junhoc_PS6

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0.1 Problem Set 6 for MACS30150

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Before proceeding, let us import the necessary packages.

```
In [129]: import pandas as pd
          import matplotlib.pyplot as plt
          from pandas.plotting import scatter_matrix
          import numpy as np
          import statsmodels.api as sm
          from sklearn.model_selection import train_test_split
          from sklearn.metrics import confusion_matrix
          from sklearn import neighbors
          import math
          from sklearn.linear_model import LogisticRegression
```

1 Problem 1

Let us import the data first. It seems that there isn't much weirdness from the first inspection.

```
In [3]: ## raw read-in of the data
        data = pd.read_csv("Auto.csv")
        data.head()
```

```
Out[3]:     mpg  cylinders  displacement horsepower  weight  acceleration  year  \
        0  18.0          8         307.0        130    3504          12.0    70
        1  15.0          8         350.0        165    3693          11.5    70
        2  18.0          8         318.0        150    3436          11.0    70
        3  16.0          8         304.0        150    3433          12.0    70
        4  17.0          8         302.0        140    3449          10.5    70

           origin                       name
        0       1  chevrolet chevelle malibu
        1       1          buick skylark 320
        2       1         plymouth satellite
```

```
3       1               amc rebel sst
4       1                 ford torino
```

Yet upon further inspection, there's that question mark for some of the values. So this must be the missing data.

```
In [4]: ## different from the first glance; there is that '?'
        print(data['horsepower'].unique())
```

```
['130' '165' '150' '140' '198' '220' '215' '225' '190' '170' '160' '95'
 '97' '85' '88' '46' '87' '90' '113' '200' '210' '193' '?' '100' '105'
 '175' '153' '180' '110' '72' '86' '70' '76' '65' '69' '60' '80' '54' '208'
 '155' '112' '92' '145' '137' '158' '167' '94' '107' '230' '49' '75' '91'
 '122' '67' '83' '78' '52' '61' '93' '148' '129' '96' '71' '98' '115' '53'
 '81' '79' '120' '152' '102' '108' '68' '58' '149' '89' '63' '48' '66'
 '139' '103' '125' '133' '138' '135' '142' '77' '62' '132' '84' '64' '74'
 '116' '82']
```

Therefore let us re-read-in the data, and designate `?` as part of the `na_values`.

