12) $E = n_0 \varepsilon_0 + n_1 \varepsilon_1$

$N = n_0 + n_1$

a)

$$\Omega(N,n_0) = \binom{N}{n_0} = \frac{N!}{n_0!(N-n_0)!} = \frac{N!}{n_0!(n_0+n_1-n_0)!} = \frac{N!}{n_0!\,n_1!}$$

B) $S(N,n_0) = K_B \ln(\Omega)$ $\qquad \ln(N!) \approx N\ln(N) - N$

$$S = K_B \ln\left(\frac{N!}{n_0!\,n_1!}\right) = K_B[\ln(N!) - \ln(n_0!) - \ln(n_1!)]$$

$$= K_B[N\ln(N) - N - \ln(n_0!) - \ln(n_1!)]$$

$$= K_B[N\ln(N) - n_0 - n_1 - \ln(n_0!) - \ln(n_1!)]$$

$$S(N,n_0,n_1) = K_B[N\ln(N) - \sum_{i=0}^{1} n_i \ln(n_i)]$$

c) $x = \frac{n_1}{N} \rightarrow n_1 = xN \qquad n_0 = N - n_1 = N - xN = N(1-x)$

$$\Omega = \frac{N!}{n_0!\,n_1!} = \frac{N!}{(xN)!\,(N(1-x))!}$$

$$S = K_B \ln\left(\frac{N!}{(xN)!\,(N(1-x))!}\right) = K_B[\ln(N!) - \ln((xN)!) - \ln((N(1-x))!)]$$

$$= K_B[N\ln(N) \cancel{-N} - xN\ln(xN) \cancel{+xN} - N(1-x)\ln(N(1-x)) + \cancel{N(1-x)}]$$

$$= K_B[N\ln(N) - xN\ln(xN) - N(1-x)\ln(N(1-x))]$$

$$= K_B N[\ln(N) - x\ln(xN) - (1-x)\ln(N(1-x))]$$

$$= K_B N[\cancel{\ln(N)} - x\ln(x) \cancel{- x\ln(N)} \cancel{- \ln(N)} \cancel{+ x\ln(N)} - (1-x)\ln(1-x)]$$

$$= K_B N[-x\ln(x) - (1-x)\ln(1-x)]$$

$$S(N,x) = -K_B N[x\ln(x) + (1-x)\ln(1-x)]$$

E) $\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_N = \left(\frac{\partial S}{\partial x}\right)_N \left(\frac{\partial x}{\partial E}\right)_N$; $x = \frac{1}{N(\varepsilon_1 - \varepsilon_0)}(E - N\varepsilon_0)$; $\Delta E = \varepsilon_1 - \varepsilon_0$

$$\frac{\partial S}{\partial x} = -K_B N [\ln(x) - \ln(1-x)] = -K_B N \ln\left(\frac{x}{1-x}\right)$$

$$\frac{\partial x}{\partial E} = \frac{1}{N\Delta E}$$

$$\frac{1}{T} = \frac{-K_B N \ln\left(\frac{x}{1-x}\right)}{N\Delta E} \rightarrow \frac{\Delta E}{-K_B T} = \ln\left(\frac{x}{1-x}\right) \rightarrow e^{\Delta E/-K_B T} = \frac{x}{1-x}$$

$$\rightarrow e^{\Delta E/-K_B T} - x e^{\Delta E/-K_B T} = x \rightarrow x + x e^{\Delta E/-K_B T} = e^{\Delta E/-K_B T} \rightarrow x\left(1 + e^{\Delta E/-K_B T}\right) = e^{\Delta E/-K_B T}$$

$$\rightarrow x = \frac{e^{\Delta E/-K_B T}}{1 + e^{\Delta E/-K_B T}} \rightarrow x = \frac{\left(e^{\Delta E/K_B T}\right)^{-1}}{1 + e^{\Delta E/-K_B T}} \rightarrow x = \frac{1}{\left(1 + e^{\Delta E/-K_B T}\right)\left(e^{\Delta E/K_B T}\right)}$$

$$\Rightarrow x(T) = \frac{1}{1 + e^{\Delta E/K_B T}}$$

F)

Cuando $T \rightarrow \infty$ entonces $x = \frac{1}{2}$

$$\lim_{T\rightarrow\infty} x(T) = \lim_{T\rightarrow\infty} \frac{1}{1 + e^{\Delta E/K_B T}} = \frac{1}{1+1} = \frac{1}{2}$$

$$S(N,x) = -K_B N [x \ln(x) + (1-x)\ln(1-x)]$$
$$= -K_B N \left[\tfrac{1}{2}\ln\left(\tfrac{1}{2}\right) + \left(1 - \tfrac{1}{2}\right)\ln\left(1 - \tfrac{1}{2}\right)\right]$$
$$= -K_B N \ln\left(\tfrac{1}{2}\right)$$
$$= K_B N \ln\left(\tfrac{1}{2}^{-1}\right) = K_B N \ln(2)$$

G) $\Delta S = nR\ln\left(\frac{V_F}{V_I}\right) = K_B N \ln\left(\frac{V_F}{V_I}\right) = K_B N \ln\left(\frac{2V}{V}\right)$ $\quad V_2 = 2V$, $V_1 = V$

$$= K_B N \ln(2)$$

Este resultado y el anterior son exactamente iguales