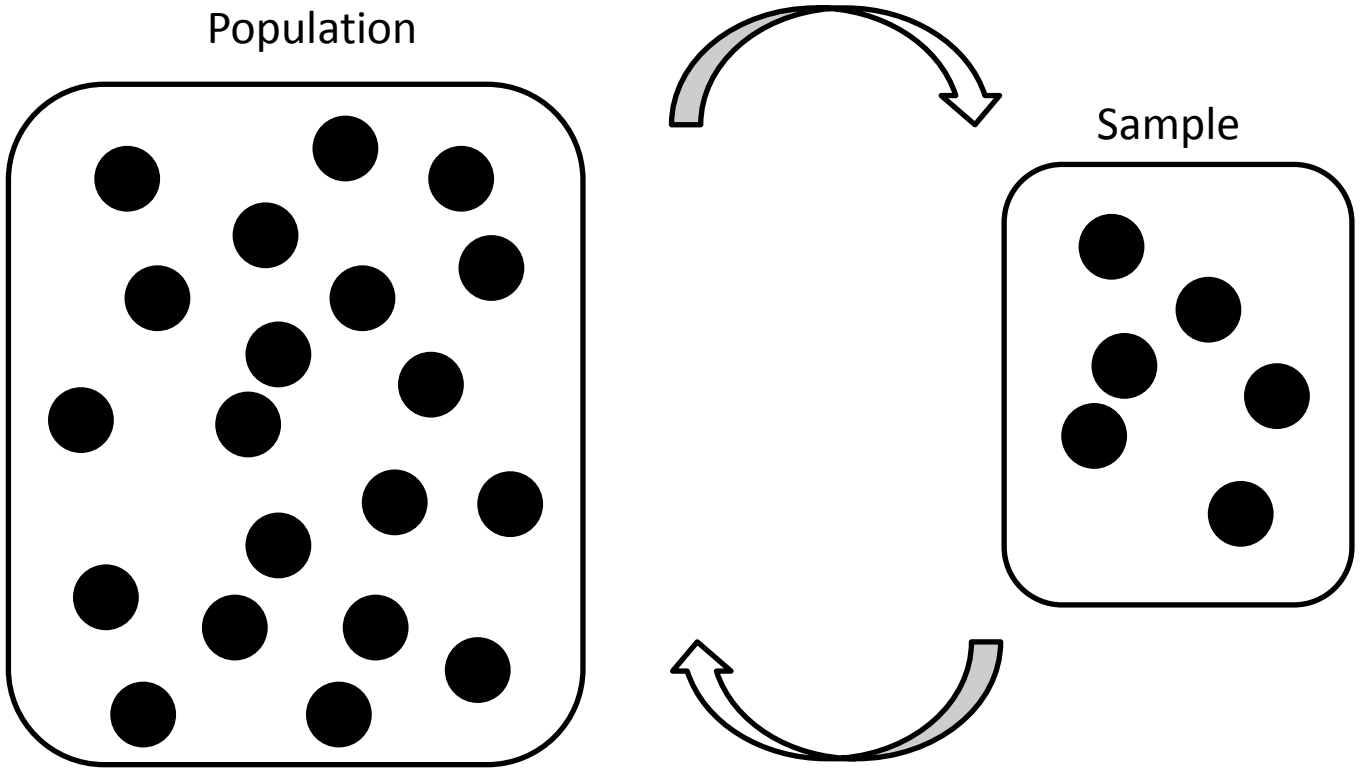
Sampling distributions

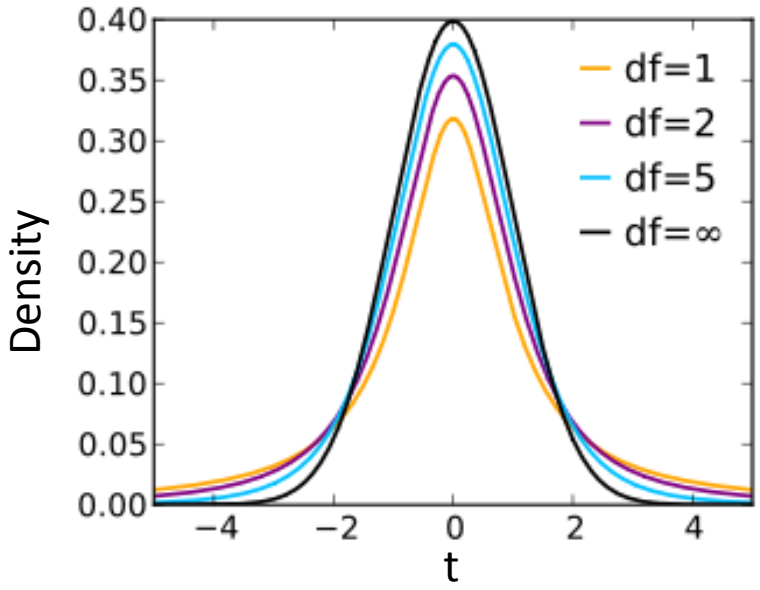


t-student distribution

This distribution is useful to study the sample mean distribution.

The probability density function (pdf) is given by

$$f(t) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\,\Gamma(\frac{\nu}{2})}\left(1+\frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}, \quad \text{with } t \in \mathbb{R} \text{ and } v > 0$$

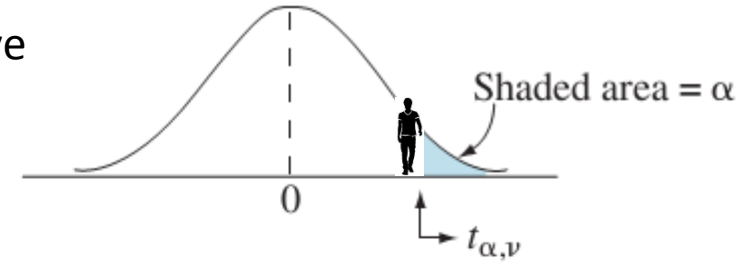


The parameter v is called degrees of freedom (df)

Where $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}\,\mathrm{d}t.$

t-student table

Areas under curve



