

Bubble Sort

Algorithm

```
1 for i = 1 to n
2   for j = 1 to n-1
3     if v[j] > v[j+1]
4       swap(v[j], v[j+1])
```

(A) (B)

Analysis

$$(A) \Rightarrow \sum_{j=i}^{n-1} c = \underbrace{c + c + \dots + c}_{n-1-i+1} = c(n-i) \quad \begin{matrix} i=1 \\ n-1 \end{matrix}$$

$$(B) \Rightarrow \sum_{i=1}^n c(n-i) = c \sum_{i=1}^n (n-i) = c \overbrace{\left[(n-1) + (n-2) + \dots + (1) \right]}^{n-1}$$

$$= c \left\{ \left[\frac{(n+1)+1}{2} \right] \cdot \frac{(n-1)}{2} \right\} = c \left(\frac{n^2-n}{2} \right) \Rightarrow O(n^2)$$

Time complexity

$O(n^2)$

$\Theta(n^2)$

$\Omega(n^2)$

Space complexity

$\Theta(1)$

Number of swaps

best case: 0

worst case: n^2

Average: n^2

Good for nothing

// There are implementations with $\Omega(n)$