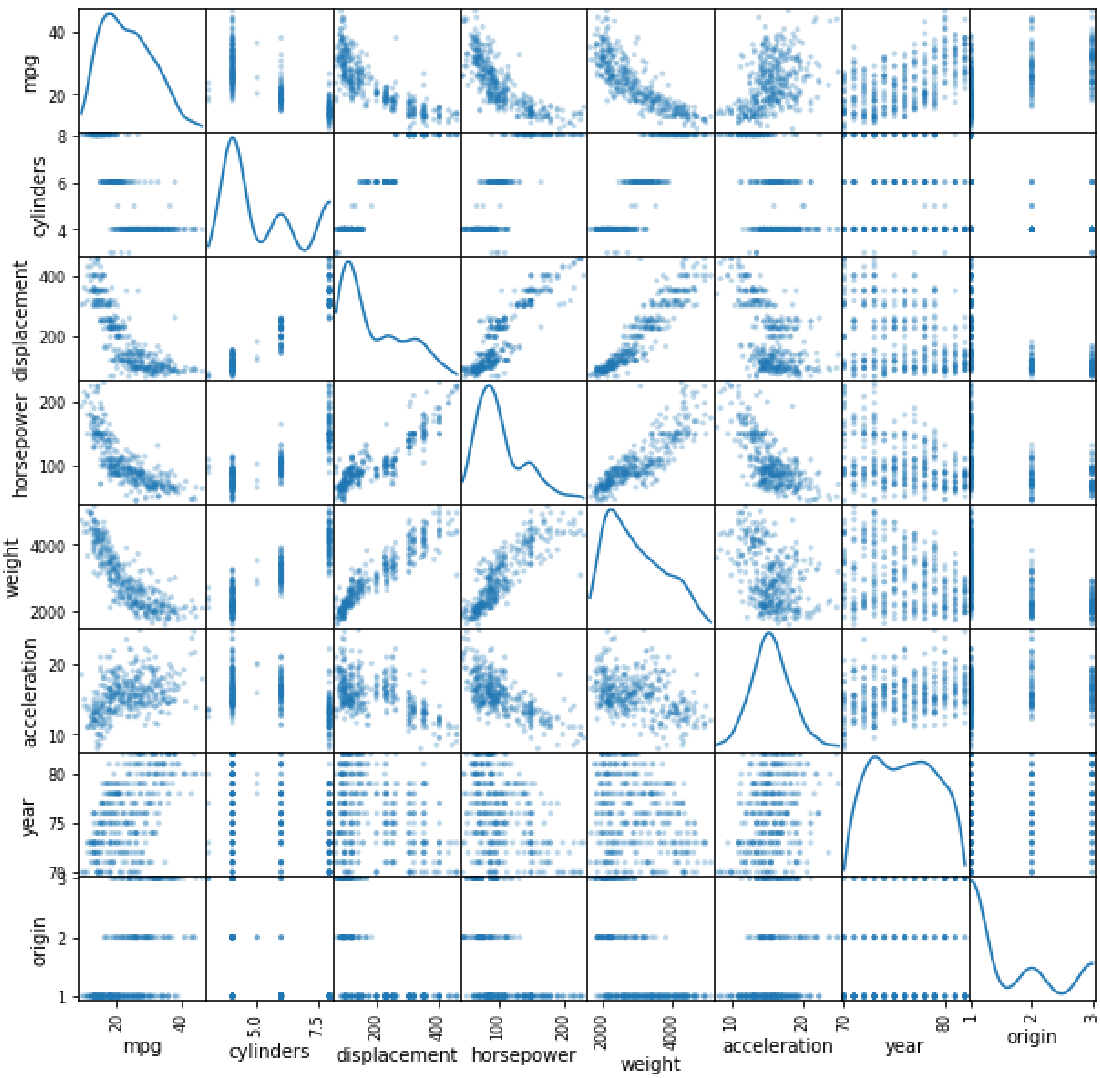
```
In [5]: ## re-read-in of the data
        data = pd.read_csv("Auto.csv", na_values='?')
        print(data['horsepower'].unique())
```

```
[ 130.  165.  150.  140.  198.  220.  215.  225.  190.  170.  160.   95.
   97.   85.   88.   46.   87.   90.  113.  200.  210.  193.   nan  100.
  105.  175.  153.  180.  110.   72.   86.   70.   76.   65.   69.   60.
   80.   54.  208.  155.  112.   92.  145.  137.  158.  167.   94.  107.
  230.   49.   75.   91.  122.   67.   83.   78.   52.   61.   93.  148.
  129.   96.   71.   98.  115.   53.   81.   79.  120.  152.  102.  108.
   68.   58.  149.   89.   63.   48.   66.  139.  103.  125.  133.  138.
  135.  142.   77.   62.  132.   84.   64.   74.  116.   82.]
```

1.1 Problem 1-(b)

Below code presents the scatterplot matrix.

```
In [6]: scatter_matrix(data, alpha=0.3, figsize=(10,10),
                       diagonal='kde')
        plt.show()
```



1.2 1-(c)

Below code presents the correlation matrix for the quantitative variables.

```
In [7]: data.corr()
```

```
Out[7]:                    mpg  cylinders  displacement  horsepower    weight  \
        mpg           1.000000  -0.776260     -0.804443   -0.778427 -0.831739
        cylinders    -0.776260   1.000000      0.950920    0.842983  0.897017
        displacement -0.804443   0.950920      1.000000    0.897257  0.933104
        horsepower   -0.778427   0.842983      0.897257    1.000000  0.864538
        weight       -0.831739   0.897017      0.933104    0.864538  1.000000
        acceleration  0.422297  -0.504061     -0.544162   -0.689196 -0.419502
        year          0.581469  -0.346717     -0.369804   -0.416361 -0.307900
```

```
origin        0.563698  -0.564972     -0.610664   -0.455171 -0.581265

              acceleration      year    origin
mpg               0.422297  0.581469  0.563698
cylinders        -0.504061 -0.346717 -0.564972
displacement     -0.544162 -0.369804 -0.610664
horsepower       -0.689196 -0.416361 -0.455171
weight           -0.419502 -0.307900 -0.581265
acceleration      1.000000  0.282901  0.210084
year              0.282901  1.000000  0.184314
origin            0.210084  0.184314  1.000000
```

