**Chaotic Dynamics – CSCI 4446/5446**
**Spring 2017**

Problem Set 6

Issued:                                                                        21 February 2017

Due:                                                                            28 February 2017

**Reading:** *Kantz & Schreiber, section 3.5; Strogatz, section 8.7 (undergrads: skip the examples in 8.7)*; Parker&Chua, chapter 2.

**Bibliography:**

- E. Bradley and D. E. Straub, "Using Chaos to Improve the Capture Range of a Phase-Locked Loop: Experimental Verification," *IEEE Trans. on Circuits and Systems*, **43**:914-922 (1996). On my webpage under publications.

- E. Bradley and J. Stuart, "Using Chaos to Generate Variations on Movement Sequences," *Chaos* **8**:800-807 (1998). On my webpage under publications.

- D. S. Dabby, "Musical Variations from a Chaotic Mapping," *Chaos* **6**:95-107 (1996).

- M. Hénon, "On the numerical computation of Poincaré maps," *Physica D* **5**:412-414 (1982). On E-reserves.

**Problems:**

1. Write a simplistic *temporal* Poincaré section program that takes a fixed trajectory (a list of state-space points, each annotated with the corresponding time), does a simple check on each point to see if the trajectory has pierced the hyperplane $\Sigma : t = nT$ with $n = 1, 2, ...$, and plots that point if such a crossing has occurred.

(a) Test this program on the trajectory from problem 2(b) on PS4. Use the same timestep as in PS4 and the surface of section $\Sigma : t = nT_0$, where $T_0$ is the natural period of the device and $n = 1, 2, ....$ (Hint: if you have trouble figuring out $T_0$, generate a trajectory starting from the initial condition $[\theta, \omega] = [0.010]$ and see how long it takes to return.) Turn in a plot of the sectioned trajectory. Remember to use $\theta$ modulo $2\pi$ on the horizontal axis. Is this what you expected to see? Why or why not?

(b) Repeat (a), but with some different $T$ that is not rationally related to $T_0$. Explain the differences between the plots in parts (a) and (b).

(c) Repeat (a) for one of the chaotic pendulum trajectories that you generated in problem 6 on PS4. Use $\Sigma : t = nT_{drive}$, where $T_{drive}$ is the drive period, and plot *at least* 1000 points.

(d) Raise the step size that you use to generate the test trajectory—keeping the timespan covered by the trajectory constant—and describe the effects on the plot in part (c). Why does this happen? (No need to turn in plots on this question.) Note: don't vary the stepsize so far as to induce numerical dynamics or you'll be seeing different effects.

2. Now make your Poincaré section algorithm more intelligent about how it finds the point of intersection with the hyperplane, using a linear interpolation between the two trajectory points that fall on either side of $\Sigma$ to approximate the actual point of piercing. Note: do *not* use this interpolated point as an initial condition for the next round of integrations! The interpolation error will severely affect the dynamics.

(a) Repeat parts (c) and (d) of problem 1 using the new algorithm. Describe and explain any differences between the two sets of plots.

3. *(Required for people enrolled in CSCI 5446; optional for others)* Modify your section algorithm from problem 2 to construct a *spatial* section. Use this code to construct sections of the $r = 50$ Lorenz attractor at (a) $y = 20$ and (b) $y = 2x$. These may look rather odd, particularly the latter.

**Final Note**: if a Poincaré section algorithm can control the numerical integrator (i.e., the trajectory isn't a fixed input), you can do much better, adjusting the stepsize so that the point falls *exactly* on the plane of section, as discussed in Henon's paper and in class.