftarrow (e_c - e_e) \cdot \dfrac{Location_i}{L_h} + e_e \\ \quad \| \quad \| \text{also if } Location_i > L - L_h \\ \quad \| \quad \| \quad \| \; ecc_i \leftarrow (e_c - e_e) \cdot \dfrac{(L - Location_i)}{L_h} + e_e \\ \quad \| \quad \| \text{else} \\ \quad \| \quad \| \quad \| \; ecc_i \leftarrow e_c \\ \text{if } drape = \text{“parabolic”} \\ \quad \| \text{“parabolic drape”} \\ \quad \| \text{for } i \in 0..Segments \\ \quad \| \quad \| \; ecc_i \leftarrow \dfrac{(e_c - e_e)}{(0.5 \cdot L)^2} \cdot \left(Location_i \cdot L - \left(Location_i\right)^2\right) + e_e \\ \text{return } ecc \end{array} \right.$$

Equation for prestress over transfer length

$PR(f_p, A_{ps}, L, L_t, L_{db}) :=$

$Lo \leftarrow Location$

if $L_{db} > 0$

  "Strands are DEBONDED"

  for $i \in 0..Segments$

    if $Lo_i \geq L_{db} + 2 \cdot L_t \wedge Lo_i \leq L - L_{db} - 2 \cdot L_t$

      $PreForce_i \leftarrow f_p \cdot A_{ps}$

    if $L_{db} < Lo_i < L_{db} + 2 \cdot L_t$

      $PreForce_i \leftarrow \dfrac{(Lo_i - L_{db})}{2 \cdot L_t} \cdot f_p \cdot A_{ps}$

    if $L - L_{db} - 2 \cdot L_t < Lo_i < L - L_{db}$

      $PreForce_i \leftarrow \dfrac{(L - L_{db} - Lo_i)}{2 \cdot L_t} \cdot f_p \cdot A_{ps}$

    if $Lo_i \leq L_{db} \vee Lo_i \geq L - L_{db}$

      $PreForce_i \leftarrow 0 \cdot kip$

else

  "Strands are FULLY BONDED"

  for $i \in 0..Segments$

    if $Lo_i \geq L_t \wedge Lo_i \leq L - L_t$

      $PreForce_i \leftarrow f_p \cdot A_{ps}$

    if $Lo_i < L_t$

      $PreForce_i \leftarrow \dfrac{Location_i}{L_t} \cdot f_p \cdot A_{ps}$

    if $Lo_i > L - L_t$

      $PreForce_i \leftarrow \dfrac{(L - Location_i)}{L_t} \cdot f_p \cdot A_{ps}$

return $PreForce$

2) Service stresses of basic beam

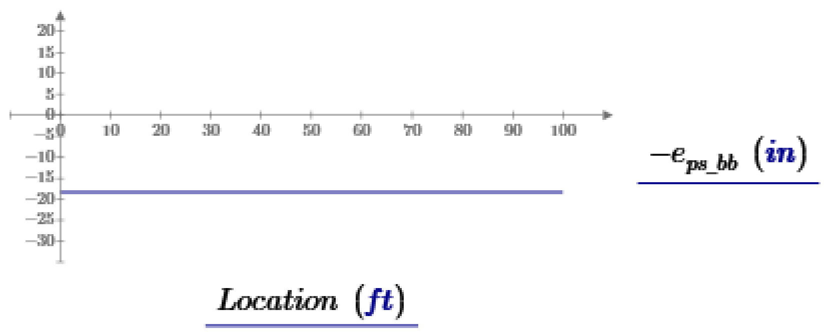
2.1) Determine transfer length of pretensioned strands

$$L_{t\_ps} := \frac{f_{se\_ps}}{3000 \cdot psi} \cdot d_{ps} \qquad L_{t\_ps} = 28.8\ in$$

2.2) Plot variation in tendon profile and prestress of basic beam

$$e_{ps\_bb} := profile\left(e_{ps\_bb}, e_{ps\_bb}, L \cdot 0.5, 0, L, \text{“harped”}\right)$$

Pretensioned strands eccentricity from centroid of basic beam

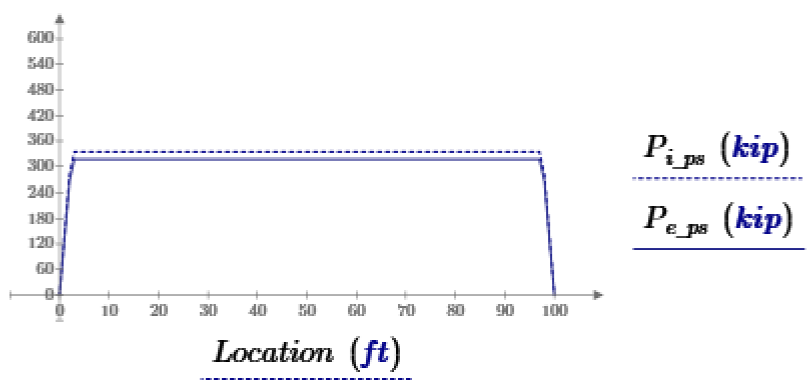


$$P_{i\_ps} := PR\left(f_{pi\_ps}, A_{ps}, L, L_{t\_ps}, 0\right)$$

Prestress force of pretensioned strands immediately after transfer

$$P_{e\_ps} := PR\left(f_{se\_ps}, A_{ps}, L, L_{t\_ps}, 0\right)$$

Prestress force of pretensioned strands after all losses



$$\max\left(P_{i\_ps}\right) = 334.6\ kip \qquad \max\left(P_{e\_ps}\right) = 316.7\ kip$$

2.3) Plot stress state for prestress transfer of pretensioned strands of the basic beam

Stress nomenclature:

f_{abc}

a = top (t) or bottom (b) of

b = precast (p) or topping (c)

c = load stage number

Span is assumed to be from end to end of beam

$$M(w) := \left\| \begin{array}{l} \text{for } i \in 0..Segments \\ \quad \left\| M_i \leftarrow 0.5 \cdot w \cdot \left(Location_i \cdot L - \left(Location_i \right)^2 \right) \right. \\ \text{return } M \end{array} \right.$$

Load Stage 1 - Prestress Transfer of pretensioned strands

Stress at top and bottom of basic beam after release

$$f_{tp1} := \overrightarrow{\left(\frac{P_{i\_ps}}{A_{bb}} - \frac{\left(P_{i\_ps} \cdot e_{ps\_bb}\right)}{S_t} + \frac{M\left(w_{0\_bb}\right)}{S_t} \right)}$$

$\max\left(f_{tp1}\right) = 834 \; psi$

$min\left(f_{tp1}\right) = -450 \; psi$

$$f_{bp1} := \overrightarrow{\left(\frac{P_{i\_ps}}{A_{bb}} + \frac{\left(P_{i\_ps} \cdot e_{ps\_bb}\right)}{S_b} - \frac{M\left(w_{0\_bb}\right)}{S_b} \right)}$$