Percentage points of Student's t distribution

df/α =	.40	.25	.10	.05	.025	.01	.005	.001	.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587

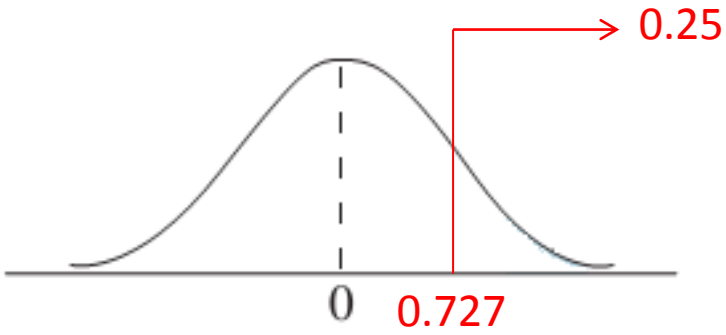
Degrees of freedom

Quantiles

Example 1

Obtain $t_{0.25,\,5}$ → α → df

df/α =	.40	.25	.10
1	0.325	1.000	3.078
2	0.289	0.816	1.886
3	0.277	0.765	1.638
4	0.271	0.741	1.533
5	0.267	0.727	1.476
6	0.265	0.718	1.440
7	0.263	0.711	1.415
8	0.262	0.706	1.397
9	0.261	0.703	1.383
10	0.260	0.700	1.372



Answer: $t_{0.25,\,5} = 0.727$

Example 2

Obtain α if $t_{\alpha, 3} = 2.353$

df/α =	.40	.25	.10	.05	.025	.01	.005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032

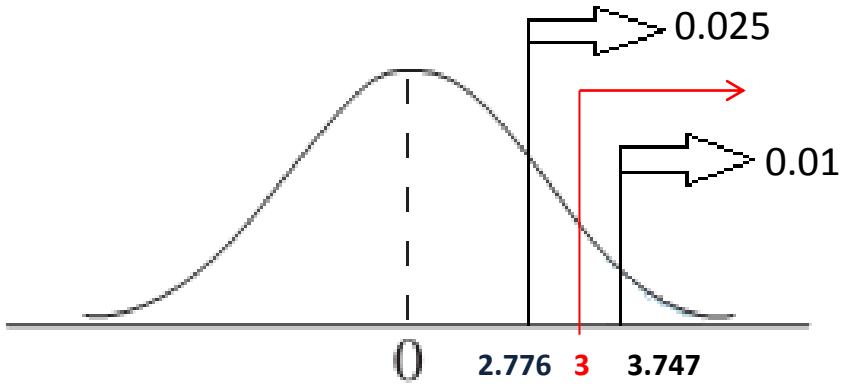
Answer: α = 0.05

Example 3

Obtain α if $t_{\alpha,4} = 3$

df/α =	.40	.25	.10	.05	.025	.01	.005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032

Answer: $\alpha \in (0.01, 0.025)$



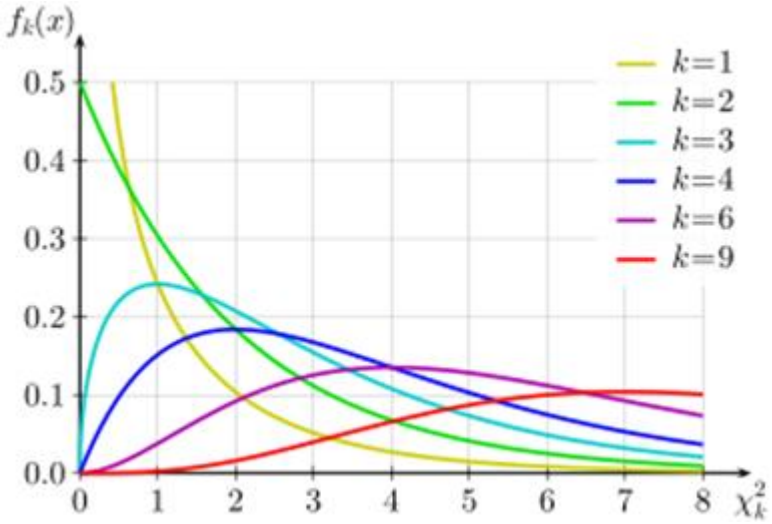
Chi-square distribution

This distribution is useful to study the sample variance distribution.

The probability density function (fdp) is given by

$$f(x;\,k) = \begin{cases} \dfrac{x^{(k/2)-1}e^{-x/2}}{2^{k/2}\Gamma\left(\frac{k}{2}\right)}, & x \geq 0; \\ 0, & \text{otherwise.} \end{cases}$$

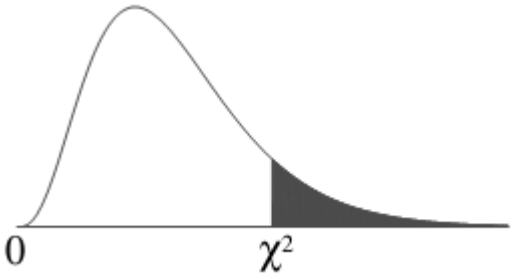
with x ∈ $[0, \infty)$ and $k > 0$



The parameter k is called degrees of freedom (df)

Chi-square table

Areas under curve



df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$
1	0.000	0.000	0.001	0.004	0.016	2.706
2	0.010	0.020	0.051	0.103	0.211	4.605
3	0.072	0.115	0.216	0.352	0.584	6.251
4	0.207	0.297	0.484	0.711	1.064	7.779
5	0.412	0.554	0.831	1.145	1.610	9.236

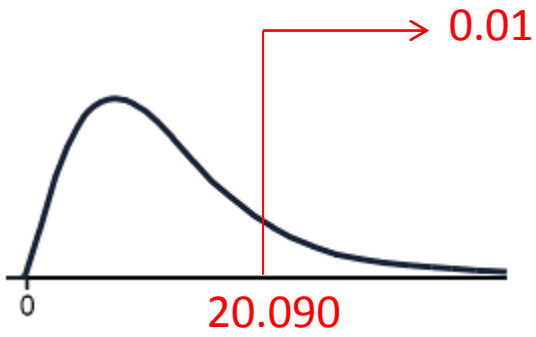
Degrees of freedom

Quantiles

Example 5

Obtain α if $x^2_{\alpha,\ 8} = 20$

df	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	3.841	5.024	6.635	7.879
2	5.991	7.378	9.210	10.597
3	7.815	9.348	11.345	12.838
4	9.488	11.143	13.277	14.860
5	11.070	12.833	15.086	16.750
6	12.592	14.449	16.812	18.548
7	14.067	16.013	18.475	20.278
8	15.507	17.535	20.090	21.955
9	16.919	19.023	21.666	23.589
10	18.307	20.483	23.209	25.188

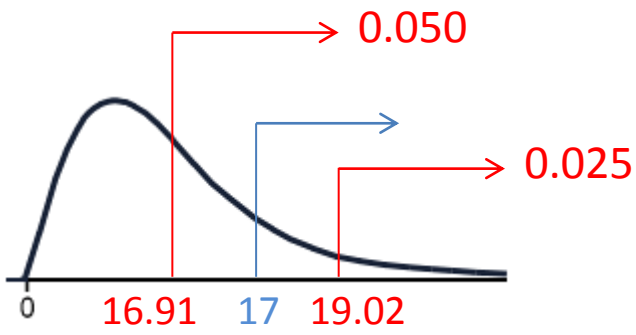


Answer: $\alpha = 0.01$

Example 6

Obtain α if $x^2_{\alpha,\ 9} = 17$

df	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	3.841	5.024	6.635	7.879
2	5.991	7.378	9.210	10.597
3	7.815	9.348	11.345	12.838
4	9.488	11.143	13.277	14.860
5	11.070	12.833	15.086	16.750
6	12.592	14.449	16.812	18.548
7	14.067	16.013	18.475	20.278
8	15.507	17.535	20.090	21.955
9	16.919	19.023	21.666	23.589
10	18.307	20.483	23.209	25.188



Answer: $\alpha \in (0.025, 0.050)$

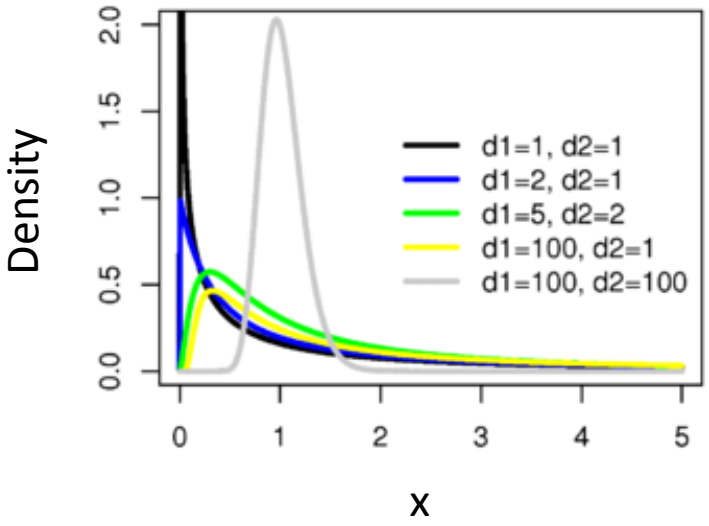
F distribution

This distribution is used in analysis of variance.

The probability density function (fdp) is given by

$$f(x; d_1, d_2) = \frac{\sqrt{\frac{(d_1\, x)^{d_1}\ d_2^{d_2}}{(d_1\, x + d_2)^{d_1+d_2}}}}{x\, \mathrm{B}\left(\frac{d_1}{2}, \frac{d_2}{2}\right)}$$

with x ∈ $[0, \infty)$ and $d_1, d_2 > 0$

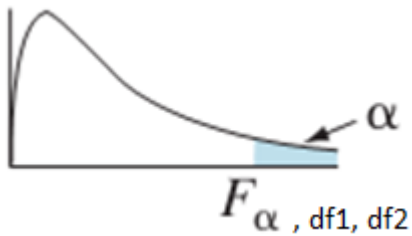


The parameter d_1, d_2 are called degrees of freedom (df)

Where $\mathrm{B}(x, y) = \frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)}$

F table

Areas under curve



df_2	α	df_1 = 1	2	3	4
1	.25	5.83	7.50	8.20	8.58
	.10	39.86	49.50	53.59	55.83
	.05	161.4	199.5	215.7	224.6
	.025	647.8	799.5	864.2	899.6
	.01	4052	5000	5403	5625
2	.25	2.57	3.00	3.15	3.23
	.10	8.53	9.00	9.16	9.24
	.05	18.51	19.00	19.16	19.25
	.025	38.51	39.00	39.17	39.25
	.01	98.50	99.00	99.17	99.25
	.005	198.5	199.0	199.2	199.2
	.001	998.5	999.0	999.2	999.2

Degrees of freedom df1

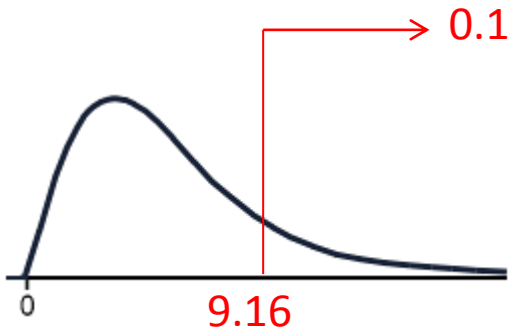
Quantiles

Degrees of freedom df2

Example 7

Obtain $f_{3,\,2,\,0.1}$

		df_1			
df_2	α	1	2	3	4
1	.25	5.83	7.50	8.20	8.58
	.10	39.86	49.50	53.59	55.83
	.05	161.4	199.5	215.7	224.6
	.025	647.8	799.5	864.2	899.6
	.01	4052	5000	5403	5625
2	.25	2.57	3.00	3.15	3.23
	.10	8.53	9.00	9.16	9.24
	.05	18.51	19.00	19.16	19.25
	.025	38.51	39.00	39.17	39.25
	.01	98.50	99.00	99.17	99.25
	.005	198.5	199.0	199.2	199.2
	.001	998.5	999.0	999.2	999.2

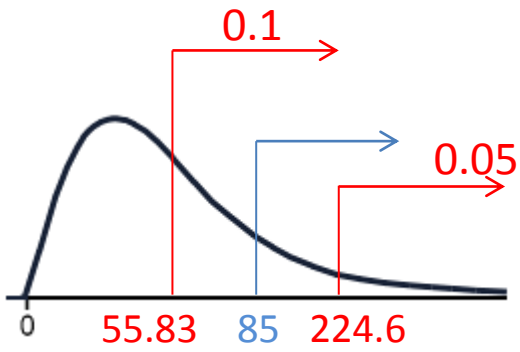


Answer: $f_{3,\,2,\,0.1} = 9.16$

Example 8

Obtain α if $f_{4,\,1,\,\alpha} = 85$

		df_1			
df_2	α	1	2	3	4
1	.25	5.83	7.50	8.20	8.58
	.10	39.86	49.50	53.59	55.83
	.05	161.4	199.5	215.7	224.6
	.025	647.8	799.5	864.2	899.6
	.01	4052	5000	5403	5625
2	.25	2.57	3.00	3.15	3.23
	.10	8.53	9.00	9.16	9.24
	.05	18.51	19.00	19.16	19.25
	.025	38.51	39.00	39.17	39.25
	.01	98.50	99.00	99.17	99.25
	.005	198.5	199.0	199.2	199.2
	.001	998.5	999.0	999.2	999.2



Answer: $\alpha \in (0.05\ 0.10)$

Tables on web

Normal	http://www.statdistributions.com/normal
t-student	http://www.statdistributions.com/t/
Chi-square	http://www.statdistributions.com/chisquare/
F	http://www.statdistributions.com/f/

Example

StatDistributions.com

Enter either the p-value (represented by the blue area on the graph) or the test statistic (the coordinate along the horizontal axis) below to have the other value computed.

Chi-square distribution Other distributions: Normal • Student's t • F

p-value: 0.05

χ^2 value: 9.488

d.f.: 4

- (•) right tail
- ( ) left tail