

perceptron

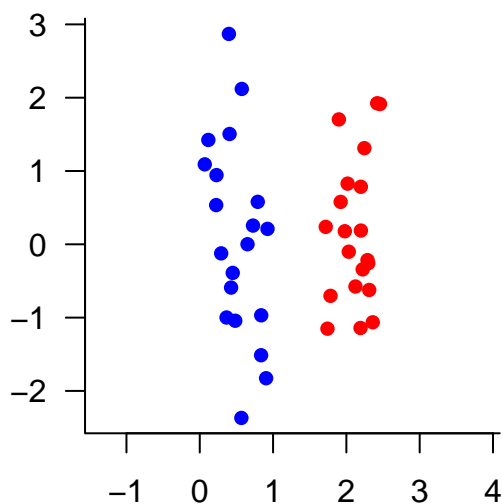
Joyce Robbins

8/13/2018

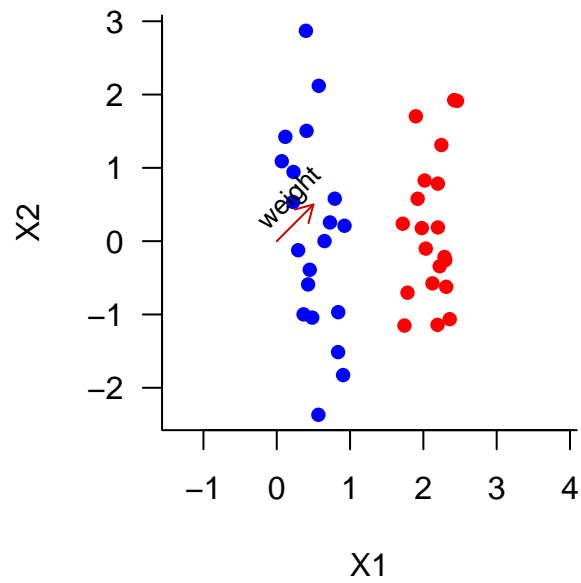
The perceptron is a simple algorithm that learns to classify inputs into two classes by adjusting the weights (w) in the equation $y_i = \text{sign}(w_i x_i)$ until all inputs in a training set are correctly classified. Here the steps of algorithm will be presented visually in two-dimensional space.

The basics

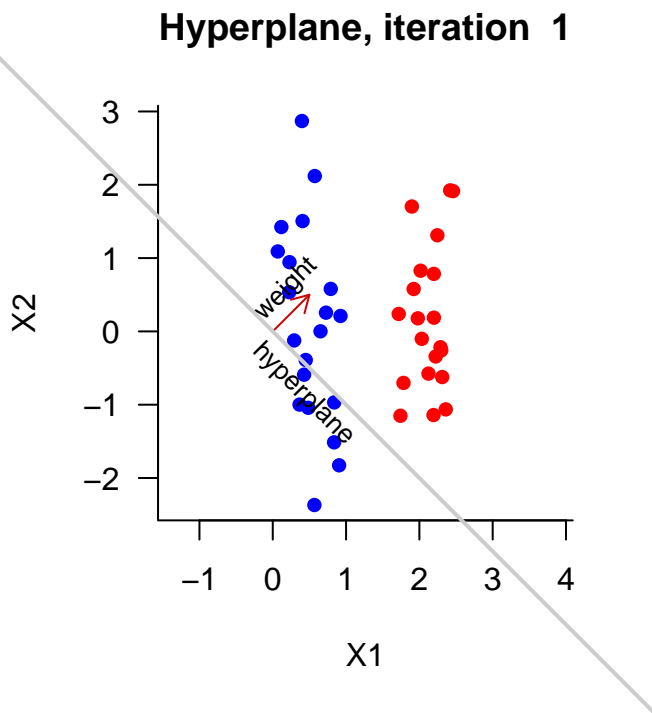
We start by plotting (x_1, x_2) , coloring each point by class. Note that the points can be separated by a line; if this is not the case, the algorithm won't work.



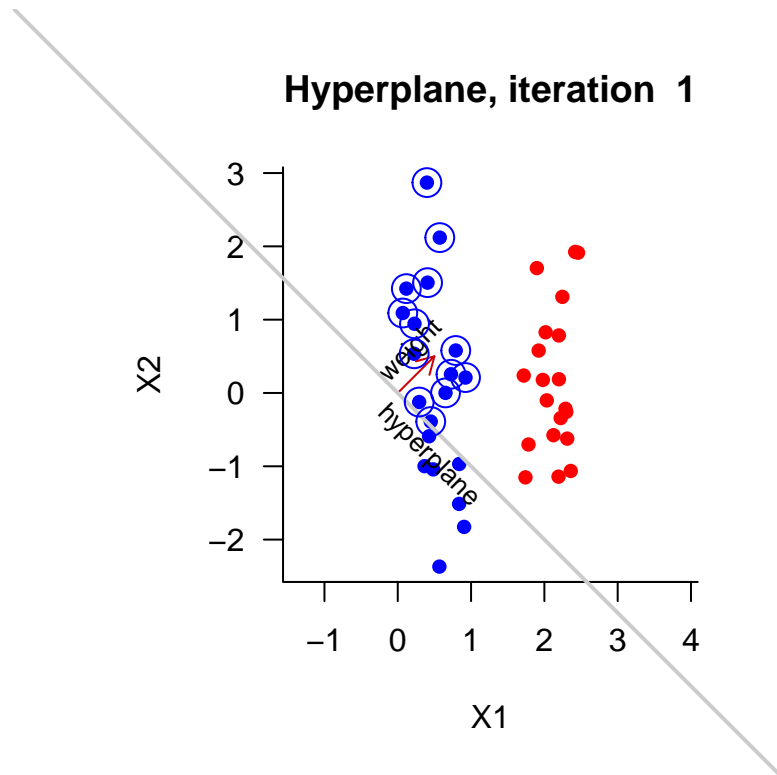
We start with an arbitrary weight vector, (w_0, w_1, w_2) . Often $(0, 0, 0)$ is used, but we'll start with $(0, 0.5, 0.5)$ so we can visualize it:



The decision boundary, or hyperplane, is the line orthogonal to the weight vector. For points on the line, the sign of $(w_i x_i)$ equals zero. On one side of the line, the sign of $(w_i x_i)$ is greater than zero whereas on the other side the sign of $(w_i x_i)$ is less than zero; hence the line serves to divide all points into two classes according to the perceptron logic.



Note the circled points – these are the misclassified points – the ones for which $y_i \neq \text{sign}(w_i x_i)$.



The Algorithm

The perceptron algorithm works by updating the weight vector based on a randomly selected misclassified point, calculating the new hyperplane, and repeating until the hyperplane separates all points into the two classes.

The formula for the new weight vector is:

$$w_{t+1} = w_t + \eta y_i x_i, \text{ where}$$

x_i = the misclassified point

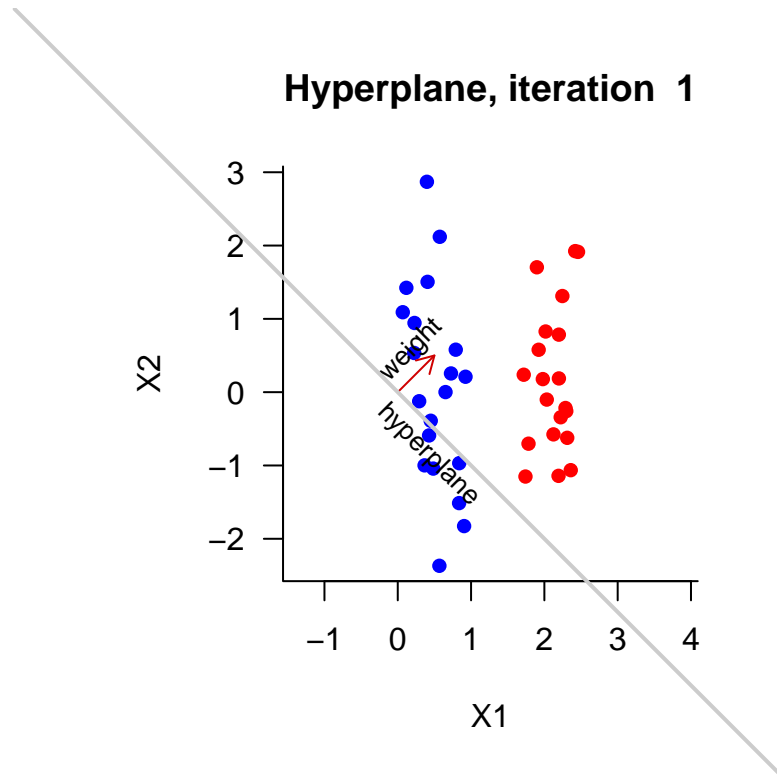
y_i = the true label of the misclassified point (-1 or 1)