$\max\left(f_{bp1}\right) = 2563 \; psi$

$min\left(f_{bp1}\right) = 0 \; psi$

Allowable stresses

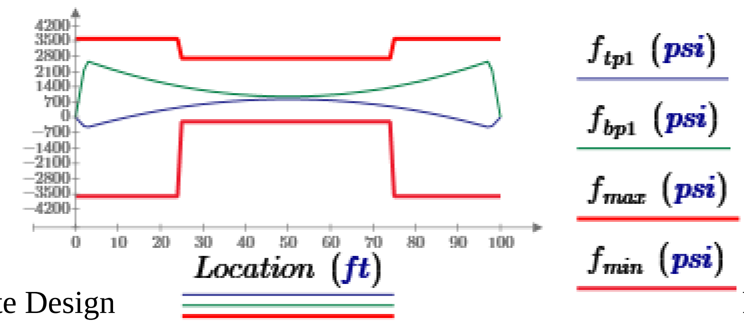
$Lend := 0.25 \cdot L$ Define "end" of beam

$$f_{max} := \left\| \begin{array}{l} \text{for } i \in 0..Segments \\ \quad \left\| \begin{array}{l} \text{if } Location_i > Lend \wedge Location_i < L - Lend \\ \quad \left\| f_i \leftarrow 0.6 \cdot f'_{ci} \right. \\ \text{else} \\ \quad \left\| f_i \leftarrow 0.8 \cdot f'_{ci} \right. \end{array} \right. \\ f \end{array} \right.$$

Allowable compression (24.5.4.1)

$$f_{min} := \left\| \begin{array}{l} \text{for } i \in 0..Segments \\ \quad \left\| \begin{array}{l} \text{if } Location_i > Lend \wedge Location_i < L - Lend \\ \quad \left\| f_i \leftarrow -3 \cdot \sqrt{f'_{ci} \cdot psi} \right. \\ \text{else} \\ \quad \left\| f_i \leftarrow -0.8 \cdot f'_{ci} \right. \end{array} \right. \\ f \end{array} \right.$$

Allowable tension (24.5.4.1)

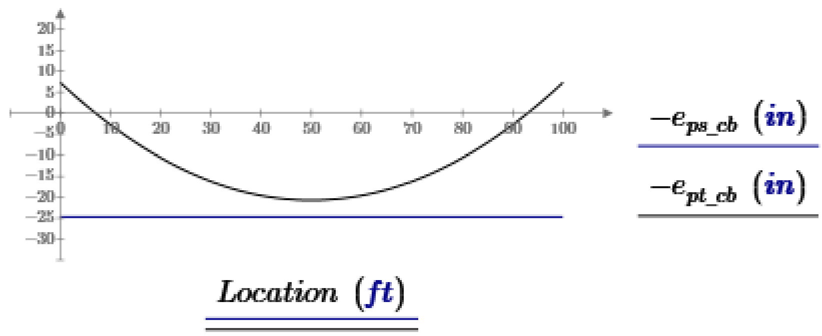


3) Service stresses of composite beam

3.1) Plot variation in tendon profile and prestress of composite beam

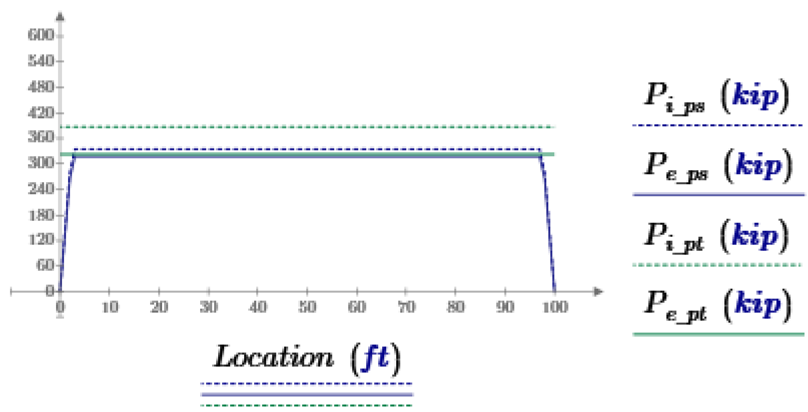
$e_{ps\_cb} := profile\left(e_{ps\_cb}, e_{ps\_cb}, L \cdot 0.5, 0, L, \text{"harped"}\right)$ Pretensioned strands eccentricity from centroid of composite beam

$e_{pt\_cb} := profile\left(e_{pt\_e\_cb}, e_{pt\_c\_cb}, L \cdot 0.5, 0, L, \text{"parabolic"}\right)$ Post-tensioned strands eccentricity from centroid of composite beam



$P_{i\_pt} := PR\left(f_{pi\_pt}, A_{pt}, L, 0, 0\right)$ Prestress force of post-tensioned strands immediately after transfer

$P_{e\_pt} := PR\left(f_{se\_pt}, A_{pt}, L, 0, 0\right)$ Prestress force of post-tensioned strands after all losses



$\max\left(P_{i\_pt}\right) = 386.7 \; kip$ $\max\left(P_{e\_pt}\right) = 322.2 \; kip$

3.2) Plot stress state for prestress transfer of pretensioned and post-tensioned strands of composite beam

Load Stage 2 - Deck cast (unshored) and prestress transfer of post-tensioned strands

Stress at top and bottom of precast and deck after deck is placed and post-tension released

$$f_{tp2} := \overrightarrow{\left(\frac{P_{e\_ps}}{A_{bb}} + \frac{P_{i\_pt}}{A_{bb}} - \frac{\left(P_{e\_ps} \cdot e_{ps\_cb}\right)}{S_t} - \frac{\left(P_{i\_pt} \cdot e_{pt\_cb}\right)}{S_t} + \frac{M\left(w_t\right)}{S_t}\right)}$$

$\max\left(f_{tp2}\right) = 1719 \; psi$

$min\left(f_{tp2}\right) = -1055 \; psi$

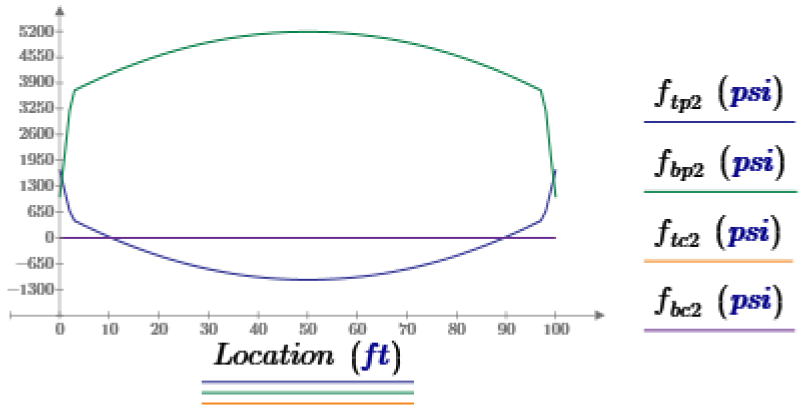
$$f_{bp2} := \overrightarrow{\left(\frac{P_{e\_ps}}{A_{bb}} + \frac{P_{i\_pt}}{A_{bb}} + \frac{\left(P_{e\_ps} \cdot e_{ps\_cb}\right)}{S_b} + \frac{\left(P_{i\_ps} \cdot e_{pt\_cb}\right)}{S_b} - \frac{M\left(w_t\right)}{S_b}\right)}$$

$\max\left(f_{bp2}\right) = 5190 \; psi$

$min\left(f_{bp2}\right) = 1023 \; psi$

$f_{tc2_j} := 0\ psi$ $\max(f_{tc2}) = 0\ psi$ $\min(f_{tc2}) = 0\ psi$

$f_{bc2_j} := 0\ psi$ $\max(f_{bc2}) = 0\ psi$ $\min(f_{bc2}) = 0\ psi$



Load Stage 3 - Erection (unshored)

Stress at top and bottom of precast and deck at the time of erection (includes long-term losses)

