An Optimal Deadlock Prevention Policy for Flexible Manufacturing Systems Modeled with Petri Nets Using Structural Analysis

Wei Duan, Chunfu Zhong, Xiang Wang, Ateekh Ur Rehman, Usama Umer, Naiqi Wu, Fellow, IEEE,

Abstract

This paper derives an iterative deadlock prevention policy for systems of simple sequential processes with resources (S^3PRs) based on structural analysis, which consists of two stages. The first stage is called siphons control. Strict minimal siphons (SMSs) in an S^3PR net are computed and control places are added by imposing P-invariants associated with the complementary sets of the SMSs, which restricts no legal system behavior. The original resource places are removed and the newly added control places are regarded as resource places, resulting in a new net which needs to add control places for its SMSs if deadlocks persist. Repeat this step until a new net without SMSs is obtained. Then an S^4PR, called the first-controlled net, is obtained by integrating all added control places into the original net. The second stage, called non-max-marked siphons control, is performed in an iterative way if the system is still not live. At each iteration, solving an mixed integer linear programming (MILP) problem is utilized to compute a non-max-marked siphon, and a control place is added for the siphon to the first-controlled net, resulting in an augmented net. The iteration is executed until a final-augmented net generates no new non-max-marked siphon. Based on above two stages, this paper can in general obtain a supervisor with more behavior permissiveness compared with the previous studies. Moreover, an optimal supervisor can be found if a first-controlled net has no non-max-marked siphon, implying that the second stage is not necessary. Finally, some examples are provided to demonstrate the proposed policy.

Flexible manufacturing system, Petri net, Deadlock prevention policy, Siphon control

I. Introduction

A flexible manufacturing system (FMS) can run in parallel to process multiple different products at the same time. However, it may lead to deadlocks, blocking the entire system or part of it, due to the inappropriate allocation of resources in the system. It is significant to develop an effective control policy to solve the problem of deadlocks [1-6],

*This work was supported in part by the National Natural Science Foundation of China under Grants 61603285, 61472295, 61873342 and 51305325, and the Science Technology Development Fund, MSAR, under Grant 122/2017/A3. (*Corresponding author*: *Chunfu Zhong*).

W. Duan, C. Zhong, and X. Wang are with the School of Electro-Mechanical Engineering, Xidian University, Xi'an 710071, China (e-mail: dwei1024@126.com; cfuzhong@gmail.com; e-mail: 1056750400@qq.com).

A. Rehman is with the Department of Industrial Engineering, College of Engineering, PO Box 800, King Saud University, Riyadh 11421, Saudi Arabia (e-mail: arehman@ksu.edu.sa).

U. Umer is with the Advance Manufacturing Institute, College of Engineering, PO Box 800, King Saud University, Riyadh 11421, Saudi Arabia (e-mail: uumer@ksu.edu.sa).

N. Wu is with the Institute of Systems Engineering, Macau University of Science and Technology, Macau SAR China (e-mail: nqwu@must.edu.mo).

[8], [14-17], [20-21], [29-30], [41-47], [50-51] in an FMS. There are mainly three approaches: deadlock detection and recovery, deadlock avoidance [30], [36], [45], and deadlock prevention [1-4], [33-34], [37-41], [46].

Petri nets can well describe concurrency, distributivity, similarity, uncertainty and randomness of a system. Thus they are often used in FMSs for modeling and analyzing them [7], [9-12], [16-19], [22-25], [28], [31-32], [43-44], [45-50], [53], [55]. There are two methods to address the deadlocks problem in an FMS by Petri nets. The first is based on the theory of regions by exploring the reachablity graph of Petri nets [10], [29], [35], [42]. The other is to design a supervisor by structural analysis, such as siphon control that eliminates deadlocks through the control of siphons [2], [7-9], [12], [13].

Since deadlock prevention is a static strategy, an off-line computing mechanism is usually used to control the allocation of resources such that an FMS does not reach deadlock states. This paper proposes a deadlock prevention policy based on siphon control for FMSs, which adds control places with related arcs to the Petri net model. It can build a control mechanism in advance by structural analysis.

In a Petri net, once a siphon loses its tokens, it remains unmarked under any subsequent marking. Therefore, the related transitions cannot fire, and the net loses liveness. In [2], Ezpeleta et al. present a method of adding control places for S^3PR nets by studying the relationship between deadlocks and siphons. However, it considerably restricts the behaviors of the nets in general. A feasible liveness-enforcing Petri net supervisor is mainly assessed from three aspects: (1) behavior permissiveness, (2) computational complexity, and (3) structural complexity. Therefore, more behavior permissiveness, lower computational and structural complexity contribute to an elegant supervisor.

However, it is in general difficult to design a supervisor satisfying the above criteria. The number of SMSs in a Petri net has an exponential relationship with the structural size of the net. As a net structure grows, the number of control places that need to be added increases quickly. To reduce structural complexity, Li and Zhou propose the concept of elementary siphons in a Petri net [39]. They prove that under certain conditions, a Petri net supervisor can be obtained by simply controlling the elementary siphons, and thereby the structural complexity of a supervisor can be greatly reduced. A drawback of the idea that the supervisor derived from elementary siphons is not maximally permissive.

An iterative siphon control can obtain more behavior permissiveness compared with other methods. Huang et al. propose an iterative deadlock prevention policy based on mixed integer linear programming (MILP) problems for S^3PR nets [40]. To enhance the modeling ability and convenience of S^3PR nets, some general subclasses of Petri nets, such as S^4PR nets, are presented for modeling and analysis of manufacturing systems. In an S^3PR, deadlocks control can be implemented by ensuring that SMSs are always marked. In an S^4PR, a deadlock can occur even if SMSs are marked at any reachable marking. Barkaoui et al. study the notion of max-controlled siphons by satisfying invariant control in an S^4PR [4]. Zhong et al. propose an iterative approach to analyze the liveness of an S^4PR by utilizing this concept and obtain decent results [1].

A maximally permissive, also called optimal, supervisor can bring in high utilization of resources in a system.

Motivated by the work in [1], [40], this paper proposes a deadlock prevention policy for S^{3}PR nets in an iterative way to obtain an optimal or more permissive supervisor. The policy includes two stages. In order not to generate new SMSs in the iterative siphon control, most policies adopt a method where the output arcs of control places are bounded to the source transitions, which restricts behavior permissiveness. We apply a novel approach called siphons control in the first stage. Specifically, given an S^{3}PR, SMSs are computed and control places are added for them in the net. Then the original resource places in the original net model as well as their corresponding arcs are removed and the added control places are regarded as new resource places, which results in a new net. Then we compute the SMSs in this new net. Continue the above processes until no SMSs can be found. An S^{4}PR, called first-controlled net, is obtained by integrating all the control places added at each iteration into the original net. This stage does not need to bound the output arcs of control places to the source transitions, thus preventing behavior permissiveness from being restricted. However, those added control places may produce new SMSs associating with resource places in the original net. As a result, the first-controlled net may not be live. In the second stage, solving an MILP problem is utilized to check whether the net has a non-max-marked siphon or not. This stage is called non-max-marked siphons control, where it does not need to compute all SMSs such that computational overheads are reduced. If there is no non-max-marked siphon, then no new siphon is generated, and the net is proved to be live and maximally permissive. Otherwise, a control place is added for the siphon to the net, resulting in a new augmented net. Repeat the above step until there does not exist a non-max-marked siphon. Eventually, a final-augmented net with more behavior permissiveness is obtained.

The rest of this paper is organized as follows. Section II reviews two special classes of Petri nets: S^{3}PR nets and S^{4}PR nets. Siphons control is presented in Section III. The corresponding notions of max-controlled siphons are utilized to design control places in Section IV. Section V proposes a deadlock prevention policy. Section VI gives several examples to demonstrate the proposed approach. Finally, conclusions and further research topics are exposed in Section VII. An appendix presents the basic definitions and properties of Petri nets used throughout the paper.

II. Preliminaries

Some basic notions of Petri nets are shown in the Appendix. This section mainly reviews the definitions of S^{3}PR nets [2] and S^{4}PR nets [1]. In what follows, $\mathbb{N}^+$ denotes the set of positive integers and $\mathbb{N}^{|P|}$ denotes the set of $|P|-$dimensional vectors.

A. S^{3}PR

Definition 1: A simple sequential process (S^{2}P) is a Petri net $N = (P_S \cup \{p^0\}, T, F)$, where (1) $P_S \neq \emptyset$ is called the set of operation places; (2) $p^0 \notin P_S$ is called the idle place; (3) N is a strongly connected state machine; (4) every circuit of N contains place p^0.

Definition 2: A simple sequential process with resources (S^{2}PR) is a Petri net $N = (\{p^0\} \cup P_S \cup P_R, T, F)$ such that:

1) The subnet generated by $X = P_S \cup \{p^0\} \cup T$ is an S^{2}P.

2) $P_R \neq \emptyset$ and $(P_S \cup \{p^0\}) \cap P_R = \emptyset$, where $r \in P_R$ is called a resource place.

3) $\forall p \in P_S$, $\forall t \in {}^\bullet p$, $\forall t' \in p^\bullet$, $\exists r_p \in P_R$, ${}^\bullet t \cap P_R = t'^\bullet \cap P_R = \{r_p\}$.

4) The following statements are verified: (a) $\forall r \in P_R$, ${}^{\bullet\bullet}r \cap P_S = r^{\bullet\bullet} \cap P_S \neq \emptyset$; (b) $\forall r \in P_R$, ${}^\bullet r \cap r^\bullet = \emptyset$.

5) ${}^{\bullet\bullet}(p^0) \cap P_R = (p^0)^{\bullet\bullet} \cap P_R = \emptyset$.

Note that ${}^\bullet r$ represents the set of input transitions of place r, ${}^{\bullet\bullet}r = \bigcup_{t \in {}^\bullet r} {}^\bullet t$ is the set of all input places of all input transitions of place r. Similarly, $r^{\bullet\bullet} = \bigcup_{t \in r^\bullet} t^\bullet$ represents the set of all output places of all output transitions of place r.

Definition 3: Let $N = (\{p^0\} \cup P_S \cup P_R, T, F)$ be an S^{2}PR. An initial marking M_0 is called an acceptable initial marking for N if (1) $M_0(p^0) \geq 1$; (2) $M_0(p) = 0$, $\forall p \in P_S$; (3) $M_0(r) \geq 1$, $\forall r \in P_R$. An S^{2}PR with such a marking is said to be acceptably marked.

Definition 4: A system of S^{2}PR, called S^{3}PR for short, is defined recursively as follows:

1) An S^{2}PR is an S^{3}PR.

2) Let $N_i = (P_{S_i} \cup \{p_i^0\} \cup P_{R_i}, T_i, F_i)$, $i \in \{1, 2\}$, be two S^{3}PR nets such that $(P_{S_1} \cup \{p_1^0\}) \cap (P_{S_2} \cup \{p_2^0\}) = \emptyset$, $P_{R_1} \cap P_{R_2} = P_C \neq \emptyset$, and $T_1 \cap T_2 = \emptyset$. Then, the net $N = (P_S \cap P^0 \cap P_R, T, F)$ resulting from the composition of N_1 and N_2 via P_C (denoted as $N = N_1 \bigcirc N_2$) defined as follows: (a) $P_S = P_{S_1} \cup P_{S_2}$; (b) $P^0 = \{p_1^0\} \cup \{p_2^0\}$; (c) $P_R = P_{R_1} \cup P_{R_2}$; (d) $T = T_1 \cup T_2$; and (e) $F = F_1 \cup F_2$, is also an S^{3}PR.