1.3 1-(d)

Let us first remove the rows with missing data.

```
In [8]: data_nomiss = data.dropna()
        data_nomiss.shape[0]
```

```
Out[8]: 392
```

```
In [9]: data_nomiss.tail()
```

```
Out[9]:       mpg  cylinders  displacement  horsepower  weight  acceleration  year  \
        392  27.0          4         140.0        86.0    2790          15.6    82
        393  44.0          4          97.0        52.0    2130          24.6    82
        394  32.0          4         135.0        84.0    2295          11.6    82
        395  28.0          4         120.0        79.0    2625          18.6    82
        396  31.0          4         119.0        82.0    2720          19.4    82

             origin             name
        392       1  ford mustang gl
        393       2        vw pickup
        394       1    dodge rampage
        395       1      ford ranger
        396       1       chevy s-10
```

Let us set up the dependent variable (`mpg_nm`) and the regressors (`X_nm`).

```
In [10]: ## nm for not missing
         mpg_nm = data_nomiss['mpg'].values
         X_nm = data_nomiss[['cylinders', 'displacement', 'horsepower', 'weight',
                             'acceleration', 'year', 'origin']].values
```

```
In [11]: num_nm = X_nm.shape[0]
         const_nm = np.ones(num_nm).reshape((num_nm, 1))
         X_nm_w_const = np.hstack((const_nm, X_nm))
         X_nm_w_const
```

```
Out[11]: array([[   1. ,    8. ,  307. , ...,   12. ,   70. ,    1. ],
                [   1. ,    8. ,  350. , ...,   11.5,   70. ,    1. ],
                [   1. ,    8. ,  318. , ...,   11. ,   70. ,    1. ],
                ...,
                [   1. ,    4. ,  135. , ...,   11.6,   82. ,    1. ],
                [   1. ,    4. ,  120. , ...,   18.6,   82. ,    1. ],
                [   1. ,    4. ,  119. , ...,   19.4,   82. ,    1. ]])
```

1.3.1 1-(d)-i.