$P_e := P_{e\_ps} + P_{e\_pt}$

$$f_{tp3} := \overrightarrow{\left(\frac{P_e}{A_{bb}} - \frac{(P_{e\_ps} \cdot e_{ps\_cb})}{S_t} - \frac{(P_{e\_pt} \cdot e_{pt\_cb})}{S_t} + \frac{M(w_t)}{S_{tp}}\right)}$$

$\max(f_{tp3}) = 1433\ psi$

$\min(f_{tp3}) = -1512\ psi$

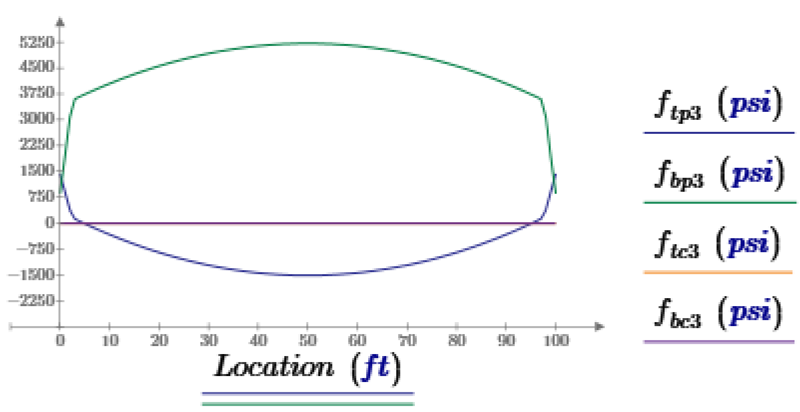
$$f_{bp3} := \overrightarrow{\left(\frac{P_e}{A_{bb}} + \frac{(P_{e\_ps} \cdot e_{ps\_cb})}{S_b} + \frac{(P_{e\_ps} \cdot e_{pt\_cb})}{S_b} - \frac{M(w_t)}{S_{bp}}\right)}$$

$\max(f_{bp3}) = 5216\ psi$

$\min(f_{bp3}) = 852\ psi$

$f_{tc3_j} := 0\ psi$ $\max(f_{tc3}) = 0\ psi$ $\min(f_{tc3}) = 0\ psi$

$f_{bc3_j} := 0\ psi$ $\max(f_{bc3}) = 0\ psi$ $\min(f_{bc3}) = 0\ psi$



Load Stage 4 - Dead + Live Load (unshored)

Stress at top and bottom of precast with all service loads in place

$$f_{tp4} := \overrightarrow{\left(\frac{P_e}{A_{bb}} - \frac{(P_{e\_ps} \cdot e_{ps\_cb})}{S_t} - \frac{(P_{e\_pt} \cdot e_{pt\_cb})}{S_t} + \frac{M(w_t)}{S_t} + \frac{M(w_{LL}+w_{DL})}{S_{tp}}\right)}$$

$\max(f_{tp4}) = 1433\ psi$

$min(f_{tp4}) = 196\ psi$

$$f_{bp4} := \overrightarrow{\left(\frac{P_e}{A_{bb}} + \frac{(P_{e\_ps} \cdot e_{ps\_cb})}{S_t} + \frac{(P_{e\_pt} \cdot e_{pt\_cb})}{S_t} - \frac{M(w_t)}{S_b} - \frac{M(w_{LL}+w_{DL})}{S_{bp}}\right)}$$

$\max(f_{bp4}) = 2827\ psi$

$min(f_{bp4}) = 272\ psi$

$$f_{tc4} := \frac{M(w_{LL}+w_{DL})}{S_{tc}} \cdot n_c$$

$\max(f_{tc4}) = 944\ psi$ $\quad min(f_{tc4}) = 0\ psi$

$$f_{bc4} := \frac{M(w_{LL}+w_{DL})}{S_{bc}} \cdot n_c$$

$\max(f_{bc4}) = 554\ psi$ $\quad min(f_{bc4}) = 0\ psi$

Allowable Stresses

$$f_{max} := \left\| \begin{array}{l} \text{for } i \in 0..Segments \\ \quad \left\| \begin{array}{l} f_{i,0} \leftarrow 0.45 \cdot f'_c \\ f_{i,1} \leftarrow 0.6 \cdot f'_c \end{array} \right. \\ f \end{array} \right.$$

Allowable compression (24.5.4.1)

Classification (ACI 24.5.2.1)

$$f_U := \left\| \begin{array}{l} \text{for } i \in 0..Segments \\ \quad \left\| f_i \leftarrow -7.5 \cdot \sqrt{f'_c \cdot psi} \right. \\ f \end{array} \right.$$

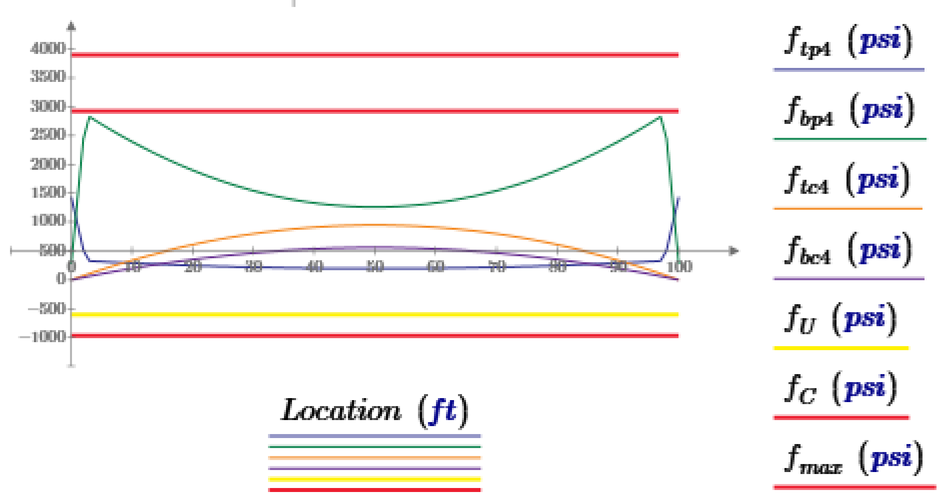
$f_{U_1} = -605\ psi$

Class U (uncracked)

$$f_C := \left\| \begin{array}{l} \text{for } i \in 0..Segments \\ \quad \left\| f_i \leftarrow -12 \cdot \sqrt{f'_c \cdot psi} \right. \\ f \end{array} \right.$$

$f_{C_1} = -967\ psi$

Class C (cracked)



Section is uncracked (Class U)

4) Flexural Strength

4.1) Check prestress to determine if empirical equation can be used or to use strain compatibility

$$CheckPrestress\_ps := \begin{array}{l} \text{if } f_{pu\_ps} \cdot 0.5 \le f_{se\_ps} \\ \quad \| \text{“Prestress is sufficient to use empirical equation”} \\ \text{if } f_{pu\_ps} \cdot 0.5 > f_{se\_ps} \\ \quad \| \text{“WARNING: Prestress is INSUFFICIENT to use empirical equation”} \end{array}$$

$$CheckPrestress\_ps = \text{“Prestress is sufficient to use empirical equation”}$$

$$CheckPrestress\_pt := \begin{array}{l} \text{if } f_{pu\_pt} \cdot 0.5 \le f_{se\_pt} \\ \quad \| \text{“Prestress is sufficient to use empirical equation”} \\ \text{if } f_{pu\_pt} \cdot 0.5 > f_{se\_pt} \\ \quad \| \text{“WARNING: Prestress is INSUFFICIENT to use empirical equation”} \end{array}$$

$$CheckPrestress\_pt = \text{“Prestress is sufficient to use empirical equation”}$$

$$\beta_{one}(f'_c) := \begin{array}{l} \text{if } f'_c \le 4000\ psi \\ \quad \| 0.85 \\ \text{also if } f'_c > 4000\ psi \wedge f'_c < 8000\ psi \\ \quad \| 0.85 - \left(0.05 \cdot \dfrac{(f'_c - 4000\ psi)}{1000\ psi}\right) \\ \text{else} \\ \quad \| 0.65 \end{array}$$

