P P ¶ P $\mathscr{P}$ P P P $\mathbf{P}$

中文示例

Theorem 1 *content...*

$\notin \beta$

Theorem 2

Definition 1

the signatures of the transition relations:

$T \in \mathbb{P}(Q \times V \times Q)$

$T \in V \to P(Q \times Q)$

$T \in Q \times Q \to P(V)$

$T \in Q \times V \to P(Q)$

$T \in Q \to P(V \times Q)$

for example, the function $T \in Q \to P(V \times Q)$ is defined as $T(p) = \{(a,q) : (p,a,q) \in T\}$

ε-transition relation:

$E \in P(Q \times Q)$

$E \in Q \to P(Q)$

$T \in P(Q \times V \times Q), T = \{(s,a,q)\}$

$T(s) \in Q \to P(V \times Q), T(s) = \{(a,q) : (s,a,q) \in T\}$

$Q_{map} : P(Q \times V), Q_{map} = \{(q,a) : (s,a,q) \in T\}$

$Q_{map}(q) = \{a : (s,a,q) \in T\}$

${Q_{map}}^{-1} : V \nrightarrow P(Q), {Q_{map}}^{-1} = \{(a,q) : (s,a,q) \in T\}$

According to Convention A.4 (Tuple projection):

$\bar{\pi}_2(T) = \{(s,q) : (s,a,q) \in T\}$

$Q_{map} = (\bar{\pi}_1(T))^R, Q_{map} = \{(a,q) : (s,a,q) \in T\}^R = \{(q,a) : (s,a,q) \in T\}$

$f(a) = (f(a^R))^R$

Prefix-closure: Let $L \subseteq V^*$,then

$$\overline{L} := \{s \in V^* : (\exists t \in V^*)[st \in L]\}$$

In words, the prefix closure of L is the language denoted by $\overline{L}$ and consisting of all the prefixes in L. In general, $L \subseteq \overline{L}$.

L is said to be prefix-closed if $L = \overline{L}$. Thus language L is prefix-closed if any prefix of any string in L is also an element of L.

$L_1 = \{\varepsilon, a, aa\}, L_1 = \overline{L_1}, L_1$ is prefix-closed.

$L_2 = \{a, b, ab\}, \overline{L_2} = \{\varepsilon, a, b, ab\}, L_2 \subset \overline{L_2}, L_2$ is not prefix closed.

Post-language: Let $L \subseteq V^*$ and $s \in L$. Then the post-language of L after s, denoted by L/s, is the language

$$L/s := \{t \in V^* : st \in L\}$$

By definition, $L/s = \emptyset$ if $s \notin \overline{L}$.

Definition 2 (Left derivatives) *: Given language $A \subseteq V^*$ and $w \in V^*$ we define the left derivative of A with respect to w as:*

$$w^{-1}A = \{x \in V^* : wx \in A\}$$

A 关于 w 的左导数，就是 A 中：$\{w$ 的后缀组成的字符串集合$\}$。

Sometimes derivatives are written as $D_w A$ or as $\frac{dA}{dw}$. Right derivatives are analogously defined. Derivatives can also be extended to $B^{-1}A$ where B is also a language.

Example 1 $A = \{a, aab, baa\}, a^{-1}A = D_a A = \frac{dA}{da} = \{\varepsilon, ab, \emptyset\} = \{\varepsilon, ab\}$

Example 2 $L = \{ba, baa, baab, ca\}, w = \{ba\},$

$w^{-1}L = \{\varepsilon, a, ab, \emptyset\} = \{\varepsilon, a, ab\}$

$(wa)^{-1}L = (baa)^{-1}L = \{\emptyset, \varepsilon, b, \emptyset\} = \{\varepsilon, b\}$

$a^{-1}(w^{-1}L) = a^{-1}\{\varepsilon, a, ab\} = \{\emptyset, \varepsilon, b\} = \{\varepsilon, b\}$

$w \in L \equiv \varepsilon \in w^{-1}L, and (wa)^{-1}L = a^{-1}(w^{-1}L)$

Example 3 $a^{-1}\{a\} = \{\varepsilon\}; \quad a^{-1}\{b\} = \emptyset, \quad \Leftarrow if(a \neq b)$

Example 4 $L_0 = \{ab\}, L_1 = \{ac\}, L_0L_1 = \{abac\}$

$a^{-1}(L_0L_1) = \{bac\}$

$a^{-1}(L_0L_1) = (a^{-1}L_0)L_1 \cup \emptyset \quad \Leftarrow (\varepsilon \notin L_0)$

$= \{b\}L_1 = \{bac\}$

Example 5 $L_0 = \{\varepsilon, ab\}, L_1 = \{ac\}, L_0L_1 = \{ac, abac\}$

$a^{-1}(L_0L_1) = \{c, bac\}$

$a^{-1}(L_0L_1) = (a^{-1}L_0)L_1 \cup a^{-1}L_1 \quad \Leftarrow (\varepsilon \in L_0)$

$= \{\emptyset, b\}L_1 \cup \{c\} = \{c, bac\}$

Proof 1 $a^{-1}(L_0L_1)$

$1.if(\varepsilon \in L_0) \Rightarrow a^{-1}(L_0L_1) = (a^{-1}L_0)L_1 \cup a^{-1}L_1$

$L_0 = (L_0 \backslash \{\varepsilon\}) \cup \{\varepsilon\}$

$a^{-1}(L_0L_1) = a^{-1}(((L_0 \backslash \{\varepsilon\}) \cup \{\varepsilon\})L_1)$

$= a^{-1}(L_0L_1 \cup L_1)$

$a^{-1}L_0 = a^{-1}((L_0 \backslash \{\varepsilon\}) \cup \{\varepsilon\})$

$= a^{-1}(L_0 \ \backslash \{\varepsilon\}) \cup a^{-1}\{\varepsilon\}$

$= a^{-1}L_0 \cup \emptyset = a^{-1}L_0$

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(1) 如果 L 是一个语言，a 是一个符号，则 L/a(称作 L 和 a 的商) 是所有满足如下条件的串 w 的集合：wa 属于 L。例如，如果 $L=\{a,aab,baa\}$，则 $L/a=\{\varepsilon,ba\}$, 证明：如果 L 是正则的，那么 L/a 也是。提示：从 L 的 DFA 出发，考虑接受状态的集合。

(2) 如果 L 是一个语言，a 是一个符号，则 $a\backslash L$ 是所有满足如下条件的串 w 的集合: aw 属于 L。例如，如果 $L=\{a,aab,baa\}$, 则 $a\backslash L=\{\varepsilon,ab\}$, 证明：如果 L 是正则的，那么 $a\backslash L$ 也是。提示：记得正则语言在反转运算下是封闭的，又由 (1) 知，正则语言的商运算下是封闭的。

Definition 3 (Kleene-closure) *: Let $L \subseteq V^*$, then*

$$L^* := \{\varepsilon\} \cup L \cup LL \cup LLL \cup \cdots$$

This is the same operation that we defined above for the set V, except that now it is applied to set L whose elements may be strings of length greater than one. An element of L^* is formed by the concatenation of a finite (but possibly arbitrarily large) number of elements of L; this includes the concatenation of "zero" elements, that is the empty string ε. Note that $*$ operation is idempotent: $(L^*)^* = L^*$.

$$\begin{aligned} L^* &= (L\backslash\{\varepsilon\})L^* \cup \{\varepsilon\} \\ &= \{\varepsilon\} \cup L \cup LL \cup LLL \cup \dots \end{aligned}$$