In the below OLS regression results table, $x1$ indicates cylinders, $x2$ displacement, $x3$ horsepower, $x4$ weight, $x5$ acceleration, $x6$ year, and $x7$ origin (*const* obviously indicates constant). Therefore, the coefficients with statistical significance at $p = 0.01$ (or 1% level) are those on displacement, weight, year, origin, and the constant (that is, $\beta_2, \beta_4, \beta_6, \beta_7$ and β_0 in the question's equation).

```
In [82]: reg_nm = sm.OLS(endog=mpg_nm, exog=X_nm_w_const)
         result1 = reg_nm.fit()
         print(result1.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.821
Model:                            OLS   Adj. R-squared:                  0.818
Method:                 Least Squares   F-statistic:                     252.4
Date:                Tue, 19 Feb 2019   Prob (F-statistic):          2.04e-139
Time:                        21:31:59   Log-Likelihood:                -1023.5
No. Observations:                 392   AIC:                             2063.
Df Residuals:                     384   BIC:                             2095.
Df Model:                           7
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const        -17.2184      4.644     -3.707      0.000     -26.350      -8.087
x1            -0.4934      0.323     -1.526      0.128      -1.129       0.142
x2             0.0199      0.008      2.647      0.008       0.005       0.035
x3            -0.0170      0.014     -1.230      0.220      -0.044       0.010
x4            -0.0065      0.001     -9.929      0.000      -0.008      -0.005
x5             0.0806      0.099      0.815      0.415      -0.114       0.275
x6             0.7508      0.051     14.729      0.000       0.651       0.851
x7             1.4261      0.278      5.127      0.000       0.879       1.973
==============================================================================
Omnibus:                       31.906   Durbin-Watson:                   1.309
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               53.100
Skew:                           0.529   Prob(JB):                     2.95e-12
Kurtosis:                       4.460   Cond. No.                     8.59e+04
==============================================================================
```

```
Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 8.59e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

1.3.2 1-(d)-ii.

Again, referring to the above OLS results table, we see that coefficients on cylinders, horsepower, and acceleration have p-values that are greater than 0.1. Therefore, the said coefficients are **not** statistically significant at the 10% level (which are β_1, β_3, and β_5 on the question's equation).

1.3.3 1-(d)-iii.

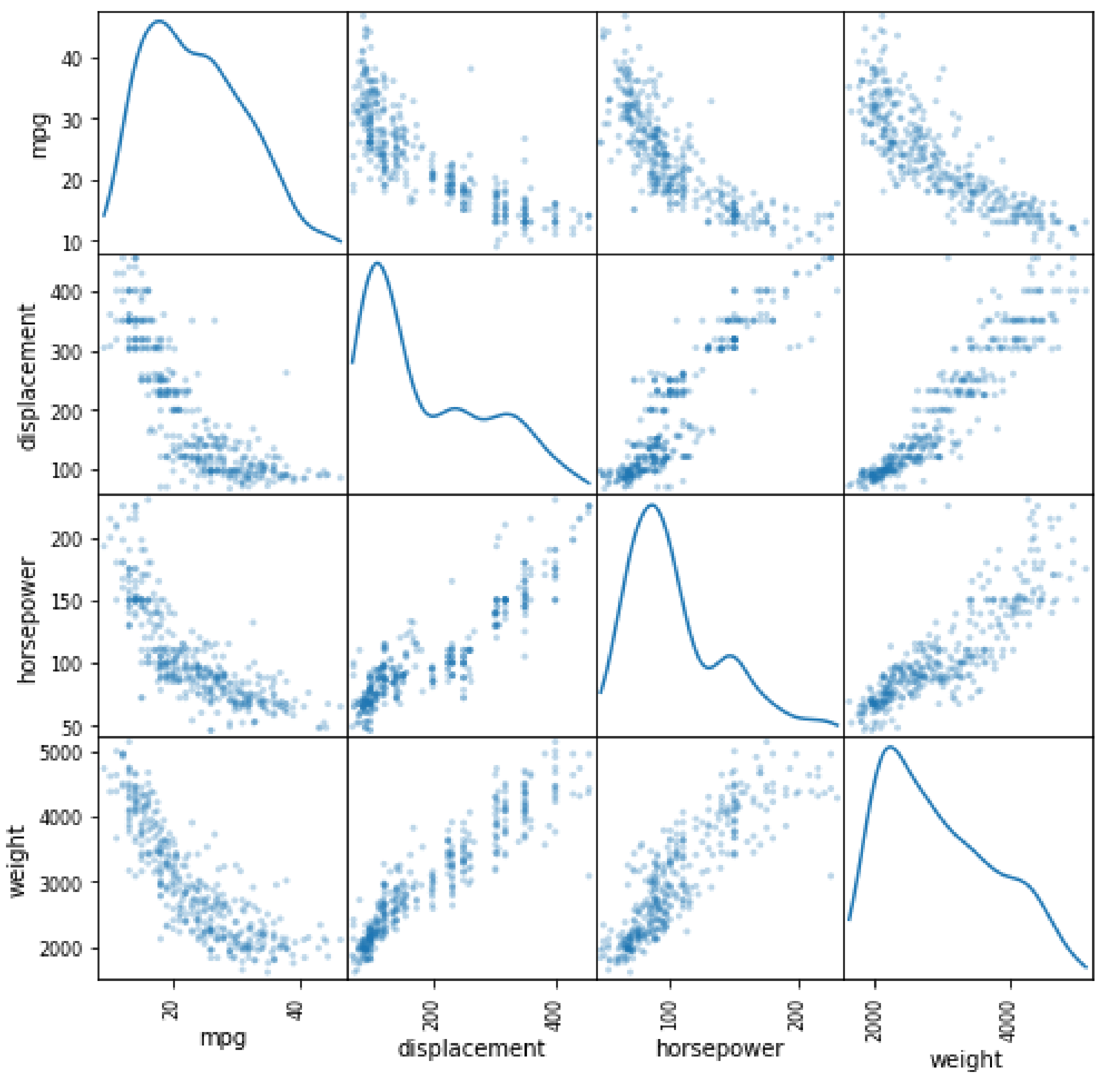
Referring to the above OLS results table once again, the coefficient on year is (approximately) 0.7508. This means that on average, one unit increase in vehicle year (i.e. if a vehicle is newer by a year) is associated with 0.7508 unit increase in miles per gallon, or mpg.

1.4 1-(e)

From the scatterplots in part (b) and also below (which is one that is partially reproduced), we can see that the three variables likely to have a nonlinear relationship with mpg is displacement, horsepower, and weight.