Let $I_m = \{1, 2, \ldots, m\}$ be a set of indices. An S^{3}PR composed of m S^{2}PR, denoted by $N = \bigcirc_{i \in I_m} N_i$, is defined as follows: $N = N_1$ if $m = 1$; $N = (\bigcirc_{i=1}^{m-1} N_i) \bigcirc N_m$ if $m > 1$. Transitions in $(P^0)^\bullet$ (${}^\bullet(P^0)$) are called source (sink) transitions that represent the entry (exit) of raw materials when a manufacturing system is modeled with an S^{3}PR.

Definition 5: Let N be an S^{3}PR. (N, M_0) is called an acceptably marked S^{3}PR if one of the two following statements is true:

1) (N, M_0) is an acceptably marked S^{2}PR.

2) $N = N_1 \bigcirc N_2$, where (N_i, M_{0_i}), $i = 1, 2$, is an acceptably marked S^{3}PR and

(a) $\forall i \in \{1, 2\}$, $\forall p \in P_{S_i} \cup \{p_i^0\}$, $M_0(p) = M_{0_i}(p)$.

(b) $\forall i \in \{1, 2\}$, $\forall r \in P_{R_i} \setminus P_C$, $M_0(r) = M_{0_i}(r)$.

(c) $\forall r \in P_C$, $M_0(r) = max\{M_{0_1}(r), M_{0_2}(r)\}$.

Let S be an SMS in an S^{3}PR $N = (P_S \cup P^0 \cup P_R, T, F)$. S can be represented by $S^S \cup S^R$, where $S^R = S \cap P_R$ and $S^S = S \cap P_S$, as shown in [2].

Definition 6: [?] Let (N, M_0) be a marked S^{3}PR. For $r \in P_R$, $H(r)$=${}^{\bullet\bullet}r \cap P_S$, the operation places that use r, is called the set of holders of r. Let $[S] = (\bigcup_{r \in S^R} H(r)) \backslash S$. $[S]$ is called the complementary set of siphon S.

Operation places in a siphon S compete for resources with operation places in $[S]$. When all tokens in resource places of S flow into operation places in $[S]$, S will be emptied, which results in dead transitions. Hence we need to construct a control place to ensure that the siphon S can be marked at any reachable marking of an S^{3}PR net.

B. S^{4}PR

Definition 7: A generalized connected self-loop-free net $N = \bigcirc_{i\in I_m} N_i = (P, T, F, W)$ is said to be an S^{4}PR if:

1) $N_i = (P_{S_i} \cup \{p_i^0\} \cup P_{R_i}, T_i, F_i, W_i)$, $i \in I_m$, $p_i^0 \notin P_{S_i} \cup P_{R_i}$, $P_{S_i} \cap P_{R_i} = \emptyset$.
2) $P = P_S \cup P^0 \cup P_R$ is a partition such that (1) $P_S = \bigcup_{i\in I_m} P_{S_i}$ is called the set of operation places, where for all $i \neq j$, $P_{S_i} \neq \emptyset$ and $P_{S_i} \cap P_{S_j} = \emptyset$; (2) $P^0 = \bigcup_{i\in I_m} \{p_i^0\}$ is called the set of idle places; (3) $P_R = \bigcup_{i\in I_m} P_{R_i}$ is called the set of resource places.
3) $T = \bigcup_{i\in I_m} T_i$ is called the set of transitions, where for all $i \neq j$, $T_i \neq \emptyset$ and $T_i \cap T_j = \emptyset$.
4) for all $i \in I_m$, the subnet $\bar{N}_i$ generated by $P_{S_i} \cup \{p_i^0\} \cup T_i$ is a strongly connected state machine such that every circuit of the state machine contains idle place p_i^0.
5) for all $r \in P_R$, there exists a unique minimal P-semiflow $I_r \in \mathbb{N}^{|P|}$ such that $\{r\} = ||I_r|| \cap P_R$, $P^0 \cap ||I_r|| = \emptyset$, $P_S \cap ||I_r|| \neq \emptyset$, and $I_r(r) = 1$.
6) $P_S = \bigcup_{r\in P_R}(||I_r|| \backslash \{r\})$.

Definition 8: An initial marking M_0 is acceptable for an S^{4}PR $N = (P_S \cup P^0 \cup P_R, T, F, W)$ if (1) $\forall i \in I_m, M_0(p_i^0) > 0$; (2) $\forall p \in P_S$, $M_0(p) = 0$; and (3) $\forall r \in P_R$, $M_0(r) \geq max_{p\in||I_r||} I_r(p)$.

Let S be an SMS in an S^{4}PR $N = (P_S \cup P^0 \cup P_R, T, F, W)$. Then $S = S^R \cup S^S$ satisfies $S \cap P_R = S^R \neq \emptyset$ and $S \cap P_S = S^S \neq \emptyset$, as shown in [1].

Definition 9: [?] Let r be a resource place, S be an SMS and I_r be a P-semiflow associated with r in an S^{4}PR. The set of holders of resource r, denoted as $H(r)$, is defined as the difference of two multisets I_r and r, i.e., $H(r) = I_r - r$. As a multiset, $Th(S) = \Sigma_{r\in S^R} H(r) - \Sigma_{r\in S^R, p\in S^S} I_r(p)p$ is called the complementary set of siphon S. $Th_S(p)$ denotes an element p in $Th(S)$.

III. Siphons control

In an S^{3}PR, the presence of unmarked siphons leads to dead transitions and makes the net not live. It is necessary to ensure that siphons are marked at any reachable markings through some external control mechanisms. It can be achieved by adding control places such that a control place and the complementary set of a siphon constitute a P-invariant. This section proposes an iterative way to control unmarked siphons. At the beginning, SMSs are computed in a marked S^{3}PR net (N, M_0) and control places are added for them. After that, resource places with their related arcs are removed and the control places with their related arcs are reserved to obtain a new net. Continue to compute SMSs by regarding the newly added control places as resource places until no new SMSs are produced. Then, a first-controlled net is obtained by integrating all added control places with their related arcs at each iteration into the net (N, M_0).

In order to obtain an optimal supervisor, we need to ensure that no legal behavior in (N, M_0) can be restricted. Hence, a method of adding control places based on the complementary sets of SMSs is presented as shown below. In what follows, we refer an S^{3}PR (S^{4}PR) with an acceptable initial marking to as a marked S^{3}PR (S^{4}PR).

Proposition 1: [?] Let S be an SMS in a marked S^{3}PR (N, M_0) with its complementary set $[S]$. A control place V_S is added such that $\sum_{p\in[S]} p + V_S$ is a P-semiflow of the resulting net (N^α, M_0^α), where $\forall p \in P_S \cup P^0 \cup P_R$, $M_0^\alpha(p) = M_0(p)$, and $M_0^\alpha(V_S) = M_0(S) - \xi_S$ ($\xi_S \in \mathbb{N}^+$). S is controlled if $1 \le \xi_S \le M_0(S) - 1$, where ξ_S is called the control depth variable for an SMS S, representing the strength of controlling S.

Theorem 1: [?] Let S be an SMS in a marked S^{3}PR (N, M_0), and a control place V_S is designed for it by Proposition 1. S is optimally controlled if $\xi_S = 1$.

An SMS can be optimally controlled by designing a control place according to Proposition 1 and Theorem 1. However, if each SMS in (N, M_0) is optimally controlled, it may produce new SMSs due to the added control places. Besides, adding those control places increases structural complexity of the resulting net. It is more difficult to compute the new SMSs. To mitigate this problem, an iterative way is used. For a marked S^{3}PR net, at each iteration, original resource places with their related arcs are removed and the added control places with their related arcs are reserved, resulting in a new net, which is a relatively efficient and accurate approach to find the SMSs derived from the added control places.

In what follows, a marked S^{3}PR (N, M_0) is represented by $N = (P_S \cup P^0 \cup P_R, T, F)$. We can also define F in another way such as $F = F_{P_1} \cup F_{P_2} \cup F_{R_1} \cup F_{R_2}$, where $F_{P_1} = \{(t,p) | t \in {}^\bullet(P^0 \cup P_S), p \in (P^0 \cup P_S)\}$, $F_{P_2} = \{(p,t) | t \in (P^0 \cup P_S)^\bullet, p \in (P^0 \cup P_S)\}$, $F_{R_1} = \{(t,p) | t \in {}^\bullet P_R, p \in P_R\}$, and $F_{R_2} = \{(p,t) | t \in P_R{}^\bullet, p \in P_R\}$.

Definition 10: Let (N, M_0) be an S^{3}PR with $N = (P_S \cup P^0 \cup P_R, T, F)$. The net $(N_{V_i}, M_{0V_i}) = (P_S \cup P^0 \cup \Phi_i, T, F_{P_1} \cup F_{P_2} \cup F_{V_i})$ is said to be the i-order controlled net of (N, M_0) if it satisfies the following statements for $i \in I_m$:

1) $\Phi_i = \{V_S | S \in \Pi_{i-1}\}$ is a set of control places, where Π_{i-1} is a set of SMSs in $(N_{V_{i-1}}, M_{0V_{i-1}})$.
2) $F_{V_i} = F_{V'} \cup F_{V''}$, where $F_{V'} = \{(V_S, t) | t \in {}^\bullet[S]\}$, and $F_{V''} = \{(t, V_S) | t \in [S]^\bullet\}$.
3) (a) $\forall p \in P_S \cup P^0$, $M_{0V_i}(p) = M_0(p)$; (b) $\forall V_S \in \Phi_i$, $M_{0V_1}(V_S) = M_0(S) - 1$ if $i = 1$ and $M_{0V_i}(V_S) = M_{0V_{i-1}}(S) - 1$ if $i > 1$.

Let (N, M_0) be the 0-order controlled net such that $(N_{V_0}, M_{0V_0}) = (N, M_0)$. We utilize the m-order controlled net (N_{V_m}, M_{0V_m}) to represent the net after the final iteration, in which no SMS exists. Note that $\Phi_1 = \{V_S | S \in \Pi_0\}$ if $i = 1$, where Π_0 is a set of SMSs in (N_{V_0}, M_{0V_0}). According to Proposition 1, we directly add a control place by the complementary set of an SMS, in stead of letting output arcs of the control place bound to the source transitions of an S^{3}PR. Let $\xi_S = 1$ by Theorem 1. We can ensure that every siphon S in N_{V_i} is optimally controlled. Then, control places are added in an iterative way until no SMS is generated. A first-controlled net (N_V, M_{0V}) is obtained by synthesizing all added control places and their corresponding arcs into the original net (N_{V_0}, M_{0V_0}). $(N_V, M_{0V}) = (P, T, F_V)$, where $P = P^0 \cup P_S \cup P_R \cup \Phi$, $\Phi = \bigcup_{i=1}^m \Phi_i$, and $F_V = (\bigcup_{i=1}^m F_{V_i}) \cup F$.

There exist three SMSs in Fig. 1: $S_{01} = \{p_3, p_8, p_{11}, p_{12}, p_{13}\}$, $S_{02} = \{p_3, p_6, p_{10}, p_{13}, p_{14}\}$, and $S_{03} = \{p_3, p_6, p_{11}, p_{12}, p_{13}, p_{14}\}$. We have $\Pi_0 = \{S_{01}, S_{02}, S_{03}\}$, $[S_{01}] = \{p_2, p_{10}\}$, $[S_{02}] = \{p_8, p_9\}$, and $[S_{03}] = \{p_2, p_8, p_9, p_{10}\}$.

According to Proposition 1 and Theorem 1, control places are added for S_{01}, S_{02}, and S_{03} as shown in Table 1. A 1-order controlled net $(N_{V_1}, M_{0V_1}) = (P^0 \cup P_S \cup \Phi_1, T, F_{P_1} \cup F_{P_2} \cup F_{V_1})$ is obtained, where $\Phi_1 = \{V_{01}, V_{02}, V_{03}\}$. Since

Fig. 1: A marked S^{3}PR (N_{V_0}, M_{0V_0}).

