## the signatures of the transition relations:

$$T \in \mathbb{P}(Q \times V \times Q)$$

$$T \in V \to P(Q \times Q)$$

$$T \in Q \times Q \rightarrow P(V)$$

$$T \in Q \times V \rightarrow P(Q)$$

$$T \in Q \to P(V \times Q)$$

for example, the function  $T \in Q \to P(V \times Q)$  is defined as  $T(p) = \{(a,q): (p,a,q) \in T\}$ 

## $\varepsilon$ -transition relation:

$$E \in P(Q \times Q)$$

$$E \in Q \rightarrow P(Q)$$

$$T \in P(Q \times V \times Q), T = \{(s, a, q)\}$$

$$T(s) \in Q \to P(V \times Q), T(s) = \{(a,q) : (s,a,q) \in T\}$$

$$Q_{map}: P(Q \times V), Q_{map} = \{(q, a): (s, a, q) \in T\}$$

$$Q_{map}(q) = \{a : (s, a, q) \in T\}$$

$$Q_{map}^{-1}: V \rightarrow P(Q), Q_{map}^{-1} = \{(a,q): (s,a,q) \in T\}$$

According to Convention A.4 (Tuple projection):

$$\bar{\pi}_2(T) = \{(s,q) : (s,a,q) \in T\}$$

$$Q_{map} = (\bar{\pi}_1(T))^R, Q_{map} = \{(a,q) : (s,a,q) \in T\}^R = \{(q,a) : (s,a,q) \in T\}$$

$$f(a) = (f(a^R))^R$$

**Prefix-closure:** Let  $L \subseteq V^*$ , then

$$\overline{L} := \{ s \in V^* : (\exists t \in V^*) [st \in L] \}$$

In words, the prefix closure of L is the language denoted by  $\overline{L}$  and consisting of all the prefixes in L. In general,  $L \subseteq \overline{L}$ .

L is said to be prefix-closed if  $L = \overline{L}$ . Thus language L is prefix-closed if any prefix of any string in L is also an element of L.

$$L_1 = \{\varepsilon, a, aa\}, L_1 = \overline{L_1}, L_1 \text{ is prefix-closed.}$$
  
 $L_2 = \{a, b, ab\}, \overline{L_2} = \{\varepsilon, a, b, ab\}, L_2 \subset \overline{L_2}, L_2 \text{ is not prefix closed.}$ 

**Post-language:** Let  $L \subseteq V^*$  and  $s \in L$ . Then the post-language of L after s, denoted by L/s, is the language

$$L/s := \{t \in V^* : st \in L\}$$

By definition,  $L/s = \emptyset$  if  $s \notin \overline{L}$ .

**Definition A.15 (Left derivatives):** Given language  $A \subseteq V^*$  and  $w \in V^*$  we define the left derivative of A with respect to w as:

$$w^{-1}A = \{x \in V^* : wx \in A\}$$

Sometimes derivatives are written as  $D_w A$  or as  $\frac{dA}{dw}$ . Right derivatives are analogously defined. Derivatives can also be extended to  $B^{-1}A$  where B is also a language.

**Kleene-closure:** Let  $L \subseteq V^*$ , then

$$L^* := \{ \varepsilon \} \cup L \cup LL \cup LLL \cup \cdots$$

This is the same operation that we defined above for the set V, except that now it is applied to set L whose elements may be strings of length greater than one. An element of  $L^*$  is formed by the concatenation of a finite (but possibly arbitrarily large) number of elements of L; this includes the concatenation of "zero" elements, that is the empty string  $\varepsilon$ . Note that \* operation is idempotent:  $(L^*)^* = L^*$ .

Theorem 1 content...

 $\notin \beta$ 

Theorem 2

Definition 1