Homework 1

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Problem 1:

Katz centrality is defined as:

$$c_{\text{Katz}} = \beta(I - \alpha A)^{-1}\vec{1}$$

First, we need to be sure that $(I - \alpha A)$ is invertible. This matrix becomees singular when:

$$\det(I - \alpha A) = 0$$

which it is equal to:

$$\det\left(A - \frac{1}{\alpha}I\right) = 0$$

And this last expression is the definition of Eigendecomposition of a matrix. So we can say that the eigenvalues (λ) is equal to:

$$\lambda = \frac{1}{\alpha} \longrightarrow \alpha = \frac{1}{\lambda}$$

So, to keep the matrix non-singular requires:

$$\alpha < \frac{1}{\lambda}$$

Now, the remain question is, which eigenvalue. And the answer is all of them, so we are going to pick the one that is most restrictive, i.e.:

$$\alpha < \frac{1}{\lambda_1}$$

where λ_1 is the fist eigenvalue.

Problem 2:

By definition we know that the number of walks of length r from node v_i to node v_j is represented by:

$$N_{ij}^{(r)} = [A^r]_{ij}$$

But in the case of the number of common neighbors between two nodes (v_i and v_j), we want the number of walks of length 2 between these two nodes because we want the intersection between the number of nodes around node v_i and node v_j. So, using the definition of walk, the number of common neighbors between v_i and v_j is:

$$n_{ij} = \sum_{k=1}^{n} A_{ik} A_{kj} = [A^2]_{ij}$$

Problem 3:

Part A

In the python code there are 2 functions: Get_Neighbors and Get_Jaccard_Matrix.

Get_Neighbors receives the node what you want to identify its neighbors and a list of all edges inside of the graph. And return a list with the neighbors of that specific node.

Get_Jaccard_Matrix receives the graph and using the list of neighbors of node i and node j, calculate the intersection and the union between the two list and finally calculate the matrix index. This function returns a list with the node i and j and its respective Jaccard matrix index.

Part B

By construction, Get_Jaccard_Matrix returns all possible combination, so we can identify when it comes to Ginori family values.