Relationship between depth of equivalent stress block and depth of N.A. is a function of concrete strength (22.2.2.4.1)

$\beta_1 := \beta_{one}(f'_{ct})$ $\qquad$ $\beta_1 = 0.85$

4.2) Determine stress in pretensioned and post-tensioned strands at flexural strength using ACI empirical equation for fps

$b_{eff} := b_{cb} \cdot n_c = 44.028\ in$

$\gamma_p := 0.28$ $\qquad$ fpy/fpu > 90 for prestressing strand

$d_{p\_ps} := e_{ps\_cb} + y_{tp}$ $\qquad$ $d_{p\_ps} := \max(d_{p\_ps}) = 36\ in$

$d_{p\_pt} := e_{pt\_cb} + y_{tp}$ $\qquad$ $d_{p\_pt} := \max(d_{p\_pt}) = 32\ in$

$$\rho_{p\_ps} := \frac{A_{ps}}{b_{eff} \cdot d_{p\_ps}}$$

$$\rho_{p\_pt} := \frac{A_{pt}}{b_{eff} \cdot d_{p\_pt}}$$

$\rho := 0$ $\qquad$ No mild steel in section, so rho and rho' are zero

$\rho' := 0$

$$CS_{ps} := \rho_{p\_ps} \cdot \frac{f_{pu\_ps}}{f'_c} \qquad CS_{ps} = 0.048$$

$$fps_{ps} := f_{pu\_ps} \cdot \left(1 - \frac{\gamma_p}{\beta_1} \cdot CS_{ps}\right) \qquad fps_{ps} = 266\ \mathbf{ksi}$$

$$CS_{pt} := \rho_{p\_pt} \cdot \frac{f_{pu\_pt}}{f'_c} \qquad CS_{pt} = 0.059$$

$$fps_{pt} := f_{pu\_pt} \cdot \left(1 - \frac{\gamma_p}{\beta_1} \cdot CS_{pt}\right) \qquad fps_{pt} = 265\ \mathbf{ksi}$$

4.3) Determine development length using fps from ACI empirical equation (25.4.8.1)

$$L_{d\_ps} := L_{t\_ps} + \frac{\left(fps_{ps} - f_{se\_ps}\right)}{1000 \cdot \mathbf{psi}} \cdot d_{ps} \qquad L_{d\_ps} = 75.4\ \mathbf{in}$$

$$L_{d\_pt} := \frac{\left(fps_{pt} - f_{se\_pt}\right)}{1000 \cdot \mathbf{psi}} \cdot d_{ps} \qquad L_{d\_pt} = 51.392\ \mathbf{in}$$

4.4) Determine f_{ps} and PHI for flexural calculations

$$
PS\left(fps, fse, A_{ps}, L, L_t, L_d, L_{db}\right) :=
\begin{array}{|l}
Lo \leftarrow Location \\
Dfps \leftarrow fps - fse \\
\text{if } L_{db} > 0 \\
\quad \begin{array}{|l} L_t \leftarrow L_t \cdot 2 \\ L_d \leftarrow L_d \cdot 2 \end{array} \\
pL_t \leftarrow L_t + L_{db} \\
pL_d \leftarrow L_d + L_{db} \\
\text{for } i \in 0 .. Segments \cdot 0.5 \\
\quad \begin{array}{|l}
\text{if } Lo_i \le L_{db} \\
\quad \begin{array}{|l} PS_i \leftarrow 0\ psi \end{array} \\
\text{if } L_{db} < Lo_i \le pL_t \\
\quad \begin{array}{|l} PS_i \leftarrow \dfrac{\left(Lo_i - L_{db}\right)}{L_t} \cdot fse \cdot A_{ps} \end{array} \\
\text{if } pL_t < Lo_i \le pL_d \\
\quad \begin{array}{|l} PS_i \leftarrow \left(\dfrac{\left(Lo_i - L_{db} - L_t\right)}{L_d - L_t} \cdot Dfps + fse\right) \cdot A_{ps} \end{array} \\
\text{if } Lo_i > pL_d \\
\quad \begin{array}{|l} PS_i \leftarrow fps \cdot A_{ps} \end{array} \\
PS_{Segments - i} \leftarrow PS_i
\end{array} \\
\text{return } PS
\end{array}
$$

$$Phi\left(L, L_t, L_d, L_{db}\right) := \begin{array}{|l} Lo \leftarrow Location \\ \text{if } L_{db} > 0 \\ \quad \begin{array}{|l} L_t \leftarrow L_t \cdot 2 \\ L_d \leftarrow L_d \cdot 2 \end{array} \\ pL_t \leftarrow L_t + L_{db} \\ pL_d \leftarrow L_d + L_{db} \\ \text{for } i \in 0 .. Segments \cdot 0.5 \\ \quad \begin{array}{|l} \text{if } Lo_i \le pL_t \\ \quad \begin{array}{|l} Phi_i \leftarrow 0.75 \end{array} \\ \text{if } pL_t < Lo_i \le pL_d \\ \quad \begin{array}{|l} Phi_i \leftarrow \dfrac{\left(Lo_i - L_t - L_{db}\right)}{L_d - L_t} \cdot (0.9 - 0.75) + 0.75 \end{array} \\ \text{if } Lo_i > pL_d \\ \quad \begin{array}{|l} Phi_i \leftarrow 0.9 \end{array} \\ Phi_{Segments - i} \leftarrow Phi_i \end{array} \\ \text{return } Phi \end{array}$$

$$f_{ps\_ps} := PS\left(fps_{ps}, f_{se\_ps}, 1, L, L_{t\_ps}, L_{d\_ps}, L_{db\_ps}\right)$$

Variation in stress of pretensioned tendon at flexural strength as a function of position along the length of the member

$$f_{ps\_pt} := PS\left(fps_{pt}, f_{se\_pt}, 1, L, 0, L_{d\_pt}, L_{db\_pt}\right)$$

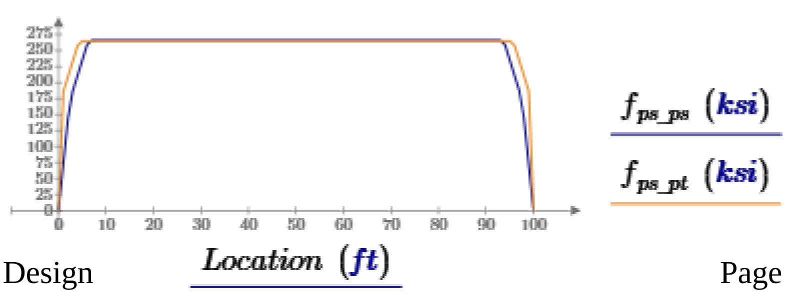
Variation in stress of post-tensioned tendon at flexural strength as a function of position along the length of the member

