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Chapter 1

Hopcroft's algorithm

1.0.1 algorithm

Member function `min_Hopcroft` implements Hopcroft's $n \log n$ minimization algorithm, as presented in [[WATSON94b], Algorithm 4.8].

The combination of the out-transitions of all of the States is stored in a **CRSet** *C*.

Set *L* from the abstract algorithm is implemented as a mapping from States to int (an array of int is used).

Array *L* should be interpreted as follows: if State *q* a representative, then the following pairs still require processing (are still in abstract set *L*):

$$([q], C_0), \dots, ([q], C_{L(q)-1})$$

The remaining pairs do not require processing:

$$([q], C_{L[q]}), \dots, ([q], C_{|C|-1})$$

This implementation facilitates quick scanning of *L* for the next valid State-CharRange pair.

1.0.2 Example

```
CRSet C; // the out labels of State's: { 'a' 'b' }
int L[|Q|]: // the index of L = q: 对应等价类[q]; L[q]表示正在处理[q]对
           应的字符在C中的index,
0 1 2 3 4 5 6 7 8
0 0 0 0 0 0 0 0 0

Initialize partitions, E_0:
{ 0 1 2 3 4 5 }
{ 6 7 8 }
```


Pick one $[q]$ in L , Processing $[q] = \text{index of } L = [6], 'b'$
 current all partitions:
 $\{ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \}$
 $\{ 6 \ 7 \ 8 \}$

\equiv **for** each $[p]$, (**split** $[p]$ w.r.t ($\text{index of } L[6], 'b'$)

\equiv **split** $[0]$ w.r.t ($\text{index of } L[6], 'b'$)

before split, partitions:

$\{ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \}$
 $\{ 6 \ 7 \ 8 \}$

new split of $[0]$ is $[1]$

after split, partitions:

$\{ 0 \ 2 \ 3 \ 4 \ 5 \}$
 $\{ 1 \}$
 $\{ 6 \ 7 \ 8 \}$

before L :

0 1 2 3 4 5 6 7 8
 0 0 0 0 0 0 1 0 0

p **and** r are the **new** representatives. Now update L with the smallest of
 $[0], [1]$

using $[r] = [1], L[r] = C.\text{size}();$

after L :

0 1 2 3 4 5 6 7 8
 0 2 0 0 0 0 1 0 0

\equiv **split** $[6]$ w.r.t ($\text{index of } L[6], 'b'$)

before split, partitions:

$\{ 0 \ 2 \ 3 \ 4 \ 5 \}$
 $\{ 1 \}$
 $\{ 6 \ 7 \ 8 \}$

new split of $[6]$ is $[8]$

after split, partitions:

$\{ 0 \ 2 \ 3 \ 4 \ 5 \}$
 $\{ 1 \}$

```
{ 6 7 }
{ 8 }
```

before L:

```
0 1 2 3 4 5 6 7 8
0 2 0 0 0 0 1 0 0
```

p and r are the new representatives. Now update L with the smallest of
[6],[8]

```
using [r] = [8], L[r]=C.size();
```

after L:

```
0 1 2 3 4 5 6 7 8
0 2 0 0 0 0 1 0 2
```

— while each [q], (split [p] w.r.t ([index of L]=[q],a))

L:

```
0 1 2 3 4 5 6 7 8
0 2 0 0 0 0 1 0 2
```

Pick one [q] in L, Processing [q]= index of L = [1], 'b'

current all partitions(eq.classes) repr:

```
{ 0 1 6 8 }
```

current all partitions:

```
{ 0 2 3 4 5 }
```

```
{ 1 }
```

```
{ 6 7 }
```

```
{ 8 }
```

== for each [p], (split [p] w.r.t (index of L)[1], 'b')

==split [0] w.r.t (index of L)[1], 'b')

before split, partitions:

StateEqRel

```
{ 0 2 3 4 5 }
```

```
{ 1 }
```

```
{ 6 7 }
```

```
{ 8 }
```

new split of [0] is [-1]

