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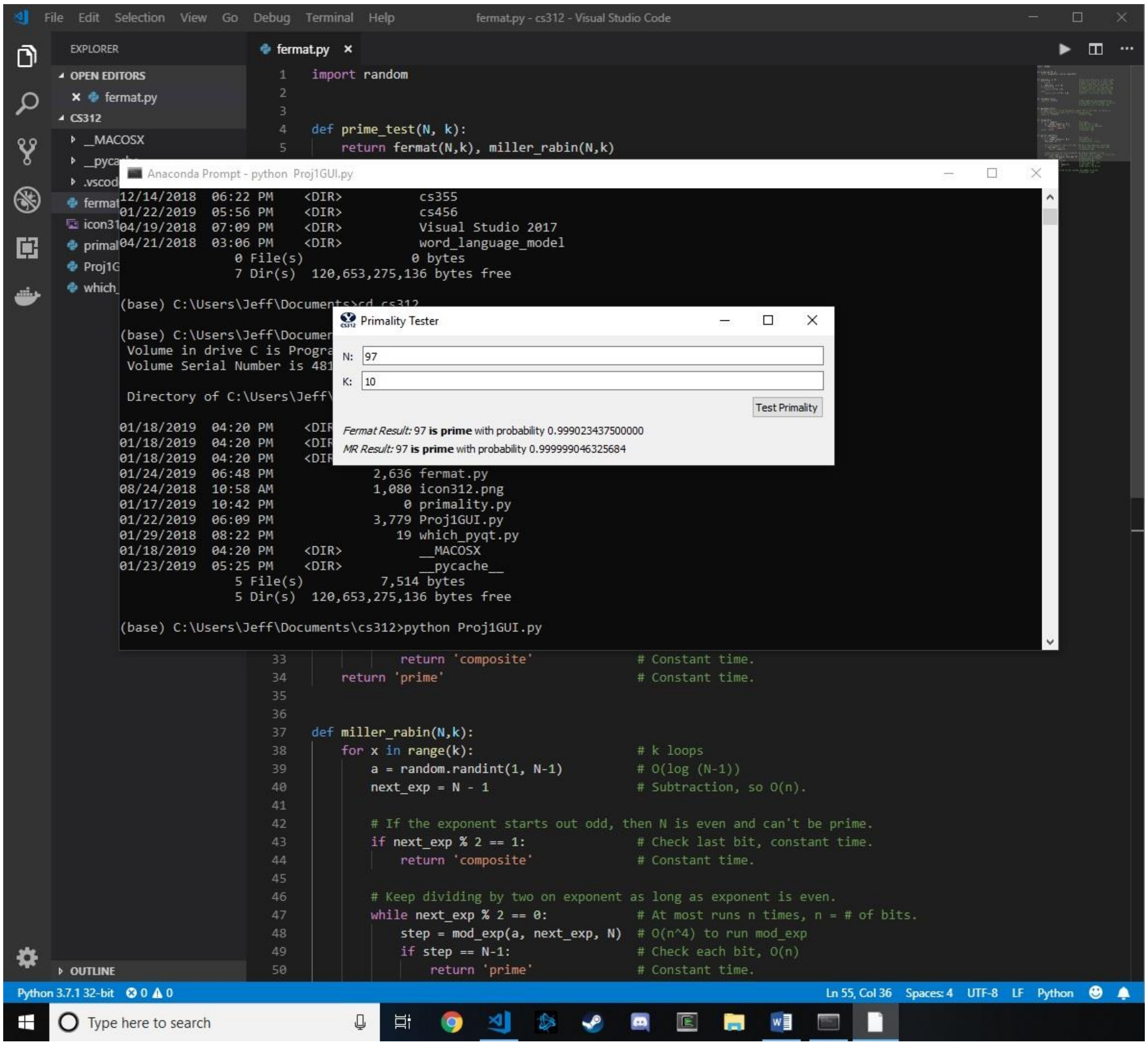
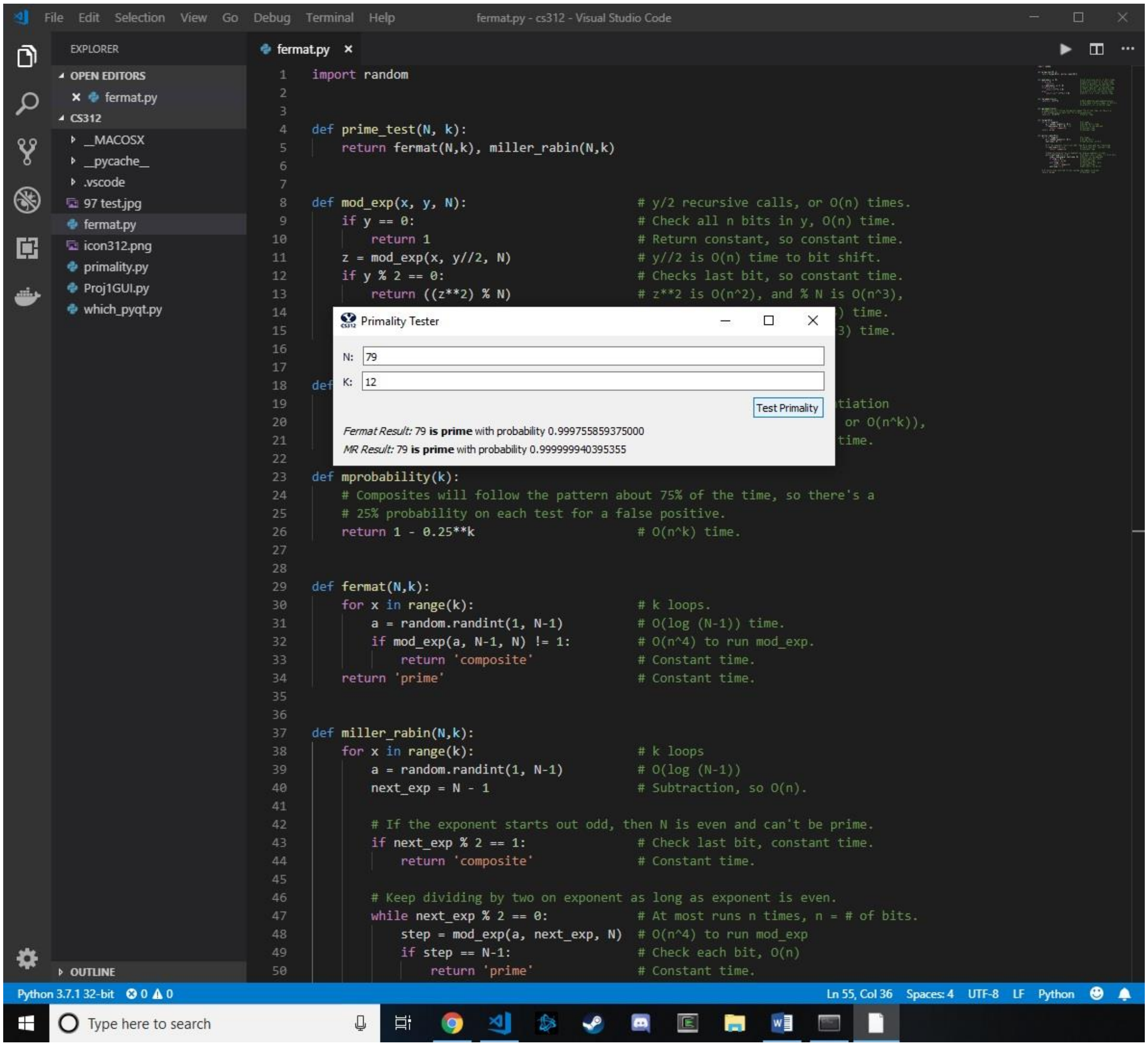
Section 2

