# 信号检测与估值

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# ch4. 信号波形的检测

## ch4-4. 一般二元信号波形的检测—充分统计量的方法

# 一般二元信号波形的检测—充分统计量的方法

- **条件:** 功率谱密度为 $P_n(\omega) = N_0/2$ 的高斯白噪声背景中一般二元信号波形检测
- **正交级数展开法:** 信道噪声是白噪声,正交函数集可任意选取。
- **充分统计量法:** 选取特定的正交函数集,使得有关发送信号的信息只包含在有限的展开系数中。

# 一般二元信号波形的检测—充分统计量的方法

信号模型

$$H_0 : x(t) = s_0(t) + n(t), 0 \le t \le T$$

$$H_1 : x(t) = s_1(t) + n(t), 0 \le t \le T$$

$n(t)$ 为零均值高斯白噪声

**正交函数集 $\{f_k(t)\}$ 的构造问题**

波形相关系数 $\rho$:

$$\rho = \frac{1}{\sqrt{E_0 E_1}} \int_0^T s_0(t) s_1(t) dt, \quad (|\rho| \le 1)$$

$\rho = 0$ 时, 信号 $s_0(t)$ 与 $s_1(t)$ 正交。

$\rho \ne 0$ 时, 信号 $s_0(t)$ 与 $s_1(t)$ 不正交。

# 一般二元信号波形的检测—充分统计量的方法

$H_0 : x(t) = s_0(t) + n(t), 0 \leq t \leq T$

$H_1 : x(t) = s_1(t) + n(t), 0 \leq t \leq T$

**(1) 选择一组完备正交函数集,构造两个坐标函数:**

**第一个坐标函数满足:**

$$f_1(t) = \frac{1}{\sqrt{E_1}} s_1(t)$$

$f_1(t)$ 为确知信号 $s_1(t)$ 的归一化函数, $E_1 = \int_0^T s_1^2(t)dt$

其余坐标函数 $f_k(t), k \geq 2$ 是与 $f_1(t)$ 正交, 且两两正交的任意归一化函数, 即

$$f_j(t) \text{和} f_k(t) \text{是正交的}, k \geq 1, j \geq 1, k \neq j$$

# 格拉姆—施密特法构造 $f_2(t)$

**格拉姆—施密特 (Gram—Schmidt) 正交化法构造第二个坐标函数:**

利用 $s_0(t)$ 构造与 $f_1(t)$ 正交的信号 $g_2(t)$, 使 $s_0(t)$ 在 $f_1(t)$ 上的投影 $s_1$ 为零。

$$
\begin{aligned}
g_2(t) &= s_0(t) - s_1 f_1(t) \\
&= s_0(t) - \left[\int_0^T s_0(t) f_1(t)\,dt\right] f_1(t) \\
&= s_0(t) - \left[\int_0^T s_0(t)\frac{1}{\sqrt{E_1}} s_1(t)\,dt\right]\frac{1}{\sqrt{E_1}} s_1(t) \quad \text{by } f_1(t) = \frac{1}{\sqrt{E_1}} s_1(t) \\
&= s_0(t) - \rho\sqrt{\frac{E_0}{E_1}} s_1(t) \quad \text{by } \rho = \frac{1}{\sqrt{E_0 E_1}}\int_0^T s_0(t) s_1(t)\,dt
\end{aligned}
$$

# 格拉姆—施密特法构造 $f_2(t)$

$$g_2(t) = s_0(t) - \rho\sqrt{\frac{E_0}{E_1}}s_1(t)$$

$$\int_0^T s_0^2(t)dt = E_0,\ \int_0^T s_1^2(t)dt = E_1,\ \int_0^T s_0(t)s_1(t)dt = \rho\sqrt{E_0E_1}$$

