```
b)

I)

re ::= union

union ::= union '|' intersect | intersect

intersect ::= intersect '&' concat | concat

concat ::= concat not | not

not ::= '~' not | star

star ::= atom '+' | atom '*' | atom '?' | atom

atom ::= '!' | '#' | '.' | 'c'
```

ii) Recursive descent parse: top to bottom, left to right.

Going from top to bottom and left to right will force the recursion to always match on the non-terminal on the left hand side and continue infinitely rather than reaching a terminal in the grammar. For the parsing to work the production must be consuming the characters from the string to match a terminal, but in this case the left recursion will continue to call the function corresponding to a non-terminal to try to match that non-terminal resulting in the infinite loop.

```
iii)

re ::= union

union ::= intersects unions

unions ::= empty | '|' intersects unions

intersect ::= concat intersects

concat ::= not concats

concats ::= empty | not concats

not ::= '~' not | star

star ::= atom stars

stars ::= empty | '*' atom stars | '+' atom stars | '?' atom stars

atom ::= '!' | '#' | '.' | 'c'
```

i) Typing Rules: Type Reg Ex Literal Type Reg Ex Test

THE: ROYEXP THE2: String
THE1: ROYEXP THE2: String
THE1: ROYEXP THE2: String

Small-step operational semantics;

Search 1 $e_i \rightarrow e_i$ e_1 test $(e_2) \rightarrow e'_1$ test (e_2)

Search 2 $V_1 = re$ $e_2 \rightarrow e_2$ V_1 , test $(e_2) \rightarrow V_1$, test (e_2)

Do b = retest (re, s) V=re V_= s v. test (v2) W b