η = the learning rate, which we'll set to 1 for the sake of simplicity

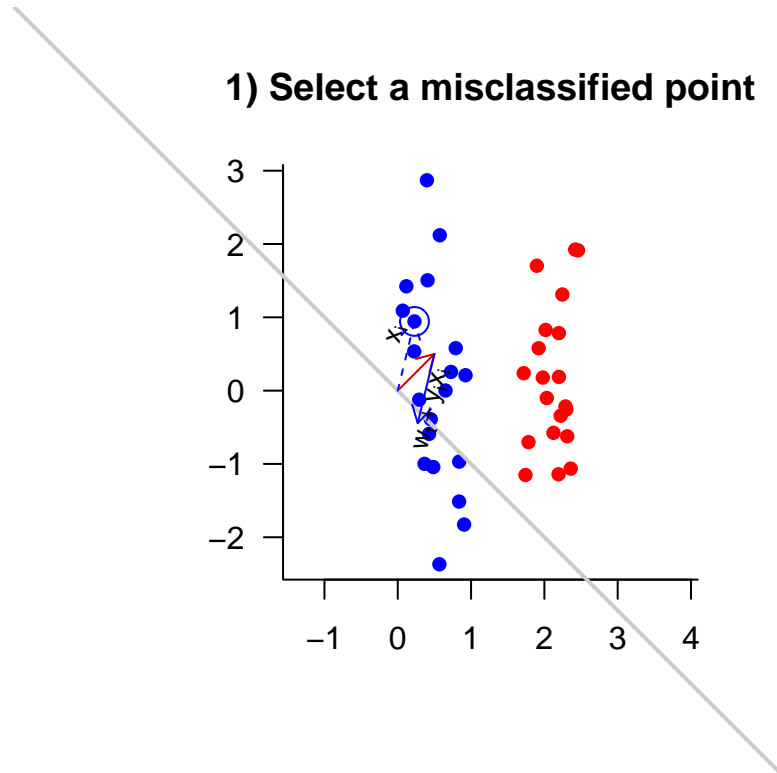
Visually, the new weight vector, w_{t+1} , is determined by adding $y_i x_i$ to w_t and then shifting by the offset $w_0 / \|w\|_2$.

We'll go through the algorithm one step at a time.

We begin with our original weight vector and hyperplane:

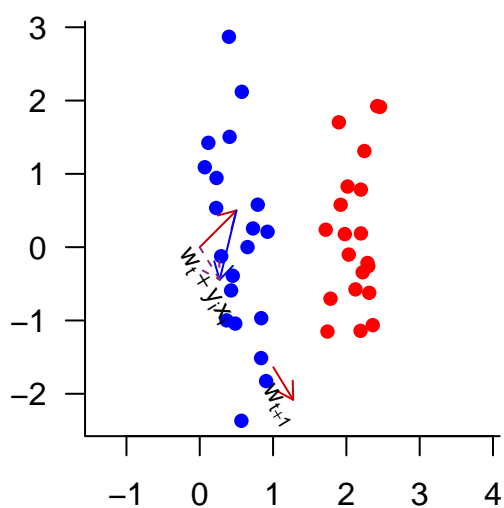


Next we randomly select a misclassified point. In the diagram below, x_i is shown as a **dashed blue arrow**, and $y_i x_i$ added to w_t as a **solid blue arrow**:



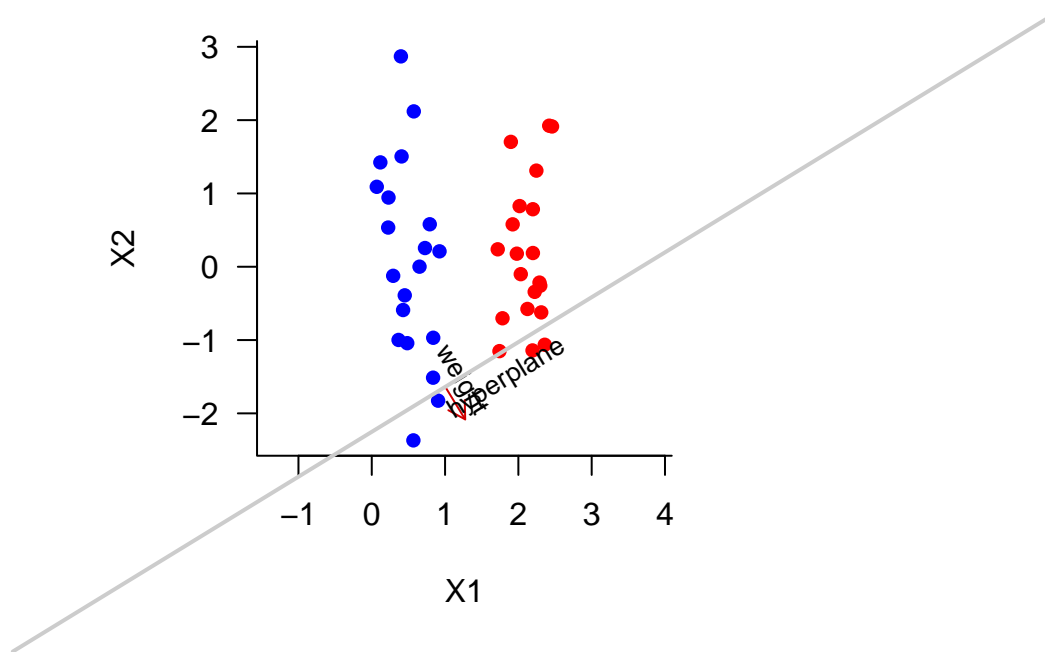
Next we determine the new weight vector by shifting the vector sum by $w_0/||w||_2$:

2) Draw new weight vector

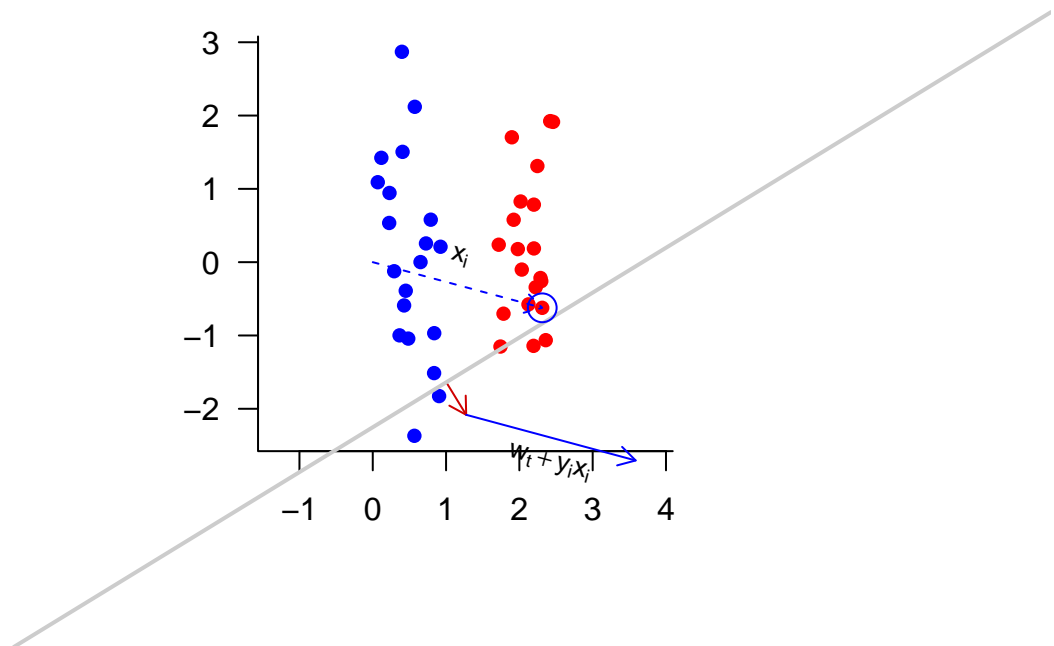


Finally, we draw the new hyperplane:

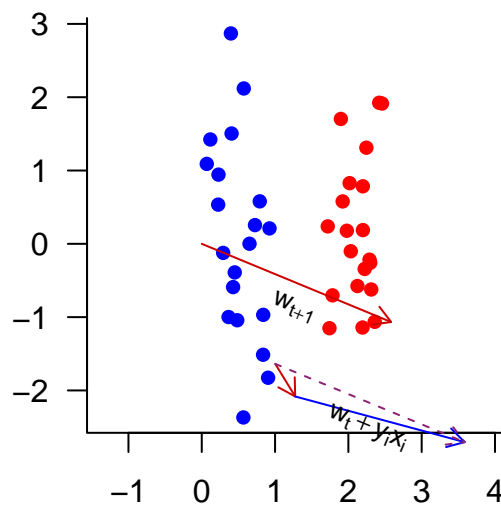
Hyperplane, iteration 2



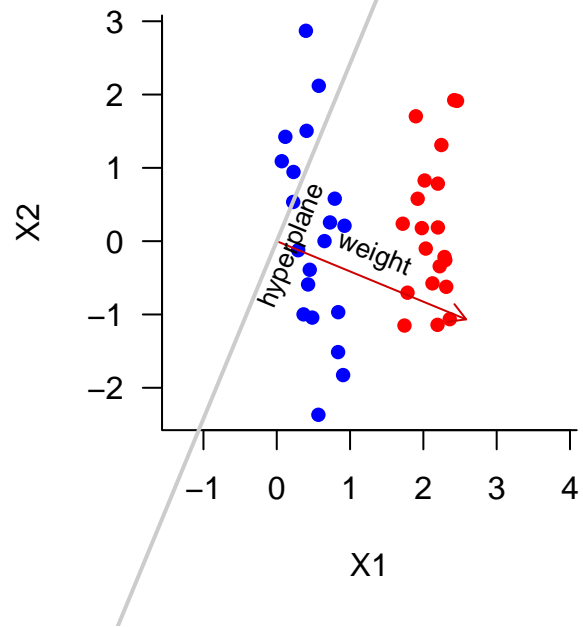
1) Select a misclassified point



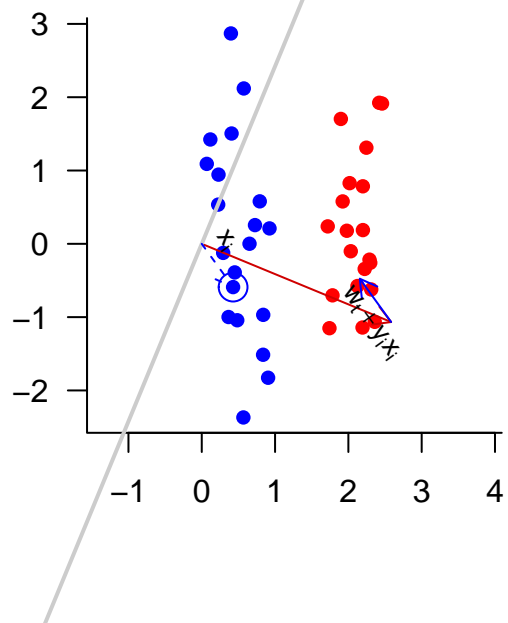
2) Draw new weight vector



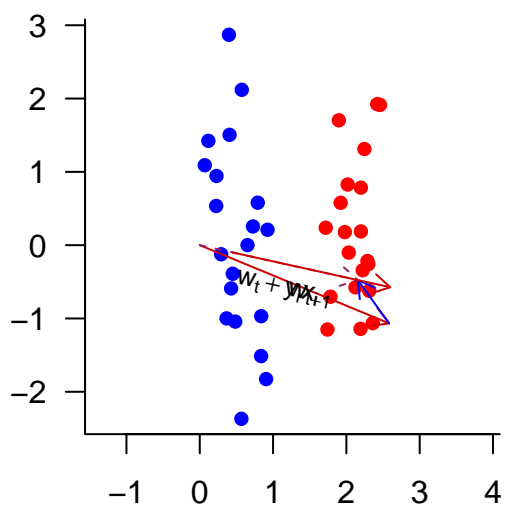
Hyperplane, iteration 3



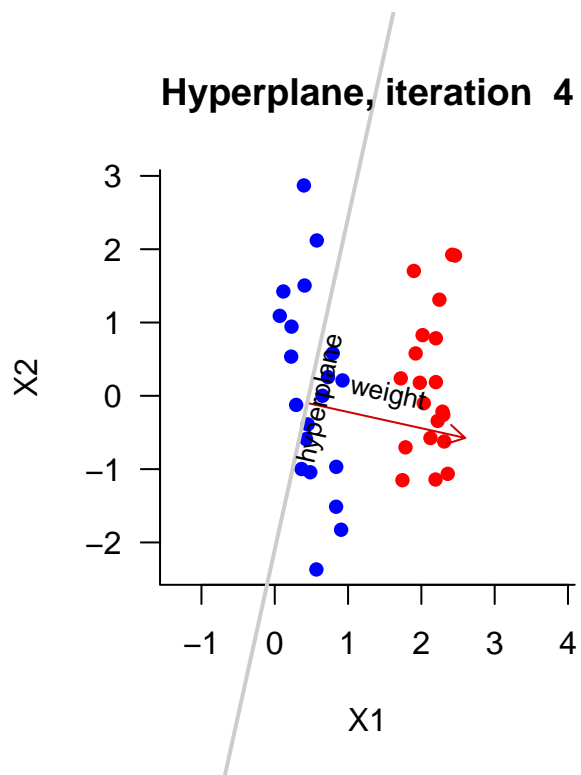
1) Select a misclassified point



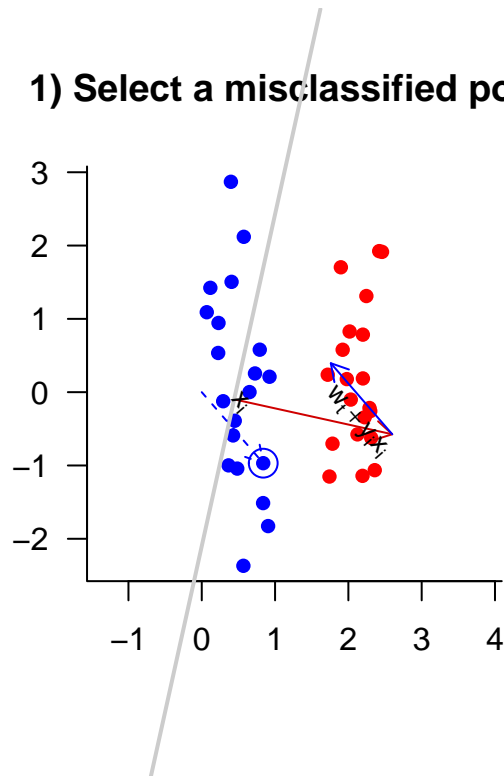
2) Draw new weight vector



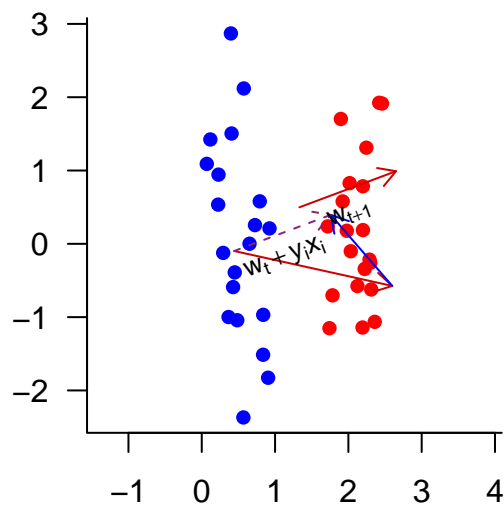
Hyperplane, iteration 4