归一化 $g_2(t)$, 得到**第二个坐标函数:**

$$f_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)dt}} = \frac{s_0(t) - \rho\sqrt{\frac{E_0}{E_1}}s_1(t)}{\sqrt{\int_0^T \left(s_0(t) - \rho\sqrt{\frac{E_0}{E_1}}s_1(t)\right)^2 dt}}$$

$$= \frac{s_0(t) - \rho\sqrt{\frac{E_0}{E_1}}s_1(t)}{\sqrt{\int_0^T \left(s_0^2(t) - 2\rho\sqrt{\frac{E_0}{E_1}}s_0(t)s_1(t) + \rho^2\frac{E_0}{E_1}s_1^2(t)\right) dt}}$$

$$= \frac{s_0(t) - \rho\sqrt{\frac{E_0}{E_1}}s_1(t)}{\sqrt{E_0 - 2\rho\sqrt{\frac{E_0}{E_1}}\sqrt{E_0E_1}\rho + \rho^2\frac{E_0}{E_1}E_1}} = \frac{1}{\sqrt{(1-\rho^2)E_0}}\left(s_0(t) - \rho\sqrt{\frac{E_0}{E_1}}s_1(t)\right)$$

# 证明 $f_1(t)$ 和 $f_2(t)$ 是正交函数集的前两个坐标函数

$$f_1(t) = \frac{1}{\sqrt{E_1}} s_1(t), \quad 0 \le t \le T$$

$$f_2(t) = \frac{1}{\sqrt{(1-\rho^2)E_0}} \left( s_0(t) - \rho\sqrt{\frac{E_0}{E_1}} s_1(t) \right), \quad 0 \le t \le T$$

$$\int_0^T s_0^2(t)dt = E_0, \int_0^T s_1^2(t)dt = E_1, \int_0^T s_0(t)s_1(t)dt = \rho\sqrt{E_0E_1}$$

证明: $f_1(t)$ 和 $f_2(t)$ 满足正交集坐标函数的定义。

先证明 $f_1(t), f_2(t)$ 是归一化函数。因为

$$\int_0^T f_1^2(t)dt = \frac{1}{E_1}\int_0^T s_1^2(t)dt = 1$$

$$\begin{aligned}
\int_0^T f_2^2(t)dt &= \frac{1}{(1-\rho^2)E_0}\int_0^T \left( s_0(t) - \rho\sqrt{\frac{E_0}{E_1}} s_1(t) \right)^2 dt \\
&= \frac{1}{(1-\rho^2)E_0}\int_0^T \left( s_0^2(t) - 2\rho\sqrt{\frac{E_0}{E_1}} s_0(t)s_1(t) + \rho^2\frac{E_0}{E_1} s_1^2(t) \right) dt \\
&= \frac{1}{(1-\rho^2)E_0}\left( E_0 - 2\rho\sqrt{\frac{E_0}{E_1}}\sqrt{E_0E_1}\rho + \rho^2\frac{E_0}{E_1}E_1 \right) = 1
\end{aligned}$$

# 证明 $f_1(t)$ 和 $f_2(t)$ 是正交函数集的前两个坐标函数

$$f_1(t) = \frac{1}{\sqrt{E_1}} s_1(t), \quad 0 \le t \le T$$

$$f_2(t) = \frac{1}{\sqrt{(1-\rho^2)E_0}} \left( s_0(t) - \rho \sqrt{\frac{E_0}{E_1}} s_1(t) \right), \quad 0 \le t \le T$$

$$\int_0^T s_0^2(t)dt = E_0, \int_0^T s_1^2(t)dt = E_1, \int_0^T s_0(t)s_1(t)dt = \rho\sqrt{E_0E_1}$$

证明: 再证明 $f_1(t)$,$f_2(t)$ 是相互正交的两个函数。因为

$$\begin{aligned}\int_0^T f_1(t)f_2(t)dt &= \int_0^T \frac{1}{\sqrt{E_1}} s_1(t) \frac{1}{\sqrt{(1-\rho^2)E_0}} \left( s_0(t) - \rho\sqrt{\frac{E_0}{E_1}} s_1(t) \right) dt \\ &= \frac{1}{\sqrt{(1-\rho)^2 E_0 E_1}} \left( \int_0^T s_0(t)s_1(t)dt - \rho\sqrt{\frac{E_0}{E_1}} \int_0^T s_1^2(t)dt \right) \\ &= \frac{1}{\sqrt{(1-\rho)^2 E_0 E_1}} \left( \rho\sqrt{E_0E_1} - \rho\sqrt{\frac{E_0}{E_1}} E_1 \right) = 0\end{aligned}$$

