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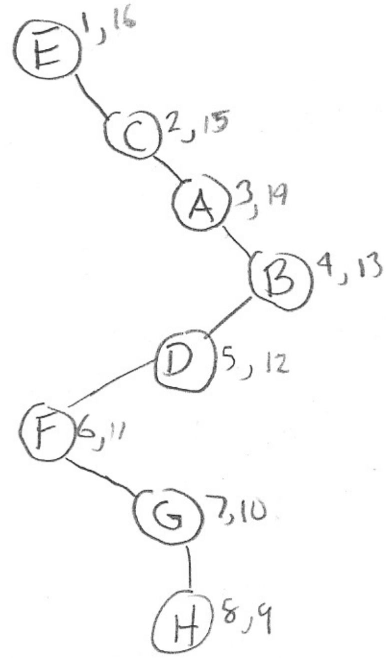
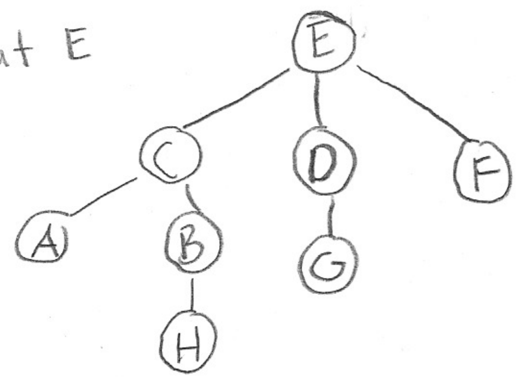
CSCI 3104 HW 4

1. a) DFS tree starting at E



b) BFS tree starting at E



c) Dijkstras algorithm

i) 1)

E:0	A:∞
C:∞	B:∞
D:∞	G:∞
F:∞	H:∞

2)

E:0	A:∞
C:3	B:∞
D:3	G:∞
F:2	H:∞

3)

E:0	A:∞
C:3	B:∞
D:3	G:8
F:2	H:∞

4)

E:0	A:∞
C:3	B:12
D:3	G:4
F:2	H:∞

5)

E:0	A:7
C:3	B:6
D:3	G:4
F:2	H:∞

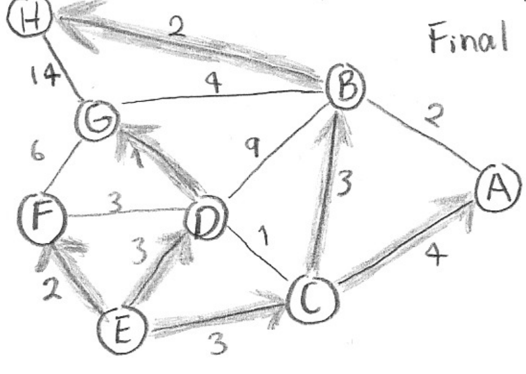
6)

E:0	A:7
C:3	B:6
D:3	G:4
F:2	H:18

7)

E:0	A:7
C:3	B:6
D:3	G:4
F:2	H:8

ii)



Final Shortest-path tree

2. Linear-time algorithm for following task:

Input: Undirected graph $G = (V, E)$ with unit edge lengths; nodes $u, v \in V$

Output: Number of distinct shortest paths from u to v.

```
function num_shortest_paths(G, u, v)
    for all x ∈ V:
        dist[x] = ∞
        shortest_paths[x] = 0

    dist[u] = 0
    Q = [u]        (will be vertex queue)
    shortest_paths[u] = 1
    while Q is not empty:
        u = eject(Q, u); if u == v: return shortest_paths[v]
        for all edges (u, x) ∈ E:
            if dist[x] = ∞:
                insert(Q, x)
                dist[x] = dist[u] + 1
            if x == v:
                shortest_paths[v] = shortest_paths[x] + 1
```

Implemented using BFS, runs in linear time because doing one extra operation per edge.

3. Input: Strongly connected directed graph $G=(V,E)$, positive length edges and node v_0

Output: Matrix M of shortest paths between all pairs of nodes passing v_0

find Shortest Path (G, l_e, v_0)

$\delta_{v_0}(v) = \text{Dijkstra}(G, l_e, v_0)$ // for length of shortest path from v_0 to all $v \in V$

$\delta^R_{v_0}(v) = \text{Dijkstra}(G^R, l_e, v_0)$ // length of shortest path from v to v_0 for all $v \in V$

for all $(u,v) \in V \times V$:

$M_{u,v} = \delta^R_{v_0}(u) + \delta_{v_0}(v)$ // gives shortest path in form of $u \rightarrow v_0 \rightarrow v$

return M

Runtime analysis:

Dijkstra's running time $= O((|V|+|E|)\cdot\log|V|)$

Adding/combining in $O(|V|^2)$ run time

Total runtime $= O((|V|+|E|)\cdot\log|V| + |V|^2)$

Honor Code Pledge: "On my honor, as a University of Colorado at Boulder Student, I have neither given nor recieved unauthorized assistance."

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