LECTURE 2. REGRETION

AI & ML, Applications in Manufacturing

(MANU 465)

Ahmad Mohammadpanah

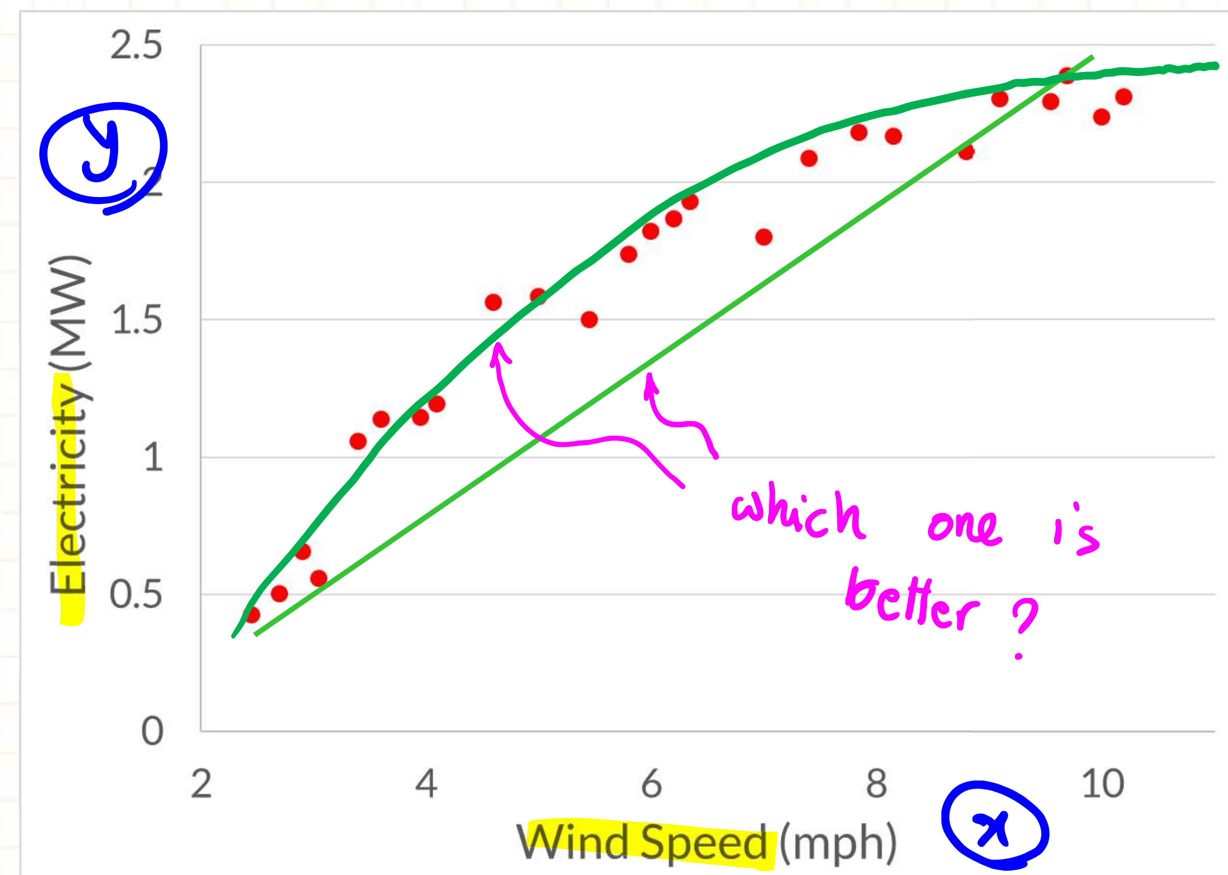
Ph.D., P.Eng.