所以,$f_1(t)$,$f_2(t)$ 是相互正交的两个函数。

综上,$f_1(t)$,$f_2(t)$ 是归一化函数, 且满足正交性, 是正交函数集的前两个坐标函数。

□

# 充分统计量的方法, 选择一组完备正交函数集

$$f_1(t) = \frac{1}{\sqrt{E_1}} s_1(t), \quad 0 \le t \le T$$

$$f_2(t) = \frac{1}{\sqrt{(1-\rho^2)E_0}} \left( s_0(t) - \rho \sqrt{\frac{E_0}{E_1}} s_1(t) \right), \quad 0 \le t \le T$$

其余坐标函数 $f_k(t), k \le 3$ 是与 $f_1(t)$ 和 $f_2(t)$ 正交, 且两两相互正交的任意归一化函数, 即 $f_j(t)$ 和 $f_k(t)$ 是正交的, $k \ge 1, j \ge 1, k \ne j$

$$\int_0^T f_j(t) f_k(t) dt = 0, \quad k \ge 1, j \ge 1, k \ne j$$

# 对接收信号进行正交展开 (假设 $H_0 : x_1$)

**假设 $H_0 : x(t) = s_0(t) + n(t)$ 下, 展开系数 $x_1$**

$$
\begin{aligned}
x_1 &= \int_0^T x(t) f_1(t) dt = \int_0^T [s_0(t) + n(t)] f_1(t) dt = \int_0^T s_0(t) f_1(t) dt + \int_0^T n(t) f_1(t) dt \\
&= \int_0^T s_0(t) [\frac{1}{\sqrt{E_1}} s_1(t)] dt + n_1 = \frac{1}{\sqrt{E_1}} \int_0^T s_0(t) s_1(t) dt + n_1 \\
&= \rho \sqrt{E_0} + n_1
\end{aligned}
$$

$$
f_1(t) = \frac{1}{\sqrt{E_1}} s_1(t), \quad \int_0^T n(t) f_1(t) dt = n_1
$$

$$
\rho = \frac{1}{\sqrt{E_0 E_1}} \int_0^T s_0(t) s_1(t) dt \implies \int_0^T s_0(t) s_1(t) dt = \rho \sqrt{E_0 E_1}
$$

# 对接收信号进行正交展开 (假设 $H_0 : x_2$)

**假设 $H_0 : x(t) = s_0(t) + n(t)$ 下, 展开系数 $x_2$**

$$
\begin{aligned}
x_2 &= \int_0^T x(t) f_2(t) dt = \int_0^T [s_0(t) + n(t)] f_2(t) dt = \int_0^T s_0(t) f_2(t) dt + \int_0^T n(t) f_2(t) dt \\
&= \int_0^T s_0(t) \frac{1}{\sqrt{(1-\rho^2)E_0}} \left( s_0(t) - \rho \sqrt{\frac{E_0}{E_1}} s_1(t) \right) dt + n_2 \\
&= \frac{1}{\sqrt{(1-\rho^2)E_0}} \left[ \int_0^T s_0^2(t) dt - \rho \sqrt{\frac{E_0}{E_1}} \int_0^T s_0(t) s_1(t) dt \right] + n_2 \\
&= \frac{1}{\sqrt{(1-\rho^2)E_0}} \left[ E_0 - \rho \sqrt{\frac{E_0}{E_1}} \rho \sqrt{E_0 E_1} \right] + n_2 = \sqrt{(1-\rho^2)E_0} + n_2
\end{aligned}
$$

$$
f_2(t) = \frac{1}{\sqrt{(1-\rho^2)E_0}} \left( s_0(t) - \rho \sqrt{\frac{E_0}{E_1}} s_1(t) \right), \quad E_0 = \int_0^T s_0^2(t) dt, \quad \int_0^T n(t) f_2(t) dt = n_2
$$

$$
\rho = \frac{1}{\sqrt{E_0 E_1}} \int_0^T s_0(t) s_1(t) dt \implies \int_0^T s_0(t) s_1(t) dt = \rho \sqrt{E_0 E_1}
$$