```
In [69]: data_2 = \
             pd.DataFrame(data[['mpg', 'displacement', 'horsepower', 'weight']])
```

```
In [71]: scatter_matrix(data_2, alpha=0.3, figsize=(8,8),
                        diagonal='kde')
         plt.show()
```



1.5 1-(e)-i.

To start, let us create the variables which are the squared terms for displacement, horsepower, weight, and acceleration; these will be denoted `disp_sq`, `hp_sq`, `weight_sq`, and `accel_sq`, respectively.

```
In [83]: ## re-read-in of the data
         data_for_sq = pd.read_csv("Auto.csv", na_values='?')
         data_nomiss_sq = pd.DataFrame(data_for_sq.dropna())
```

```
In [84]: data_nomiss_sq['disp_sq'] = (data_nomiss_sq['displacement']) ** 2
         data_nomiss_sq['hp_sq'] = (data_nomiss_sq['horsepower']) ** 2
         data_nomiss_sq['weight_sq'] = (data_nomiss_sq['weight']) ** 2
         data_nomiss_sq['accel_sq'] = (data_nomiss_sq['acceleration']) ** 2
```

Let us now estimate a new multiple regression model by OLS using the above-created dataset. First we need to designate regressors and the dependent variables, and add the constant term.

```
In [85]: X_nm_sq = data_nomiss_sq[['cylinders', 'displacement', 'horsepower',
                                    'weight', 'acceleration', 'year', 'origin',
                                    'disp_sq', 'hp_sq', 'weight_sq', 'accel_sq']].values
         y_nm_sq = data_nomiss_sq['mpg'].values
```

```
In [89]: num_nm_sq = X_nm_sq.shape[0]
         const_nm_sq = np.ones(num_nm_sq).reshape((num_nm_sq, 1))
         X_nm_w_const_sq = np.hstack((const_nm_sq, X_nm_sq))
         X_nm_w_const_sq
```

```
Out[89]: array([[  1.00000000e+00,   8.00000000e+00,   3.07000000e+02, ...,
                   1.69000000e+04,   1.22780160e+07,   1.44000000e+02],
                [  1.00000000e+00,   8.00000000e+00,   3.50000000e+02, ...,
                   2.72250000e+04,   1.36382490e+07,   1.32250000e+02],
                [  1.00000000e+00,   8.00000000e+00,   3.18000000e+02, ...,
                   2.25000000e+04,   1.18060960e+07,   1.21000000e+02],
                ...,
                [  1.00000000e+00,   4.00000000e+00,   1.35000000e+02, ...,
                   7.05600000e+03,   5.26702500e+06,   1.34560000e+02],
                [  1.00000000e+00,   4.00000000e+00,   1.20000000e+02, ...,
                   6.24100000e+03,   6.89062500e+06,   3.45960000e+02],
                [  1.00000000e+00,   4.00000000e+00,   1.19000000e+02, ...,
                   6.72400000e+03,   7.39840000e+06,   3.76360000e+02]])
```

Below is the regression result. Note that $x1$ indicates cylinders, $x2$ displacement, $x3$ horsepower, $x4$ weight, $x5$ acceleration, $x6$ year, $x7$ origin, $x8$ displacement squared, $x9$ horsepower squared, $x10$ weight squared, and $x11$ acceleration squared (*const* obviously indicates constant).