Project 1

Screenshots




File Edit Selection View Go Debug Terminal Help
fermat.py - cs312 - Visual Studio Code
EXPLORER
OPEN EDITORS
fermat.py
CS312
__MACOSX
__pycache__
.vscode
97 test.jpg
fermat.py
icon312.png
primality.py
Proj1GUI.py
which_pyqt.py
OUTLINE
import random
def prime_test(N, k):
return fermat(N,k), miller_rabin(N,k)
def mod_exp(x, y, N): # y/2 recursive calls, or O(n) times.
if y == 0: # Check all n bits in y, O(n) time.
return 1 # Return constant, so constant time.
z = mod_exp(x, y//2, N) # y//2 is O(n) time to bit shift.
if y % 2 == 0: # Checks last bit, so constant time.
return ((z**2) % N) # z**2 is O(n^2), and % N is O(n^3),
) time.
3) time.
def
tiation
or O(n^k)),
time.
Primality Tester
N: 79
K: 12
Test Primality
Fermat Result: 79 is prime with probability 0.999755859375000
MR Result: 79 is prime with probability 0.999999940395355
def mprobability(k):
# Composites will follow the pattern about 75% of the time, so there's a
# 25% probability on each test for a false positive.
return 1 - 0.25**k # O(n^k) time.
def fermat(N,k):
for x in range(k): # k loops.
a = random.randint(1, N-1) # O(log (N-1)) time.
if mod_exp(a, N-1, N) != 1: # O(n^4) to run mod_exp.
return 'composite' # Constant time.
return 'prime' # Constant time.
def miller_rabin(N,k):
for x in range(k): # k loops
a = random.randint(1, N-1) # O(log (N-1))
next_exp = N - 1 # Subtraction, so O(n).
# If the exponent starts out odd, then N is even and can't be prime.
if next_exp % 2 == 1: # Check last bit, constant time.
return 'composite' # Constant time.
Keep dividing by two on exponent as long as exponent is even.
while next_exp % 2 == 0: # At most runs n times, n = # of bits.
step = mod_exp(a, next_exp, N) # O(n^4) to run mod_exp
if step == N-1: # Check each bit, O(n)
return 'prime' # Constant time.
Python 3.7.1 32-bit
Ln 55, Col 36 Spaces: 4 UTF-8 LF Python
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Time complexity of key portions

mod_exp:

There are y/2 recursive calls of function, so runs in log y, or n times through (where n is the number of bits in y).

1. Checks all n bits, so O(n) for time.
2. Return constant, so constant time.
3. O(n) time because it’s bit shifting in a parameter for the function call.
4. Simply check the last bit, so constant time.
5. The z**2 is O(n^2) because it’s simply z * z, or integer multiplication. But the modulo N takes O(n^3) time, which dominates, so O(n^3) time.
7. There are two multiplications, and one modulo, so O(n^3 + 2n^2), or O(n^3) time.

It takes O(n^3) to run O(n) times, meaning overall mod_exp has a O(n^4) time complexity.

fermat:

There are k loops.

Runs the mod_exp in O(n^4) time, which is the most dominant part.

Assuming k is a constant much smaller than N, overall a O(k*n^4) or O(n^4) time complexity.

miller_rabin:

There are k loops.

1. The while loop runs at most n times, where n = # bits of next_exp.
2. Takes O(n^4) to run mod_exp.
3. Checks each bit, so O(n).

The rest are checking each bit, returning a constant, or a bit shift. So O(n) for the checks, constant time for returning a constant, and O(n^2) for bit shifts.

Assuming k is a constant much smaller than N, overall a O(k*n^4) or O(n^4) time complexity.

Space complexity of key portions

mod_exp:

- O(n) space for checks.
- z, at worst, stores the value N-1, so O(n) space
- The bit shifts take O(n) space.

- Multiplications cost O(n^2) space

Has O(n) recursive calls. Overall O(n^3) space complexity.

fermat:

Has k loops. Because mod_exp has O(n^3) space complexity, has overall a O(k*n^3) or O(n^3) space complexity.

miller_rabin:

Same as fermat; space complexity is dominated by mod_exp's O(n^3).

Equation for calculating probabilities of fprobability & mprobability

The Fermat theorem stated that roughly half of the numbers that followed the pattern would be Carmichael numbers. Therefore, when running *k* tests, there is a 50% chance of getting a false positive for each test. The overall accuracy after running *k* tests would be 1 - .5^*k*, where .5^*k* represents the probability that all of the values for a that suggested the number is prime were actually Carmichael numbers.

Similarly, the Miller Rabin probability states roughly 75% of numbers would not give a false positive in detecting a composite number. Therefore, the probability that each test integer *a* in *k* tests were actually Carmichael numbers is 1 - .25^*k.*

Code

```
def mod_exp(x, y, N):                          # y/2 recursive calls, or O(n) times.
    if y == 0:                                 # Check all n bits in y, O(n) time.
        return 1                               # Return constant, so constant time.
    z = mod_exp(x, y//2, N)                    # y//2 is O(n) time to bit shift.
    if y % 2 == 0:                             # Checks last bit, so constant time.
        return ((z**2) % N)                    # z**2 is O(n^2), and % N is O(n^3),
    else:                                      # so O(n^3 + n^2), or O(n^3) time.
        return ((x * (z**2)) % N)              # O(n^2 + n^2 + n^3) = O(n^3) time.


def fprobability(k):
    return 1 - 0.5**k                          # Both addition and exponentiation
                                               # (multiply n bits k times, O(n^k)),
                                               # so O(n^k + n), or O(n^k) time.

def mprobability(k):
    # Composites will follow the pattern about 75% of the time, so there's a
    # 25% probability on each test for a false positive.
    return 1 - 0.25**k                         # O(n^k) time.


def fermat(N,k):
    for x in range(k):                         # k loops.
        a = random.randint(1, N-1)             # O(log (N-1)) time.
        if mod_exp(a, N-1, N) != 1:            # O(n^4) to run mod_exp.
            return 'composite'                 # Constant time.
    return 'prime'                             # Constant time.


def miller_rabin(N,k):
    for x in range(k):                         # k loops
        a = random.randint(1, N-1)             # O(log (N-1))
        next_exp = N - 1                       # Subtraction, so O(n).

        # If the exponent starts out odd, then N is even and can't be prime.
        if next_exp % 2 == 1:                  # Check last bit, constant time.
            return 'composite'                 # Constant time.

        # Keep dividing by two on exponent as long as exponent is even.
        while next_exp % 2 == 0:               # At most n times, n = # of bits.
            step = mod_exp(a, next_exp, N)     # O(n^4) to run mod_exp
            if step == N-1:                    # Check each bit, O(n)
```

```
            return 'prime'                # Constant time.
        elif step != 1:                   # Check each bit, O(n)
            return 'composite'            # Constant time.
        next_exp //= 2                    # Bit shift, so O(n^2)

    # If every step resulted in one, assume the number is prime.
    return 'prime'                        # Constant time.
```