TABLE I: Control places for an S^{3}PR (N_{V_0}, M_{0V_0}) shown in Fig. 1

V_S	preset	postset	$M_{0V}(V_{1i})$
V_{01}	t_2, t_{11}	t_1, t_{10}	3
V_{02}	t_{10}	t_8	3
V_{03}	t_2, t_{11}	t_1, t_8	5

there is no new SMS, a first-controlled net (N_V, M_{0V}) is obtained, where $(N_V, M_{0V}) = (P^0 \cup P_S \cup P_R \cup \Phi_1, T, F \cup F_{V_1})$. It is verified that the net is live and maximally permissive. For some nets, the redundancy problem of control places may arise in the process of computation. The two following properties in the i-order controlled net are found, which are useful to reduce unnecessary computation.

Property 1: Let S_1, S_2, and S_3 be three SMSs in (N_{V_i}, M_{0V_i}), where $i \geq 0$. If $[S_3] = [S_1] \cup [S_2]$, $M_{0V_i}(S_3) - 1 = M_{0V_i}(S_1) - 1 + M_{0V_i}(S_2) - 1$, and S_1 and S_2 are controlled by Proposition 1 and Theorem 1, then S_3 is always marked at any reachable marking in (N_{V_i}, M_{0V_i}).

If S_1 and S_2 are controlled by Proposition 1 and Theorem 1, then we have $M_{0V_i}(V_{S_1}) = M_{0V_i}(S_1) - 1$ and $M_{0V_i}(V_{S_2}) = M_{0V_i}(S_2) - 1$. Since $[S_3] = [S_1] \cup [S_2]$ and $M_{0V_i}(S_3) - 1 = M_{0V_i}(S_1) - 1 + M_{0V_i}(S_2) - 1$, we have $M([S_3]) = M([S_1]) + M([S_2])$. Note that $[S_1] \cup \{V_{S_1}\}$ is the support of a P-semiflow in (N_{V_i}, M_{0V_i}), $\forall M \in R(N_{V_i}, M_{0V_i})$, we have $M(V_{S_1}) + M([S_1]) = M_{0V_i}(V_{S_1})$. By $M(V_{S_1}) \geq 0$, we have $M([S_1]) \leq M_{0V_i}(V_{S_1})$. Similarly, $M([S_2]) \leq M_{0V_i}(V_{S_2})$ holds. Therefore, $M([S_3]) \leq M_{0V_i}(V_{S_1}) + M_{0V_i}(V_{S_2}) = M_{0V_i}(S_1) - 1 + M_{0V_i}(S_2) - 1 = M_{0V_i}(S_3) - 1$. We conclude that S_3 is always marked at any reachable marking M.

Definition 11: Let P_R be the set of resource places in (N_{V_0}, M_{0V_0}) and S be an SMS in an i-order controlled net (N_{V_i}, M_{0V_i}). S_β is called the storer of S in (N_{V_0}, M_{0V_0}) if $S_\beta = S^S \cup P_{SR}$, where $P_{SR} = \bigcup_{p \in S^S} p^{\bullet\bullet} \cap P_R$.

Property 2: Let S be an SMS in a net system (N_{V_i}, M_{0V_i}) and S_β be the storer of S in (N_{V_0}, M_{0V_0}), where $i \geq 1$. If $M_{0V_i}(S) \geq M_{0V_0}(S_\beta) + 1$, then for all $M \in R(N_V, M_{0V})$, $M(S) > 0$.

A first-controlled net (N_V, M_{0V}) is synthesized by all added control places with their corresponding arcs to (N, M_0). In other words, we have S and S_β in (N_V, M_{0V}), where $M_{0V}(S) = M_{0V_i}(S)$ and $M_{0V}(S_\beta) = M_{0V_0}(S_\beta)$. Since $S = S^S \cup S^R$ and $S_\beta = S^S \cup P_{SR}$ by Definition 11, for all $M \in R(N_V, M_{0V})$, we have

$$M(S^S) + M(S^R) = M_{0V}(S), \tag{1}$$

and

$$M(S^S) + M(P_{SR}) = M_{0V}(S_\beta). \tag{2}$$

Thus $M(S^R) - M(P_{SR}) = M_{0V}(S) - M_{0V}(S_\beta)$ is obtained by (3) minus (4). If $M_{0V_i}(S) \geq M_{0V_0}(S_\beta) + 1$, it means $M_{0V}(S) \geq M_{0V}(S_\beta) + 1$, then $M(S^R) - M(P_{SR}) \geq 1$. Since $M(P_{SR}) \geq 0$, $M(S^R) \geq 1 > 0$, $M(S) > 0$.

To a certain extent, the computation process can be simplified according to Properties 1 or 2. Let $\Pi_{F_i} = \Pi_i - \Pi_{C_i}$, where Π_{C_i} is a set of siphons in Π_i that do not need to be explicitly controlled. At each iteration, control places are only added for the siphons in Π_{F_i}.

Fig. 2: A marked S^{3}PR (N_{V_0}, M_{0V_0}).

For a marked S^{3}PR (N_{V_0}, M_{0V_0}) as shown in Fig. 2, $\Pi_0 = \{S_{01}, S_{02}, S_{03}, S_{04}\}$, where S_{01}=$\{p_5, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}\}$, S_{02}=$\{p_5,\ p_{10}, p_{13}, p_{14}, p_{15}, p_{16}\}$, $S_{03} = \{p_5, p_9, p_{14}, p_{16}\}$, and $S_{04} = \{p_3, p_{11}, p_{12}, p_{13}\}$. Their complementary sets are $[S_{01}] = \{p_2, p_3, p_4,\ p_6, p_8, p_9, p_{10}\}$, $[S_{02}] = \{p_3, p_4, p_6, p_8, p_9\}$, $[S_{03}] = \{p_6, p_8\}$, and $[S_{04}] = \{p_2, p_{10}\}$, respectively. Since $[S_{01}] = [S_{02}] \cup [S_{04}]$, and $M_0(S^R_{01}) - 1 = M_0(S^R_{02}) - 1 + M_0(S^R_{04}) - 1 = 5$, S_{01} is implicitly controlled by Property 1. Then we have $\Pi_{F_0} = \{S_{02}, S_{03}, S_{04}\}$.

After adding three control places for each siphon in Π_{F_0} as shown in Table 2, we have $\Phi_1 = \{V_{02}, V_{03}, V_{04}\}$. The 1-order controlled net (N_{V_1}, M_{0V_1}) is obtained in Fig. 3. Let us continue to compute SMSs. There exists one siphon $S_{11} = \{p_3, p_4, p_6, p_{10}, V_{02}, V_{03}\}$ in Π_1, and a storer $S_\beta = \{p_3, p_4, p_6, p_{10}, p_{13}, p_{14}, p_{15}\}$ is found in (N_{V_0}, M_{V_0}), where $S^S = \{p_3, p_4, p_6, p_{10}\}$ and $P_{SR} = \{p_{13}, p_{14}, p_{15}\}$. Since $M_{0V_1}(S_{11}) = 5 > M_{0V_0}(S_\beta) + 1 = 4$, S_{11} can be marked at any reachable marking after a first-controlled net is synthesized according to Property 2. We can conclude that no SMS is generated in this iteration since S_{11} does not need to be explicitly controlled.

TABLE II: Adding a control place for each SMS

control places	$M_{0V}(V_{0i}), i = 2, 3, 4$	preset	postset
V_{02}	2	t_2, t_{11}	t_1, t_{10}
V_{03}	3	t_4, t_7, t_{10}	t_2, t_8
V_{04}	1	t_7, t_9	t_6, t_8

Fig. 3: The 1-order controlled net (N_{V_1}, M_{0V_1}).

Therefore, the first stage is finished. V_{02}, V_{03}, and V_{04} with their corresponding arcs are integrated into the net (N_{V_0}, M_{V_0}) and a first-controlled net (N_V, M_{0V}) is obtained. By using the approach of controlling unmarked siphons from Proposition 1 and Theorem 1 in an iterative way, the net is still not live. A way to figure it out is explained in the next section.

IV. Non-max-marked siphons control

For some S^{3}PR nets such as the one shown in Fig. 1, optimal Petri net supervisors can be obtained by only integrating all control places in the first stage. While others like the net in Fig. 2, they are not live after controlling all SMSs. Nevertheless, all of them are converted to S^{4}PR nets after the first stage, since there exists a place in P_S possesses two or more resources in P_R and Φ at the same time. In S^{3}PR nets, deadlocks can be prevented by making all SMSs marked. Comparing with S^{3}PR nets, deadlocks can occur even if all SMSs are marked in S^{4}PR nets. Hence, this section introduces the concept of non-max-controlled siphons, and we need to detect whether there exist non-max-marked siphons in the net by solving MILP problems. In what follows, Definitions 12-14 and Theorem 3 are from [?]. In the sequel, for a given place p, we denote $max_{t\in p^\bullet}\{W(p,t)\}$ by $max_{p^\bullet}$. Since (N_V, M_{0V}) is a marked S^{4}PR after the siphons control stage, (N_V, M_{0V}) can be updated as $N_V = (P, T, F_V, W_V)$, where $W_V(f) = 1$ if for all f in F_V.

Definition 12: Let (N_V, M_{0V}) be a marked S^{4}PR net and S be a siphon of N_V. S is said to be max-marked at a marking $M \in R(N_V, M_{0V})$ if there exists a place $p \in S$ such that $M(p) \geq max_{p^\bullet}$.

Definition 13: Let (N_V, M_{0V}) be a marked S^{4}PR net and S be a siphon of N_V. S is said to be max-controlled if S is max-marked at any reachable marking.

Definition 14: An S^{4}PR net (N_V, M_{0V}) is said to satisfy the max-cs property (controlled-siphon property) if each minimal siphon of N_V is max-controlled.

Theorem 2: Let (N_V, M_{0V}) be a marked S^{4}PR net. It is live if it satisfies max-cs property.

Lemma 1: [?] Let (N_V, M_{0V}) be a marked S^{4}R net and S be a siphon of N_V. S is max-controlled if there exists a P-invariant I such that $\forall p \in (||I||^- \cap S)$, $max_{p^\bullet} = 1$, $||I||^+ \subseteq S$, $\sum_{p\in P} I(p)M_{0V}(p) > \sum_{p\in S} I(p)(max_{p^\bullet} - 1)$.

Definition 15: Let (N_V, M_{0V}) be a marked S^{4}PR. $(N_{V'}, M_{0V'}) = (P \cup \{V_n\}, T, F_{V'}, W_{V'})$ is said to be the final-augmented net of (N_V, M_{0V}) if:

1) V_n is a control place for a non-max-marked siphon S.

2) $F_{V'} = F_V \cup F_{V_{n_1}} \cup F_{V_{n_2}}$, where $F_{V_{n_1}} = \{(V_n, t) | t \in {}^\bullet Th(S)\}$ and $F_{V_{n_2}} = \{(t, V_n) | t \in Th(S)^\bullet\}$.

3) $W_{V'} : F_{V'} \to \mathbb{N}^+$ is a mapping that assigns a weight to any arc in $F_{V'}$.

4) $\forall p \in P \cup \{V_n\}$, $M_{0V'}(p) = M_{0V}(p)$, and $M_{0V'}(V_n) = M_{0V}(S) - \xi_{S_n}$ ($\xi_{S_n} \in \mathbb{N}^+$).