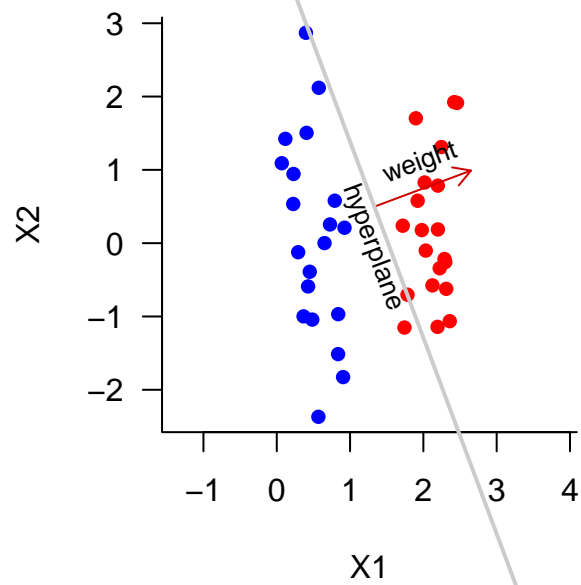
1) Select a misclassified point



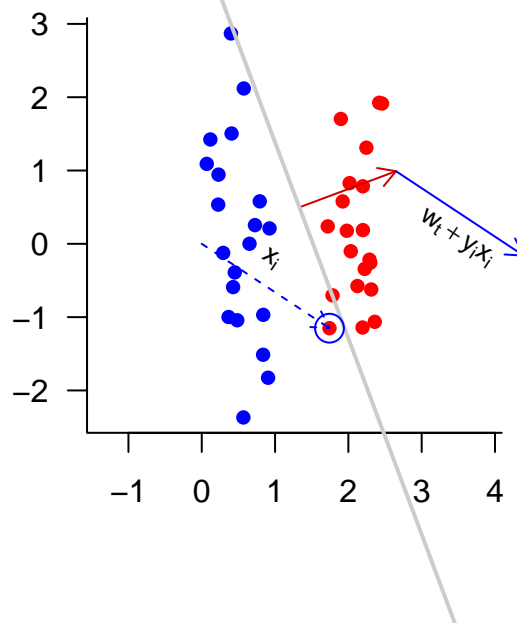
2) Draw new weight vector



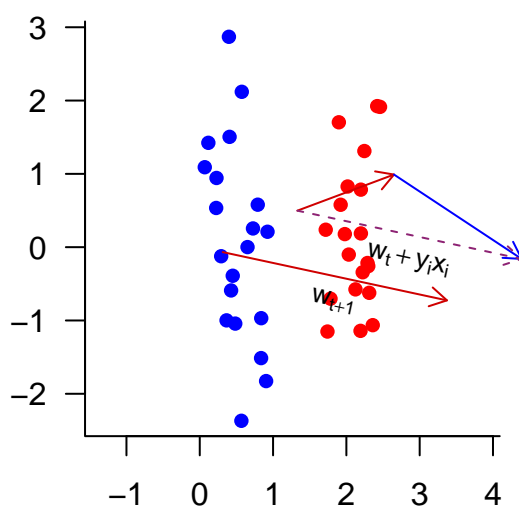
Hyperplane, iteration 5



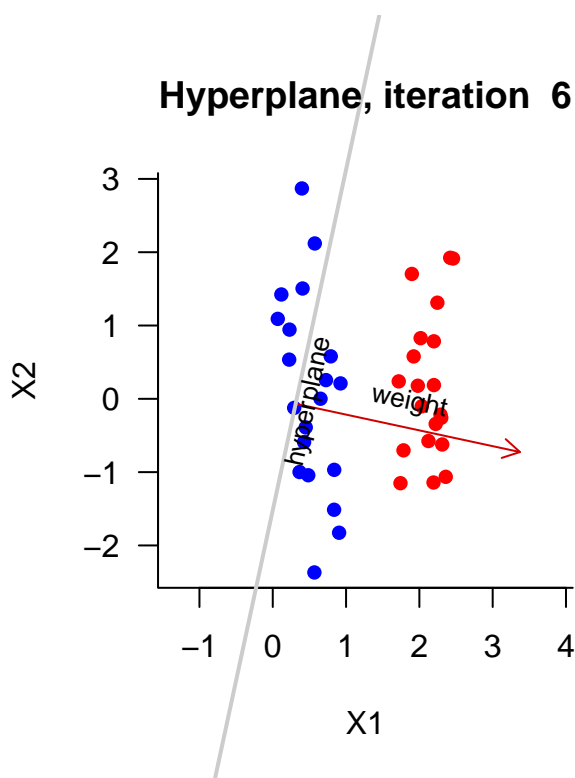
1) Select a misclassified point



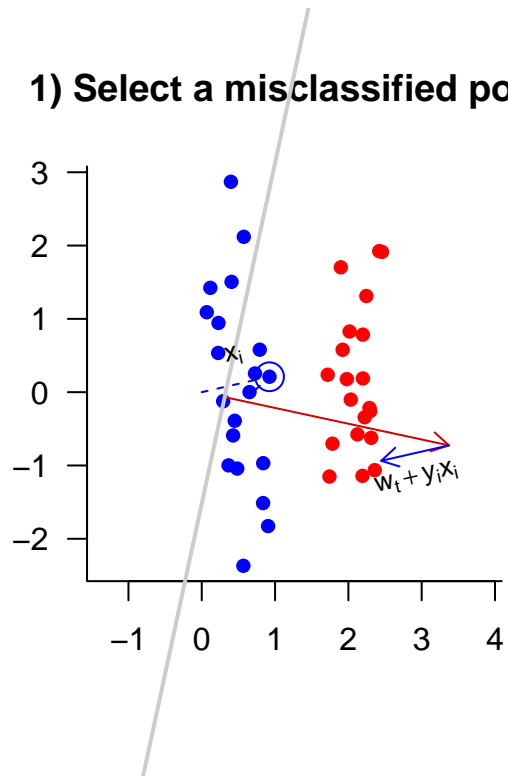
2) Draw new weight vector



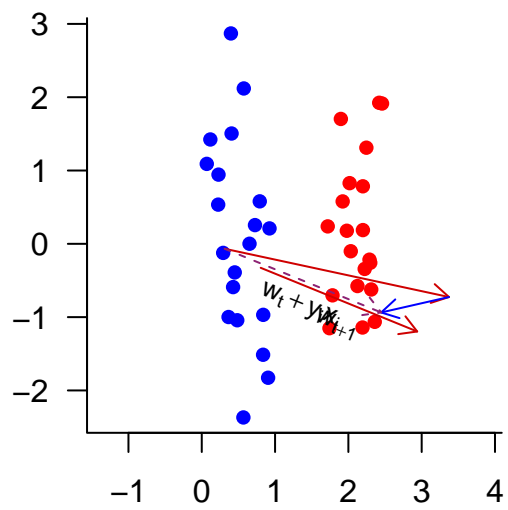
Hyperplane, iteration 6



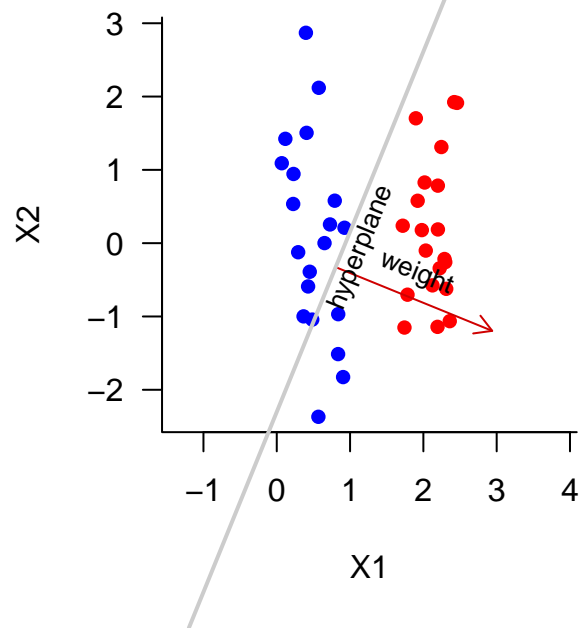
1) Select a misclassified point



