8) $\chi^2(a_0,a_1) = \sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i))^2$

i) $\frac{d(\chi^2(a_0,a_1))}{d a_0} = \sum_{i=1}^{n} -2[y_i - (a_0 + a_1 x_i)] = 0$ $\quad \sum_{i=1}^{n} 1 = n$

$$\sum_{i=1}^{n} y_i - \sum_{i=1}^{n} a_0 - a_1 \sum_{i=1}^{n} x_i = 0 \rightarrow \sum_{i=1}^{n} y_i - a_0 n - a_1 \sum_{i=1}^{n} x_i = 0$$

$$\rightarrow \sum_{i=1}^{n} y_i - a_1 \sum_{i=1}^{n} x_i = a_0 n$$ $\quad \bar{x} = \sum_{i=1}^{n} x_i \div n \; ; \; \bar{y} = \sum_{i=1}^{n} y_i \div n$

$$\bar{y} - a_1 \bar{x} = a_0 \rightarrow \boxed{a_0^* = \bar{y} - a_1 \bar{x}}$$

ii) $\frac{d(\chi^2(a_0,a_1))}{d a_1} = \sum_{i=1}^{n} -2x_i [y_i - (a_0 + a_1 x_i)] = 0$

$$\sum_{i=1}^{n} x_i y_i - a_0^* \sum_{i=1}^{n} x_i - a_1 \sum_{i=1}^{n} x_i^2 = 0 \rightarrow \sum_{i=1}^{n} x_i y_i - (\bar{y} - a_1 \bar{x}) \sum_{i=1}^{n} x_i - a_1 \sum_{i=1}^{n} x_i^2 = 0$$

$$\rightarrow \sum_{i=1}^{n} x_i y_i - \left(\frac{\sum_{i=1}^{n} y_i}{n} - a_1 \frac{\sum_{i=1}^{n} x_i}{n} \right) \sum_{i=1}^{n} x_i - a_1 \sum_{i=1}^{n} x_i^2 = 0$$

$$\rightarrow \sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_i}{n} - a_1 \left[\frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n} + \sum_{i=1}^{n} x_i^2 \right] = 0$$

$$\rightarrow \boxed{a_1 = \frac{\sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n}}{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}}$$

B) $\chi^2(a_0, a_1, a_2) = \sum_{i=1}^{n} \left(y_i - (a_0 + a_1 x_i + a_2 x_i^2)\right)^2$

$$\frac{d\chi^2(a_0, a_1, a_2)}{d a_0} \Rightarrow \sum_{i=1}^{n} -2\left(y_i - (a_0 + a_1 x_i + a_2 x_i^2)\right) = 0$$

$$\rightarrow \sum_{i=1}^{n} \left[y_i - (a_0 + a_1 x_i + a_2 x_i^2)\right] = 0 \rightarrow \sum_{i=1}^{n} \left[a_0 + a_1 x_i + a_2 x_i^2 = y_i\right]$$

ii) $$\frac{d\chi^2(a_0, a_1, a_2)}{d a_1} \Rightarrow \sum_{i=1}^{n} -2x_i\left(y_i - (a_0 + a_1 x_i + a_2 x_i^2)\right) = 0$$

$$\rightarrow \sum_{i=1}^{n} x_i y_i - (a_0 x_i + a_1 x_i^2 + a_2 x_i^3) = 0 \rightarrow \sum_{i=1}^{n} \left[a_0 x_i + a_1 x_i^2 + a_2 x_i^3 = x_i y_i\right]$$

iii) $$\frac{d\chi^2(a_0, a_1, a_2)}{d a_2} \Rightarrow \sum_{i=1}^{n} -2x_i^2\left(y_i - (a_0 + a_1 x_i + a_2 x_i^2)\right) = 0$$

$$\rightarrow \sum_{i=1}^{n} x_i^2 y_i - (a_0 x_i^2 + a_1 x_i^3 + a_2 x_i^4) = 0 \rightarrow \sum_{i=1}^{n} \left[a_0 x_i^2 + a_1 x_i^3 + a_2 x_i^4 = x_i^2 y_i\right]$$

- para minimizar las funciones, se tienen que derivar con respecto a los parámetros e igualar a 0 para poder despejar los parámetros a_0 y a_1, y en el punto B encontrar el sistema de ecuaciones.

- La regularidad que encontramos es que si hubiera un cuarto parámetro (a_3) e hiciéramos la derivada con respecto a ese parámetro, el grado x_i de la ecuación aumentaría en 1