Proposition 2: Let S be an SMS in a marked S^{4}PR net (N_V, M_{0V}). A control place V_n is added to (N_V, M_{0V}) by imposing that $g_S = Th(S) + V_n$ is a P-invariant of the final-augmented net $(N_{V'}, M_{0V'})$. Let $h_S = \sum_{r\in S^R} I_r - g_S$ and $M_{0V'}(V_n) = M_{0V}(S) - \xi_{S_n}$. S is max-controlled if $\xi_{S_n} > \sum_{p\in S} h_S(p)(max_{p^\bullet} - 1)$.

Since g_S and $\sum_{r\in S^R} I_r$ are P-invariants of $N_{V'}$, $h_S = \sum_{p\in S} h_S(p)p - \sum_{p\in Th(S)} Th_S(p)p - V_n$ is also a P-invariants of $N_{V'}$. $Th(S) \subseteq P_S$, $\forall p \in Th(S)$, $M_{0V'}(p) = M_{0V}(p) = 0$.

$\sum_{p\in(P\cup\{V_n\})} h_S(p)M_{0V'}(p) = \sum_{p\in S} h_S(p)M_{0V'}(p) - \sum_{p\in Th(S)} Th_S(p)M_{0V'}(p) - M_{0V'}(V_S)$

$\geq M_{0V}(S) - \sum_{p\in Th(S)} Th_S(p)M_{0V}(p) - M_{0V'}(V_S)$

$= M_{0V}(S) - (M_{0V}(S) - \xi_{S_n})$

$= \xi_{S_n} > \sum_{p\in S} h_S(p)(max_{p^\bullet} - 1)$.

Otherwise, $h_S = \sum_{r\in S^R} I_r - g_S$ so that $||h_S||^- \cap S = \emptyset$ and $||h_S||^+ = S$. Therefore, S is max-controlled from Lemma 1.

As for ξ_{S_n}, it has the same function as ξ_S in the siphons control. For more permissive behavior in a final-augmented net, ξ_{S_n} is expected to be minimal under the constraint condition in Proposition 2. When $p \in P_S$, $max_{p^\bullet} - 1 = 0$.

Therefore, $\xi_{S_n} > \sum_{p\in S} h_S(p)\ (max_{p^\bullet}-1) = \sum_{p\in S^R}(max_{p^\bullet}-1)$. Let $\xi_{S_n} = \sum_{p\in S^R}(max_{p^\bullet}-1)+1$ be a minimum value to ensure that S is max-controlled.

The control policy in this stage is called non-max-marked siphons control. For a first-controlled net (N_V, M_{0V}), a non-max-marked siphon is computed by solving an MILP problem. Then, a control place is added by Proposition 2 to the net, which makes the siphon max-controlled. Repeat the above two steps until no non-max-marked siphon can be found in a final-augmented net $(N_{V'}, M_{0V'})$. As a result, each siphon is max-controlled, which means that the net is live because it satisfies the max cs-property. It is worth noting that a supervisor can be obtained without computing all siphons by this stage.

The method of determining whether there is a non-max-marked siphon in (N_V, M_{0V}) by solving an MILP problem is shown below [?]:

$$\begin{aligned} \textbf{min} \quad & z = 1^T s \\ \textbf{s.t} \quad & K_1 Pre^T s \geq Post^T s \\ & X^T M = k \\ & K_2 s + M - L \leq K_2 1 \\ & 1^T s \geq 2 \end{aligned} \tag{3}$$

where $s \in \{0,1\}^m$, $M \in R(N_V, M_{0V})$, $Pre: P\times T \to \mathbb{N}$, $Post: P\times T \to \mathbb{N}$, and $k = X^T M_{0V}$. Here the three constants K_1, K_2 and L are defined as $K_1 = max\{1^T Post(\bullet, t) | t \in T\}$, $K_2 = max\{M(p) | p \in P, M \in R(N_V, M_{0V})\}$ and $L(i) = max_{p_i^\bullet} - 1 (i \in I_m, p_i \in P)$.

The first constraint ensures that s is the characteristic vector of a siphon S. Let X be a matrix where each column is a P-semiflow of (N_V, M_{0V}), and the set of invariant markings is denoted by $I_X(N_V, M_{0V}) = \{M \in \mathbb{N}^{|P|} | X^T M = X^T M_0\}$. The second equation ensures that M belongs to the set $I_X(N_V, M_{0V})$. The third guarantees that for all $p_i \in P$, $K_2 s(i) + M(p_i) - L(i) \leq K_2$ holds. The last one ensures that there are at least two places in a siphon. The objective function ensures that only non-max-marked siphons are computed.

Theorem 3: [?] Let (N_V, M_{0V}) be a marked Petri net. A siphon S in (N_V, M_{0V}) is a non-max-marked siphon if its characteristic vector s satisfies (5).

For the net (N_{V_0}, M_{0V_0}) as shown in Fig. 2, (N_V, M_{0V}) is obtained as shown in Fig. ?? after the siphons control stage. We decide whether there exists a non-max-marked siphon in (N_V, M_{0V}) by solving an MILP problem (5).

Fig. 4: The first-controlled net (N_V, M_{0V}).

Let $s^T = [x_1, x_2, ..., x_{19}]$ and $M^T = [y_1, y_2, ..., y_{19}]$, where $x_i = \{0,1\}$ and $y_i \geq 0$ for $i = 1, 2, ..., 19$. Then, $M_{0V}^T = [10, 0, 0, 0, 0, 0, 10, 0, 0, 0, 0, 2, 1, 1, 1, 1, 2, 3]$. $k = X^T M_{0V} = [10, 2, 1, 1, 1, 1, 2, 3, 10, 1]^T$, $K_1 = 4$, $K_2 = 10$ and $L^T = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$, respectively. They lead to the following MILP problem:

min $x_1 + x_2 + \cdots + x_{18} + x_{19}$

subject to

$4x_1 + 4x_{12} + 4x_{17} \geq x_2$

$4x_2 + 4x_{13} + 4x_{18} \geq x_3 + x_{12} + x_{17}$

$4x_3 + 4x_{15} \geq x_4 + x_{13}$

$4x_4 + 4x_{16} \geq x_5 + x_{15} + x_{18}$

$4x_5 \geq x_1 + x_{17}$

$4x_3 + 4x_{14} + 4x_{19} \geq x_6 + x_{13}$

$4x_6 + 4x_{16} \geq x_5 + x_{14} + x_{18} + x_{19}$

$4x_7 + 4x_{16} + 4x_{18} + 4x_{19} \geq x_8$

$4x_8 + 4x_{14} \geq x_9 + x_{16} + x_{19}$

$4x_9 + 4x_{13} + 4x_{17} \geq x_{10} + x_{14} + x_{18}$

$4x_{10} + 4x_{12} \geq x_{11} + x_{13} + x_{17}$

$4x_{11} \geq x_7 + x_{12}$

$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 = 10$

$y_7 + y_8 + y_9 + y_{10} + y_{11} = 10$

$y_2 + y_{11} + y_{12} = 2$

$y_3 + y_{10} + y_{13} = 1$

$y_3 + y_4 + y_6 + y_8 + y_9 + y_{18} = 3$

$y_2 + y_{10} + y_{17} = 2$

$y_6 + y_9 + y_{14} = 1$

$y_4 + y_{15} = 1$

$y_5 + y_8 + y_{16} = 1$

$y_6 + y_8 + y_{19} = 1$

$10x_i + y_i \leq 10 \ (i = 1, 2, \ldots 19)$

$x_1 + x_2 + \cdots + x_{18} + x_{19} \geq 2$

It gives $min(x_1 + x_2 + ... + x_{19}) = 6$, where $x_5, x_{10}, x_{14}, x_{16}, x_{17}$ and x_{18} are all equal to 1. The corresponding siphon is $S = \{p_5, p_{10}, p_{14}, p_{16}, V_{02}, V_{03}\}$. Then, S is max-controlled after adding V_n by Proposition 2. Since $Th(S) = p_2 + p_3 + p_4 + 2p_6 + 2p_8 + 2p_9$, it is easy to find $g_S = Th(S) + V_n = p_2 + p_3 + p_4 + 2p_6 + 2p_8 + 2p_9 + V_n$, and $\xi_{S_n} = \sum_{p \in S^R}(max_{p^\bullet} - 1) + 1 = 1$. $M_{0V'}(V_n) = M_{0V}(S) - \xi_{S_n} = 3 + 2 + 1 + 1 - 1 = 6$. A final-augmented net $(N_{V'}, M_{0V'})$ is obtained after adding this control place V_n to (N_V, M_{0V}) due to the fact that no non-max-marked siphon exists in $(N_{V'}, M_{0V'})$. As a result, $(N_{V'}, M_{0V'})$ with 218 states is maximally permissive.

V. Deadlock Prevention Policy

This section develops a deadlock prevention policy by synthesizing the two above stage for S^3PR nets. It is synthesized as follows:

Algorithm 1: A liveness-enforcing supervisor for an S^3PR

Input: A marked S^3PR $(N_{V_0}, M_{0V_0}) = (P^0 \cup P_S \cup P_R, T, F)$.

Output: A final-augmented net $(N_{V'}, M_{0V'}) = (P \cup \Phi, T, F_{V'}, W_{V'})$.

```
/*******Stage One: Siphons Control*******/;
m := 0;
Compute Π_m in (N_{V_0}, M_{0V_0}) and the set of siphons that do not need to be explicitly controlled Π_{C_m} by
    Property 1;
Π_{F_m} = Π_m − Π_{C_m};
while (Π_{F_m} ≠ ∅) do
    Add a control place V_S for each SMS in Π_{F_m} by Proposition 1 and Theorem 1;
    m++;
    Φ_m = {V_S | S ∈ Π_{F_{m−1}}};
    Let (N_{V_m}, M_{0V_m}) = (P^0 ∪ P_S ∪ Φ_m, T, F_{V_m} ∪ F_{P_1} ∪ F_{P_2});
    Compute Π_m in (N_{V_m}, M_{0V_m}) and Π_{C_m} by Properties 1 or 2;
    Π_{F_m} = Π_m − Π_{C_m};
end
Φ = ⋃_{i=1}^{m} Φ_i, P = P^0 ∪ P_S ∪ P_R ∪ Φ, F_V = (⋃_{i=1}^{m} F_{V_i}) ∪ F, ∀f ∈ F_V, W_V(f) = 1;
Let (N_V, M_{0V}) = (P, T, F_V, W_V);
/*******Stage Two: Non-max-marked Siphons Control*******/;
Φ = ∅;
Compute a non-max-marked siphon S in (N_V, M_{0V}) by solving an MILP problem (5);
while there exists such a siphon S do
    Add a control place V_n by Proposition 2;
    Φ := Φ ∪ {V_n};
    Let (N_{V'}, M_{0V'}) = (P ∪ Φ, T, F_{V'}, W_{V'});
    Compute a non-max-marked siphon S in (N_{V'}, M_{0V'});
end
Output (N_{V'}, M_{0V'}).
```

Algorithm 1 can synthesize a liveness-enforcing supervisor for an S^3PR model (N_{V_0}, M_{0V_0}) if some conditions are satisfied. The first stage is to compute the set of SMSs Π_0 in (N_{V_0}, M_{0V_0}) and select the set of uncontrollable SMSs

Π_{F_0} from Π_0 by Property 1, and then a control place V_S is added for each SMS in Π_{F_0}. Reserving newly added control places and removing all resource places produce a 1-order controlled net (N_{V_1}, M_{0V_1}). Continue to compute SMSs and add control places for them. Repeat the above steps until no SMSs are generated. A first-controlled net (N_V, M_{0V}) is obtained by synthesizing all added control places at each iteration into the original net (N_{V_0}, M_{0V_0}). Next, in the second stage, we check whether (N_V, M_{0V}) has a non-max-marked siphon or not. Note that if it has none, the net will be proved to be optimal. Otherwise, a non-max-marked siphon is computed by solving an MILP problem at each iteration. A final-augmented net $(N_{V'}, M_{0V'})$ is obtained until no non-max-marked siphon exists.

