hw5-4710

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1.1

The separating line is y = x+1 because both supporting vector from negatives and positives are 0.5 away from the line from (2,4),(6,8) or (6,7).

1.2

No because when it is soft margin, the separating line moves to x=4, because the prior supporting vector(6,8) is discounted from the region.

1.3

The general idea is to transform this 2-dimension data structure into higher degree data structure. In this case, it can be transformed into 3-dimension data structure as it would be easier and more precisely(hard margin) cut by a hyper-plane to separate as now it can't be easily solved by a linear or quadratic solution. Specifically in this case, we can transform the data points to have Z coordinate for $Z = (Y - X)^2$ so the positive points can be more easily linearly separable from negatives.

2

2.1

We can substitute $|2x_1 - x_2|$ with a new variable a but try to minimize it, and add two constraints: $a \geq 2x_1 - x_2$ and $a \geq -(2x_1 - x_2)$ along with two original constraints. Thus this problem becomes:

min(a)
subject to $a \geq 2x_1 - x_2$
$a \geq -(2x_1 - x_2)$
$x_1 + x_2 \geq 5$
$x_2 \leq 2$

2.2

We can formulate it to be $min \sum_{i=1}^{m} x_i$

subject to:

$\sum_{i=1}^{m} x_i >= \sum_{i=1}^{m} \theta^T * f^i + y^i$

$\sum_{i=1}^{m} x_i <= \sum_{i=1}^{m} \theta^T * f^i + y^i$

along with the original constraints

3

3.1

To prove Nash Equilibrium, we first calculate the expected value for player 2:
q(rock) = 0*p(rock) + (-1)*p(paper) + 1*(p(scissors))
q(paper) = 1*p(rock) + 0*p(paper) + (-1)*(p(scissors))
q(scissors) = (- 1)*p(rock) + 1*p(paper) + 0*(p(scissors))
and p(rock) + p(paper) + p(scissors) = 1
similarly, player 1 has p(rock) = 1/3, p(paper) = 1/3, and p(scissors) = 1/3, and similarly,
For player 2, q(rock) = 1/3, q(paper) = 1/3, and q(scissors) = 1/3. Thus prove there is one and the only one Nash Equilibrium.

3.2

Because in case, player 2 will not play scissors at all time, thus we will have to consider mixed-strategy. We assume the probability of player 2 to play rock is x.Then we can get:
$E(P_1(rock) = 0 * P_2(rock) + -1 * P_2(paper) = -1 * (1 - x) = x - 1$
$E(P_1(paper) = 1 * P_2(rock) + 0 * P_2(paper) = x + 0 = x$
$E(P_1(scissor) = -1 * P_2(rock) + 1 * P_2(paper) = -x + 1 - x = 1 - 2x$
To optimize the utility, Player 1 will not be playing rock because if $0 \leq 1$, then $x - 1 \leq 0$, and we can also infer than $x \geq 1 - 2x$, thus $x \geq 1/3$. Then the second player should play rock accordingly(P=1/3), otherwise, choose to play paper(P=2/3). And since the first player won't be playing rock even though he could but has negative utility, it will play paper with P=2/3, and scissor with P=1/3.

4

4.1

Because of the Second-Auction Mechanism, the highest bidder will always pay the minimal currency higher than the second price. And because of the uniformed distributed price range for the unknown buyer, the probability of bidding 4000 falls in the probability of 2/3 and bidding between 3000-4000 falls in the probability of 1/3. Thus if the first buyer always bid 4000 its utility revenue is 2/3*4000 and for second buyer, utility revenue of bidding 3500 is 3500*1/3. And we combine each to get the expected $Revenue = 4000 * 2/3 + 3500 * 1/3 = 2666.67 + 1166.67 = 3833.33$.

4.2

a. Because of the posted-price mechanism, if the bid price is higher than 5000, then the item won't be sold. In addition, the uniformed distribution decided the probability to falls in 5000 is 1/3. Thus, expected revenue = 5000*1/3 =1666.67

b. Because the item will not be sold at price that is higher than our reserved price, thus setting the reserved price higher than existed posted price of 4000 will yield worse revenue. So we should set the reserved price to be 4000.