<u>**Lab 4 Write Up**</u>

1. Provide a brief write-up showing your data collected in (2) and your solution (equations) to (3).

   Data collected in (2):

   10 CM

   14 14 12 13 12 12 12 11 11 11 11 11 11 11 11 11 11 11 10 11 10 11 11 11 11 11 11 12 11 12 12 12 12 12 13 13
   Variance: 0.88

   20 CM

   23 22 22 22 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 21 21 22 23 23
   Variance: 1.13

   30 CM
   33 32 32 32 31 31 31 32 30 30 30 30 30 30 30 30 30 30 30 31 32 31 32 31 32 32 33 33
   Variance: 1.13

   40 CM

   43 42 40 41 41 41 41 40 40 40 40 40 40 40 40 41 40 40 40 41 42 43 43
   Variance = 1.15

   Question #3

   Readings from (0,0)

   22 21 21 21 22 20 20 20 20 20 20 20 20 20 21 21 21 22 22 22 22 24 21 22 21 22 20 20 20 20 20 20 20 20 20 20 20 20 21 22 22 23 23 24

   Readings from (30,0)

   27 27 27 26 26 26 27 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 27 27 27 28 28 28 27 26 26 26 26 27 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 27 27 27 27 28 28 29 27 27 27 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 27 27 27 27 27 28 28

Solution equations to (3):

Variables & Derivatives:

$$x = \left(\frac{L^2 + r_1^2 - r_2^2}{2L}\right)$$

$$y = \sqrt{\left(r_1^2 - \left(\frac{L^2 + r_1^2 - r_2^2}{2L}\right)^2\right)}$$

$$\frac{\partial x}{\partial r_1} = \frac{r_1}{L}$$

$$\frac{\partial x}{\partial r_2} = -\frac{r_2}{L}$$

$$\frac{\partial y}{\partial r_1} = -\frac{r_1\left(L^2+{r_1}^2-{r_2}^2\right)}{2L^2\sqrt{{r_2}^2-\frac{(L^2+{r_1}^2-{r_2}^2)^2}{4L^2}}}$$

$$\frac{\partial y}{\partial r_2} = \frac{2r_2+\frac{r_2\left(L^2+{r_1}^2-{r_2}^2\right)}{L^2}}{2\sqrt{{r_2}^2-\frac{(L^2+{r_1}^2-{r_2}^2)^2}{4L^2}}}$$

Equation:

$$\begin{bmatrix} {\sigma_x}^2 & 0 \\ 0 & {\sigma_y}^2 \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial r_1} & \frac{\partial x}{\partial r_2} \\ \frac{\partial y}{\partial r_1} & \frac{\partial y}{\partial r_2} \end{bmatrix} \times \begin{bmatrix} {\sigma_{r_1}}^2 & 0 \\ 0 & {\sigma_{r_2}}^2 \end{bmatrix} \times \begin{bmatrix} \frac{\partial x}{\partial r_1} & \frac{\partial y}{\partial r_1} \\ \frac{\partial x}{\partial r_2} & \frac{\partial y}{\partial r_2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{r_1}{L} & -\frac{r_2}{L} \\ -\frac{r_1\left(L^2+{r_1}^2-{r_2}^2\right)}{2L^2\sqrt{{r_2}^2-\frac{(L^2+{r_1}^2-{r_2}^2)^2}{4L^2}}} & \frac{2r_2+\frac{r_2\left(L^2+{r_1}^2-{r_2}^2\right)}{L^2}}{2\sqrt{{r_2}^2-\frac{(L^2+{r_1}^2-{r_2}^2)^2}{4L^2}}} \end{bmatrix} \times \begin{bmatrix} {\sigma_{r_1}}^2 & 0 \\ 0 & {\sigma_{r_2}}^2 \end{bmatrix} \times \begin{bmatrix} \frac{r_1}{L} & -\frac{r_1\left(L^2+{r_1}^2-{r_2}^2\right)}{2L^2\sqrt{{r_2}^2-\frac{(L^2+{r_1}^2-{r_2}^2)^2}{4L^2}}} \\ -\frac{r_2}{L} & \frac{2r_2+\frac{r_2\left(L^2+{r_1}^2-{r_2}^2\right)}{L^2}}{2\sqrt{{r_2}^2-\frac{(L^2+{r_1}^2-{r_2}^2)^2}{4L^2}}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{{r_1}^2{\sigma_{r_1}}^2}{L^2} & 0 \\ 0 & \frac{\left(2r_2+\frac{r_2\left(L^2+{r_1}^2-{r_2}^2\right)}{L^2}\right)^2{\sigma_{r_2}}^2}{4\left({r_2}^2-\frac{(L^2+{r_1}^2-{r_2}^2)^2}{4L^2}\right)} \end{bmatrix}$$

Therefore...

$$\sigma_x{}^2 = \frac{r_1{}^2 \sigma_{r_1}{}^2}{L^2}$$

$$\sigma_y{}^2 = \frac{\left(2r_2 + \frac{r_2\left(L^2 + r_1{}^2 - r_2{}^2\right)}{L^2}\right)^2 \sigma_{r_2}{}^2}{4\left(r_2{}^2 - \frac{\left(L^2 + r_1{}^2 - r_2{}^2\right)^2}{4L^2}\right)}$$

2. What happens to the variance of x and y when L increases?

   As L increases the variance of x and y decreases since x and y are indirectly proportional to L. This makes sense cause if the distance is further then the scanner captures less of the object, especially its curvature, creating less of a change in the data produced, as opposed to if the beacon were closer and senses the object for a wider scan range.

3. When would you rely on this position estimate over your odometry estimate? When would you rather trust your odometry?

   It is better to use this position estimate when the uncertainty of odometry exceeds that of ultrasonic position estimate. By having multiple methods of calculating positions we have the advantage of taking the one with the smallest variance, and therefore most accurate reading. As odometry goes on and on its variance will start smaller and more accurate, but continue to exponentially get larger. In this case, position estimates puts a cap on how large that variance can be.

4. Describe how you could estimate your bearing θ from your position estimate.

   In order to calculate theta we will need to take multiple position estimates. This is because a single estimate will only provide a location with variance, nothing about bearing. By taking another estimate we could compare the new and old estimated positions and calculate theta as the bearing between those.