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CSCI 3104: Algorithms HW 8

1. Canine Products; linear programming

a) 2 products: Frisky Pup (f) and Husky Hound (h)

f = number of Frisky Pup packages

h = number of Husky Hound packages

$\max (7-1.40)f + (6-.60)h - (1)f - (1.5\times 2)f - (2)h - (2)h = \max 1.60f + 1.40h$

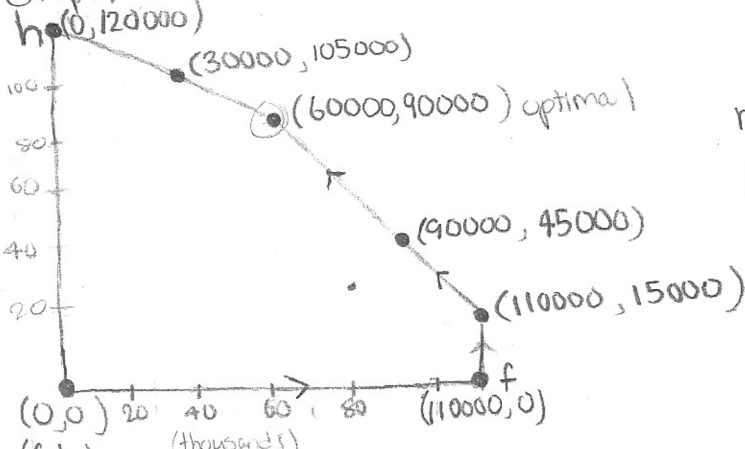
with constraints: $f \le 110{,}000$ (supply constraints $\Rightarrow h \le 120{,}000$)

$f + 2h \le 240{,}000$

$1.5f + h \le 180{,}000$

$f, h \ge 0$

b) Graph



Maximum profit:

$f = 60{,}000$; $h = 90{,}000$

$\max = 1.60(60000) + 1.4(90{,}000) = 222{,}000$

Profit = \$222,000

2. Salad problem: variables T = grams of tomatoes per 100grams (ie. $1g \Rightarrow .01$), S = spinach/100grams, C = carrot/100grams, O = oil/100grams, L = grams lettuce/100grams

Objective minimize calories = $\min 21T + 17L + 370S + 345C + 883O$

under constraints: A) $.85T + 1.63L + 12.79S + 8.38C \ge 15$

B) $2 \le .33T + .20L + 1.58S + 1.39C + 100.0O \le 6$

C) $4.65T + 2.37L + 73.68S + 80.70C \ge 4$

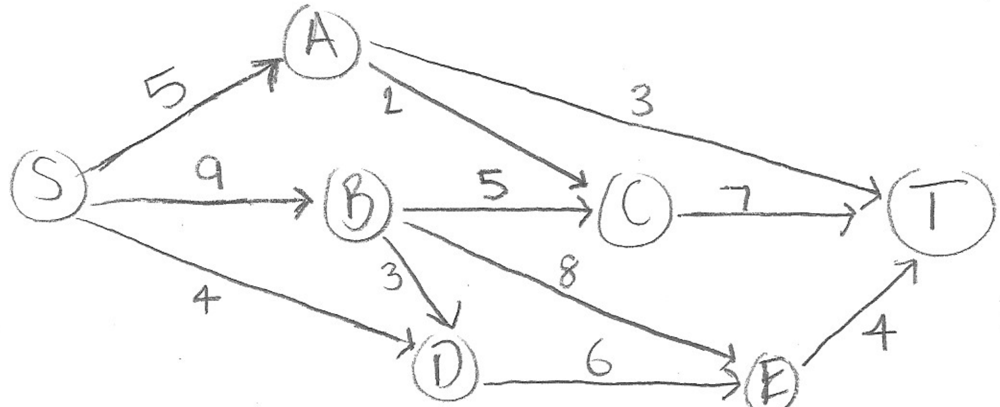
D) $9.0T + 8.0L + 7.0S + 506.40C \le 100$

E) $L + S \le T + C + O$; $T, L, S, C, O \ge 0$

The linear program wouldn't accept case E, so I made an approximation of the average salad mass then split two new cases: $L + S \le (\text{avgMass}/2)$ and $T + C + O \ge (\text{avgMass}/2)$ which produced the optimal solution min calories = 240.235 (kcal) with $T = 6 \Rightarrow 600g$, $L = 5.71 \Rightarrow 571g$, $S = .046 \Rightarrow 4.6g$ and $C = 0g$ and $O = 0g$.

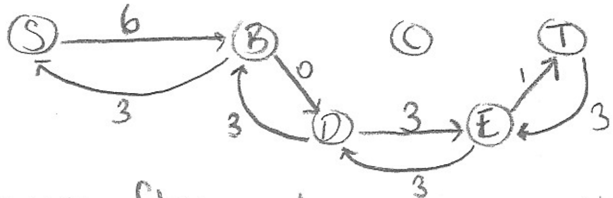
Website: Simplex method tool www.zweigmedia.com/RealWorld/simplex.html

3.



a) Augmenting path $S \to B \to D \to E \to T$: Residual capacity = 3 ($B \to D$)

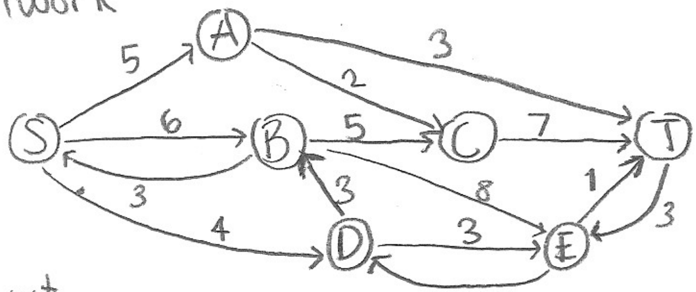
(A)



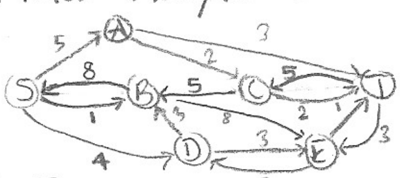
(C)

b) Maximum flow along this path = residual capacity = 3

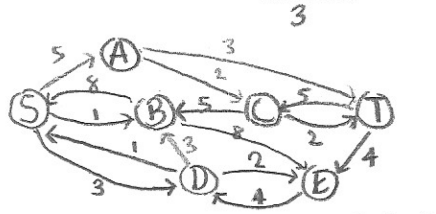
c) Residue network



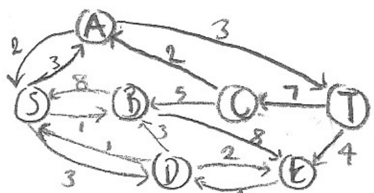
d) Max flow/min cut



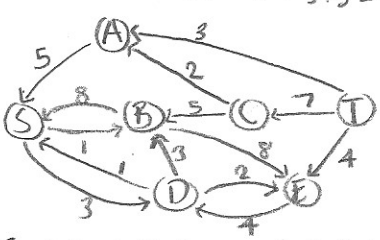
$S \to B \to C \to T$ Residual: $3+5=8$



$S \to D \to E \to T$: Residual: $3+5+1=9$

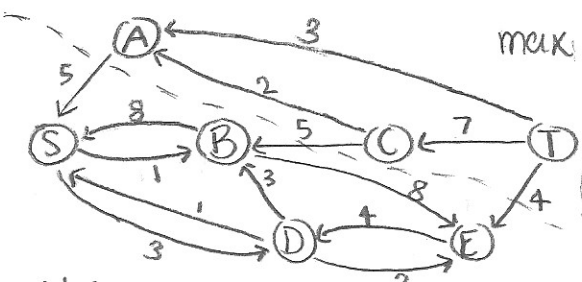


$S \to A \to C \to T$ Residual: $3+5+1+2=11$



$S \to A \to T$ Residual: $3+5+1+2+3=14$

Final Residual $\Longrightarrow$



max flow = residual cap.

thus,

$\boxed{\text{max flow} = 14}$

L = reachable nodes in G^f ; $R = V - L$, so minimum cut $L = \{S, B, D, E\}$ and $R = \{A, C, T\}$

verifying $\underbrace{(5+5+4)}_{\text{min cut}} = 14$ = max flow

Honor Code Pledge: