CS 355 Homework #7:
3D Rendering Geometry

Questions 1, 3, and 4 will walk you through the numerical computations for Lab 7.

Question 2 reviews the Gram-Schmidt orthogonalization we talked about in class.

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1. A vertex of a polygonal model is located at $(10, 15, 0)$ in the model's object space. The model is rotated by 45 degrees around the y-axis and placed so that it is centered at (its origin maps to) position $(30, 0, 40)$ in the world.

   (a) Write out the model's object-to-world transformation as a sequence of matrix operations. (You do not have to multiply out the matrices. You may also leave your answer in terms of trig functions.)

   (b) Where is that vertex now in world coordinates?

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2. A camera is located at position $(25, 20, 5)$ in the 3D world and is looking at the point $(25, 40, 25)$ so that the direction $[0, 1, 0]$ points (roughly!) up.

   (a) Use the process we covered in class (a 3D variant of Gram-Schmidt orthogonalization using cross products) to calculate the camera's x, y, and z axis directions.

   (b) Write this camera's world-to-camera transformation as the composition of a rotation matrix and translation matrix. (You again do not have to multiply out this matrix.)

   (c) What are the camera-space coordinates of the point $\mathbf{p}_w = (5, 6, 7)$?

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3. A camera is located at position $(20, 5, -40)$ and oriented so that it is pointing parallel to the x-z plane at an angle of 30 degrees off the z axis. (This is the basic setup for Labs 5–7.)

   (a) Write this camera's world-to-camera transformation using the composition of a 3D rotation matrix (around the y axis) and a translation matrix. (You again do not have to multiply out this matrix. You may also leave your answer in terms of trig functions.)

   (b) What are the camera-space coordinates of the point $\mathbf{p}_w = (5, 6, 7)$?

4. A virtual camera has the following parameters:
   - vertical field of view of 60 degrees
   - aspect ratio of 16:9 (horizontal to vertical)
   - near plane $n = 10$
   - far plane $f = 1000$

   (a) What is the clip matrix for this camera?

   (b) What are the clip-space coordinates of the camera-space point $\mathbf{p}_c = (5, -5, 50)$?

   (c) Is this point $\mathbf{p}_c = (5, -5, 50)$ within the view frustum of this camera? How can you tell directly from the clip-space coordinates without doing a division operation?

   (d) What are the canonical coordinates of this point $\mathbf{p}_c = (5, -5, 50)$?

   (e) If rendered to a high-definition display (1920×1080), what are the screen coordinates of this point?