AIntelligentManufacturing.com

$x_1, x_2, \dots x_{100}$

For example, Let's say we are interested in modeling the relation between wind speed and Electrical output

Location	Wind Speed(mph) x	Electricity (MW) y
Manitoba	2.45	0.423
Manitoba	2.7	0.5
Manitoba	2.9	0.653
Manitoba	3.05	0.558
Manitoba	3.4	1.057
Newfoundland	3.6	1.137
Newfoundland	3.95	1.144
Newfoundland	4.1	1.194
Newfoundland	4.6	1.562
Alberta	5	1.582
Newfoundland	5.45	1.501
Ontario	5.8	1.737
Alberta	6	1.822
Newfoundland	6.2	1.866
Newfoundland	6.35	1.93
Newfoundland	7	1.8
Newfoundland	7.4	2.088
Saskatchewan	7.85	2.179
Ontario	8.15	2.166
Ontario	8.8	2.112
Newfoundland	9.1	2.303
Saskatchewan	9.55	2.294
Saskatchewan	9.7	2.386
Saskatchewan	10	2.236
Saskatchewan	10.2	2.31



Regression: Modeling the relation between variables.

In this lecture, we learn how to implement the following techniques in Python

- Simple Linear Regression

- Polynomial Linear Regression

- Support Vector Regression

How does Regression Work?

let's say we have N data points.

$$L = \sum \left(y_i - (A + B x_i) \right)^2$$

$$\frac{\partial L}{\partial A} = 0 \rightarrow \sum 2 \left(y_i - (A + B x_i) \right)(-1) = 0$$

$$\sum_{i=1}^{N} y_i - \sum_{i=1}^{N} A - \sum_{i=1}^{N} B x_i = 0$$

$$\boxed{\sum y_i - NA - B \sum x_i = 0}$$

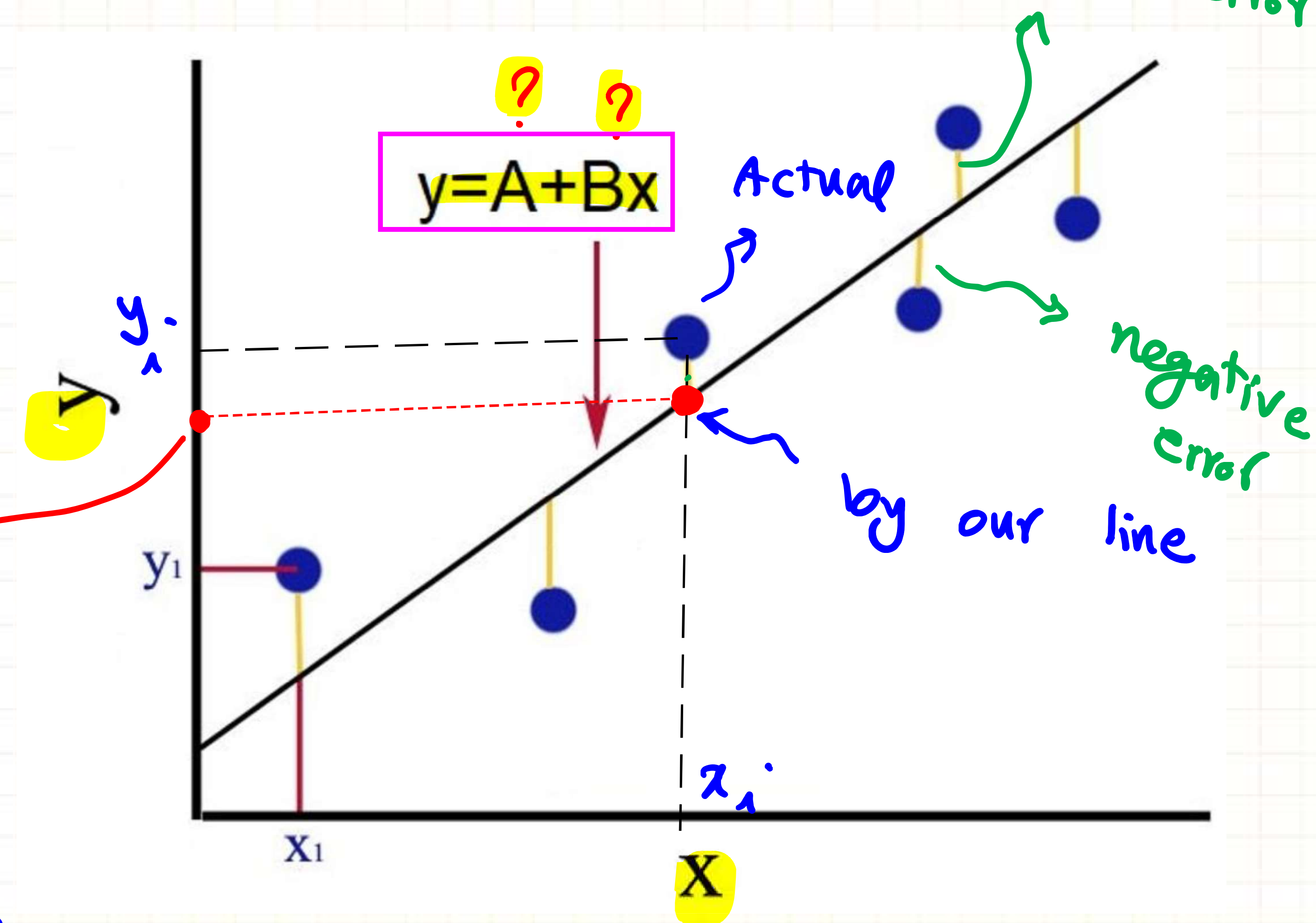
$$\hat{y} = A + B x_i$$

$$\frac{\partial L}{\partial B} = 0 \longrightarrow \sum 2 \left(y_i - (A + B x_i) \right)(-x_i) = 0$$

$$\boxed{-\sum x_i y_i + A \sum x_i + B \sum x_i^2 = 0}$$

$$\begin{cases} NA + B \sum x = \sum y \\ A \sum x + B \sum x^2 = \sum xy \end{cases} \longrightarrow$$

Positive error



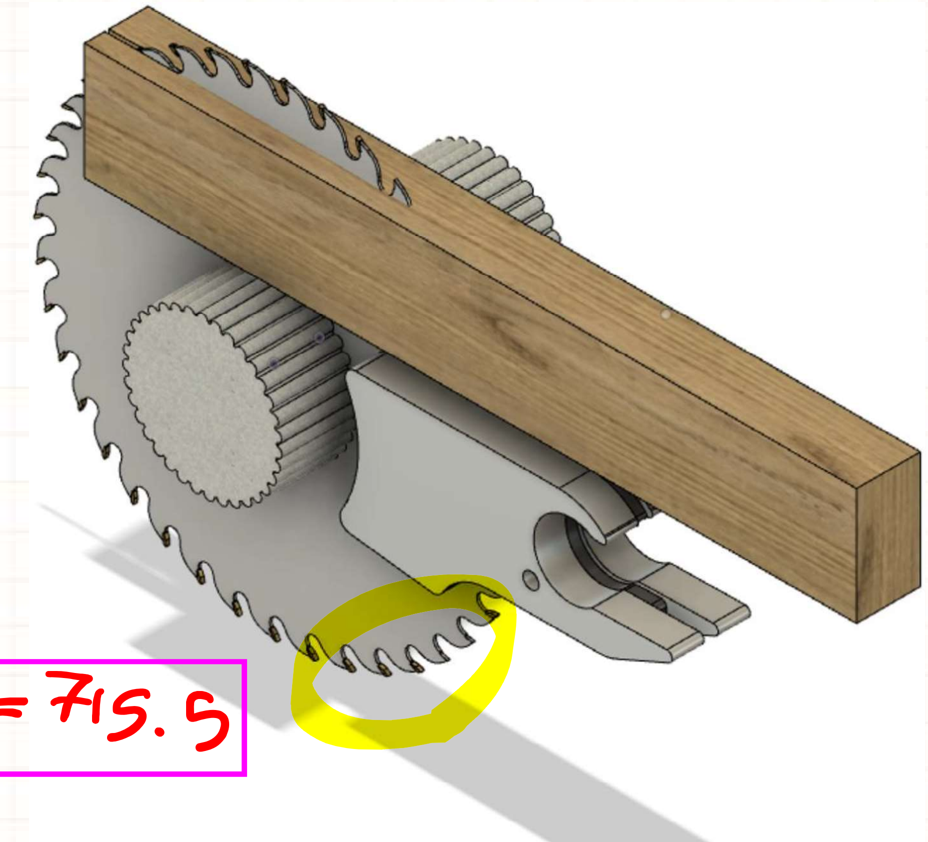
Least_squared_Method

$$\boxed{A = \frac{\sum x^2 \sum y - \sum x \sum xy}{N \sum x^2 - (\sum x)^2}}$$

$$\boxed{B = \frac{N \sum xy - \sum x \sum y}{N \sum x^2 - (\sum x)^2}}$$

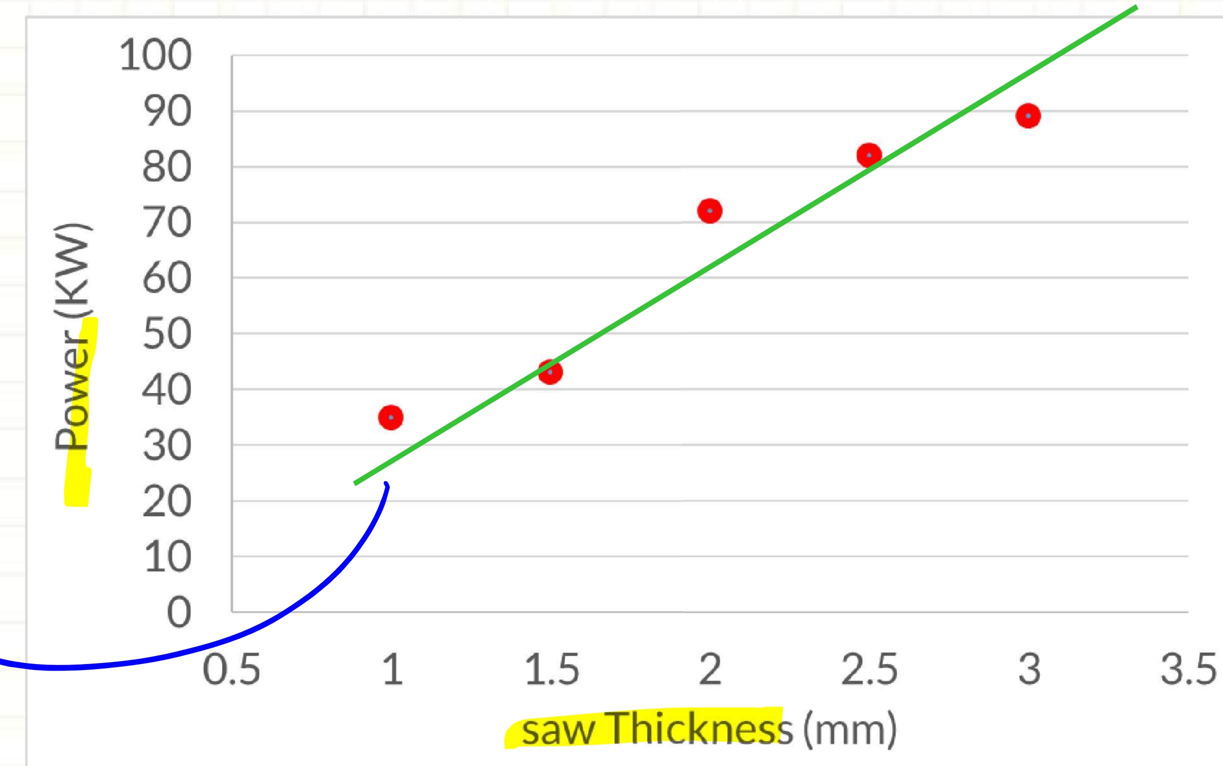
Example) Required Power for Cutting Oak, and Saw Blade Thickness.

Blade Thickness (mm) x	Required Power (KW) y	x^2	Σxy
1	35	1	35
1.5	43	2.25	64.5
2	72	4	144
2.5	82	6.25	205
3	89	9	267
Σ $\boxed{\Sigma x = 10}$	$\boxed{\Sigma y = 321}$	$\boxed{\Sigma x^2 = 22.5}$	$\boxed{\Sigma xy = 715.5}$



$$\boxed{A = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{N \Sigma x^2 - (\Sigma x)^2}} = 5.4$$

$$\boxed{B = \frac{N \Sigma xy - \Sigma x \Sigma y}{N \Sigma x^2 - (\Sigma x)^2}} = 29.4$$



$$\boxed{y = 5.4 + 29.4x}$$

Power

Blade thickness

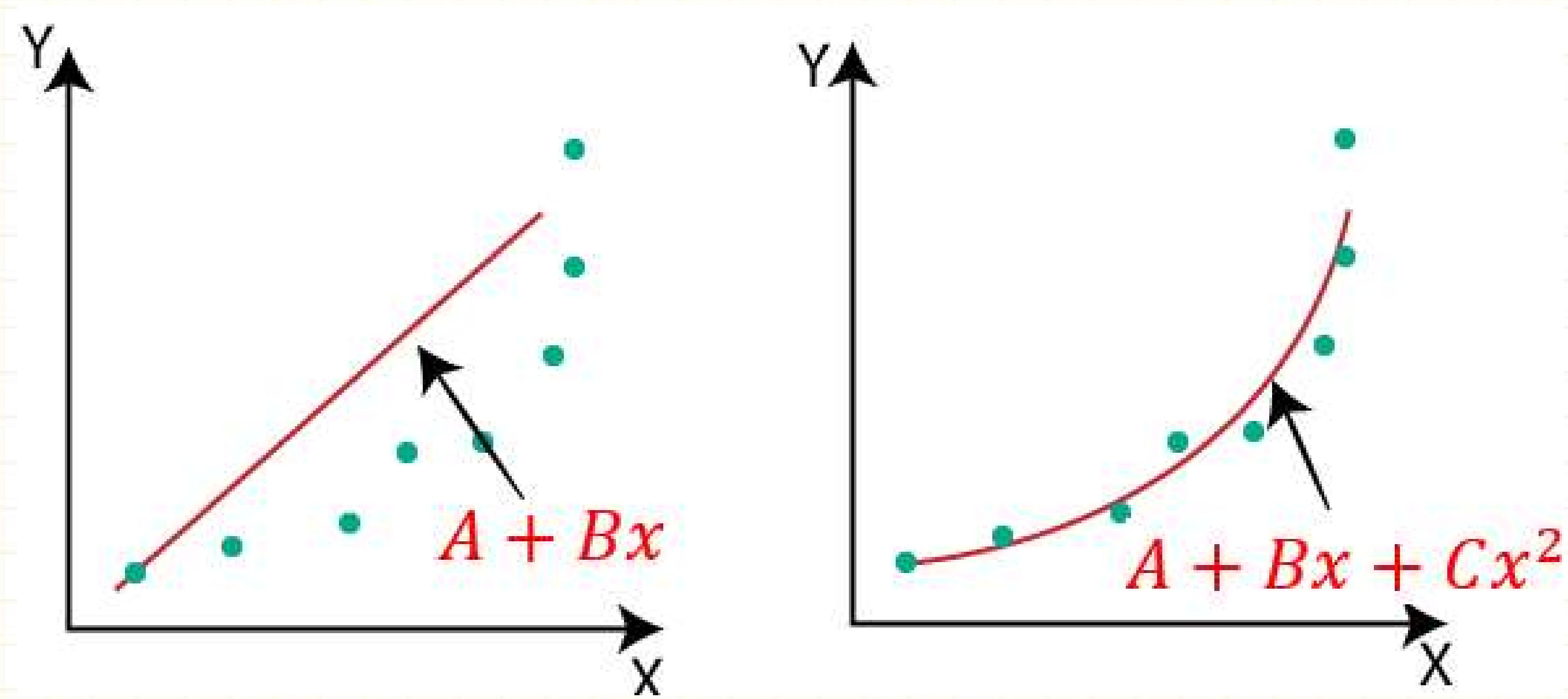
The same approach can be used to fit a polynomial, for example, fitting a second order polynomial to the fowling data

$$\boxed{L = \sum_{i=1}^{N}(y_i - (\overset{?}{A} + \overset{?}{B}x_i + \overset{?}{C}x_i^2))^2}$$

$\frac{\partial L}{\partial A} = 0$ . . . . $A = \ldots$

$\frac{\partial L}{\partial B} = 0$ . . . . $B = \ldots$

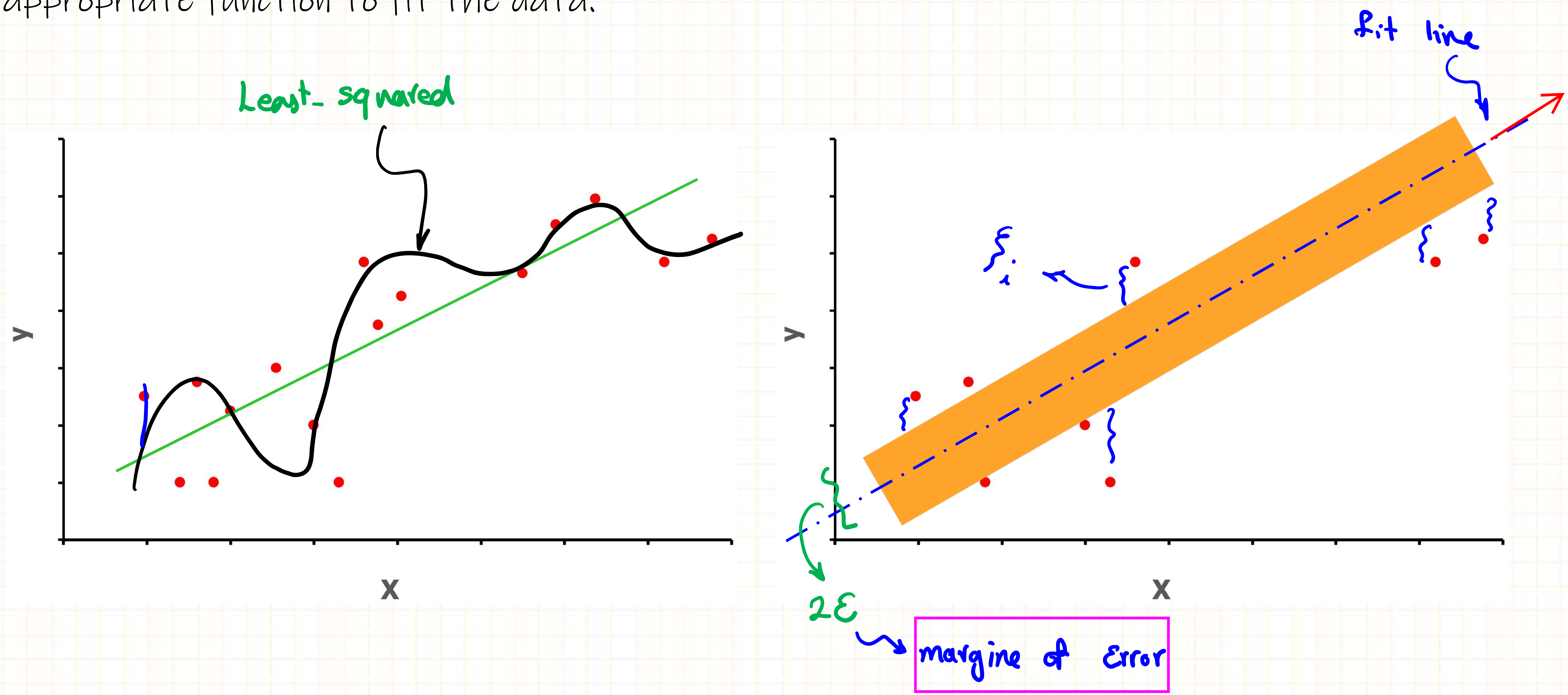
$\frac{\partial L}{\partial C} = 0$ . . . . $C = \ldots$



Support Vector Regression (SVR):

As you saw above, in Least-Square-Method our objective was to minimize the sum of squared errors. But what if we don't care how large our errors are, as long as they fall within an acceptable range?

SVR gives us the flexibility to define how much error is acceptable in our model and will find an appropriate function to fit the data.



Implement in Python

○ Linear Regression Method:

Class

library Module

```
from sklearn.linear_model import LinearRegression
Model = LinearRegression()
Model.fit(X, y)
```

object

○ Polynomial Regression Method:

```
from sklearn.preprocessing import PolynomialFeatures
poly_reg = PolynomialFeatures(degree = 2)
X_poly = poly_reg.fit_transform(X)
Model= LinearRegression()
Model.fit(X_poly, y)
```

obj.

new variable

○ SVR Method:

```
from sklearn.svm import SVR
Model= SVR(kernel = 'rbf')
Model.fit(X[redacted], y[redacted])
```

!

How to check the accuracy of a Regression?

R^2 → 1 ✓ perfect

→ 0 X bad :(

(Please find out, how to compute R^2 in python)

Example) The Windmill data above (in class).

To Practice

- Canvas/Practice Examples 2-5
- Tutorial 2 (California House Price)