# 对接收信号进行正交展开 (假设 $H_0 : x_k$)

**假设 $H_0 : x(t) = s_0(t) + n(t)$ 下, 展开系数 $x_k$**

$$x_k = \int_0^T x(t)f_k(t)dt = \int_0^T [s_0(t) + n(t)]f_k(t)dt = \int_0^T s_0(t)f_k(t)dt + \int_0^T n(t)f_k(t)dt$$
$$= 0 + \int_0^T n(t)f_k(t)dt = n_k \quad k \geq 3$$

$$f_2(t) = \frac{1}{\sqrt{(1-\rho^2)E_0}}\left(s_0(t) - \rho\sqrt{\frac{E_0}{E_1}}s_1(t)\right), s_1(t) = \sqrt{E_1}f_1(t)$$

$$\Longrightarrow s_0(t) = \left(\sqrt{(1-\rho^2)E_0}\right)f_2(t) + \left(\rho\sqrt{E_0}\right)f_1(t)$$

$$\int_0^T f_j(t)f_k(t)dt = 0, \quad k \geq 1, j \geq 1, k \neq j$$

# 对接收信号进行正交展开 (假设 $H_1$)

**假设 $H_1 : x(t) = s_1(t) + n(t)$ 下，展开系数**

$$
\begin{aligned}
x_1 &= \int_0^T x(t) f_1(t)dt = \int_0^T [s_1(t) + n(t)] f_1(t)dt = \int_0^T s_1(t) f_1(t)dt + \int_0^T n(t) f_1(t)dt \\
&= \int_0^T s_1(t)[\frac{1}{\sqrt{E_1}} s_1(t)]dt + n_1 = \frac{1}{\sqrt{E_1}} \int_0^T s_1^2(t)dt + n_1 \\
&= \sqrt{E_1} + n_1 \quad (\text{by } f_1(t) = \frac{1}{\sqrt{E_1}} s_1(t), E_1 = \int_0^T s_1^2(t)dt) \\
x_2 &= \int_0^T x(t) f_2(t)dt = \int_0^T [s_1(t) + n(t)] f_2(t)dt = \int_0^T s_1(t) f_2(t)dt + \int_0^T n(t) f_2(t)dt \\
&= \int_0^T [\sqrt{E_1} f_1(t)] f_2(t)dt + n_2 = 0 + n_2 = n_2 \\
x_k &= \int_0^T x(t) f_k(t)dt = \int_0^T [s_1(t) + n(t)] f_k(t)dt = \int_0^T [\sqrt{E_1} f_1(t) + n(t)] f_k(t)dt \\
&= \int_0^T n(t) f_k(t)dt = n_k \quad k \geq 3 \quad (by \quad s_1(t) = \sqrt{E_1} f_1(t), \int_0^T f_1(t) f_k(t)dt = 0, k \geq 3)
\end{aligned}
$$

# 充分量统计法

**(2) 利用构造的正交函数集 $f_1(t), f_2(t)$ 和 $\{f_k(t)|k \geq 3\}$ 对接收信号进行正交展开**

## 两个假设下展开系数 $x_1, x_2$

$$H_0 : x_1 = \int_0^T x(t)f_1(t)dt = \int_0^T [s_0(t) + n(t)]f_1(t)dt = \rho\sqrt{E_0} + n_1$$

$$H_0 : x_2 = \int_0^T x(t)f_2(t)dt = \int_0^T [s_0(t) + n(t)]f_2(t)dt = \sqrt{(1-\rho^2)E_0} + n_2$$

$$H_1 : x_1 = \int_0^T x(t)f_1(t)dt = \int_0^T [s_1(t) + n(t)]f_1(t)dt = \sqrt{E_1} + n_1$$

$$H_1 : x_2 = \int_0^T x(t)f_2(t)dt = \int_0^T [s_1(t) + n(t)]f_2(t)dt = n_2$$

$$H_0, H_1 : x_k = \int_0^T x(t)f_k(t)dt = n_k \quad (k \geq 3) \implies \text{不含确知信号 } s_0(t), s_1(t) \text{ 信息}$$

