Problem Set 7

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Household problem: The main problem of interest I'd like to look at is the following household problem:

$$\max_{c_t} \sum_{t=0}^{T} \beta^t u(c_t) \tag{1}$$
$$\text{subject to } c_t \leq \frac{m_t}{P_t} - \frac{m_{t+1}}{P_t}$$

In this maximization problem, households are choosing consumption in each period. The constraint is a cash in advance (CIA) constraint, which restricts consumption c_t to only the real money holdings that the household has on hand m_t but isn't saving for next period, m_{t+1}. The environment is as follows:

1. Population: I am modeling households.

2. Preferences: I will use log utility (i.e. $u_t = \beta^t \log(c_t)$)

   - $0 < \beta < 1$
   - log utility gives:
     - $u'(.) > 0, u''(.) < 0$
     - and Indada condition $u'(0) = \infty$

3. Technology: There is no production. Price level P follows an AR(1) process:

   $$P_{t+1} = \alpha + \rho P_t + \varepsilon_t \text{ where } \varepsilon_t \sim N(0, \sigma_\varepsilon)$$

   The households know everything except for the price level in the next period when making consumption and savings decisions.

4. Endowments: $m_0 > 0$ and $P_0 \neq 0$ are given.

The Bellman equation for this problem is given by:

$$V(P,m) = \max_{c}\{\log(c) + \beta E_0[V(P',m')]\} \tag{2}$$
$$\text{subject to } c = \frac{m}{P} - \frac{m'}{P}$$

The state variables for this equation are P, the price level and m, the household's money holdings. The control variables are c, consumption, or m, real money holdings. You can think of the household as either choosing how much to consume, or how much cash they want to not spend this period. Substituting the CIA constraint for c, we get,

$$V(P,m) = \max_{m'}\{\log\left(\frac{m}{P} - \frac{m'}{P}\right) + \beta E_0[V(P',m')]\} \tag{3}$$

The FONCs for this problem are:

$$\frac{1}{P}u'_{m'}(\frac{m}{P} - \frac{m'}{P}) = \beta E[V'_{m'}(P',m')]$$

and with $E[V'_{m'}] = \frac{1}{\alpha+\rho P}u'_{m'}(\frac{m'}{\alpha+\rho P} - \frac{m''}{\alpha+\rho P})$, we have:

$$\frac{1}{P}u'_{m'}(\frac{m}{P} - \frac{m'}{P}) = \frac{\beta}{\alpha+\rho P}u'_{m'}(\frac{m'}{\alpha+\rho P} - \frac{m''}{\alpha+\rho P}) \tag{4}$$