Chapter 1

Finite automata minimization algorithms

1.1 Introduction

1.2 Brzozowski's algorithm

$\varepsilon - free$ FA: $M_0 = (Q_0, V, T_0, \emptyset, S_0, F_0)$
to be minimized DFA: $M_2 = (Q_2, V, T_2, \emptyset, S_2, F_2)$
intermediate NFA: $M_1 = (Q_1, V, T_1, \emptyset, S_1, F_1)$

NFA: $M_1 \rightarrow$ DFA: $M_2, M_2 = suseful_s \circ subsetopt(M_1)$

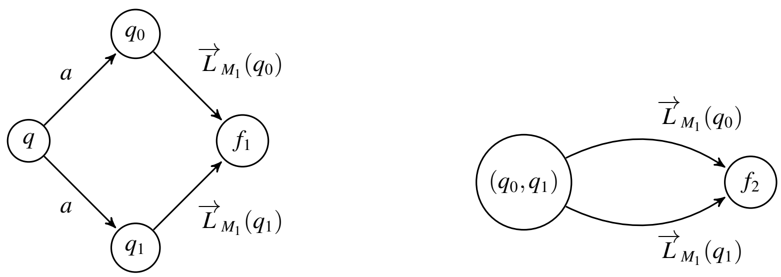
$$
\begin{aligned}
& q_0, q_1 \in Q_1, Q_2 \subseteq \mathbb{P}(Q_1), \forall p \in Q_2, p = (q_0, q_1) \\
& \overrightarrow{L}_{M_2}(p) = \overrightarrow{L}_{M_1}(q_0) \cup \overrightarrow{L}_{M_1}(q_1) \\
& \Rightarrow \\
& \overrightarrow{L}_{M_2}(p) = \bigcup_{q \in p} \overrightarrow{L}_{M_1}(q) \\
& \Rightarrow
\end{aligned}
$$

1.3 Minimization by equivalence of states

Let $M = (Q, V, T, F)$ be a deterministic finite automaton, where Q is a finite set of states, V is a finite set of input symbols, T is a mapping from $Q \times V$ into Q, and $F \subseteq Q$ is the set of final states. No initial state is

$$\begin{tikzcd} Q_1 \arrow[r, "subsetopt"] \arrow[dr] & P(Q_1) \arrow[d, "useful_s"] \\ & D(Q_2) \end{tikzcd}$$

图 1.1: $M_2 = suseful_s \circ subsetopt(M_1)$



(a) $M_1 = (Q_1, V, T_1, \emptyset, S_1, F_1)$ (b) $M_2 = (Q_2, V, T_2, \emptyset, S_2, F_2)$

图 1.2: $M_2 = suseful_s \circ subsetopt(M_1)$

specified since it is of no importance in what follows. The mapping T is extended to $T \times V^*$ in the usual manner where V^* denotes the set of all finite strings (including the empty string ε) of symbols from I.

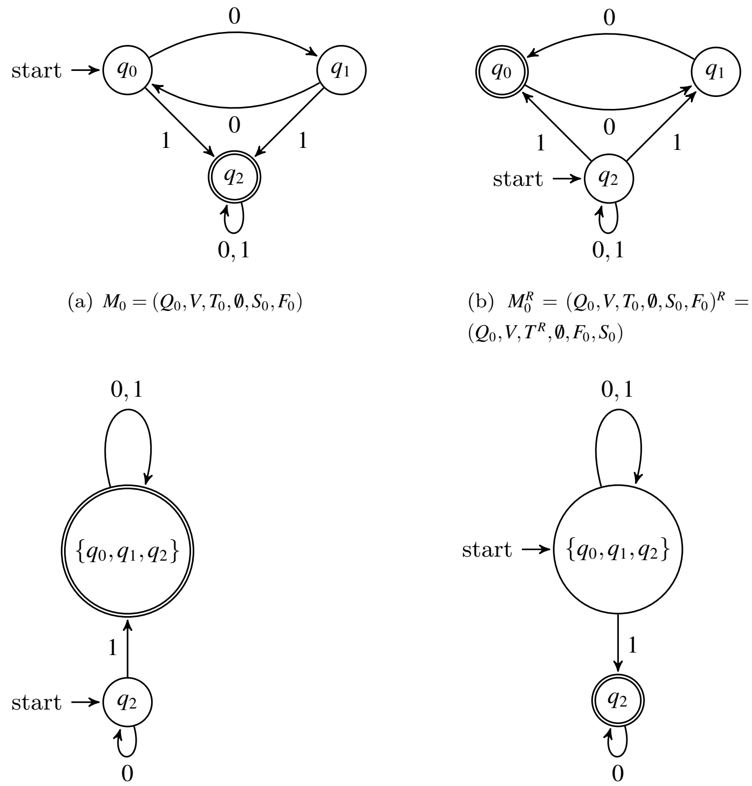
Definition 1.1 (equivalent states). States s and t are said to be equivalent if for each $x \in V^*, T(s,x) \in F$ if and only if $T(t,x) \in F$.

Example 1.1. Consider the automaton with $Q = \{a,b,c,d,e\}, V = 0,1, F = \{d,e\}$, and T is given by the arcs of diagram of Fig. (1.7).

{a,b} is not equivalent, since $T(a,0) \in F$ but $T(b,0) \notin F$.

{d,e}is not equivalent, since $T(d,0) \in F$ but $T(e,0) \notin F$

Sets of equivalent states: {a,c},{b},{d},{e} □



(a) $M_0 = (Q_0, V, T_0, \emptyset, S_0, F_0)$

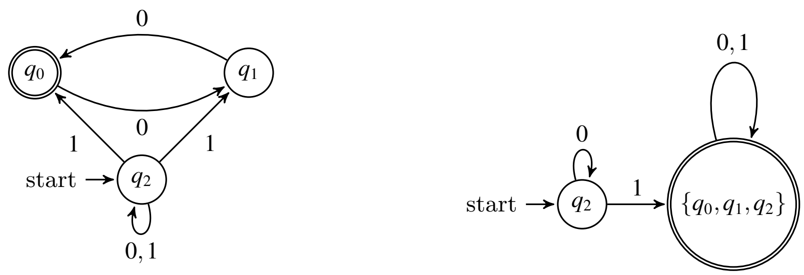
(b) $M_0^R = (Q_0, V, T_0, \emptyset, S_0, F_0)^R = (Q_0, V, T^R, \emptyset, F_0, S_0)$

(c) $useful_s \circ subsetopt \circ R(M_0)$

(d) $M_1 = R \circ useful_s \circ subsetopt \circ R(M_0)$

图 1.3: $M_1 = R \circ useful_s \circ subsetopt \circ R(M_0)$

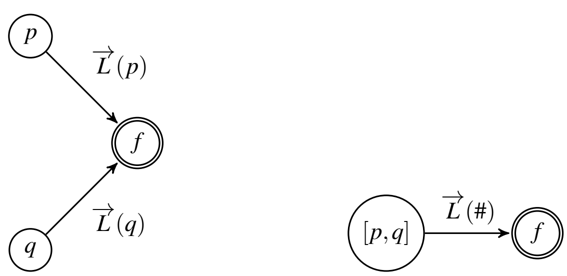
$$
\begin{aligned}
\text{start: } & U=\{q_2\} \\
u=q_2:\; & T(q_2,0)=\{q_2\}, T(q_2,1)=\{q_0,q_1,q_2\} \\
& \text{add new start to } D,\ D=\{q_2,\{q_0,q_1,q_2\}\} \\
u=\{q_0,q_1,q_2\}:\; & T(\{q_0,q_1,q_2\},0)=T(q_0,0)\cup T(q_1,0)\cup T(q_2,0)=\{q_1\}\cup\{q_0\}\cup\{q_2\}=\{q_0,q_1,q_2\} \\
& T(\{q_0,q_1,q_2\},1)=T(q_0,1)\cup T(q_1,1)\cup T(q_2,1)=\emptyset\cup\emptyset\cup\{q_0,q_1,q_2\}=\{q_0,q_1,q_2\}
\end{aligned}
$$



(a) M (b) $useful_s \circ subsetopt(M)$

图 1.4: $useful_s \circ subsetopt(M)$

Equivalence relation $E \subseteq Q \times Q$
$(p,q) \in E \equiv (\overrightarrow{L}(p) = \overrightarrow{L}(q))$



(a) $(p,q) \in E$ (b) $(p,q) \in E$

图 1.5: Equivalence relation $E \subseteq Q \times Q$

$\overrightarrow{L}(p) = \bigcup_{a \in V}(\{a\} \cdot \overrightarrow{L}(T(p,a)) \cup \{\varepsilon | p \in F\}$

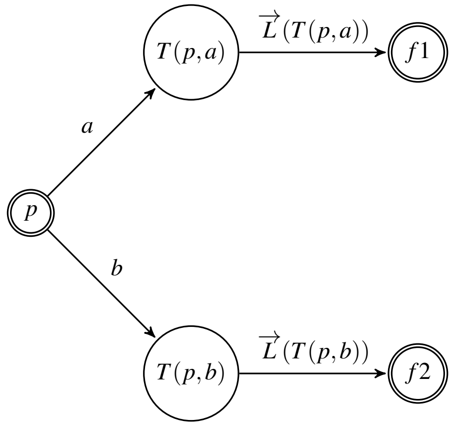


图 1.6: $L(p)$

{a,b},{d,e}is not equivalent states.

Sets of equivalent states: {a,c},{b},{d},{e}

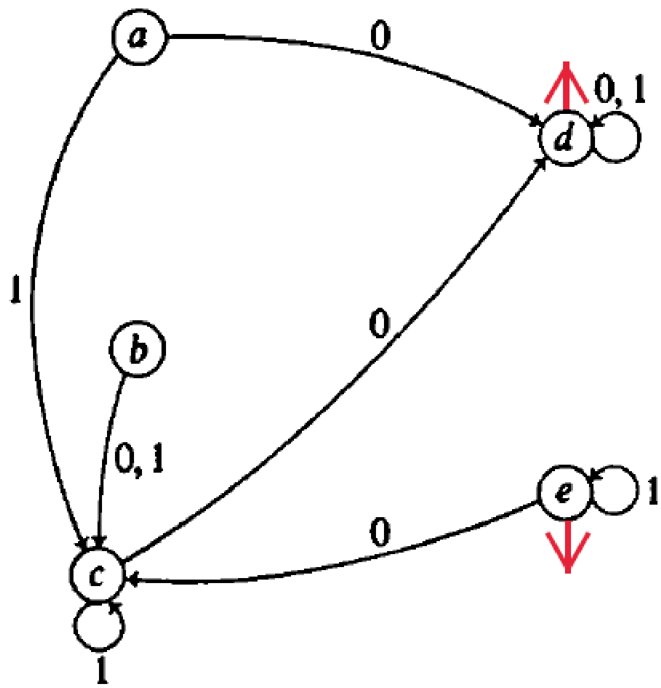


图 1.7: Finite state automaton