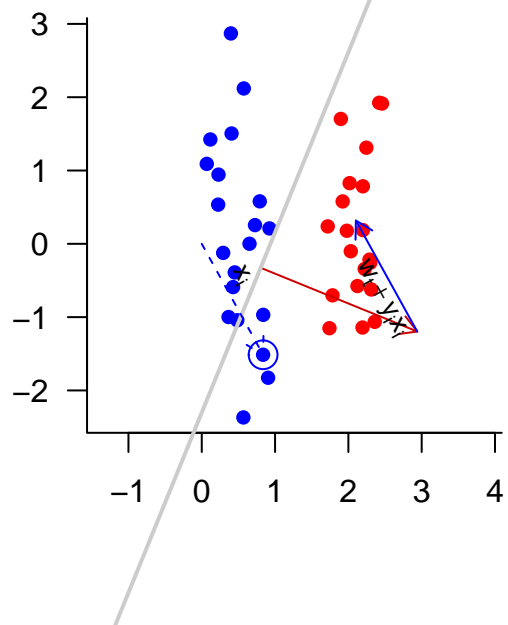
2) Draw new weight vector



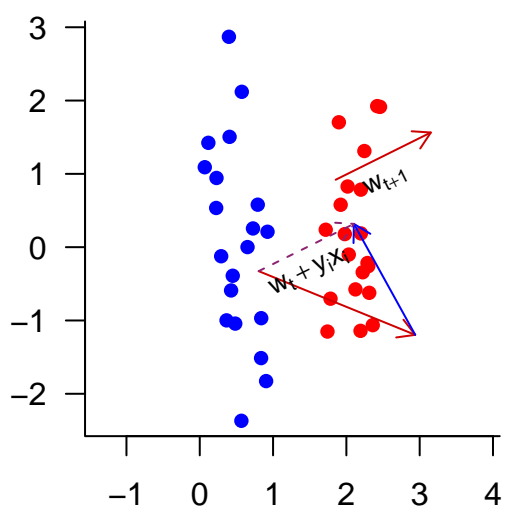
Hyperplane, iteration 7



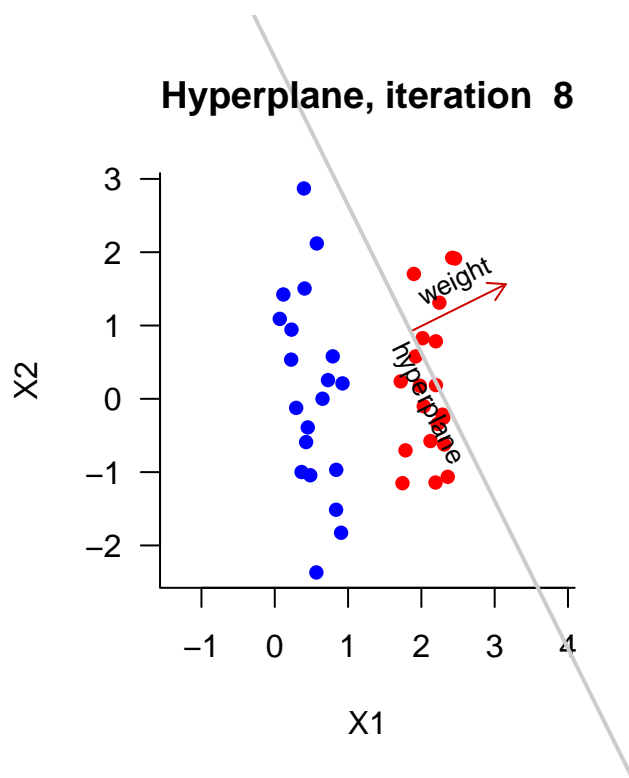
1) Select a misclassified point



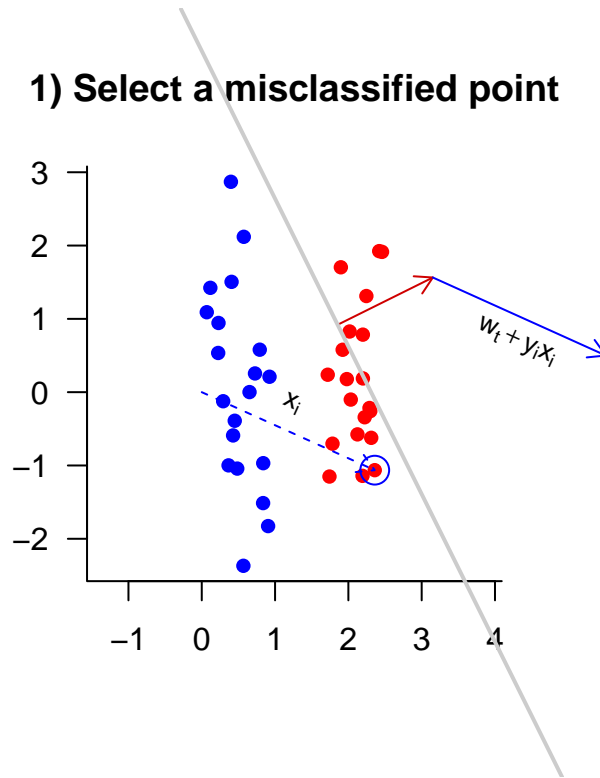
2) Draw new weight vector



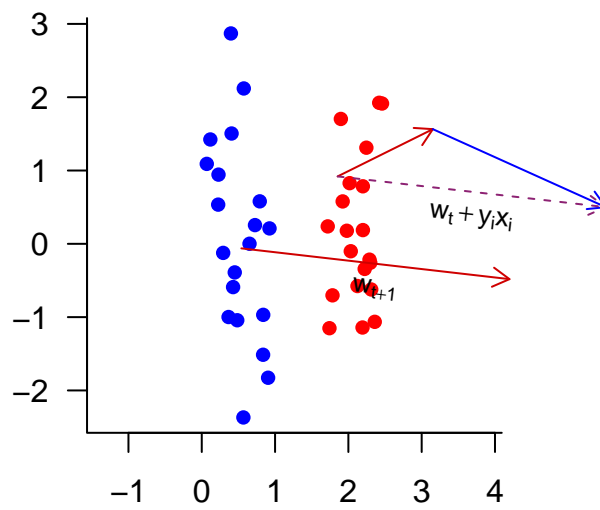
Hyperplane, iteration 8

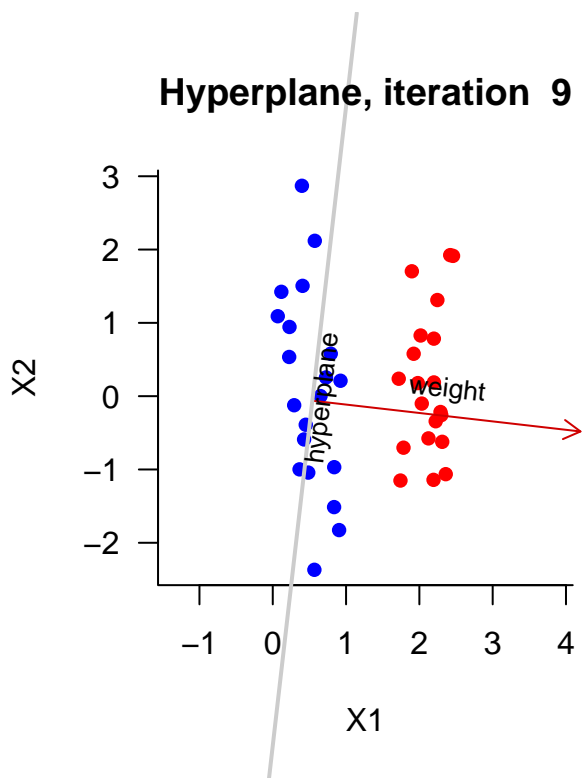


1) Select a misclassified point

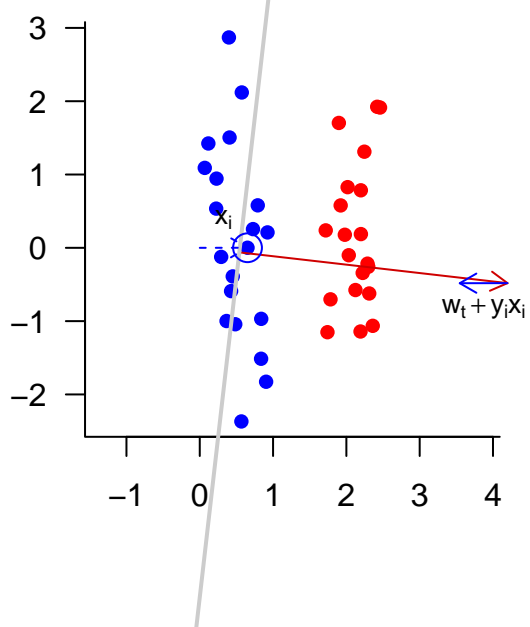


2) Draw new weight vector