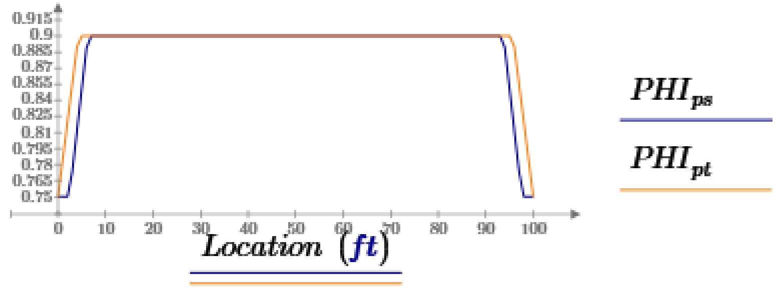
$$PHI_{ps} := Phi\left(L, L_{t\_ps}, L_{d\_ps}, L_{db\_ps}\right)$$

Variation in phi factor for pretensioned tendon as a function of position along the length of the member within the development length

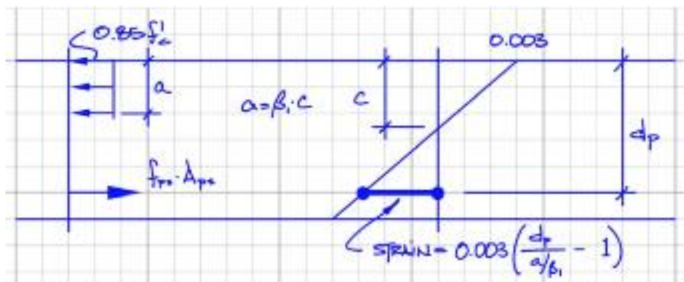
$$PHI_{pt} := Phi\left(L, 0, L_{d\_pt}, L_{db\_pt}\right)$$

Variation in phi factor for post-tensioned tendon as a function of position along the length of the member within the development length





4.5) Use equilibrium to determine flexural strength



$$a_{ps} := \frac{(f_{ps\_ps} \cdot A_{ps})}{0.85 \cdot f'_c \cdot b_{eff}} \quad \max(a_{ps}) = 2.01\ \boldsymbol{in}$$

$$a_{pt} := \frac{(f_{ps\_pt} \cdot A_{pt})}{0.85 \cdot f'_c \cdot b_{eff}} \quad \max(a_{pt}) = 2.17\ \boldsymbol{in}$$

$$a := \max(a_{ps}) + \max(a_{pt}) = 4.171\ \boldsymbol{in} \qquad h_f := 5.5\ \boldsymbol{in}$$

Check if depth of the stress block (a) is less than the thickness of the deck (hf)

$$\varepsilon_{empirical\_ps} := 0.003 \cdot \left(\frac{(\beta_1 \cdot \max(d_{p\_ps}))}{\max(a_{ps})} - 1 \right) \qquad \varepsilon_{empirical\_ps} = 0.0428$$

Strain in the prestressing pretensioned steel is greater than 0.005 so the phi factor is 0.9 (ACI 21.2.2)

$$\varepsilon_{empirical\_pt} := 0.003 \cdot \left(\frac{(\beta_1 \cdot \max(d_{p\_pt}))}{\max(a_{pt})} - 1 \right) \qquad \varepsilon_{empirical\_pt} = 0.0347$$

Strain in the prestressing post-tensioned steel is greater than 0.005 so the phi factor is 0.9 (ACI 21.2.2)

$$\phi M_{n\_ps} := \overrightarrow{\left(PHI_{ps} \cdot f_{ps\_ps} \cdot A_{ps} \cdot \left(d_{p\_ps} - \frac{a_{ps}}{2}\right)\right)}$$ $\max\left(\phi M_{n\_ps}\right) = 1281\ \boldsymbol{kip \cdot ft}$

Design moment strength due to pretnesioned tendon at midspan

$$\phi M_{n\_pt} := \overrightarrow{\left(PHI_{pt} \cdot f_{ps\_pt} \cdot A_{pt} \cdot \left(d_{p\_pt} - \frac{a_{pt}}{2}\right)\right)}$$ $\max\left(\phi M_{n\_pt}\right) = 1221\ \boldsymbol{kip \cdot ft}$

Design moment strength due to post-tensioned tendon at midspan

$$\phi M_n := \phi M_{n\_ps} + \phi M_{n\_pt}$$ $\max\left(\phi M_n\right) = 2502\ \boldsymbol{kip \cdot ft}$

Design moment strength at midspan

4.6) Plot design flexural strength along the length of the member and compare it to the factored moment and minimum reinforcement requirements of ACI

Span is assumed to be from end to end of beam

$$M(w) := \left\| \begin{array}{l} \text{for } i \in 0 \,..\, Segments \\ \quad \left\| M_i \leftarrow 0.5 \cdot w \cdot \left(Location_i \cdot L - \left(Location_i\right)^2\right) \right. \\ M \end{array} \right.$$

$$w_u := 1.2 \cdot \left(w_{DL} + w_{0\_cb}\right) + 1.6 \cdot w_{LL}$$ $w_u = 1.97\ \boldsymbol{klf}$

Factored uniform load

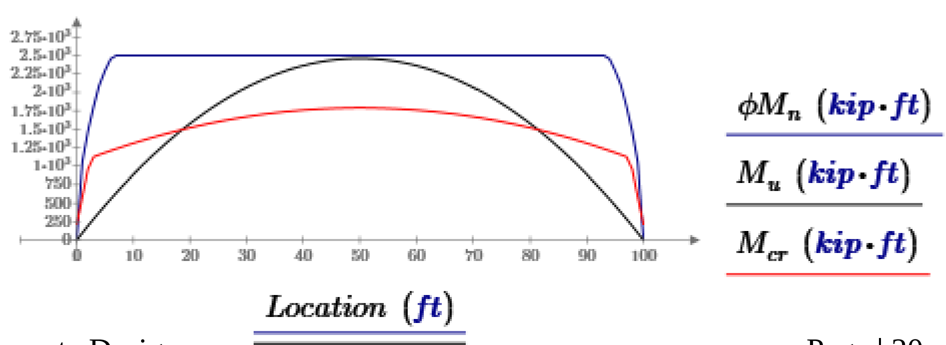
$$M_u := M\left(w_u\right)$$ $\max\left(M_u\right) = 2462\ \boldsymbol{kip \cdot ft}$

Moment strength (phiMn) is greater than factored moment (Mu). Moment capacity of memeber is SUFFIECIENT.

$$f_r := 7.5 \cdot \sqrt{f'_c \cdot \boldsymbol{psi}}$$

$$M_{cr} := \overrightarrow{\left(S_{bp} \cdot \left(\frac{P_e}{A_{cb}} + \frac{\left(P_{e\_ps} \cdot e_{ps\_cb}\right)}{S_{bp}} + \frac{\left(P_{e\_pt} \cdot e_{pt\_cb}\right)}{S_{bp}} + f_r\right)\right)}$$ $\max\left(1.2 \cdot M_{cr}\right) = 2148\ \boldsymbol{kip \cdot ft}$

Design moment strength is greater than max(1.2*Mcr), which satisfies the minimum reinforcement requirements of ACI



