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CSCI 3302 HW 1

1. The standard lawnmower has 2 degrees of freedom, being the translation back and forward and the yaw rotation (assuming you're not pulling wheelies with your mower, elsewise it would include the pitch degree). You can still manage to mow your entire lawn because the yaw freedom; you are able to push or pull (move forward or back) while rotating the direction of the mower to cover all ground. If you need to mow to the left of you but don't have the ability to sway left you can still rotate (yaw) that direction then move forward.
2. Maximum degrees of freedom for objects driving on the plane is three; the object can move forward and back (translation 1 – surging) or skid left or right (translation 2 – swaying) and can rotate or turn left and right (rotation – yawing).
3. Vectors
   a. Angle between v1: (cos45, -sin45, 0)^T and v2: (sin45, cos45, 0)^T
      i. Dot product: v1*v2 = cos45*sin45 + -sin45*cos45 + 0 = root(2)/2*root(2)/2 + -root(2)/2*root(2)/2 = 2/4 – 2/4 = 0
      ii. Magnitudes: ||v1||^2 = 2/4 + 2/4 = 1 -> ||v1|| = 1, ||v2|| = 1
      iii. Angle: cos(theta) = (v1*v2)/(||v1||*||v2||) = 0, thus theta = arccos(0) = pi/2, so the angle is 90 degrees
   b. Third vector v3: (0, 0, 1)^T creates a coordinate system with v1 and v2
4. Matrix

a) Basis vectors $\hat{X}_A, \hat{Y}_A, \hat{Z}_A$ and $\hat{X}_B, \hat{Y}_B, \hat{Z}_B$

$$^A_BR = \left[\,^A\hat{X}_B \quad ^A\hat{Y}_B \quad ^A\hat{Z}_B\,\right] = \left[\begin{pmatrix}\hat{X}_B\cdot\hat{X}_A\\ \hat{X}_B\cdot\hat{Y}_A\\ \hat{X}_B\cdot\hat{Z}_A\end{pmatrix}, \begin{pmatrix}\hat{Y}_B\cdot\hat{X}_A\\ \hat{Y}_B\cdot\hat{Y}_A\\ \hat{Y}_B\cdot\hat{Z}_A\end{pmatrix}, \begin{pmatrix}\hat{Z}_B\cdot\hat{X}_A\\ \hat{Z}_B\cdot\hat{Y}_A\\ \hat{Z}_B\cdot\hat{Z}_A\end{pmatrix}\right]$$

b) $\hat{X}_B = \begin{bmatrix}0\\1\\0\end{bmatrix}$ in frame $\{A\}$

$$^BQ = {^B_AT}\,^AQ \Rightarrow \,^BQ = \left[\begin{array}{ccc|c} & ^B_AR & & ^BP \\ \hline 0 & 0 & 0 & 1\end{array}\right]\begin{bmatrix}^AQ\\ 1\end{bmatrix}$$

$$= \begin{bmatrix}\hat{X}_B\cdot\hat{X}_A\\ \hat{X}_B\cdot\hat{Y}_A\\ \hat{X}_B\cdot\hat{Z}_A\end{bmatrix} = 0\cdot\hat{X}_A + 1\cdot\hat{Y}_A + 0\cdot\hat{Z}_A = \hat{Y}_A$$

c) $$^B_AR = \left[\,^B\hat{X}_A \quad ^B\hat{Y}_A \quad ^B\hat{Z}_A\,\right] = \left[\begin{pmatrix}\hat{X}_A\cdot\hat{X}_B\\ \hat{X}_A\cdot\hat{Y}_B\\ \hat{X}_A\cdot\hat{Z}_B\end{pmatrix}, \begin{pmatrix}\hat{Y}_A\cdot\hat{X}_B\\ \hat{Y}_A\cdot\hat{Y}_B\\ \hat{Y}_A\cdot\hat{Z}_B\end{pmatrix}, \begin{pmatrix}\hat{Z}_A\cdot\hat{X}_B\\ \hat{Z}_A\cdot\hat{Y}_B\\ \hat{Z}_A\cdot\hat{Z}_B\end{pmatrix}\right]$$

5. Tricycle Forward Kinematics