Theorem 4: $(N_{V'}, M_{0V'})$ is live by Algorithm 1.

In an $\mathrm{S^3PR}$, deadlocks stem from the existence of a least one unmarked siphon S ($[S]$ competes resources with S^S and finally holds all resource units). A approach that designs a control place by Proposition 1 and Theorem 1 can prevent S from being unmarked. During the siphons control stage, original resources places are removed and added control places are regarded as new resources places at each iteration. Therefore, the operation places in siphons of (N_{V_i}, M_{0V_i}) may be contained in the complementary sets of new SMSs of $(N_{V_{i+1}}, M_{0V_{i+1}})$. Let P_{t_i} $(i = 0, 1, \ldots, m)$ be the subset of operation places in (N_{V_i}, M_{0V_i}), satisfying

$P_{t_0} = ^{\bullet\bullet}(P^0) \cap P_S$ in (N_{V_0}, M_{0V_0}),

$P_{t_1} = ^{\bullet\bullet}(P_{t_0}) \cap P_S$ in (N_{V_1}, M_{0V_1}),

......

$P_{t_i} = ^{\bullet\bullet}(P_{t_{i-1}}) \cap P_S$ in (N_{V_i}, M_{0V_i}),

......

$P_{t_m} = ^{\bullet\bullet}(P_{t_{m-1}}) \cap P_S$ in (N_{V_0}, M_{0V_0}).

Suppose that $p \in P_{t_i}$ contains at least one token. Then $t \in P_{t_i}^{\bullet}$ will definitely fire. We conclude that the firing of $t \in P_{t_i}^{\bullet}$ due to $p \in P_{t_i}$ cannot lead to a deadlock and $p \in P_{t_i}$ is no longer the holder of resources in $(N_{V_{i+1}}, M_{0V_{i+1}})$, which means that the places in P_{t_i} do not compete for resources in $(N_{V_{i+1}}, M_{0V_{i+1}})$, and then they cannot be in the complementary sets of new SMSs. Thus, the number of SMSs can decrease after each iteration. The siphons control stage will be terminated. All added control places with their corresponding arcs are synthesized into (N_{V_0}, M_{0V_0}), resulting in a first-controlled net (N_V, M_{0V}). As for the second stage, suppose that there are finite dead states in (N_V, M_{0V}). Once we make a non-max-marked siphon controlled in terms of a control place designed by Proposition 2, some dead states can be removed. Since the number of dead states is limited, by repeating the above steps, if every siphon in $(N_{V'}, M_{0V'})$ is max-controlled, the net is live by Theorem 2.

Theorem 5: In Algorithm 1, (N_V, M_{0V}) is optimal if there does not exist no non-max-marked siphon.

According to the siphons control stage, we can ensure that each siphon S is optimally controlled through adding control places by Proposition 1 and Theorem 1. If no non-max-marked siphon exists, the non-max-marked siphons control stage is not necessary. As a result, (N_V, M_{0V}) is optimal.

VI. Examples

A. Example 1

The net system in Fig. ?? consists of 19 places and 14 transitions, where $P^0 = \{p_{101}, p_{108}\}$, $P_R = \{p_{114}, p_{115}, p_{116}, p_{117}, p_{118}, p_{119}\}$, and $P_S = \{p_{102}, p_{103}, p_{104}, p_{105}, p_{106}, p_{107}, p_{109}, p_{110}, p_{111}, p_{112}, p_{113}\}$. In the first iteration, SMSs are computed in the N_{V_0}, M_{0V_0}. There are five SMSs: $S_{01} = \{p_{107}, p_{113}, p_{114}, p_{115}, p_{116}, p_{117}, p_{118}, p_{119}\}$, $S_{02} = \{p_{105}, p_{113}, p_{114}, p_{115}, p_{118}\}$, $S_{03} = \{p_{102}, p_{107}, p_{113}, p_{115}, p_{116}, p_{117}, p_{118}, p_{119}\}$, $S_{04} = \{p_{102}, p_{107}, p_{111}, p_{113}, p_{116}, p_{117}, p_{118}, p_{119}\}$ and $S_{05} = \{p_{102}, p_{105}, p_{113}, p_{115}, p_{118}\}$. $\Pi_0 = \{S_{01}, S_{02}, S_{03}, S_{04}, S_{05}\}$ and the complementary sets of the SMSs in Π_0 are computed by Definition 6, i.e, $[S_{01}] = \{p_{102}, p_{103}, p_{104}, p_{105}, p_{106}, p_{109}, p_{110}, p_{111}, p_{112}\}$, $[S_{02}] = \{p_{102}, p_{103}, p_{104}, p_{111}, p_{112}\}$, $[S_{03}] = \{p_{103}, p_{105}, p_{106}, p_{109}, p_{110}, p_{111}, p_{112}\}$, $[S_{04}] = \{p_{105}, p_{106}, p_{109}, p_{110}\}$, and $[S_{05}] = \{p_{103}, p_{111}, p_{112}\}$. Control places are added for them according to Proposition 1 and Theorem 1 as shown in Table 3.

Fig. 5: A marked S^{3}PR (N_{V_0}, M_{0V_0}).

TABLE III: Control places are added in the first iteration

V_S	preset	postset	$M_{0V_1}(V_{0i}), (i = 1, 2, 3, 4, 5)$
V_{01}	t_7, t_{13}	t_1, t_9	5
V_{02}	t_4, t_5, t_{13}	t_1, t_{11}	2
V_{03}	t_7, t_{13}	t_2, t_4, t_9	4
V_{04}	t_7, t_{11}	t_4, t_5, t_9	3
V_{05}	t_5, t_{13}	t_1, t_{11}	1

Hence, we have $\Phi_1 = \{V_{01}, V_{02}, V_{03}, V_{04}, V_{05}\}$ and $(N_{V_1}, M_{0V_1}) = \{P^0 \cup P_S \cup \Phi_1, T, F_{V_1} \cup F_{P_1} \cup F_{P_2}\}$. Then $\Pi_1 = \{S_{11}, S_{12}, S_{13}\}$ is computed in (N_{V_1}, M_{0V_1}), where $S_{11} = \{p_{105}, p_{106}, p_{111}, p_{112}, p_{117}, p_{118}\}$, $S_{12} = \{p_{105}, p_{106}, p_{111}, p_{112}, p_{115}, p_{117}\}$, and $S_{13} = \{p_{103}, p_{105}, p_{106}, p_{111}, p_{112}, p_{115}, p_{116}\}$. The complementary sets of these SMSs $[S_{11}] = \{p_3, p_{10}\}$, $[S_{12}] = \{p_3, p_4, p_{10}\}$, and $[S_{13}] = \{p_4\}$ are computed by Definition 6. The corresponding control places are added by Proposition 1 and Theorem 1 as shown in Table 4.

TABLE IV: Control places are added in the second iteration

V_S	preset	postset	$M_{0V_2}(V_{1i}), i = 1, 2, 3$
V_{11}	t_5, t_{11}	t_2, t_{10}	3
V_{12}	t_4, t_5, t_{11}	t_2, t_3, t_{10}	4
V_{13}	t_4	t_3	4

Now $\Phi_2 = \{V_{11}, V_{12}, V_{13}\}$, and we obtain a 2-order controlled net $(N_{V_2}, M_{0V_2}) = (P^0 \cup P_S \cup \Phi_2, T, F_{V_2} \cup F_{P_1} \cup F_{P_2})$. As no SMSs can be computed in (N_{V_2}, M_{0V_2}), we integrate all control places with their related arcs to (N_{V_0}, M_{0V_0}). Thus, $(N_V, M_{0V}) = (P^0 \cup P_R \cup P_S \cup \Phi, T, F_V)$ is obtained, where $\Phi = \Phi_1 \cup \Phi_2$, $F_V = F_{V_1} \cup F_{V_2} \cup F$.

In the second stage, the net can be updated into $(N_V, M_{0V}) = (P, T, F_V, W_V)$, where $P = P^0 \cup P_R \cup P_S \cup \Phi$. A non-max-marked siphon S_1 is found by solving an MILP problem (5) in (N_V, M_{0V}), where $S_1 = \{p_{107}, p_{111}, p_{112}, p_{117}, p_{119}, p_{121}, p_{123}\}$. Its complementary set $Th(S_1) = p_{102} + p_{103} + p_{104} + p_{105} + p_{106} + 2p_{109} + 2p_{110}$. A control place is added for it by Proposition 2, and we have a P-invariant $g_{S_1} = p_{102} + p_{103} + p_{104} + p_{105} + p_{106} + 2p_{109} + 2p_{110} + V_1$, and $M_{0V'}(V_1) = M_{0V}(S_1) - \sum_{p \in S_1^R}(max_{p^\bullet} - 1) - 1 = 7 - 1 = 6$. An augmented net $(N_{V'}, M_{0V'}) = (P \cup \{V_1\}, T, F_{V'}, W_{V'})$ is obtained, then another non-max-marked siphon S_2 is found by solving an MILP problem (5) in the net, where $S_2 = \{p_{107}, p_{112}, p_{115}, p_{117}, p_{119}, p_{123}, p_{124}\}$. Its complementary set $Th(S_2) = 2p_{103} + p_{105} + p_{106} + 2p_{109} + 2p_{110} + p_{111} + p_{112}$. A control place is added for it by Proposition 2, we have $g_{S_2} = 2p_{103} + p_{105} + p_{106} + 2p_{109} + 2p_{110} + p_{111} + p_{112} + V_2$ and $M_{0V'}(V_2) = 6$. Then a final-augmented net $(N_{V'}, M_{0V'}) = (P \cup \Phi, T, F_{V'}, W_{V'})$ is obtained due to the fact that no non-max-marked siphon can be found in the net, where $\Phi = \{V_1, V_2\}$. It is live and maximally permissive with 205 states. According to Algorithm 1, an optimal supervisor is obtained by adding 10 control places.

B. Example 2

Fig. 6 shows a model of an FMS with three production routings. Places p_{120}, p_{121} and p_{122} represent three robots. Places p_{123}, p_{124}, p_{125}, and p_{126} represent four machines. This model belongs to S^{3}PR, where p_{101}, p_{105} and p_{114} are idle process places, p_{120} to p_{126} are resource places, and the others are operation places. The S^{3}PR has 26750 reachable states, 21581 of which are safe, while 5169 states should be forbidden.

Fig. 6: The Petri net model (N_{V_0}, M_{0V_0}) of a flexible manufacturing system.

