16.216: ECE Application Programming
Fall 2013

Programming Assignment #5: Integral Approximation with Functions
Due **Wednesday, 10/16/13**, 11:59:59 PM

1. Introduction

In this program, you will design functions that allow you to approximate the integral of a function, $f(x)$, using the trapezoidal rule. To find this integral—the area under the curve—we can approximate the area as a series of trapezoids, as shown below:
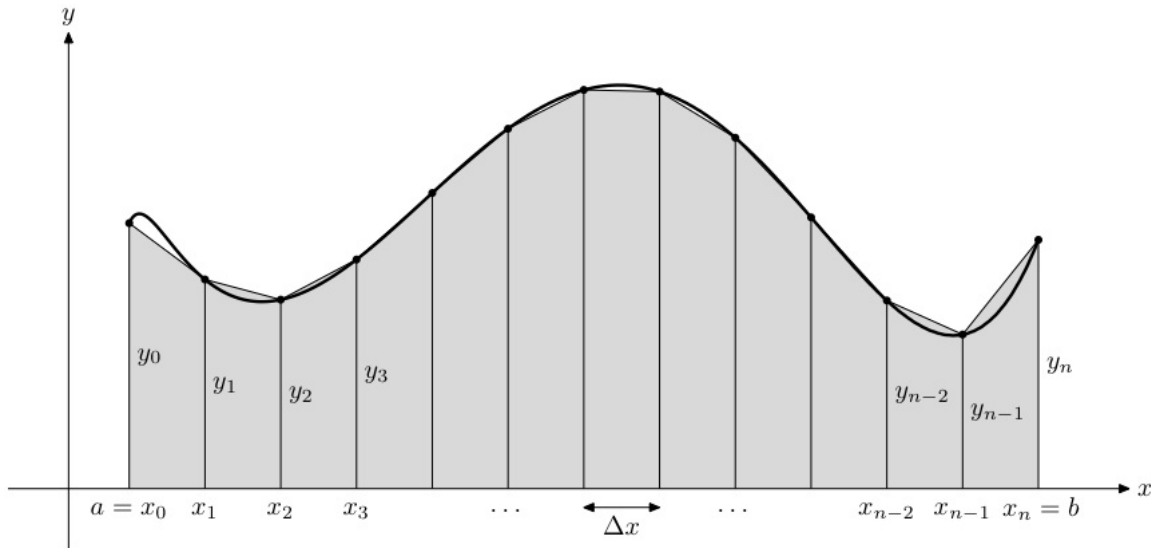
In the figure, the range [a, b] has been divided into *n* different trapezoids, each of which has the same base, $\Delta x = (b - a)/n$. Recall that a trapezoid with base b and sides $h1$ and $h2$ has area:

$$0.5 \times b \times (h1 + h2)$$

Therefore, the area of trapezoid number *k* (1 ≤ *k* ≤ *n*) from the figure above is:

$$0.5 \times \Delta x \times (y_{k-1} + y_k) = 0.5 \times \Delta x \times (f(x_{k-1}) + f(x_k))$$

To find the total area under the curve—and therefore the approximate integral—sum the areas of all trapezoids:

$$
\begin{aligned}
Area &= 0.5 \times \Delta x \times (y_0 + y_1) + 0.5 \times \Delta x \times (y_1 + y_2) + \cdots + 0.5 \times \Delta x \times (y_{n-1} + y_n) \\
&= 0.5 \times \Delta x \times (y_0 + y_1 + y_1 + y_2 + \cdots + y_{n-1} + y_n) \\
&= 0.5 \times \Delta x \times (y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n) \\
&= \mathbf{0.5} \times \Delta x \times \left(y_0 + 2 \sum_{k=1}^{n-1} y_k + y_n \right) \approx \int_a^b f(x)dx
\end{aligned}
$$

Your integral function will use the equation shown in bold above to approximate the integral, given the endpoints of the interval [a, b] and the number of trapezoids, *n*.

2. Deliverables

Submit your source file directly to Dr. Geiger (Michael_Geiger@uml.edu) as an e-mail attachment. Ensure your source file name is ***prog5_integral.c***. You should submit only the .c file. Failure to meet this specification will reduce your grade, as described in the program grading guidelines.

3. Specifications

Program structure: Your program should follow the general outline below:

1. Prompt for and read the following values:

- The low and high points of the interval $[a, b]$, over which *f(x)* is to be integrated.
- The number of trapezoids, n, to be used in that integration.

If an input error occurs, print an error message and repeat the prompt <u>for that input</u>. Input errors are as follows:

- `scanf()` cannot read the input values (for example, if the user types "A 3" for the interval endpoints).
    - o In this case, you must clear the rest of the line before retrying `scanf()`.
    - o See Lectures 10-11: PE2 for a reminder of how to check that the input is properly formatted and how to clear the rest of the line if it is not.
- The "low" interval endpoint is greater than or equal to the "high" endpoint.
- The number of trapezoids is less than 1.

2. Once the user has entered error-free input values, call the `integrate()` function (described below), which will use the trapezoidal method to approximate the integral of $f(x)$ over the interval $[a, b]$ using n trapezoids.

3. After printing the results, ask the user if he or she wants to repeat the program, and then read a single character that serves as the response to that question. If the user enters:

- `'Y'` or `'y'`   → Return to Step 1.
- `'N'` or `'n'`   → End the program.
- Any other character → Print an error message and repeat the question.

See Section 4 for test cases that demonstrate the proper format for input and output.

Functions: Your program should contain the functions with the prototypes shown below, as well as any other functions you choose to add:

- `double f(double x);`

The function being integrated, which should calculate the value:

$$f(x) = \sin(x) + \frac{x^2}{10}$$

Note that you should include the math library `<math.h>` in order to use the sin() function; this function takes a single argument and returns the sine of that value.

- `double integrate(double a, double b, int n);`

This function should use the trapezoidal method to approximate the integral of $f(x)$ over the interval [a, b] using n trapezoids, as described above. The return value of the function is the result of the approximation. Note that the function should <u>not</u> print any values to the screen—the output should be handled in the main function.

4. Test Cases

Your output should match these test cases exactly for the given input values. I will use these test cases in grading of your lab, but will also generate additional cases that will not be publicly available. <u>Note that these test cases do not cover all possible program outcomes.</u> You should create your own tests to help debug your code and ensure proper operation for all possible inputs.

```
C:\Windows\system32\cmd.exe                                          □  □

Enter endpoints of interval to be integrated (low hi): -2.1 1.5
Enter number of trapezoids to be used: 15
Using 15 trapezoids, integral between -2.100000 and 1.500000 is -0.148162
Evaluate another interval (Y/N)? Y

Enter endpoints of interval to be integrated (low hi): -2.1 1.5
Enter number of trapezoids to be used: 45
Using 45 trapezoids, integral between -2.100000 and 1.500000 is -0.153692
Evaluate another interval (Y/N)? Y

Enter endpoints of interval to be integrated (low hi): A 3
Error: Improperly formatted input

Enter endpoints of interval to be integrated (low hi): 5 4
Error: low must be < hi

Enter endpoints of interval to be integrated (low hi): 0 0.1
Enter number of trapezoids to be used: Q
Error: Improperly formatted input
Enter number of trapezoids to be used: 0
Error: numT must be >= 1
Enter number of trapezoids to be used: 10
Using 10 trapezoids, integral between 0.000000 and 0.100000 is 0.005029
Evaluate another interval (Y/N)? Q
Error: must enter Y or N
Evaluate another interval (Y/N)? N
```