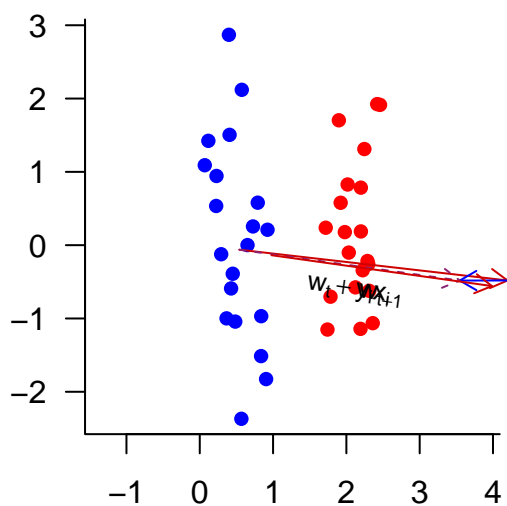




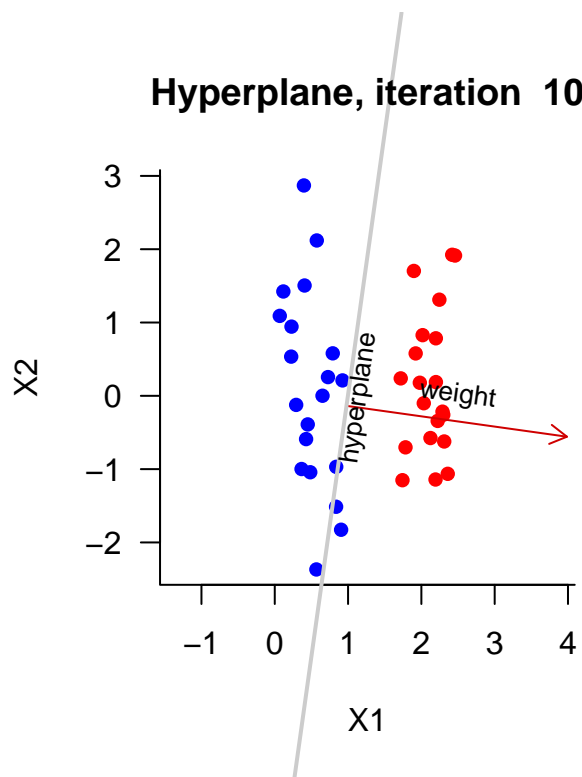
1) Select a misclassified point



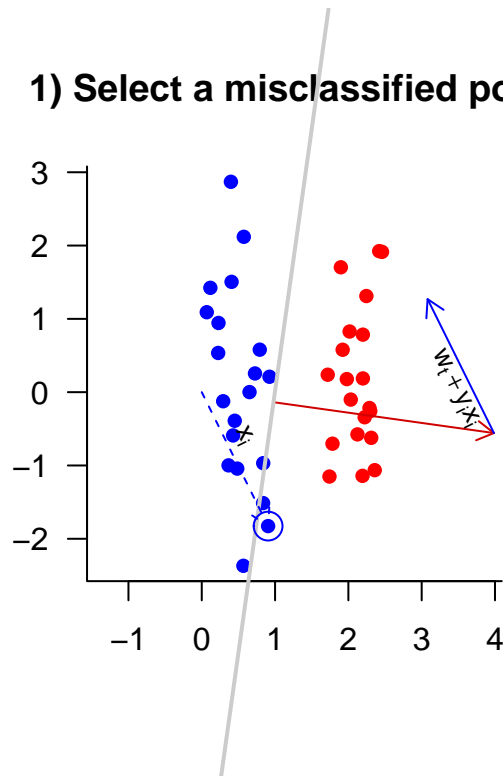
2) Draw new weight vector



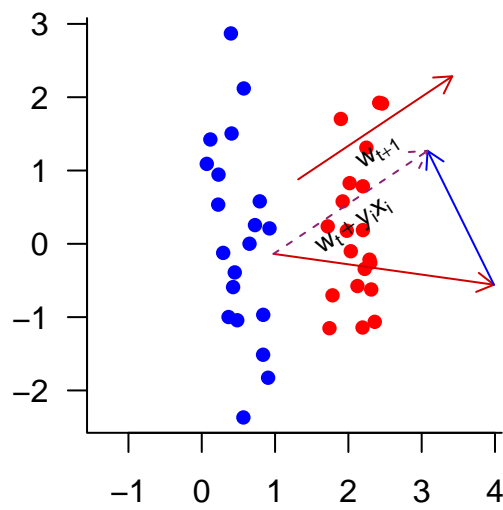
Hyperplane, iteration 10



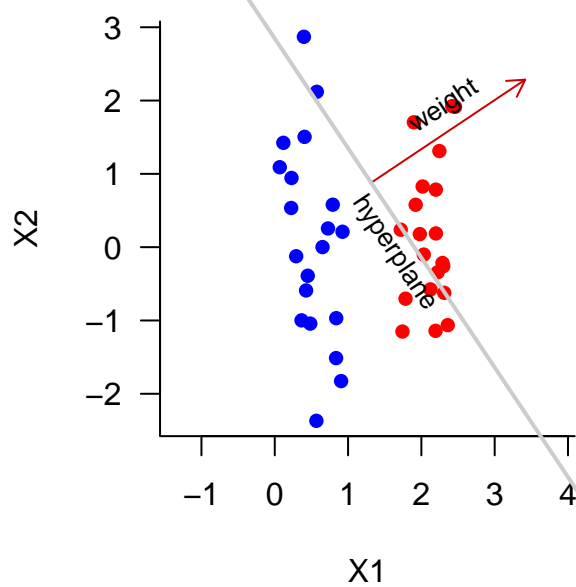
1) Select a misclassified point



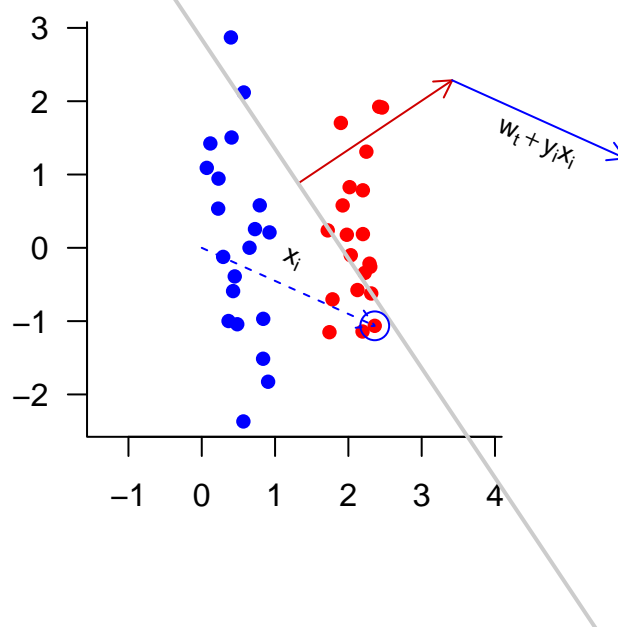
2) Draw new weight vector



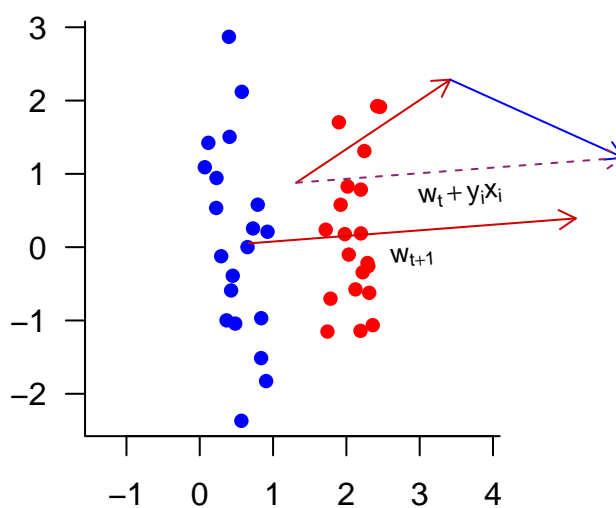
Hyperplane, iteration 11



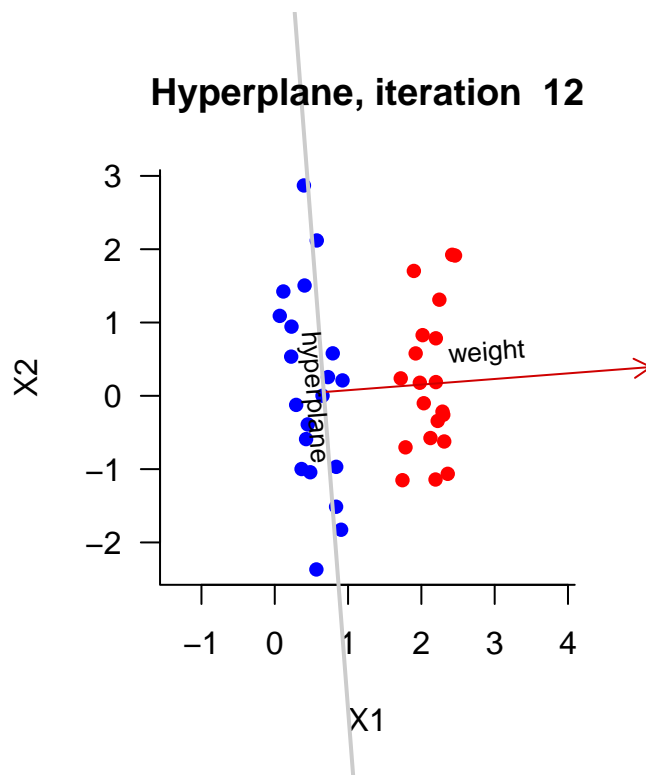