First of all, every SMS in Π_0 and its complementary set are computed as shown in Tables 5 and 6. However, the complementary sets of SMSs, including $[S_{06}]$, $[S_{08}]$, $[S_{010}]$, $[S_{013}]$, $[S_{014}]$, $[S_{015}]$, $[S_{016}]$, and $[S_{018}]$, have the following relationships:

$[S_{06}] = [S_{02}] \cup [S_{04}],\ M_{0V_0}(S_{06}) - 1 = 4 = M_{0V_0}(S_{02}) - 1 + M_{0V_0}(S_{04}^R) - 1 = 3 - 1 + 3 - 1 = 4;$

$[S_{07}] = [S_{02}] \cup [S_{03}],\ M_{0V_0}(S_{07}) - 1 = 4 = M_{0V_0}(S_{02}) - 1 + M_{0V_0}(S_{03}) - 1 = 3 - 1 + 3 - 1 = 4;$

$[S_{08}] = [S_{03}] \cup [S_{04}],\ M_{0V_0}(S_{08}) - 1 = 4 = M_{0V_0}(S_{03}) - 1 + M_{0V_0}(S_{04}) - 1 = 3 - 1 + 3 - 1 = 4;$

$[S_{010}] = [S_{02}] \cup [S_{03}] \cup [S_{04}],\ M_{0V_0}(S_{010}) - 1 = 6 = M_{0V_0}(S_{02}) - 1 + M_{0V_0}(S_{03}) - 1 + M_{0V_0}(S_{04}) - 1 = 3 - 1 + 3 - 1 + 3 - 1 = 6;$

$[S_{011}] = [S_{04}] \cup [S_{05}],\ M_{0V_0}(S_{011}) - 1 = 5 = M_{0V_0}(S_{04}) - 1 + M_{0V_0}(S_{05}) - 1 = 3 - 1 + 4 = 5;$

$[S_{013}] = [S_{04}] \cup [S_{09}],\ M_{0V_0}(S_{013}) - 1 = 7 = M_{0V_0}(S_{04}) - 1 + M_{0V_0}(S_{09}) - 1 = 3 - 1 + 6 - 1 = 7;$

$[S_{014}] = [S_{02}] \cup [S_{012}],\ M_{0V_0}(S_{014}) - 1 = 7 = M_{0V_0}(S_{02}) - 1 + M_{0V_0}(S_{012}) - 1 = 3 - 1 + 6 - 1 = 7;$

$[S_{015}] = [S_{03}] \cup [S_{012}],\ M_{0V_0}(S_{015}) - 1 = 7 = M_{0V_0}(S_{03}) - 1 + M_{0V_0}(S_{012}) - 1 = 3 - 1 + 6 - 1 = 7;$

$[S_{016}] = [S_{02}] \cup [S_{03}] \cup [S_{012}],\ M_{0V_0}(S_{016}) - 1 = 9 = M_{0V_0}(S_{02}) - 1 + M_{0V_0}(S_{03}) - 1 + M_{0V_0}(S_{012}) - 1 = 3 - 1 + 3 - 1 + 6 - 1 = 9;$

$[S_{017}] = [S_{05}] \cup [S_{012}],\ M_{0V_0}(S_{017}) - 1 = 8 = M_{0V_0}(S_{05}) - 1 + M_{0V_0}(S_{012}) - 1 = 6 - 1 + 6 - 1 = 8;$

$[S_{018}] = [S_{09}] \cup [S_{012}],\ M_{0V_0}(S_{018}) - 1 = 10 = M_{0V_0}(S_{09}) - 1 + M_{0V_0}(S_{012}) - 1 = 6 - 1 + 6 - 1 = 10.$

TABLE V: SMSs and their complementary set of (N_{V_0}, M_{0V_0}) as shown in Fig. 6

	SMSs in the model (N_{V_0}, M_{0V_0})
S_{01}	$p_{110}, p_{118}, p_{122}, p_{126}$
S_{02}	$p_{104}, p_{109}, p_{112}, p_{117}, p_{121}, p_{124}$
S_{03}	$p_{102}, p_{104}, p_{108}, p_{113}, p_{117}, p_{121}, p_{126}$
S_{04}	$p_{102}, p_{104}, p_{108}, p_{112}, p_{116}, p_{121}, p_{125}$
S_{05}	$p_{102}, p_{104}, p_{108}, p_{110}, p_{117}, p_{121}, p_{122}, p_{126}$
S_{06}	$p_{104}, p_{109}, p_{112}, p_{116}, p_{121}, p_{124}, p_{125}$
S_{07}	$p_{104}, p_{109}, p_{113}, p_{117}, p_{121}, p_{124}, p_{126}$
S_{08}	$p_{102}, p_{104}, p_{108}, p_{113}, p_{116}, p_{121}, p_{125}, p_{126}$
S_{09}	$p_{104}, p_{110}, p_{117}, p_{121}, p_{122}, p_{124}, p_{126}$
S_{010}	$p_{104}, p_{109}, p_{113}, p_{116}, p_{121}, p_{124}, p_{125}, p_{126}$
S_{011}	$p_{102}, p_{104}, p_{108}, p_{110}, p_{116}, p_{121}, p_{122}, p_{125}, p_{126}$
S_{012}	$p_{102}, p_{104}, p_{108}, p_{112}, p_{115}, p_{120}, p_{121}, p_{123}, p_{125}$
S_{013}	$p_{104}, p_{110}, p_{116}, p_{121}, p_{122}, p_{124}, p_{125}, p_{126}$
S_{014}	$p_{104}, p_{109}, p_{112}, p_{115}, p_{120}, p_{121}, p_{123}, p_{124}, p_{125}$
S_{015}	$p_{102}, p_{104}, p_{108}, p_{113}, p_{115}, p_{120}, p_{121}, p_{123}, p_{125}, p_{126}$
S_{016}	$p_{104}, p_{109}, p_{113}, p_{115}, p_{120}, p_{121}, p_{123}, p_{124}, p_{125}, p_{126}$
S_{017}	$p_{102}, p_{104}, p_{108}, p_{110}, p_{115}, p_{120}, p_{121}, p_{122}, p_{123}, p_{125}, p_{126}$
S_{018}	$p_{104}, p_{110}, p_{115}, p_{120}, p_{121}, p_{122}, p_{123}, p_{124}, p_{125}, p_{126}$

TABLE VI: SMSs and their complementary set of (N_{V_0}, M_{0V_0}) as shown in Fig. 6

	The complementary sets of the strict minimal siphons
$[S_{01}]$	p_{113}, p_{119}
$[S_{02}]$	$p_{102}, p_{103}, p_{108}$
$[S_{03}]$	p_{112}, p_{118}
$[S_{04}]$	p_{111}, p_{117}
$[S_{05}]$	$p_{112}, p_{113}, p_{118}, p_{119}$
$[S_{06}]$	$p_{102}, p_{103}, p_{108}, p_{111}, p_{117}$
$[S_{07}]$	$p_{102}, p_{103}, p_{108}, p_{112}, p_{118}$
$[S_{08}]$	$p_{111}, p_{112}, p_{117}, p_{118}$
$[S_{09}]$	$p_{102}, p_{103}, p_{108}, p_{109}, p_{112}, p_{113}, p_{118}, p_{119}$
$[S_{010}]$	$p_{102}, p_{103}, p_{108}, p_{111}, p_{112}, p_{117}, p_{118}$
$[S_{011}]$	$p_{111}, p_{112}, p_{113}, p_{117}, p_{118}, p_{119}$
$[S_{012}]$	$p_{106}, p_{107}, p_{111}, p_{116}, p_{117}$
$[S_{013}]$	$p_{102}, p_{103}, p_{108}, p_{109}, p_{111}, p_{112}, p_{113}, p_{117}, p_{118}, p_{119}$
$[S_{014}]$	$p_{102}, p_{103}, p_{106}, p_{107}, p_{108}, p_{111}, p_{112}, p_{116}, p_{117}, p_{118}$
$[S_{015}]$	$p_{106}, p_{107}, p_{111}, p_{112}, p_{116}, p_{117}, p_{118}$
$[S_{016}]$	$p_{102}, p_{103}, p_{106}, p_{107}, p_{108}, p_{111}, p_{112}, p_{116}, p_{117}, p_{118}$
$[S_{017}]$	$p_{106}, p_{107}, p_{111}, p_{112}, p_{113}, p_{116}, p_{117}, p_{118}, p_{119}$
$[S_{018}]$	$p_{102}, p_{103}, p_{106}, p_{107}, p_{108}, p_{109}, p_{111}, p_{112}, p_{113}, p_{116}, p_{117}, p_{118}, p_{119}$

According to Property 1, siphons $S_{06}, S_{08}, S_{010}, S_{013}, S_{014}, S_{015}, S_{016}$ are implicitly controlled. Hence, $\Pi_{F_0} = \{S_{01}, S_{02}, S_{03}, S_{04}, S_{05}, S_{09}, S_{012}\}$. Control places are added for these SMSs in Π_{F_0} by Proposition 1 and Theorem 1, as shown in Table 7. A 1-order controlled net $(N_{V_1}, M_{0V_1}) = (P^0 \cup P_S \cup \Phi_1, T, F_{V_1} \cup F_{P_1} \cup F_{P_2})$ is obtained, where $\Phi_1 = \{V_{01}, V_{02}, V_{03}, V_{04}, V_{05}, V_{09}, V_{012}\}$.

TABLE VII: Added control places in the first iteration

V_S	preset	postset	$M_{0V_1}(V_{1i})$
V_{01}	t_{10}, t_{16}	t_9, t_{15}	2
V_{02}	t_4, t_{13}	t_3, t_{11}	2
V_{03}	t_9, t_{17}	t_8, t_{16}	2
V_{04}	t_8, t_{18}	t_7, t_{17}	2
V_{05}	t_{10}, t_{17}	t_8, t_{15}	3
V_{09}	$t_5, t_{10}, t_{13}, t_{17}$	t_3, t_8, t_{11}, t_{15}	5
V_{012}	t_3, t_8, t_{19}	t_1, t_{17}	5

TABLE VIII: SMSs and their complementary of (N_{V_1}, M_{0V_1})

	SMSs
S_{11}	$p_{113}, p_{118}, p_{120}, p_{124}$
S_{12}	$p_{113}, p_{117}, p_{120}, p_{124}, p_{126}$
S_{13}	$p_{112}, p_{113}, p_{117}, p_{123}, p_{124}$
S_{14}	$p_{112}, p_{117}, p_{122}, p_{123}$
S_{15}	$p_{106}, p_{107}, p_{113}, p_{116}, p_{117}, p_{120}, p_{124}, p_{125}$
S_{16}	$p_{106}, p_{107}, p_{112}, p_{113}, p_{116}, p_{117}, p_{124}, p_{127}$
S_{17}	$p_{106}, p_{107}, p_{112}, p_{116}, p_{117}, p_{122}, p_{127}$
S_{18}	$p_{102}, p_{103}, p_{108}, p_{109}, p_{113}, p_{116}, p_{117}, p_{120}, p_{122}, p_{125}, p_{127}$
S_{19}	$p_{102}, p_{103}, p_{108}, p_{109}, p_{112}, p_{113}, p_{116}, p_{117}, p_{125}, p_{127}$
S_{110}	$p_{102}, p_{103}, p_{108}, p_{109}, p_{112}, p_{113}, p_{117}, p_{123}, p_{125}$
	Complementary sets of SMSs
$[S_{11}]$	p_{112}, p_{119}
$[S_{12}]$	$p_{111}, p_{112}, p_{118}, p_{119}$
$[S_{13}]$	$p_{111}, p_{118}, p_{119}$
$[S_{14}]$	p_{111}, p_{118}
$[S_{15}]$	$p_{111}, p_{112}, p_{118}, p_{119}$
$[S_{16}]$	$p_{111}, p_{118}, p_{119}$
$[S_{17}]$	p_{111}, p_{118}
$[S_{18}]$	$p_6, p_7, p_{111}, p_{112}, p_{118}, p_{119}$
$[S_{19}]$	$p_{106}, p_{107}, p_{111}, p_{118}, p_{119}$
$[S_{110}]$	$p_{111}, p_{118}, p_{119}$

Continue to compute SMSs in (N_{V_1}, M_{0V_1}). Details about SMSs and their complementary sets are shown in Table 8. We have $\Pi_{F_1} = \{S_{12}, S_{14}\}$ according to Properties 1 and 2. Control places are added for S_{12} and S_{14} by Proposition

1 and Theorem 1. V_{12} and V_{14} are obtained with $M_{0V_2}(V_{12}) = 3$, ${}^\bullet V_{12} = \{t_8, t_{17}\}, V_{12}^\bullet = \{t_7, t_{16}\}$. $M_{0V_2}(V_{14}) = 5$, ${}^\bullet V_{14} = \{t_8, t_{17}\}, V_{14}^\bullet = \{t_7, t_{16}\}$. Thus, we have $\Phi_2 = \{V_{12}, V_{14}\}$. By removing Φ_1 and reserving Φ_2, we can obtain the 2-order controlled net (N_{V_2}, M_{0V_2}), where there is no more new SMS generated. A first-controlled net $(N_V, M_{0V}) = (P, T, F_V)$ is obtained, where $P = P^0 \cup P_R \cup P_S \cup \Phi$, $\Phi = \Phi_1 \cup \Phi_2$, and $F_V = F_{V_1} \cup F_{V_2} \cup F$. And we update the net into $(N_V, M_{0V}) = (P, T, F_V, W_V)$, where W_V is a mapping from F_V to $\mathbb{N}^+$.