==split [1] w.r.t (index of L)[1], 'b')

before split, partitions:

```
{ 0 2 3 4 5 }
```

```

{ 1 }
{ 6 7 }
{ 8 }

new split of [1] is [-1]
==split [6] w.r.t (index of L)[1], 'b')
before split, partitions:
{ 0 2 3 4 5 }
{ 1 }
{ 6 7 }
{ 8 }

new split of [6] is [-1]
==split [8] w.r.t (index of L)[1], 'b')
before split, partitions:
StateEqRel
{ 0 2 3 4 5 }
{ 1 }
{ 6 7 }
{ 8 }

new split of [8] is [-1]
— while each [q], (split [p] w.r.t ([index of L]=[q],a))
L:
0 1 2 3 4 5 6 7 8
0 1 0 0 0 0 1 0 2
Pick one [q] in L, Processing [q]= index of L = [1], 'a'
current all partitions(eq.classes) repr:
{ 0 1 6 8 }
current all partitions:
{ 0 2 3 4 5 }
{ 1 }
{ 6 7 }
{ 8 }

== for each [p], (split [p] w.r.t (index of L)[1], 'a')
==split [0] w.r.t (index of L)[1], 'a')
before split, partitions:
{ 0 2 3 4 5 }
{ 1 }
{ 6 7 }

```

```
{ 8 }
```

```
new split of [0] is [2]
after split, partitions:
```

```
StateEqRel
```

```
{ 0 }
```

```
{ 1 }
```

```
{ 2 3 4 5 }
```

```
{ 6 7 }
```

```
{ 8 }
```

```
before L:
```

```
L:
```

```
0 1 2 3 4 5 6 7 8
```

```
0 0 0 0 0 0 1 0 2
```

```
p and r are the new representatives. Now update L with the smallest of
[0],[2]
```

```
using [p] = [0], L[r]=L[p]; L[p]=C.size();
```

```
after L:
```

```
L:
```

```
0 1 2 3 4 5 6 7 8
```

```
2 0 0 0 0 0 1 0 2
```

```
==split[1] w.r.t (index of L)[1], 'a')
```

```
before split, partitions:
```

```
StateEqRel
```

```
{ 0 }
```

```
{ 1 }
```

```
{ 2 3 4 5 }
```

```
{ 6 7 }
```

```
{ 8 }
```

```
new split of [1] is [-1]
```

```
==split[6] w.r.t (index of L)[1], 'a')
```

```
before split, partitions:
```

```
StateEqRel
```

```
{ 0 }
```

```
{ 1 }
```

```
{ 2 3 4 5 }
```

```
{ 6 7 }
```

```
{ 8 }
```



```

new split of [6] is [-1]
==split [8] w.r.t (index of L)[1], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 3 4 5 }
{ 6 7 }
{ 8 }

new split of [8] is [-1]
— while each [q], (split [p] w.r.t ([index of L]=[q],a))
L:
0 1 2 3 4 5 6 7 8
2 0 0 0 0 0 1 0 2
Pick one [q] in L, Processing [q]= index of L = [0], 'b'
current all partitions(eq.classes) repr:
{ 0 1 2 6 8 }
current all partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 3 4 5 }
{ 6 7 }
{ 8 }

== for each [p], (split [p] w.r.t (index of L)[0], 'b')
==split [0] w.r.t (index of L)[0], 'b')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 3 4 5 }
{ 6 7 }
{ 8 }

new split of [0] is [-1]
==split [1] w.r.t (index of L)[0], 'b')
before split, partitions:
StateEqRel
{ 0 }

```

```

{ 1 }
{ 2 3 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [1] is [-1]
==split [2] w.r.t (index of L)[0], 'b')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 3 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [2] is [-1]
==split [6] w.r.t (index of L)[0], 'b')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 3 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [6] is [-1]
==split [8] w.r.t (index of L)[0], 'b')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 3 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [8] is [-1]
— while each [q], (split [p] w.r.t ([index of L]=[q],a))
L:
0 1 2 3 4 5 6 7 8
1 0 0 0 0 0 1 0 2
Pick one [q] in L, Processing [q]= index of L = [0], 'a'