1) Select a misclassified point



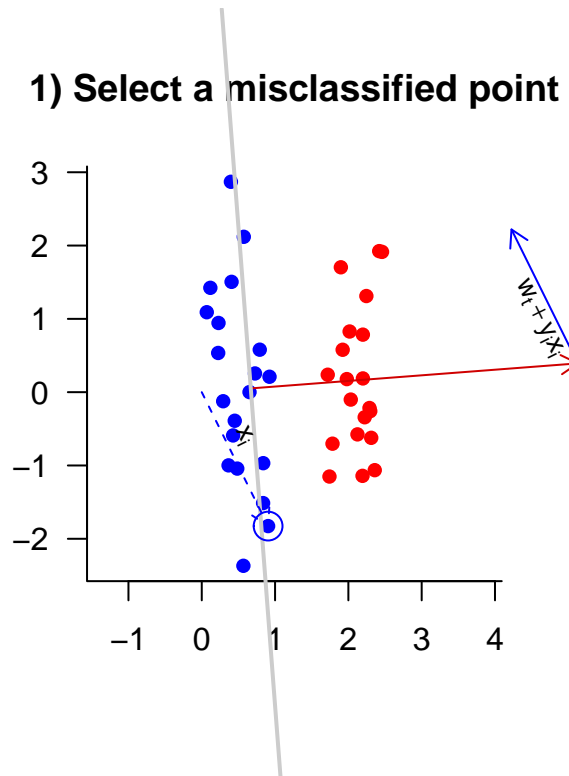
2) Draw new weight vector



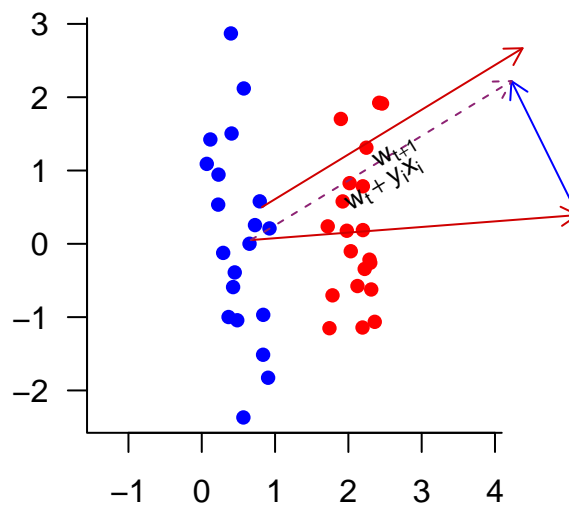
Hyperplane, iteration 12



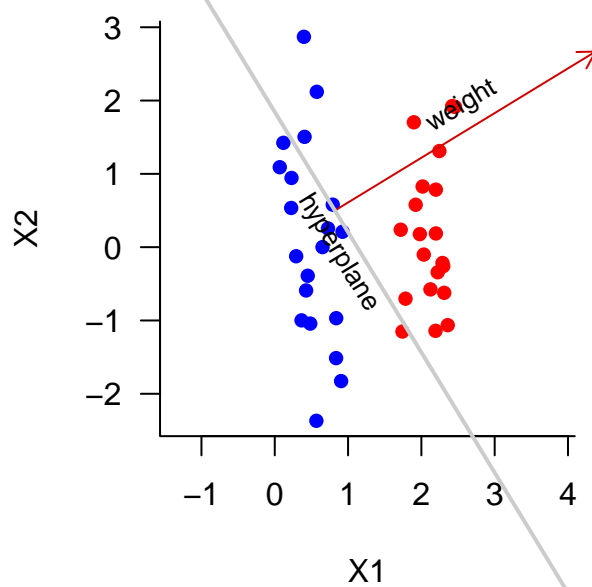
1) Select a misclassified point



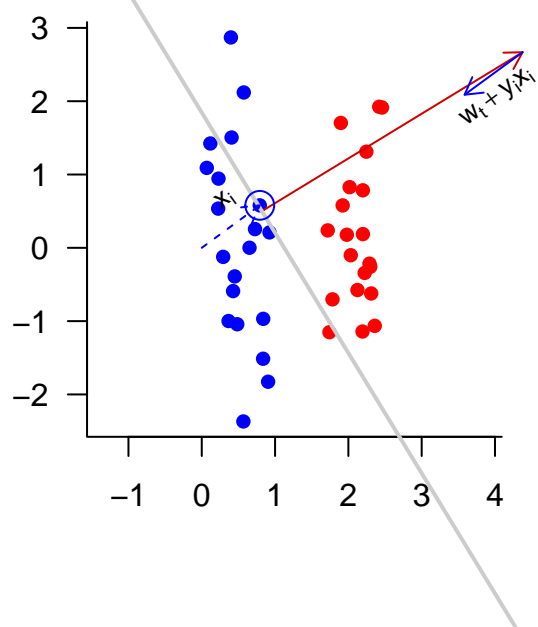
2) Draw new weight vector



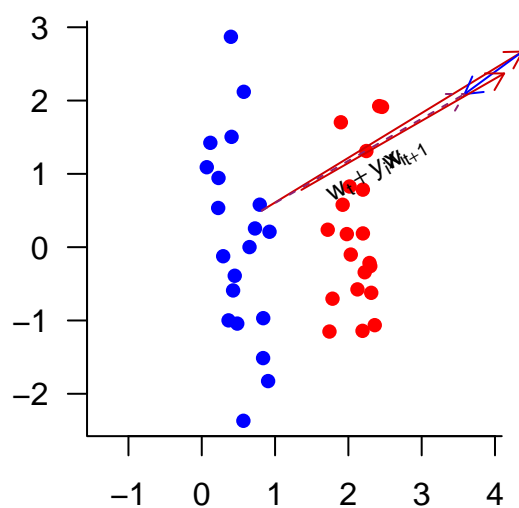
Hyperplane, iteration 13



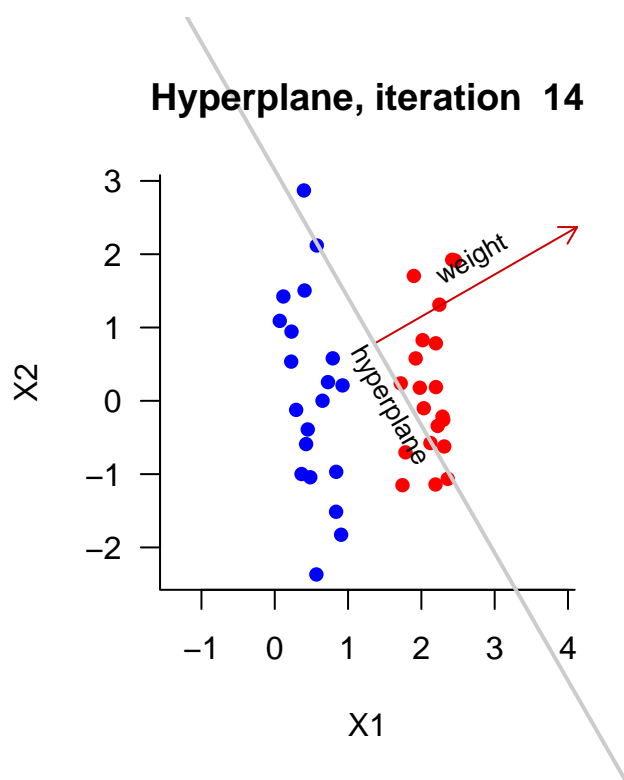
1) Select a misclassified point