5) Shear Strength

$\phi := 0.75$ Strength reduction factor for shear

5.1) Determine transfer length

$L_{t\_ps} := 50 \cdot d_{ps}$ $L_{t\_ps} = 25\ in$ ACI 22.5.9.1

5.2) Determine shear envelope

$$V(w) := \left\| \begin{array}{l} \text{for } i \in 0..Segments \\ \quad \left\| V_i \leftarrow \dfrac{(w \cdot L)}{2} - w \cdot Location_i \right. \\ V \end{array} \right.$$

$$V_{Env}(w) := \left\| \begin{array}{l} \text{for } i \in 0..Segments \\ \quad \left\| V_{Env_i} \leftarrow \left| \dfrac{(w \cdot L)}{2} - w \cdot Location_i \right| \right. \\ V_{Env} \end{array} \right.$$

$V_u := V(w_u)$

$V_{uEnv} := V_{Env}(w_u)$

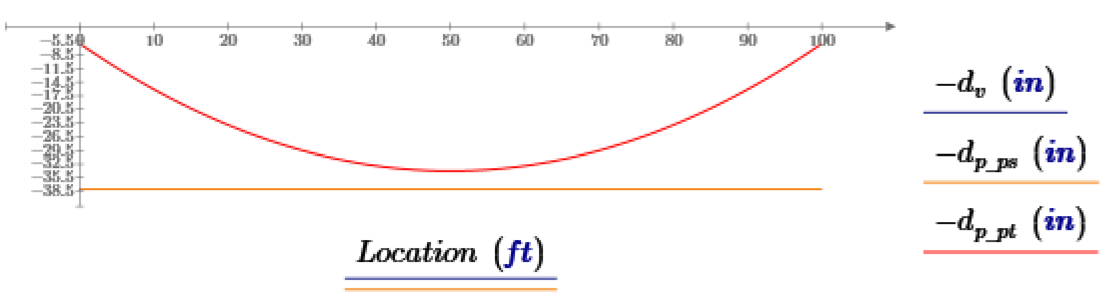
5.3) Determine effective depth

$d_{p\_ps} := e_{ps\_cb} + y_{tc}$

$d_{p\_pt} := e_{pt\_cb} + y_{tc}$

$$d_v := \left\| \begin{array}{l} \text{for } i \in 0..Segments \\ \quad \left\| dp_i \leftarrow \max\left(d_{p\_ps_i}, d_{p\_pt_i}, 0.8 \cdot h_{cb}\right) \right. \\ \text{return } dp \end{array} \right.$$

Effective depth for calculating shear strength (not taken less than 0.8h)



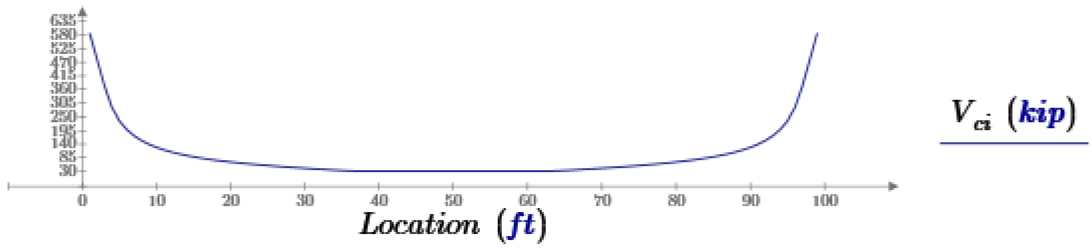
$\max(d_v) = 38\ in$

5.4) Determine concrete contribution considering flexure-shear cracking (Vci)

$f_r := 6 \cdot \sqrt{f'_c \cdot psi} = 484\ psi$ Modulus of rupture

$V_{c2} := 1.7 \cdot \sqrt{f'_c \cdot psi} \cdot b_w \cdot d_v$ Minimum shear strength contributed by concrete

$$V_{ci} := \left\| \begin{array}{l} \text{for } i \in 0..Segments \\ \quad \left\| \begin{array}{l} \text{if } M_{u_i} > 0 \vee M_{u_i} < 0 \\ \quad \left\| \begin{array}{l} V\_over\_M \leftarrow \dfrac{\left|V_{u_i}\right|}{\left|M_{u_i}\right|} \\ Vc_i \leftarrow \max\left(0.6 \cdot \lambda \cdot \sqrt{f'_c \cdot psi} \cdot b_w \cdot d_{v_i} + V\_over\_M \cdot M_{cr_i}, V_{c2_i}\right) \end{array} \right. \\ \text{else} \\ \quad \left\| Vc_i \leftarrow NaN \right. \end{array} \right. \\ \text{return } Vc \end{array} \right.$$



5.5) Determine concrete contribution considering web-shear cracking (Vcw)

$$V_p := \left\| \begin{array}{l} d_p(x) \leftarrow \text{linterp}\left(Location, e_{pt\_cb} + y_{tc}, x\right) \\ slope(x) \leftarrow \dfrac{d}{dx} d_p(x) \\ \text{for } i \in 0..Segments \\ \quad \left\| VP_i \leftarrow P_{e\_pt_i} \cdot \left|\sin\left(\text{atan}\left(slope\left(Location_i\right)\right)\right)\right| \right. \\ \text{return } VP \end{array} \right.$$

Calculates the vertical component of the prestress force in sloped tendons. Assumes that the tendon contributes to shear strength by taking the absolute value of the slope.

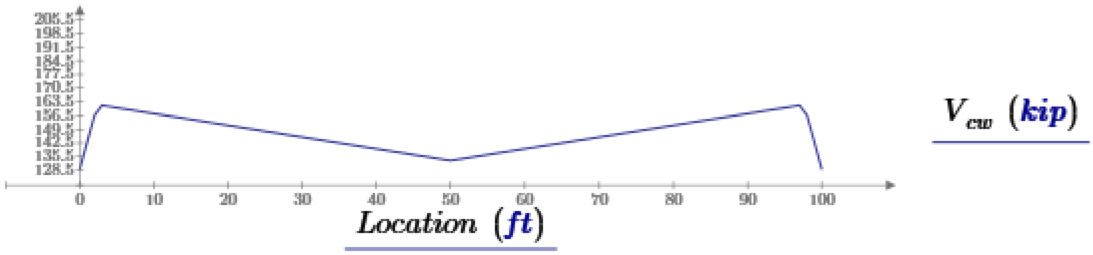
Check location of critical section according to 11.4.3

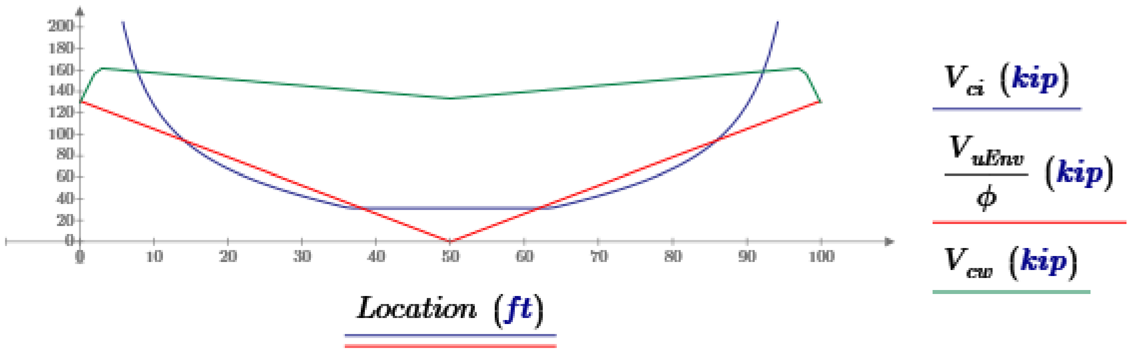
$\dfrac{h_{cb}}{2} = 21\ in$ $L_{t\_ps} = 25\ in$ Critical section lies within the transfer length (h/2 < Lt)