TABLE IX: Non-max-marked siphons

	non-max-marked siphons
S_{n_1}	$p_{110}, p_{117}, p_{122}, p_{130}, p_{131}, p_{138}$
S_{n_2}	$p_{104}, p_{110}, p_{112}, p_{117}, p_{121}, p_{122}, p_{124}, p_{138}$
S_{n_3}	$p_{102}, p_{103}, p_{110}, p_{115}, p_{120}, p_{122}, p_{123}, p_{132}, p_{134}, p_{138}$
S_{n_4}	$p_{102}, p_{103}, p_{110}, p_{115}, p_{120}, p_{122}, p_{123}, p_{125}, p_{126}, p_{132}, p_{134}$
S_{n_5}	$p_{104}, p_{110}, p_{116}, p_{121}, p_{122}, p_{124}, p_{129}, p_{144}$

TABLE X: Added control places for each corresponding non-max-marked siphons in TABLE ??

V	preset	postset	$M_{0V(V_i)}, i =$ $1, 2, 3, 4, 5$
V_1	$t_8, t_{10}, 2t_{17}$	$2t_7, 2t_{15}$	8
V_2	t_5, t_8, t_{13}, t_{17}	t_3, t_7, t_{11}, t_{15}	6
V_3	$t_3, t_5, t_8, t_{10}, t_{17}, t_{19}$	$2t_1, 2t_{15}$	16
V_4	$t_3, t_5, t_8, 2t_{10}, t_{17}, 2t_{19}$	$2t_1, t_9, 2t_{15}, t_{18}$	17
V_5	$2t_5, t_9, 2t_{10}, t_{13}, t_{17}, t_{18}, t_{19}$	$2t_1, t_8, t_{11}, 3t_{15}$	22

TABLE XI: Comparison of control policies

Control Criteria	[2]	[39]	[40]	[38]	Our method
Number of monitors	18	6	17	13	15
Number of reachable states	6287	6287	12256	21581	21581

In the second stage, we need to determine whether there exist non-max-marked siphons in the net by solving MILP problems. The related information is shown in Table 9. After five siphons are controlled successively, i.e, $\Phi = \{V_1, V_2, V_3, V_4, V_5\}$, there is no non-max-marked siphon and finally we obtain a final-augmented net $(N_{V'}, M_{0V'}) = (P \cup \Phi, T, F_{V'}, W_{V'})$. Table 11 shows a comparison among several control policies. Compared with [2], [40], we obtain an optimal supervisor with less control places. Although the method in [39] adds less control places than our method, we obtain an optimal supervisor. As for [38], we do not need to consider the reachability graphs.

VII. Conclusions

This paper develops a deadlock prevention policy for FMSs by using structural analysis techniques, which includes two stages. The first stage is called siphons control, which aims to obtain an optimal supervisor since each siphon is optimally controlled. If the first-controlled net is still not live after the first stage, then the second stage, called non-max-marked siphons control, is carried out. A non-max-marked siphon is computed by solving an MILP problem, and then the siphon is max-controlled by adding a control place. Repeat the above steps until no max-marked siphon is found in a final-augmented net. In this stage, we do not need a complete siphon enumeration by utilizing MILP problems to compute siphons. In some cases, it is shown that the proposed structure-based analysis method can lead to an optimal supervisor, which, as far as the authors know, is not exposed in the existing methods in the case that there exist ξ-resources [52]. Our future work will consider extending the policy to automata based methods for the control of FMSs [26-27], [47-48].

Acknowledgements

The authors extend their appreciation to the Deanship of Scientific Research at King Saud University for funding this work through research group number RG-1439-005.

References

[1] C. F. Zhong and Z. W. Li, “A deadlock prevention approach for flexible manufacturing systems without complete siphon enumeration of their Petri net models,” Engineering with Computers, vol. 20, pp. 269–278, 2009.

[2] J. Ezpeleta, M. J. Colom, and J. Martinez, “A Petri net based deadlock prevention policy for flexible manufacturing system,” IEEE Transactions on Robotics and Automation, vol. 11, no. 2, pp. 173–184, 1995.

[3] T. Murata, “Petri nets: Properties analysis and applications,” in Proceedings of the IEEE, vol. 77, no. 4, pp. 541–580, 1989.

[4] K. Barkaoui and J. F. Pradat-Peyre, “On liveness and controlled siphons in Petri nets,” Lecture Notes in Computer Science, vol. 1996, pp. 57–72, 1901.

[5] C. F. Zhong, Z. W. Li and K. Barkaoui, “Monitor design for siphon control in S^4PR nets: from structure analysis points of view,” International Journal of Innovative Computing, Information and Control, vol. 7, no. 1, pp. 1–22, 2011.

[6] C. F. Zhong and Z. W. Li, “Self-liveness of a class of Petri net models for flexible manufacturing systems,” IET Control Theory & Applications, vol. 4, no. 3, pp. 403–410, 2010.

[7] S. G. Wang, Y. Dan and S. Carla, “A novel approach for constraint transformation in Petri nets with uncontrollable transitions,” IEEE Transactions on Systems, Man, and Cybernetics, vol. 48, no. 8, pp. 1403–1410, 2018.

[8] Y. Dan, S. G. Wang, W. Z. Dai, W. H. Wu and Y. S. Jia, “An approach for enumerating minimal siphons in a subclass of Petri nets,” IEEE Access, vol. 12, no. 6, pp. 4255–4265, 2018.

[9] N. Q. Wu and M. C. Zhou, “Schedulability analysis and optimal scheduling of dual-arm cluster tools with residency time constraint and activity time variation,” IEEE Transactions on Automation Science and Engineering, vol. 9, no. 1, pp. 203–209, 2012.

[10] N. Q. Wu and M. C. Zhou, “Modeling, analysis and control of dual-arm cluster tools with residency time constraint and activity time variation based on Petri nets,” IEEE Transactions on Automation Science and Engineering, vol. 9, no. 2, pp. 446–454, 2012.

[11] N. Q. Wu, F. Chu, C. B. Chu, and M. C. Zhou, “Petri net modeling and cycle time analysis of dual-arm cluster tools with wafer revisiting,” IEEE Transactions on Systems, Man and Cybernetics, vol. 43, no. 1, pp. 196–207, 2013.

[12] N. Q. Wu, M. C. Zhou and Z. W. Li, “Short-term scheduling of crude-oil operations: Petri net-based control-theoretic approach,” IEEE Robotics and Automation Magazine, vol. 22, no. 2, pp. 64–76, 2015.

[13] J. F. Zhang, M. Khalgui, Z. W. Li, G. Frey, O. Mosbahi and H. B. Salah, “Reconfigurable coordination of distributed discrete event control systems,” IEEE Transactions on Control Systems Technology, vol. 23, no. 1, pp. 323–330, 2015.

[14] A. Giua, F. DiCesare, and M. Silva, “Generalized mutual exclusion constraints on nets with uncontrollable transitions,” In Process , vol. 2, pp. 974-979, 1992.

[15] Z. Y. Ma, Z. W. Li and A. Giua, "Design of optimal petri net controllers for disjunctive generalized mutual exclusion constraints," IEEE Transactions on Automatic Control, vol. 60, no. 7, pp. 1774–1785, 2015.

[16] J. H. Ye, Z. W. Li and A. Giua, "Decentralized supervision of petri nets with a coordinator," IEEE Transactions on Systems, Man and Cybernetics, vol. 45, no. 6, pp. 955–966, 2015.

[17] Y. F. Chen, Z. W. Li, K. Barkaoui and A. Giua, "On the enforcement of a class of nonlinear constraints on Petri nets," Automatica, vol. 55, pp. 116–124, 2015.

[18] Y. Tong, Z. W. Li and A. Giua, "On the equivalence of observation structures for Petri net generators," IEEE Transactions on Automatic Control, vol. 61, no. 9, pp. 2448-¨C2462, 2016.

[19] Y. Tong, Z. W. Li, C. Seatzu and A. Giua, "Verification of state-based opacity using Petri nets," IEEE Transactions on Automatic Control, vol. 62, no. 6, pp. 2823-¨C2837, 2017.

[20] Z. Y. Ma, Z. W. Li and A. Giua, "Petri net controllers for generalized mutual exclusion constraints with floor operators," Automatica, vol. 74, pp. 238–246, 2016.

[21] M. Uzam, Z. W. Li, G. Gelen and R. S. Zakariyya, "A divide-and-conquer-method for the synthesis of liveness enforcing supervisors for flexible manufacturing systems," Journal of Intelligent Manufacturing, vol. 27, no. 5, pp. 1111–1129, 2016.

[22] H. C. Liu, J. X. You, Z. W. Li and G. D. Tian, "Fuzzy Petri nets for knowledge representation and reasoning: A literature review," Engineering Applications of Artificial Intelligence, vol. 60, pp. 45–56, 2017.

[23] Z. Y. Ma, Z. W. Li and A. Giua, "Characterization of admissible marking sets in Petri nets with conflicts and synchronizations," IEEE Transactions on Automatic Control, vol. 62, no. 3, pp. 1329–1341, 2017.

[24] Z. Y. Ma, Y. Tong, Z. W. Li and A. Giua, "Basis marking representation of Petri net reachability spaces and its application to the reachability problem," IEEE Transactions on Automatic Control, vol. 62, no. 3, pp. 1078–1093, 2017.

[25] X. Y. Cong, M. P. Fanti, A. M. Mangini and Z. W. Li, "Decentralized diagnosis by Petri nets and integer linear programming," IEEE Transactions on Systems, Man, and Cybernetics, vol. 48, no. 10, pp. 1689–1700, 2017.

[26] H. M. Zhang, L. Feng, N. Q. Wu and Z. W. Li, "Integration of learning-based testing and supervisory control for requirements conformance of black-box reactive systems," IEEE Transactions on Automation Science and Engineering, vol. 15, no. 1, pp. 2–15, 2018.

[27] H. Zhang, L. Feng, and Z. W. Li, "A learning-based synthesis approach to the supremal nonblocking supervisor of discrete-event systems," IEEE Transactions on Automatic Control, vol. 63, no. 10, pp. 3345–3360, 2018.

[28] G. H. Zhu, Z. W. Li, N. Q. Wu and A. M. Al-Ahmari, "Fault identification of discrete event systems modeled by Petri nets with unobservable transitions," IEEE Transactions on Systems, Man, and Cybernetics, DOI: 10.1109/TSMC.2017.2762823, 2018.

[29] R. A. Wysk, N. S. Yang and S. Joshi, "Resolution of deadlock in flexible manufacturing system: avoidance and recovery approaches," Journal of Manufacturing Systems, vol. 13, no. 2, pp. 128–128, 1994.

[30] F. S. Hsieh and S. C. Chang, "Dispatching driven deadlock avoidance controller synthesis for flexible manufacturing systems," IEEE Transactions on Robotics and Automation, vol. 10, no. 2, pp. 196–209, 1994.

