Problem Set 7

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Part I

Household problem: The main problem of interest I'd like to look at is the following household problem:

$$\max_{c_t} \sum_{t=0}^{T} \beta^t u(c_t) \tag{1}$$
$$\text{subject to } c_t \leq \frac{m_t}{P_t} - \frac{m_{t+1}}{P_t}$$

In this maximization problem, households are choosing consumption in each period. The constraint is a cash in advance (CIA) constraint, which restricts consumption c_t to only the real money holdings that the household has on hand m_t but isn't saving for next period, m_{t+1}. The environment is as follows:

1. Population: I am modeling households.
2. Preferences: I will use CRRA utility (i.e. $u_t = \frac{c_t^{1-\sigma}}{1-\sigma}$)
   - $0 < \beta < 1$
   - log utility gives:
     - $u'(.) > 0, u''(.) < 0$
     - and Indada condition $u'(0) = \infty$
3. Technology: There is no production. Price level P follows an AR(1) process:

   $$\ln(P_{t+1}) = (1-\rho)\alpha + \rho \ln(P_t) + \varepsilon_t \text{ where } \varepsilon_t \sim N(0, \sigma_\varepsilon)$$

   The households know everything except for the price level in the next period when making consumption and savings decisions.
4. Endowments: $m_0 > 0$ and $P_0 \neq 0$ are given.

The Bellman equation for this problem is given by:

$$V(P,m) = \max_{c}\{\frac{c_t^{1-\sigma}}{1-\sigma} + \beta E_{P'|P}[V(P',m')]\} \tag{2}$$
$$\text{subject to } c = \frac{m}{P} - \frac{m'}{P}$$

The state variables for this equation are P, the price level and m, the household's money holdings. The control variables are c, consumption, or m, real money holdings. You can think of the household as either choosing how much to consume, or how much cash they want to not spend this period. Substituting the CIA constraint for c, we get,

$$V(P,m) = \max_{m'}\{\log\left(\frac{m}{P} - \frac{m'}{P}\right) + \beta E_{P'|P}[V(P',m')]\} \tag{3}$$

Part II

As of yet, I have been unable to get my value function to converge. Unfortunately, using the @numba.jit() decorator hasn't been working on my bellman operator function because there is a function defined within this function and numba does not seem to like that.