**$\boldsymbol{x} = (x_1, x_2)^T$ 是充分统计量。且 $x_1$ 和 $x_2$ 为高斯随机变量，相互统计独立。**

# 充分量统计法: $x_1, x_2$ 的均值和方差

$$E[x_1|H_0] = E\left[\rho\sqrt{E_0} + n_1\right] = \rho\sqrt{E_0}$$

$$E[x_2|H_0] = E\left[\sqrt{(1-\rho^2)E_0} + n_2\right] = \sqrt{(1-\rho^2)E_0}$$

$$Var[x_1|H_0] = Var[x_2|H_0] = E[n_1^2] = E[n_2^2] = \frac{N_0}{2}$$

$$E[x_1|H_1] = E\left[\sqrt{E_1} + n_1\right] = \sqrt{E_1}$$

$$E[x_2|H_1] = E[n_2] = 0$$

$$Var[x_1|H_1] = Var[x_2|H_1] = E[n_1^2] = E[n_2^2] = \frac{N_0}{2}$$

# 充分量统计法—构建似然比

**(3) 利用得到的展开系数,构建似然比表达式**

$$\boldsymbol{x} = (x_1, x_2)^T$$

$$\lambda(\boldsymbol{x}) = \frac{p(\boldsymbol{x}|H_1)}{p(\boldsymbol{x}|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

$$\lambda(\boldsymbol{x}) = \frac{p(x_1, x_2|H_1)}{p(x_1, x_2|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

$$\frac{\frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{\left(x_1-\sqrt{E_1}\right)^2}{N_0}\right) \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{x_2^2}{N_0}\right)}{\frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{\left(x_1-\rho\sqrt{E_0}\right)^2}{N_0}\right) \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{\left(x_2-\sqrt{(1-\rho^2)E_0}\right)^2}{N_0}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

# 充分量统计法—充分统计量 $x_1$

**(3) 利用得到的展开系数,构建似然比表达式**

展开系数中只有 $x_1$ 含有接收信号的信息,因此展开系数 $x_1$ 是一个**充分统计量**。利用 $x_1$ 构成的似然比检验,表示为

$$\lambda(x_1) = \frac{p(x_1|H_1)}{p(x_1|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

因为

$$f_1(t) = \frac{1}{\sqrt{E_s}} s(t), \quad (x_1|H_0) = n_1, \quad (x_1|H_1) = \sqrt{E_s} + n_1$$

$$x_1 = \int_0^T x(t) f_1(t) dt = \frac{1}{\sqrt{E_s}} \int_0^T x(t) s(t) dt$$

因为充分统计量 $x_1$ 是高斯随机变量, 可用假设 $H_0$ 和假设 $H_1$ 下的均值和方差表示。

# 充分量统计法—充分统计量 $x_1$

**(3) 利用得到的展开系数，构建似然比表达式**

展开系数中只有 $x_1$ 含有接收信号的信息，因此展开系数 $x_1$ 是一个充分统计量。利用 $x_1$ 构成的似然比检验，表示为

$$\lambda(x_1) = \frac{p(x_1|H_1)}{p(x_1|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

因为

$$f_1(t) = \frac{1}{\sqrt{E_s}} s(t), \quad (x_1|H_0) = n_1, \quad (x_1|H_1) = \sqrt{E_s} + n_1$$

$$x_1 = \int_0^T x(t) f_1(t) dt = \frac{1}{\sqrt{E_s}} \int_0^T x(t) s(t) dt$$

所以充分统计量 $x_1$ 是高斯随机变量，可用假设 $H_0$ 和假设 $H_1$ 下的均值和方差表示。

# 充分量统计法—充分统计量 $x_1$

$x_1$ 是高斯变量, 均值和方差

$$E[x_1|H_0] = 0, \qquad Var[x_1|H_0] = \frac{N_0}{2}$$

$$E[x_1|H_1] = \sqrt{E_s}, \qquad Var[x_1|H_1] = \frac{N_0}{2}$$

概率密度函数

$$p(x_1|H_0) = \left(\frac{1}{\pi N_0}\right)^{1/2} \exp\left(-\frac{x_1^2}{N_0}\right)$$

$$p(x_1|H_1) = \left(\frac{1}{\pi N_0}\right)^{1/2} \exp\left(-\frac{(x_1 - \sqrt{E_s})^2}{N_0}\right)$$

# 推导 $E[x_1|H_0]$

$$f_1(t) = \frac{1}{\sqrt{E_s}} s(t), \quad (x_1|H_0) = n_1, \quad (x_1|H_1) = \sqrt{E_s} + n_1$$

$$x_1 = \int_0^T x(t) f_1(t) dt = \frac{1}{\sqrt{E_s}} \int_0^T x(t) s(t) dt$$

$$E[x_1|H_0] = E[n_1] = 0$$

或:

$$\begin{aligned} E[x_1|H_0] &= E\left[\frac{1}{\sqrt{E_s}} \int_0^T x(t) s(t) dt\right] && \text{by } H_0 : x(t) = n(t) \\ &= E\left[\frac{1}{\sqrt{E_s}} \int_0^T n(t) s(t) dt\right] \\ &= \frac{1}{\sqrt{E_s}} \int_0^T E[n(t)] s(t) dt = 0 && \text{by } E[n(t)] = 0 \end{aligned}$$