$f_{pc} := \dfrac{P_e}{A_{cb}}$ Prestress considered for reduced stress over the transfer length (22.5.9.3)

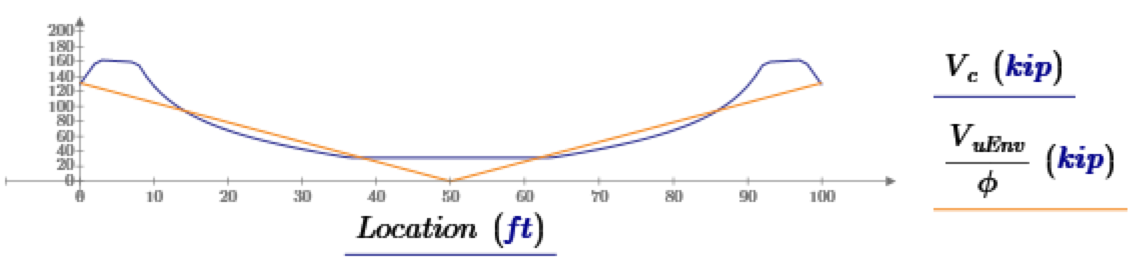
$V_{cw} := \overrightarrow{\left(\left(3.5 \cdot \lambda \cdot \sqrt{f'_c \cdot psi} + 0.3 \cdot f_{pc}\right) \cdot b_w \cdot d_v + V_p\right)}$

Reduced shear strength provided by concrete where diagonal cracking occurs in the web of the critical section (22.5.8.3)





$$V_c := \left\| \begin{array}{l} \text{for } i \in 0..Segments \\ \quad \left\| \begin{array}{l} \text{if } V_{cw_i} > V_{ci_i} \\ \quad \left\| Vmin_i \leftarrow V_{ci_i} \right. \\ \text{else} \\ \quad \left\| Vmin_i \leftarrow V_{cw_i} \right. \end{array} \right. \\ Vmin \end{array} \right.$$



$$V_{uDesign} := \overrightarrow{\left(\frac{V_{uEnv}}{\phi} - V_c\right)}$$

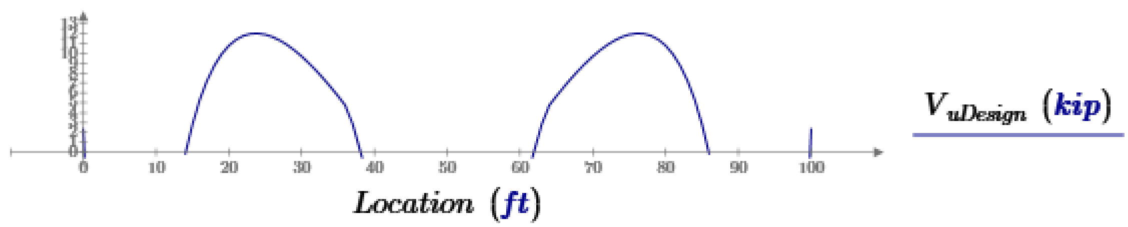
$\max\left(V_{uDesign}\right) = 12$ ***kip*** Maximum required shear

$$CheckSection := \left\| \begin{array}{l} \text{for } i \in 0..Segments \\ \quad \left\| \begin{array}{l} \text{if } V_{uDesign_i} \le 8 \cdot \lambda \cdot \sqrt{f'_c \cdot psi} \cdot b_w \cdot d_{v_i} \\ \quad \left\| SectionSize \leftarrow \text{“Section size is adequate”} \right. \\ \text{if } V_{uDesign_i} > 8 \cdot \lambda \cdot \sqrt{f'_c \cdot psi} \cdot b_w \cdot d_{v_i} \\ \quad \left\| SectionSize \leftarrow \text{“WARNING: section size is INSUFFICIENT”} \right. \end{array} \right. \\ \text{return } SectionSize \end{array} \right.$$

$CheckSection = \text{“Section size is adequate”}$

$V_{max} := 8 \cdot \lambda \cdot \sqrt{f'_c \cdot psi} \cdot b_w \cdot d_v$ $\max\left(V_{max}\right) = 147.1\ kip$

Check to ensure section size is adequate based on the required shear strength being less than the maximum shear strength that can be provided by stirrups according to ACI 22.5.1.2.



5.6) Determine shear reinforcement required for strength

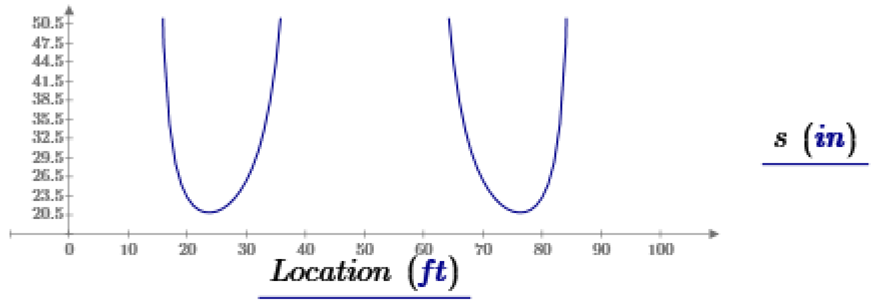
$A_v := 0.11 \cdot in^2$ #3 mild steel reinforcing bars (PCI Design Manual Table 15.4.1)

$b_v := 30\ in$ Width of contact area

$f_{yt} := 60 \cdot ksi$ ASTM A706 Grade 60 Deformed Bar (Table 20.2.2.4a)

$$s := \left\| \begin{array}{l} \text{for } i \in 0..Segments \\ \quad \left\| \begin{array}{l} \text{if } V_{uDesign_i} > 0 \\ \quad \left\| smax_i \leftarrow \dfrac{\left(A_v \cdot f_{yt} \cdot d_{v_i}\right)}{V_{uDesign_i}} \right. \\ \text{else} \\ \quad \left\| smax_i \leftarrow NaN \right. \end{array} \right. \\ \text{return } smax \end{array} \right.$$

Rearrange ACI equation 22.5.10.5.3 to calculate longitudinal spacing of shear reinforcement



5.7) Determine shear spacing limits and minimum shear reinforcement requirements

Spacing limits Table 9.7.6.2.2

$$
\begin{aligned}
s_{max1} := \;& \text{for } i \in 0..Segments \\
& \quad \text{if } V_{uDesign_i} \le 4 \cdot \sqrt{f'_c \cdot psi} \cdot b_w \cdot d_{v_i} \\
& \quad\quad smax_i \leftarrow min(0.75 \cdot h_{cb}, 24 \cdot in) \\
& \quad \text{if } V_{uDesign_i} > 4 \cdot \sqrt{f'_c \cdot psi} \cdot b_w \cdot d_{v_i} \\
& \quad\quad smax_i \leftarrow 0.5 \cdot min(0.75 \cdot h_{cb}, 24 \cdot in) \\
& \quad \text{if } \frac{V_{uEnv_i}}{\phi} \le 0.5 \cdot V_{c_i} \\
& \quad\quad smax_i \leftarrow NaN \\
& \text{return } smax
\end{aligned}
$$