[31] Z. W. Li, H. S. Hu, and A. R. Wang, "Design of liveness-enforcing supervisors for flexible manufacturing systems using Petri nets," IEEE Transactions on Systems, Man, and Cybernetics, Part C, vol. 37, no. 4, pp. 517–526, 2007.

[32] Z. W. Li and M. C. Zhou, "Control of elementary and dependent siphons in Petri nets and their application," IEEE Transactions on Systems, Man, and Cybernetics, Part A, vol. 38, no. 1, pp. 133–148, 2008.

[33] Z. W. Li, M. C. Zhou, and N. Q. Wu, "A survey and comparison of petri net-based deadlock prevention policies for flexible manufacturing systems," IEEE Transactions on Systems, Man, and Cybernetics, Part C, vol. 38, no. 2, pp. 173–188, 2008.

[34] Z. W. Li and M. Zhao, "On controllability of dependent siphons for deadlock prevention in generalized Petri nets," IEEE Transactions on Systems, Man, and Cybernetics, Part A, vol. 38, no. 2, pp. 369–384, 2008.

[35] Y. F. Chen and Z.W. Li, "Design of a maximally permissive liveness-enforcing supervisor with a compressed supervisory structure for flexible manufacturing systems," Automatica, vol. 47, no. 5, pp. 1028–1034, 2011.

[36] M. P. Fanti and M. C. Zhou, "Deadlock control methods in automated manufacturing systems," IEEE Transaction on Systems, Man and Cyberneties, vol. 34, no. 1, pp. 5–22, 2004.

[37] Z. W. Li, M. C. Zhou and M. D. Jeng. “A maximally permissive deadlock prevention policy for FMS based on Petri net siphon control and the theory of regions,” IEEE Transactions on Automation Science and Engineering, vol. 5, no. 1, pp. 182–188, 2008.

[38] L. Piroddi, R. Cordon and I. Fumagalli, “Selective siphon control for deadlock prevention in Petri nets,” IEEE Transactions on Systems, Man and Cybernetics, vol. 38, no. 6, pp. 1337–1348, 2008.

[39] Z. W. Li and M. C. Zhou, “Elementary siphons of Petri nets and their application to deadlock prevention in flexible manufacturing systems,” IEEE Transactions on Systems, Man and Cybernetics, vol. 34, no. 1, pp. 38–51, 2004.

[40] Y. S. Huang, M. D. Jeng and X. L. Xie, “Deadlock prevention policy based on Petri net and siphons,” International Journal of Production Research, vol. 39, no. 2, pp. 283–305, 2001.

[41] Z. W. Li, J. Zhang and M. Zhao, “Liveness-enforcing supervisor design for a class of generalised petri net models of flexible manufacturing systems,” LET Control Theory & Applications, vol. 1, no. 4, pp. 955–967, 2007.

[42] G. Y. Liu, P. Li, Z. W. Li and N. Q. Wu, “Robust deadlock control for automated manufacturing systems with unreliable resources based on Petri net reachability graphs,” IEEE Transactions on Systems, Man and Cybernetics, to be published, DOI: 10.1109/TSMC.2018.2815618, 2018.

[43] G. H. Zhu, Z. W. Li and N .Q. Wu, “Model-based fault identification of discrete event systems using partially observed Petri nets,” Automatica, vol. 96, pp. 201–212, 2018.

[44] X. Y. Cong, M. P. Fanti, A. M. Mangini and Z .W. Li, “On-line verification of current-state opacity by Petri nets and integer linear programming,” Automatica, vol. 94, pp. 205–213, 2018.

[45] C. Gu, Z. W. Li, N. Q. Wu, M. Khalgui, T. Qu, and A. Al-Ahmarimm, “Improved multi-step look-ahead control policies for automated manufacturing systems,” IEEE Access, vol. 6, no. 1, pp. 68824-68838, 2018.

[46] Z. W. Li, G. Y. Liu, M-H. Hanisch and M. C. Zhou, “Deadlock prevention based on structure reuse of Petri net supervisors for flexible manufacturing systems,” IEEE Transactions on Systems, Man and Cybernetics, vol. 42, no. 1, pp. 178–191, 2012.

[47] X. Wang, I. Khemaissia, M. Khalgui, Z. W. Li, O. Mosbahi and M. C. Zhou, “Dynamic low-power reconfiguration of real-time systems with periodic and probabilistic tasks,” IEEE Transactions on Automation Science and Engineering, vol. 12, no. 1, pp. 258–271, 2015.

[48] X. Wang, Z. W. Li, W. M. Wonham, “Dynamic multiple-period reconfiguration of real-time scheduling based on timed DES supervisory control,” IEEE Transactions on Industrial Informatics, vol. 12, no. 1, pp. 101–111, 2016.

[49] H. Grichi, O. Mosbahi, M. Khalgui, and Z. W. Li, “RWiN: New methodology for the development of reconfigurable WSN,” IEEE Transactions on Automation Science and Engineering, vol. 14, no. 1, pp. 109–125, 2017.

[50] M. Gasmi, O. Mosbahi, M. Khalgui, L. Gomes, and Z. W. Li, “R-Node: New pipelined approach for an effective reconfigurable wireless sensor node,” IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 48, no. 6, pp. 892–905, 2018.

[51] Z. W. Li and M. C. Zhou, Deadlock Resolution in Automated Manufacturing Systems: A Novel Petri Net Approach. New York. NY, USA: Springer, 2009.

[52] K. Y. Xing, M. C. Zhou, H. X. Liu, and F. Tian, “Optimal Petri-Net-Based Polynomial-Complexity Deadlock-Avoidance Policies for Automated Manufacturing Systems,” IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 39, no. 1, pp. 188–199, 2009.

[53] J. Ye, M. C. Zhou, Z. W. Li, and A. Al-Ahmari, “Structural decomposition and decentralized control of Petri nets,” IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 48, no. 8, pp. 1360–1369, 2018.

[54] J. F. Zhang, M. Khalgui, Z. W. Li, G. Frey, O. Mosbahi, and H. B. Salah, “Reconfigurable coordination of distributed discrete event control systems,” IEEE Transactions on Control Systems Technology, vol. 23, no. 1, pp. 323–330, 2015.

[55] Y. F. Chen, Z. W. Li, and K. Barkaoui, “New Petri net structure and its application to optimal supervisory control: Interval inhibitor arcs” IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 44, no. 10, pp. 1384–1400, 2014.

[56] C. Gu, X. Wang, and Z. W. Li , “Synthesis of supervisory control with partial observation on normal state tree structures,” IEEE Transactions on Automation Science and Engineering, DOI: 10.1109/TASE.2018.2880178, 2018.

[57] N. Q. Wu, Z. W. Li, and T. Qu, “Energy efficiency optimization in scheduling crude oil operations of refinery based on linear programming,” Journal of Cleaner Production, vol. 166, pp. 49–57, 2017.

Appendix

A generalized Petri net [4] is a 4-tuple $N = (P, T, F, W)$ where P and T are finite, non-empty, and disjoint sets. P is the set of places and T is the set of transitions with $P \cup T \neq \emptyset$ and $P \cap T = \emptyset$. $F \subseteq (P \times T) \cup (T \times P)$ is called a flow relation of the net, represented by arcs with arrows from places to transitions or from transitions to places. $W : (P \times T) \cup (T \times P) \to N$ is a mapping that assigns a weight to an arc: $W(x, y) > 0$ if $(x, y) \in F$, and $W(x, y) = 0$, otherwise, where $x, y \in P \cup T$. $N = (P, T, F, W)$ is called an ordinary net, denoted by $N = (P, T, F)$, if $\forall f \in F$, $W(f) = 1$. Note that ordinary and generalized Petri nets have the same modeling power. The only difference is that the latter may have improved modeling efficiency and convenience.

A net $N = (P, T, F, W)$ is pure (self-loop free) if for all $x, y \in P \cup T$, $W(x, y) > 0$ implies $W(y, x) = 0$. A pure net $N = (P, T, F, W)$ can be represented by its incidence matrix $[N]$, where $[N]$ is a $|P| \times |T|$ integer matrix with $[N](p, t) = W(t, p) - W(p, t)$. For a place p (transition t), its incidence vector, a row (column) in $[N]$, is denoted by $[N](p, \bullet)([N](\bullet, t))$. The incidence matrix $[N]$ of a net N can be naturally divided into two parts Pre and $Post$ according to the token flow by defining $[N] = Post - Pre$, where $Pre : P \times T \to \mathbb{N}$ and $Post : P \times T \to \mathbb{N}$, respectively.

Let $x \in P \cup T$ be a node of net $N = (P, T, F, W)$. The preset of x is defined as ${}^\bullet x = \{y \in P \cup T \mid (y, x) \in F\}$. While the postset of x is defined as $x^\bullet = \{y \in P \cup T \mid (x, y) \in F\}$. For $t \in T$, $p \in {}^\bullet t$ is called an input place of t and $p \in t^\bullet$ is called an output place of t. For $p \in P$, $t \in {}^\bullet p$ is called an input transition of p and $t \in p^\bullet$ is called an output transition of p.

A marking M of a Petri net N is a mapping from P to $\mathbb{N}$. $M(p)$ denotes the number of tokens in place p. A place p is marked by a marking M if $M(p) > 0$. The sum of tokens of all places in S is denoted by $M(S)$, *i.e.*, $M(S) = \sum_{p \in S} M(p)$. S is said to be empty at M if $M(S) = 0$. (N, M_0) is called a net system or a marked net and M_0 is called an initial marking of N.

A transition $t \in T$ is enabled at a marking M if for all $p \in {}^\bullet t$, $M(p) \geq W(p, t)$. This fact is denoted by $M[t\rangle$. Firing it yields a new marking M' such that for all $p \in P$, $M'(p) = M(p) - W(p, t) + W(t, p)$, as denoted by $M[t\rangle M'$. M' is called an immediately reachable marking from M. Marking M'' is said to be reachable from M if there exists a sequence of transitions $\sigma = t_0 t_1 \cdots t_n$ and markings M_1, M_2, $\cdots$, M_n such that $M[t_0\rangle M_1[t_1\rangle M_2 \cdots M_n[t_n\rangle M''$ holds. The set of markings reachable from M in N is called the reachability set of Petri net (N, M) and denoted by $R(N, M)$.

A transition t is live if for all $M \in R(N, M_0)$, there exists a marking $M' \in R(N, M)$ such that $M'[t\rangle$ holds. A net is live if every transition is live. A transition t is dead at a marking $M \in R(N, M_0)$ if $\forall M' \in R(N, M), M'[t\rangle$ does not hold.

A P-vector is a column vector $I : P \to \mathbb{Z}$ indexed by P, where $\mathbb{Z}$ is the set of integers. P-vector I is called a P-invariant (place invariant) if $I \neq 0$ and $I^T[N] = 0^T$. P-invariant I is a P-semiflow if every element of I is non-negative. $||I|| = \{p | I(p) = 0\}$ is called the support of I. $||I||^+ = \{p | I(p) > 0\}$ denotes the positive support of P-invariant I and $||I||^- = \{p | I(p) < 0\}$ denotes the negative support of I. If I is a P-invariant of (N, M_0), for all $M \in R(N, M_0)$, $I^T M = I^T M_0$.

A nonempty set $S \subseteq P$ is a siphon (trap) if ${}^{\bullet}S \subseteq S^{\bullet}$ ($S^{\bullet} \subseteq {}^{\bullet}S$) holds. A siphon is minimal if there is no siphon contained in it as a proper subset. A minimal siphon is called strict if it does not contain a trap, denoted as SMS for short. A siphon S can also be described by its characteristic vector $s \in \{0,1\}^m$ such that $s_i = 1$ if $p_i \in S$, else $s_i = 0$; thus $M(S) = s^T M$.