# 推导 $Vax[x_1|H_0]$(方法 1)

$$H_0 : x(t) = n(t), \quad E(x_1|H_0) = 0, \quad E_s = \int_0^T s^2(t)dt$$

$$E[n(t)n(u)] = r_n(t-u) = \frac{N_0}{2}\delta(t-u) = \frac{N_0}{2}, (t = u, \delta(t-u) = 1)$$

$$f_1(t) = \frac{1}{\sqrt{E_s}}s(t), \quad (x_1|H_0) = n_1, \quad (x_1|H_1) = \sqrt{E_s} + n_1$$

$$x_1 = \int_0^T x(t)f_1(t)dt = \frac{1}{\sqrt{E_s}}\int_0^T x(t)s(t)dt, \quad (x_1|H_0) = \frac{1}{\sqrt{E_s}}\int_0^T n(t)s(t)dt$$

$$Var[x_1|H_0] = E[((x_1|H_0) - E(x_1|H_0))^2] = E[(x_1|H_0)^2] = E\left[\left(\frac{1}{\sqrt{E_s}}\int_0^T n(t)s(t)dt\right)^2\right]$$

$$= \frac{1}{E_s}E\left[\int_0^T n(t)s(t)dt \int_0^T n(t)s(t)dt\right] = \frac{1}{E_s}E\left[\int_0^T n(t)s(t)dt \int_0^T n(u)s(u)du\right]$$

$$= \frac{1}{E_s}\int_0^T s(t)\left\{\int_0^T E[n(u)n(t)]s(u)du\right\}dt = \frac{1}{E_s}\int_0^T s(t)\left[\int_0^T \frac{N_0}{2}\delta(t-u)s(u)du\right]dt$$

$$= \frac{N_0}{2E_s}\int_0^T s(t)\left(\int_0^T s(u)du\right)dt = \frac{N_0}{2E_s}\int_0^T s^2(t)dt = \frac{N_0}{2}$$

# 推导 $Vax[x_1|H_0]$(方法 2)

$$H_0 : x(t) = n(t), \quad E(x_1|H_0) = 0, \quad E_s = \int_0^T s^2(t)dt$$

$$E[n(t)n(u)] = r_n(t-u) = \frac{N_0}{2}\delta(t-u) = \frac{N_0}{2}, (t = u, \delta(t-u) = 1)$$

$$f_1(t) = \frac{1}{\sqrt{E_s}}s(t), \quad (x_1|H_0) = n_1, \quad (x_1|H_1) = \sqrt{E_s} + n_1$$

$$x_1 = \int_0^T x(t)f_1(t)dt = \frac{1}{\sqrt{E_s}}\int_0^T x(t)s(t)dt, \quad (x_1|H_0) = \frac{1}{\sqrt{E_s}}\int_0^T n(t)s(t)dt$$

$$n_1 = \int_0^T n(t)f_1(t)dt = \int_0^T n(t)\frac{1}{\sqrt{E_s}}s(t)dt = \frac{1}{\sqrt{E_s}}\int_0^T n(t)s(t)dt$$

$$Var[x_1|H_0] = E[((x_1|H_0) - E(x_1|H_0))^2] = E[(x_1|H_0)^2] = E[n_1^2]$$

$$= E\left[\left(\frac{1}{\sqrt{E_s}}\int_0^T n(t)s(t)dt\right)^2\right] = \frac{N_0}{2} \qquad \text{(同方法 1)}$$

# 推导 $p(x_1|H_0)$

$$E[x_1|H_0] = 0, \quad Var[x_1|H_0] = \frac{N_0}{2}$$

$$\begin{aligned} p(x_1|H_0) &= \left(\frac{1}{2\pi Var[x_1|H_0]}\right)^{1/2} \exp\left(-\frac{(x_1 - E[x_1|H_0])^2}{2Var[x_1|H_0]}\right) \\ &= \left(\frac{1}{\pi N_0}\right)^{1/2} \exp\left(-\frac{x_1^2}{N_0}\right) \end{aligned}$$