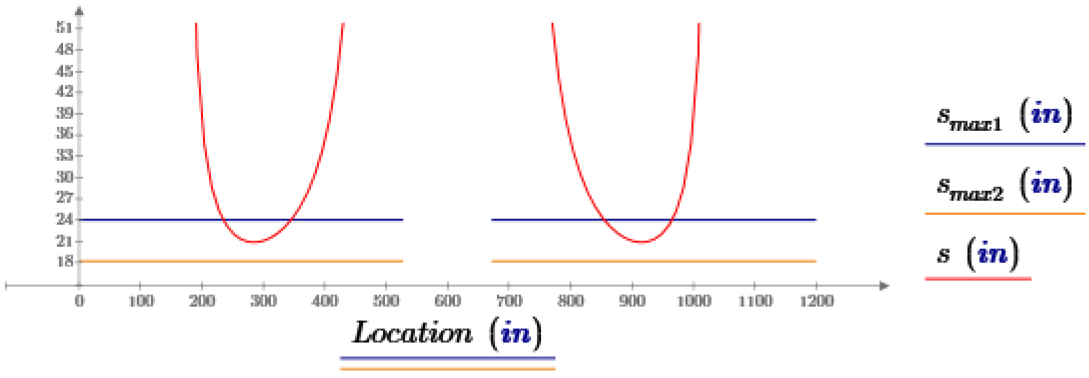
$$f_{pu} := 270\ ksi$$

$$A_{prestressed} := A_{ps} + A_{pt}$$

Minimum reinforcement Table 9.6.3.3

$$
\begin{aligned}
s_{max2} := \;& \text{for } i \in 0..Segments \\
& \quad \text{if } \frac{V_{uEnv_i}}{\phi} > 0.5 \cdot V_{c_i} \\
& \quad\quad smax_i \leftarrow \max\left(\frac{(A_v \cdot f_{yt})}{0.75 \cdot b_w \cdot \sqrt{f'_c \cdot psi}}, \frac{(80 \cdot f_{yt} \cdot d_{v_i} \cdot A_v)}{A_{prestressed} \cdot f_{pu}} \cdot \sqrt{\frac{b_w}{d_{v_i}}}\right) \\
& \quad \text{if } \frac{V_{uEnv_i}}{\phi} \le 0.5 \cdot V_{c_i} \\
& \quad\quad smax_i \leftarrow NaN \\
& \text{return } smax
\end{aligned}
$$

$$\frac{(A_v \cdot f_{yt})}{0.75 \cdot b_w \cdot \sqrt{f'_c \cdot psi}} = 18.19\ in \qquad \frac{(80 \cdot f_{yt} \cdot \max(d_v) \cdot A_v)}{A_{prestressed} \cdot f_{pu}} \cdot \sqrt{\frac{b_w}{\max(d_v)}} = 7.72\ in$$



5.8) Lay out stirrups

$L_{bL} := 0\ in$

$L_{bR} := 0\ in$

$s := [12\ \ 18\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 18\ \ 12] \cdot in \qquad s := s^{T}$

$ns := [2\ \ 32\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 32\ \ 2] \qquad ns := ns^{T}$

$Ls := \overrightarrow{(s \cdot ns)}$

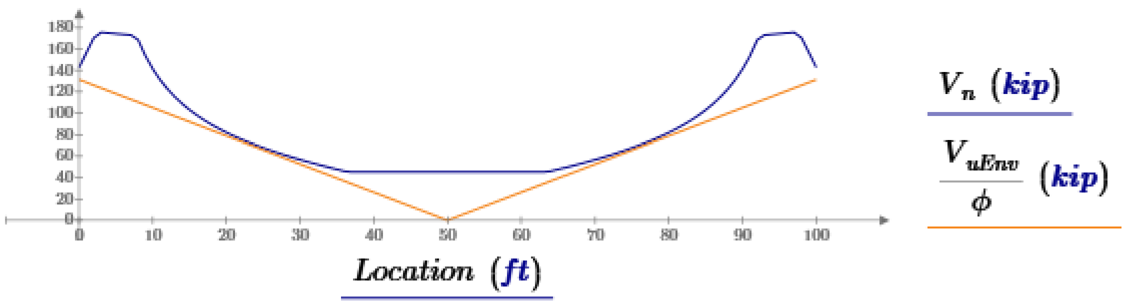
$\sum Ls = 100\ ft \qquad L = 100\ ft$

```
Ls := | sum ← 0
      | for i ∈ 0..10
      |   | sum ← Ls_i + sum
      |   | Ls_i ← sum
      | return Ls
```

$$V_n := \left\| \begin{array}{l} \text{for } i \in 0..Segments \\ \quad \left\| \begin{array}{l} \text{if } Location_i < Ls_1 \\ \quad \left\| Vn_i \leftarrow V_{c_i} + \dfrac{\left(A_v \cdot f_{yt} \cdot d_{v_i}\right)}{s_1} \right. \\ \text{for } j \in 1..8 \\ \quad \left\| \begin{array}{l} \text{if } Location_i < Ls_{j+1} \wedge Location_i \ge Ls_j \\ \quad \left\| Vn_i \leftarrow V_{c_i} + \dfrac{\left(A_v \cdot f_{yt} \cdot d_{v_i}\right)}{s_{j+1}} \right. \end{array} \right. \\ \text{if } Location_i \ge Ls_8 \\ \quad \left\| Vn_i \leftarrow V_{c_i} + \dfrac{\left(A_v \cdot f_{yt} \cdot d_{v_i}\right)}{s_9} \right. \end{array} \right. \\ \text{return } Vn \end{array} \right.$$

$$CheckShear(Vn, VuPhi) := \left\| \begin{array}{l} DRmin \leftarrow 1 \\ \text{for } i \in 0..Segments \\ \quad \left\| \begin{array}{l} \text{if } Vn_i < VuPhi_i \wedge Location_i > L_{bL} \wedge Location_i < L - L_{bR} \\ \quad \left\| \begin{array}{l} \text{if } \dfrac{Vn_i}{VuPhi_i} < DRmin \\ \quad \left\| \begin{array}{l} Lmin \leftarrow Location_i \\ DRmin \leftarrow \dfrac{Vn_i}{VuPhi_i} \end{array} \right. \\ \text{continue} \end{array} \right. \\ \text{continue} \end{array} \right. \\ \text{if } DRmin < 1 \\ \quad \left\| \begin{array}{l} CS \leftarrow \text{“Shear strength is INADEQUATE. DR = ”} \\ CS \leftarrow \text{concat}\left(CS, \text{substr}\left(\text{num2str}\left(DRmin\right), 0, 5\right)\right) \\ \text{CS} \leftarrow \text{concat}\left(CS, \text{“ at Location = ”}\right) \\ CS \leftarrow \text{concat}\left(CS, \text{substr}\left(\text{num2str}\left(\dfrac{Lmin}{ft}\right), 0, 5\right), \text{“ft”}\right) \\ \text{return } CS \end{array} \right. \\ \text{else} \\ \quad \left\| \text{“Shear strength is adequate.”} \right. \end{array} \right.$$

$$
DR(Vn, VuPhi) := \left\| \begin{array}{l}
\text{for } i \in 0 \,..\, Segments \\
\quad \left\| \begin{array}{l}
\text{if } Location_i > L_{bL} \wedge Location_i < L - L_{bR} \\
\quad \left\| \; DR_i \leftarrow \dfrac{Vn_i}{VuPhi_i} \right. \\
\text{else} \\
\quad \left\| \; DR_i \leftarrow NaN \right.
\end{array} \right. \\
\text{return } DR
\end{array} \right.
$$



$$CheckShear\left(V_n, \frac{V_{uEnv}}{\phi}\right) = \text{“Shear strength is adequate.”}$$