```

```

current all partitions(eq.classes) repr:
{ 0  1  2  6  8 }
current all partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2  3  4  5 }
{ 6  7 }
{ 8 }

== for each [p], (split [p] w.r.t (index of L)[0], 'a')
==split [0] w.r.t (index of L)[0], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2  3  4  5 }
{ 6  7 }
{ 8 }

new split of [0] is [-1]
==split [1] w.r.t (index of L)[0], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2  3  4  5 }
{ 6  7 }
{ 8 }

new split of [1] is [-1]
==split [2] w.r.t (index of L)[0], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2  3  4  5 }
{ 6  7 }
{ 8 }

new split of [2] is [-1]
```

```

==split [6] w.r.t (index of L) [0], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 3 4 5 }
{ 6 7 }
{ 8 }

new split of [6] is [-1]
==split [8] w.r.t (index of L) [0], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 3 4 5 }
{ 6 7 }
{ 8 }

new split of [8] is [-1]
— while each [q], (split [p] w.r.t ([index of L]=[q],a))
L:
0 1 2 3 4 5 6 7 8
0 0 0 0 0 0 1 0 2
Pick one [q] in L, Processing [q]= index of L = [6], 'a'
current all partitions(eq.classes) repr:
{ 0 1 2 6 8 }
current all partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 3 4 5 }
{ 6 7 }
{ 8 }

== for each [p], (split [p] w.r.t (index of L) [6], 'a')
==split [0] w.r.t (index of L) [6], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }

```

```

{ 2 3 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [0] is [-1]
==split [1] w.r.t (index of L)[6], 'a')

```

before split, partitions:

StateEqRel

```

{ 0 }
{ 1 }
{ 2 3 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [1] is [-1]
==split [2] w.r.t (index of L)[6], 'a')

```

before split, partitions:

StateEqRel

```

{ 0 }
{ 1 }
{ 2 3 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [2] is [4]
after split, partitions:

```

StateEqRel

```

{ 0 }
{ 1 }
{ 2 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

before L:

L:

```

0 1 2 3 4 5 6 7 8
0 0 0 0 0 0 0 0 2

```

p and r are the new representatives. Now update L with the smallest of [2], [4]

```

using [p] = [2], L[r]=L[p]; L[p]=C.size();

```

```

after L:
L:
0 1 2 3 4 5 6 7 8
0 0 2 0 0 0 0 0 2
==split [6] w.r.t (index of L) [6], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

new split of [6] is [-1]
==split [8] w.r.t (index of L) [6], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

new split of [8] is [-1]
— while each [q], (split [p] w.r.t ([index of L]=[q],a))
L:
0 1 2 3 4 5 6 7 8
0 0 2 0 0 0 0 0 2
Pick one [q] in L, Processing [q]= index of L = [2], 'b'
current all partitions(eq.classes) repr:
{ 0 1 2 4 6 8 }
current all partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

```

== for each [p], (split [p] w.r.t (index of L)[2], 'b')
==split [0] w.r.t (index of L)[2], 'b')

```

before split, partitions:

StateEqRel

{ 0 }

{ 1 }

{ 2 3 }

{ 4 5 }

{ 6 7 }

{ 8 }

new split of [0] is [-1]

```

==split [1] w.r.t (index of L)[2], 'b')

```

before split, partitions:

StateEqRel

{ 0 }

{ 1 }

{ 2 3 }

{ 4 5 }

{ 6 7 }

{ 8 }

new split of [1] is [-1]

```

==split [2] w.r.t (index of L)[2], 'b')

```

before split, partitions:

StateEqRel

{ 0 }

{ 1 }

{ 2 3 }

{ 4 5 }

{ 6 7 }

{ 8 }

new split of [2] is [-1]

```

==split [4] w.r.t (index of L)[2], 'b')

```

before split, partitions:

StateEqRel

{ 0 }

{ 1 }

{ 2 3 }

```
{ 4 5 }
{ 6 7 }
{ 8 }
```

```
new split of [4] is [-1]
==split [6] w.r.t (index of L)[2], 'b')
```

before split, partitions:

StateEqRel

```
{ 0 }
{ 1 }
{ 2 3 }
{ 4 5 }
{ 6 7 }
{ 8 }
```

```
new split of [6] is [-1]
==split [8] w.r.t (index of L)[2], 'b')
```

before split, partitions:

StateEqRel

```
{ 0 }
{ 1 }
{ 2 3 }
{ 4 5 }
{ 6 7 }
{ 8 }
```

```
new split of [8] is [-1]
— while each [q], (split [p] w.r.t ([index of L]=[q],a))
```

L:

```
0 1 2 3 4 5 6 7 8
0 0 1 0 0 0 0 0 2
```

Pick one [q] in L, Processing [q]= index of L = [2], 'a'

current all partitions(eq.classes) repr:

```
{ 0 1 2 4 6 8 }
```

current all partitions:

StateEqRel

```
{ 0 }
{ 1 }
{ 2 3 }
{ 4 5 }
{ 6 7 }
```


{ 8 }

== for each [p], (split [p] w.r.t (index of L)[2], 'a')

==split [0] w.r.t (index of L)[2], 'a')

before split, partitions:

StateEqRel

{ 0 }

{ 1 }

{ 2 3 }

{ 4 5 }

{ 6 7 }

{ 8 }

new split of [0] is [-1]

==split [1] w.r.t (index of L)[2], 'a')

before split, partitions:

StateEqRel

{ 0 }

{ 1 }

{ 2 3 }

{ 4 5 }

{ 6 7 }

{ 8 }

new split of [1] is [-1]

==split [2] w.r.t (index of L)[2], 'a')

before split, partitions:

StateEqRel

{ 0 }

{ 1 }

{ 2 3 }

{ 4 5 }

{ 6 7 }

{ 8 }

new split of [2] is [3]

after split, partitions:

StateEqRel

{ 0 }

{ 1 }

{ 2 }

```

{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

before L:

L:

```

0 1 2 3 4 5 6 7 8
0 0 0 0 0 0 0 0 2

```

p and r are the new representatives. Now update L with the smallest of [2], [3]

```
using [p] = [2], L[r]=L[p]; L[p]=C.size();
```

after L:

L:

```

0 1 2 3 4 5 6 7 8
0 0 2 0 0 0 0 0 2

```

胡

==split[4] w.r.t (index of L)[2], 'b')

before split, partitions:

StateEqRel

```

{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

new split of [4] is [-1]

==split[6] w.r.t (index of L)[2], 'b')

before split, partitions:

StateEqRel

```

{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

new split of [6] is [-1]

```

==split [8] w.r.t (index of L) [2], 'b')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

new split of [8] is [-1]
— while each [q], (split [p] w.r.t ([index of L]=[q],a))
L:
0 1 2 3 4 5 6 7 8
0 0 1 0 0 0 0 0 2
Pick one [q] in L, Processing [q]= index of L = [2], 'a'
current all partitions(eq.classes) repr:
{ 0 1 2 3 4 6 8 }
current all partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

== for each [p], (split [p] w.r.t (index of L) [2], 'a')
==split [0] w.r.t (index of L) [2], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [0] is [-1]
==split [1] w.r.t (index of L)[2], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [1] is [-1]
==split [2] w.r.t (index of L)[2], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [2] is [-1]
==split [3] w.r.t (index of L)[2], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [3] is [-1]
==split [4] w.r.t (index of L)[2], 'a')
before split, partitions:
StateEqRel
{ 0 }

```

```
{ 1 }
```

```
{ 2 }
```

```
{ 3 }
```

```
{ 4 5 }
```

```
{ 6 7 }
```

```
{ 8 }
```

```
new split of [4] is [-1]
```

```
==split [6] w.r.t (index of L)[2], 'a')
```

```
before split, partitions:
```

```
StateEqRel
```

```
{ 0 }
```

```
{ 1 }
```

```
{ 2 }
```

```
{ 3 }
```

```
{ 4 5 }
```

```
{ 6 7 }
```

```
{ 8 }
```

```
new split of [6] is [-1]
```

```
==split [8] w.r.t (index of L)[2], 'a')
```

```
before split, partitions:
```

```
StateEqRel
```

```
{ 0 }
```

```
{ 1 }
```

```
{ 2 }
```

```
{ 3 }
```

```
{ 4 5 }
```

```
{ 6 7 }
```

```
{ 8 }
```

```
new split of [8] is [-1]
```

```
— while each [q], (split [p] w.r.t ([index of L]=[q],a))
```

```
L:
```

```
0 1 2 3 4 5 6 7 8
```

```
0 0 0 0 0 0 0 0 2
```

```
Pick one [q] in L, Processing [q]= index of L = [8], 'b'
```

```
current all partitions(eq.classes) repr:
```

```
{ 0 1 2 3 4 6 8 }
```

```
current all partitions:
```

```
StateEqRel
```

```

{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

```

== for each [p], (split [p] w.r.t (index of L)[8], 'b')

```

```

==split[0] w.r.t (index of L)[8], 'b')

```

```

before split, partitions:

```

```

StateEqRel

```

```

{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [0] is [-1]

```

```

==split[1] w.r.t (index of L)[8], 'b')

```

```

before split, partitions:

```

```

StateEqRel

```

```

{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [1] is [-1]

```

```

==split[2] w.r.t (index of L)[8], 'b')

```

```

before split, partitions:

```

```

StateEqRel

```

```

{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }

```

```
{ 6 7 }
{ 8 }
```

```
new split of [2] is [-1]
==split [3] w.r.t (index of L)[8], 'b')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }
```

```
new split of [3] is [-1]
==split [4] w.r.t (index of L)[8], 'b')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }
```

```
new split of [4] is [-1]
==split [6] w.r.t (index of L)[8], 'b')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }
```

```
new split of [6] is [-1]
==split [8] w.r.t (index of L)[8], 'b')
```

before split, partitions:

StateEqRel

{ 0 }

{ 1 }

{ 2 }

{ 3 }

{ 4 5 }

{ 6 7 }

{ 8 }

new split of [8] is [-1]

— while each [q], (split [p] w.r.t ([index of L]=[q],a))

L:

0 1 2 3 4 5 6 7 8

0 0 0 0 0 0 0 0 1

Pick one [q] in L, Processing [q]= index of L = [8], 'a'

current all partitions(eq.classes) repr:

{ 0 1 2 3 4 6 8 }

current all partitions:

StateEqRel

{ 0 }

{ 1 }

{ 2 }

{ 3 }

{ 4 5 }

{ 6 7 }

{ 8 }

== for each [p], (split [p] w.r.t (index of L)[8], 'a')

==split [0] w.r.t (index of L)[8], 'a')

before split, partitions:

StateEqRel

{ 0 }

{ 1 }

{ 2 }

{ 3 }

{ 4 5 }

{ 6 7 }

{ 8 }

new split of [0] is [-1]


```

==split [1] w.r.t (index of L) [8], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [1] is [-1]
==split [2] w.r.t (index of L) [8], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [2] is [-1]
==split [3] w.r.t (index of L) [8], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [3] is [-1]
==split [4] w.r.t (index of L) [8], 'a')
before split, partitions:
StateEqRel
{ 0 }
{ 1 }

```

```

{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [4] is [-1]
==split [6] w.r.t (index of L) [8], 'a')

```

before split, partitions:

StateEqRel

```

{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [6] is [-1]
==split [8] w.r.t (index of L) [8], 'a')

```

before split, partitions:

StateEqRel

```

{ 0 }
{ 1 }
{ 2 }
{ 3 }
{ 4 5 }
{ 6 7 }
{ 8 }

```

```

new split of [8] is [-1]
—— while each [q], (split [p] w.r.t ([index of L]=[q],a))

```

L:

```

0 1 2 3 4 5 6 7 8
0 0 0 0 0 0 0 0 0

```

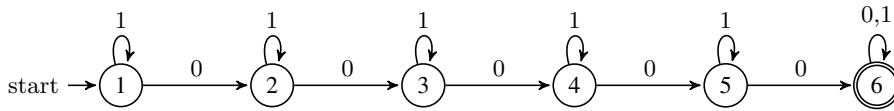


图 1.2: Minimizing example

$\{a,b\},\{d,e\}$ is not equivalent states.

Sets of equivalent states: $\{a,c\},\{b\},\{d\},\{e\}$

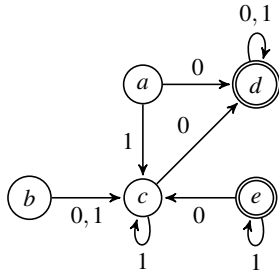


图 1.3: Finite state automaton

1.1 Minimization by equivalence of states

References

- Hopcroft2008. John E. Hopcroft, Rajeev Motwani, Jeffrey D. Ullman 著, 孙家骕等译, 自动机理论、语言和计算机导论, Third Edition, 机械工业出版社, 2008.7
- WATSON93a. WATSON, B. W. *A taxonomy of finite automata construction algorithms*, Computing Science Note 93/43, Eindhoven University of Technology, The Netherlands, 1993. Available by ftp from ftp.win.tue.nl in pub/techreports/pi.
- WATSON93b. WATSON, B. W. *A taxonomy of finite automata minimization algorithms*, Computing Science Note 93/44, Eindhoven University of Technology, The Netherlands, 1993. Available by ftp from ftp.win.tue.nl in pub/techreports/pi.
- WATSON94a. WATSON, B. W. *An introduction to the FIRE engine: A C++ toolkit for FInite automata and Regular Expressions*, Computing Science Note 94/21, Eindhoven University of Technology, The Netherlands, 1994. Available by ftp from ftp.win.tue.nl in pub/techreports/pi
- WATSON94b. WATSON, B.W. *The design. and implementation of the FIRE engine: A C++ toolkit for FInite automata and Regular Expressions*, Computing Science Note 94/22, Eindhoven University of Technology, The Netherlands, 1994. Available by ftp from ftp.win.tue.nl in pub/techreports/pi.
- Chrison2007. Christos G. Cassandras and Stéphane Lafortune, *Introduction to Discrete Event Systems*, Second Edition, New York, Springer, 2007
- Wonham2018. W. M. Wonham and Kai Cai, *Supervisory Control of Discrete-Event Systems*, Revised 2018.01.01
- Jean2018. Jean-Éric Pin, *Mathematical Foundations of Automata Theory*, Version of June 15, 2018
- 蒋宗礼 2013. 蒋宗礼, 姜守旭, 形式语言与自动机理论 (第 3 版), 清华大学出版社, 2013.05
- Lipschutz2007. S. Lipschutz and M. L. Lipson, *Schaum's Outline of Theory and Problems of Discrete Mathematics*, Third Edition, New York: McGraw-Hill, 2007.
- Rosen2007. K. H. Rosen, *Discrete Mathematics and Its Applications*, Seventh Edition, New York: McGraw-Hill, 2007.
- R.Su and Wonham2004. R. Su and W. M. Wonham, *Supervisor reduction for discrete-event systems*, Discrete Event Dyn. Syst., vol. 14, no. 1, pp. 31-53, Jan. 2004.

- Hopcroft71. Hopcroft, J.E. *An $n \log n$ algorithm for minimizing states in a finite automaton*, in The Theory of Machines and Computations (Z. Kohavi, ed.), pp.180-196, Academic Press, New York, 1971.
- Gries73. Gries, D. *Describing an Algorithm by Hopcroft*, Acta Inf. 2:97 109, 173. © by Springer-Verlag 1973
- Knuutila2001. Knuutila, T. *Re-describing an Algorithm by Hopcroft*. Theoret. Computer Science 250 (2001) 333–363.
- Ratnesh95. Ratnesh Kumar, *Modeling and Control of Logical Discrete Event Systems*, © 1995 by Springer Science+Business Media New York.
- Jean2011. Jean Berstel, Luc Boasson, Olivier Carton, Isabelle Fagnot , *Minimization of automata*, Université Paris-Est Marne-la-Vallée 2010 Mathematics Subject Classification: 68Q45, 2011.
- Kenneth2012. Kenneth H. Rosen 著, 徐六通译, 离散数学及其应用 *Discrete Mathematics and Its Applications*, seventh Edition, 2012, 机械工业出版社, 北京, 2014.