# 推导 $E[x_1|H_1]$ 和 $Var[x_1|H_1]$

$$f_1(t) = \frac{1}{\sqrt{E_s}}s(t), \quad E_s = \int_0^T s^2(t)dt, \quad n_1 = \int_0^T n(t)f_1(t)dt$$

$$(x_1|H_1) = \int_0^T x(t)f_1(t)dt = \int_0^T [s(t)+n(t)]f_1(t)dt = \sqrt{E_s} + n_1$$

$$\begin{aligned} E[x|H_1] &= E\left[\sqrt{E_s} + n_1\right] \\ &= E[\sqrt{E_s}] + E[n_1] \\ &= E[\sqrt{E_s}] = \sqrt{E_s} \\ Var[x_1|H_1] &= E[((x|H_1) - E(x|H_1))^2] = E[(\sqrt{E_s} + n_1 - \sqrt{E_s})] \\ &= E[n_1^2] = Var[x_1|H_0] \\ &= \frac{N_0}{2} \end{aligned}$$

# 推导 $p(x_1|H_1)$

$$E[x_1|H_1] = \sqrt{E_s}, \quad Var[x_1|H_1] = \frac{N_0}{2}$$

$$\begin{aligned} p(x_1|H_1) &= \left(\frac{1}{2\pi Var[x_1|H_1]}\right)^{1/2} \exp\left(-\frac{(x_1 - E[x_1|H_1])^2}{2Var[x_1|H_1]}\right) \\ &= \left(\frac{1}{\pi N_0}\right)^{1/2} \exp\left(-\frac{(x_1 - \sqrt{E_s})^2}{N_0}\right) \end{aligned}$$

# 充分量统计法—充分统计量 $x_1$

$x_1$ 是高斯变量, 均值和方差

$$E[x_1|H_0] = 0, \qquad Var[x_1|H_0] = \frac{N_0}{2}$$
$$E[x_1|H_1] = \sqrt{E_s}, \qquad Var[x_1|H_1] = \frac{N_0}{2}$$

概率密度函数

$$p(x_1|H_0) = \left(\frac{1}{\pi N_0}\right)^{1/2} \exp\left(-\frac{x_1^2}{N_0}\right)$$
$$p(x_1|H_1) = \left(\frac{1}{\pi N_0}\right)^{1/2} \exp\left(-\frac{(x_1 - \sqrt{E_s})^2}{N_0}\right)$$

# 充分量统计法—判决表达式

$$\lambda(x_1) = \frac{p(x_1|H_1)}{p(x_1|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

$$\frac{\left(\frac{1}{\pi N_0}\right)^{1/2} \exp\left(-\frac{(x_1-\sqrt{E_s})^2}{N_0}\right)}{\left(\frac{1}{\pi N_0}\right)^{1/2} \exp\left(-\frac{x_1^2}{N_0}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta \implies x_1 \underset{H_0}{\overset{H_1}{\gtrless}} \frac{N_0}{2\sqrt{E_s}} \ln\eta + \frac{\sqrt{E_s}}{2}$$

代入展开系数 $x_1 = \int_0^T x(t) f_1(t) dt = \frac{1}{\sqrt{E_s}} \int_0^T x(t) s(t) dt$

**化简得, 判决表达式:**

$$l[x(t)] \overset{def}{=} \int_0^T x(t) s(t) dt \underset{H_0}{\overset{H_1}{\gtrless}} \frac{N_0}{2} \ln\eta + \frac{E_s}{2} \overset{def}{=} \gamma$$

**结论**

由任意正交函数集对 $x(t)$ 进行正交级数展开法与由充分统计量法导出的判决表达式是完全一样的,因而也具有相同的检测系统结构和相同的检测性能。

# 简单二元信号波形的检测—总结

1. 首先, 利用随机过程的正交级数展开, 将随机过程用一组随机变量来表示;
2. 然后, 针对展开得到的随机变量, 利用第三章的统计检测方法, 构建贝叶斯检测表达式;
3. 最后, 利用展开系数与随机过程之间的表示关系, 构建波形信号的检测表达式。
4. **两种方法: 正交级数展开和充分统计量, 所得结果相同。**

欢迎批评指正！