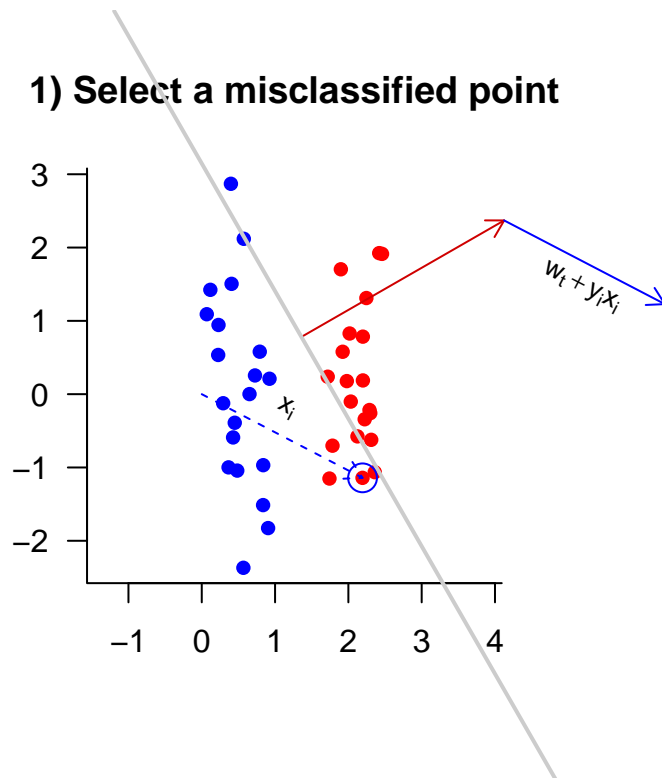
2) Draw new weight vector



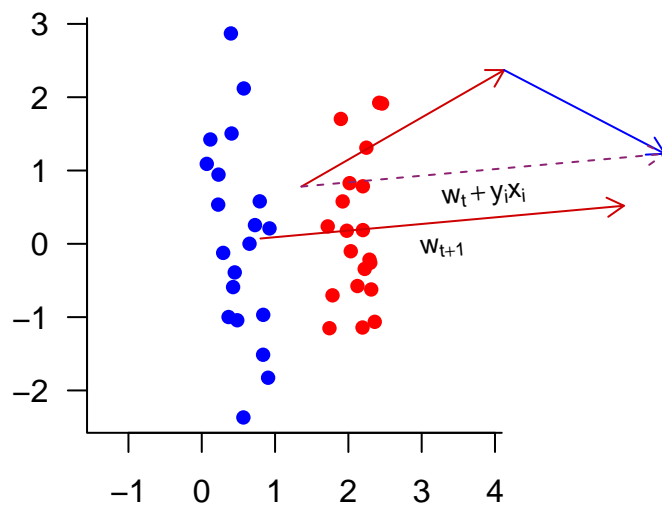
Hyperplane, iteration 14



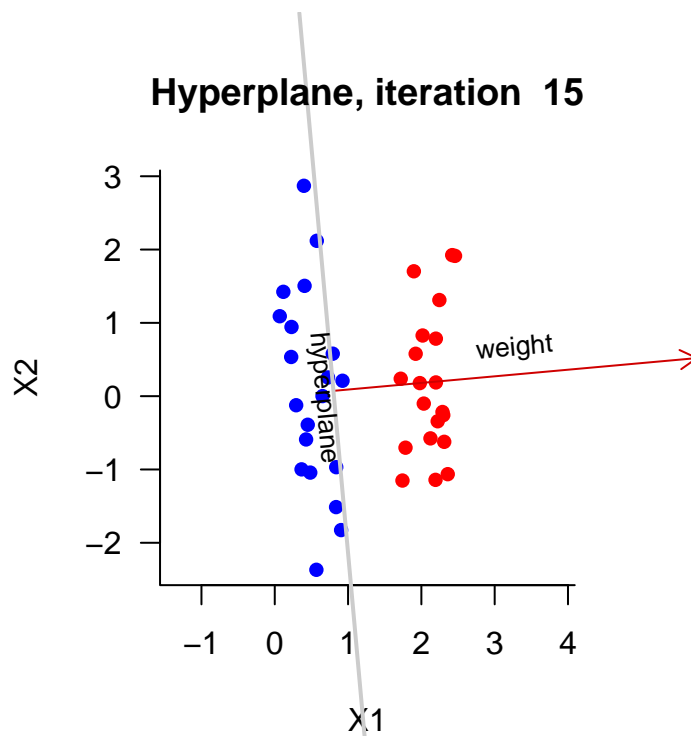
1) Select a misclassified point



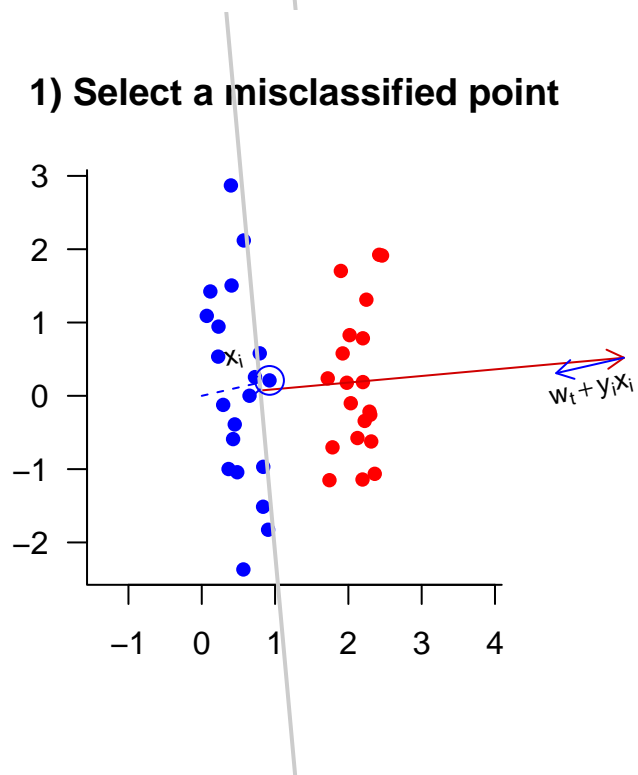
2) Draw new weight vector



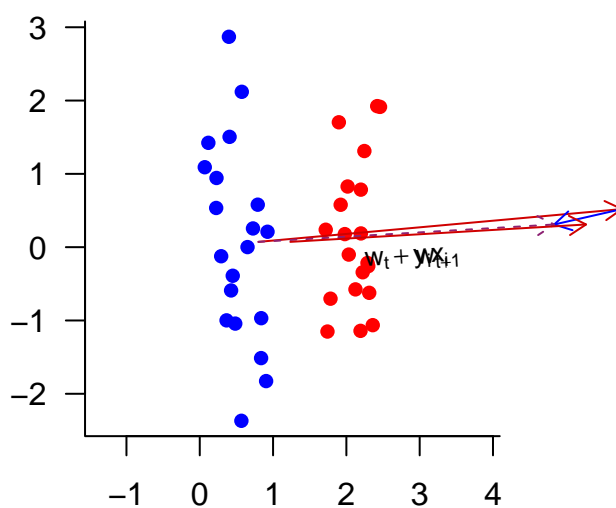
Hyperplane, iteration 15



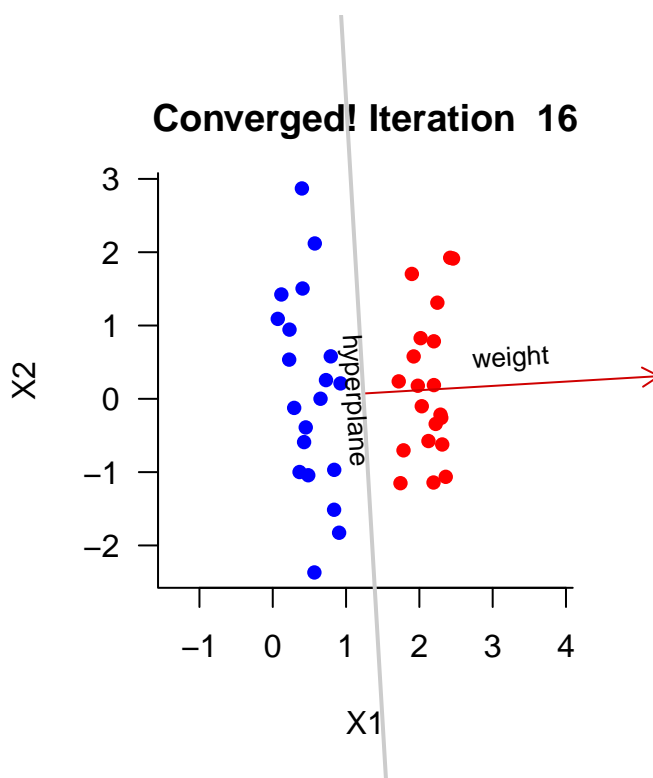
1) Select a misclassified point



2) Draw new weight vector



Converged! Iteration 16



Summary of 16 iterations:

