Homework2

Wang Taiwu, NO. 18S151560

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1.29. 使用泵引理证明下述语言不是正则的.

1.29. Using the pump principle to prove that the language is not regular.

(b). $A_2 = \{www | w \in \{a, b\}*\}$

Proof:

Using the proof of contradiction. Assuming A_2 is regular. p is the length given by the pump principle. Let s be a string $a^p b a^p b a^p b$. Because s is a member of the A_2, and the lenght of s is greater than p, so the pump lemma to ensure that s can be divided into 3 sections, s = xyz. Considering the third condition of the pump lemma, which is said | xy | < p, so y only containes a. Because $A_2 = \{www | w \in \{a, b\}\}$ xyyz is not a member of A_2. There was a contradiction here. That is to say, the language A_2 is not regular.

1.36. Let B_n={a^k | k is an integer multiple of n}. Proof: For each n $\geq$ 1, the language B_n is regular.

Proof:

For any $n \geq 1$, if B_n can be identified by a DFA, then B_n is regular.

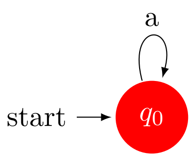

Figure 1: DFA when n is 1.

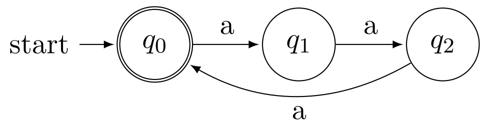

Figure 3: DFA when n is 3.

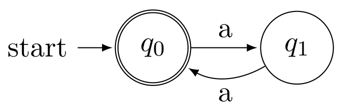

Figure 2: DFA when n is 2.

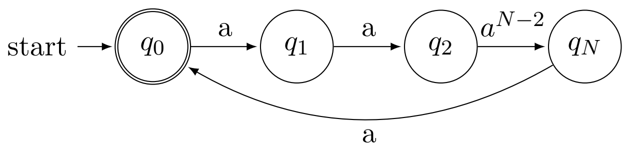

Figure 4: GNFA when n is N+1.

From the figures above, we can see that for each n, we can find a DFA recognize the language B_n. So, the language B_n is regular.

1.49. Prove the following proposition:

a. Let B={$1^k | y \in$ {0, 1}* and y contains at least k 1s, for k $\geq$1}.

b. Let C={ $1^k y | y \in${0, 1}* and y contains at most k 1s, for k $\geq$1}.

Proof:

a.

For any $k \geq 1$, we can construct DFA as belows:

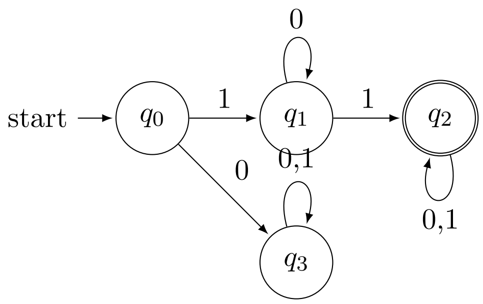

Figure 5: DFA when k is 1.

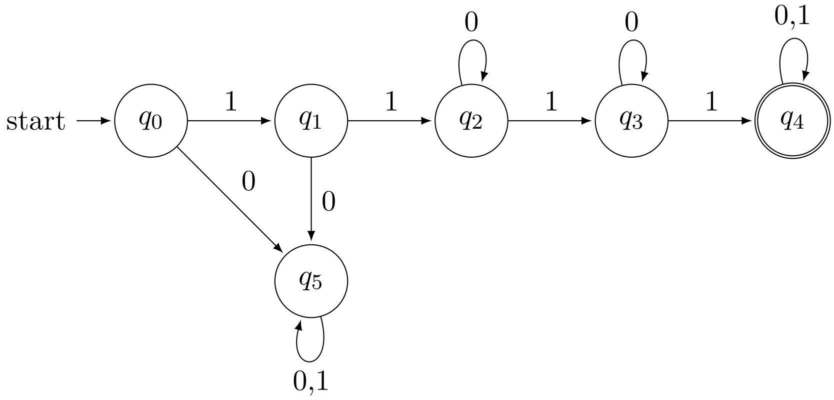

Figure 6: DFA when k is 2.

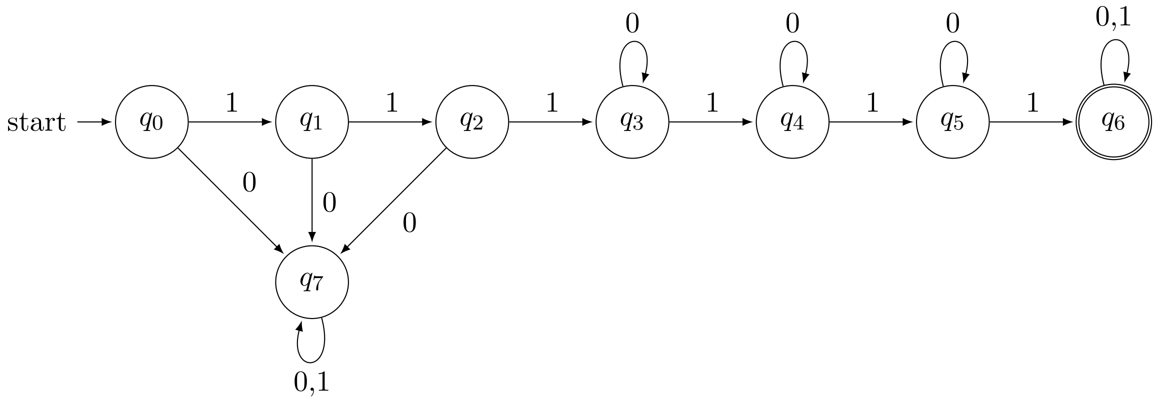

Figure 7: DFA when k is 3.

As shown in the figures above, for any k≥1, we can construct DFA, that there are k states like state q_2 and k states like state q_3 in figure 7.

b.

Using proof of contradiction. Assuming C is regular, p is the length given by the pump principle. Let s be a string 1^p01^p. Because s is a member of C, and the length of s is grater than p, so the pump lemma to ensure that s can be divided into 3 parts, s=xyz. Considering the third condition of the pump lemma, whilch is said | xy | < p. So y only contains 1. Assume the number of 1 which y ontains is m. Then xyyz=$1^{p-m}1^m1^m01^p = 1^{p+m}01^p$. Because xyyz is not a member of C. There was a contradiction here. So the assumption is not true, C is not regular.

2.10. Given an context free grammar that produces the language $A = \{a^ib^jc^k|i,j,k \geq$

$0\ and\ i = j\ or\ j = k\}$. **Is it ambiguous? why?**

Answer:

We can give a CFG that can generate A as the flowing form:

$$G = \{\{S, A, B, X, Y\}, \{a, b, c, d\}, R, S\}$$

The rules set R is:

$$S \to XA|BY \tag{1}$$

$$X \to Xa|a \tag{2}$$

$$A \to bAc|\varepsilon \tag{3}$$

$$B \to aBb|\varepsilon \tag{4}$$

$$Y \to Yc|c \tag{5}$$

The context-free grammar given by me is ambiguous. Because the string abc have two different leftmost derivations.

$$a.\ \ S \Rightarrow XA \Rightarrow aA \Rightarrow abAc \Rightarrow abc \tag{6}$$

$$b.\ \ S \Rightarrow BY \Rightarrow Bc \Rightarrow aBbc \Rightarrow abc \tag{7}$$

So, the context-free grammar G is ambiguous.