```
In [90]: reg_nm_sq = sm.OLS(endog=y_nm_sq, exog=X_nm_w_const_sq)
         result2 = reg_nm_sq.fit()
         print(result2.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.870
Model:                            OLS   Adj. R-squared:                  0.866
Method:                 Least Squares   F-statistic:                     230.2
Date:                Tue, 19 Feb 2019   Prob (F-statistic):         1.75e-160
Time:                        21:39:59   Log-Likelihood:                -962.02
No. Observations:                 392   AIC:                             1948.
Df Residuals:                     380   BIC:                             1996.
Df Model:                          11
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
```

```
const         20.1084      6.696      3.003      0.003       6.943      33.274
x1             0.2519      0.326      0.773      0.440      -0.389       0.893
x2            -0.0169      0.020     -0.828      0.408      -0.057       0.023
x3            -0.1635      0.041     -3.971      0.000      -0.244      -0.083
x4            -0.0136      0.003     -5.069      0.000      -0.019      -0.008
x5            -2.0884      0.557     -3.752      0.000      -3.183      -0.994
x6             0.7810      0.045     17.512      0.000       0.693       0.869
x7             0.6104      0.263      2.320      0.021       0.093       1.128
x8         2.257e-05   3.61e-05      0.626      0.532   -4.83e-05    9.35e-05
x9             0.0004      0.000      2.943      0.003       0.000       0.001
x10        1.514e-06   3.69e-07      4.105      0.000    7.89e-07    2.24e-06
x11            0.0576      0.016      3.496      0.001       0.025       0.090
==============================================================================
Omnibus:                       33.614   Durbin-Watson:                   1.576
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               77.985
Skew:                           0.438   Prob(JB):                     1.16e-17
Kurtosis:                       5.002   Cond. No.                     5.13e+08
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 5.13e+08. This might indicate that there are
strong multicollinearity or other numerical problems.
```

1.6 1-(e)-ii.

Before, the adjusted R^2 statistic was found to be 0.818. With the squared terms added, the adjusted R^2 statistic is found to be 0.866. There is a slight improvement, therefore, for adjusted R^2.

1.7 1-(e)-iii.

Prior to adding the squared terms, the statistical significance on the coefficient for displacement was found to be 0.008 (in terms of p-value). However, after adding the squared terms, that changed to 0.408, meaning that it is no longer significant at the 1% level (not even at 10% level as well). In addition, the statistical significance on the coefficient for displacement squared ($x8$ above) is found to be 0.532, which is not significant at the 10% level as well.

1.8 1-(e)-iv.

Prior to adding the squared terms, the statistical significance on the coefficient for cylinders was found to be 0.128 (in terms of p-value). Post-addition of squared terms, that changed to 0.440. Both are not significant at the 10% level, but one can say that the p-value worsened for the cylinders variable after adding the squared terms.

1.9 1-(f)

From below, it is seen that with the parameters specified in the question the predicted mpg will be approximately 38.7321111. Note that model year's input has been written as 99 instead of 1999 as all the observations for the year variable in the dataset are double-digited.

```
In [93]: X_pred = [1, 6, 200, 100, 3100, 15.1, 99, 1, 200 ** 2, 100 ** 2, 3100 ** 2, 15.1 ** 2]
         pred_mpg = result2.predict(X_pred)
         pred_mpg

Out[93]: array([ 38.7321111])
```

2 Problem 2

2.1 2-(a)

Let us create the function for calculating the Euclidean distance.

```
In [64]: def EuclideanDist(pt1, pt2):
             ## note that pt1 and pt2 have to be in numpy array format

             dist = (pt1 - pt2) * (pt1 - pt2)
             dist = dist.sum()
             dist = dist ** 0.5

             return dist
```

```
In [113]: origin = np.array([0, 0, 0])
          obs1 = np.array([0, 3, 0])
          obs2 = np.array([2, 0, 0])
          obs3 = np.array([0, 1, 3])
          obs4 = np.array([0, 1, 2])
          obs5 = np.array([-1, 0, 1])
          obs6 = np.array([1, 1, 1])
          obs_lst = [obs1, obs2, obs3, obs4, obs5, obs6]

          for obs in obs_lst:
              print()
              print(EuclideanDist(origin, obs))
```

```

3.0

2.0

3.16227766017

2.2360679775

1.41421356237

1.73205080757
```

As seen from above, the Euclidean distance from $X_1 = X_2 = X_3 = 0$ to the observations 1 through 6 are 3, 2, $\sqrt{10}$ (or approximately 3.16), $\sqrt{5}$ (or approximately 2.24), $\sqrt{2}$ (or approximately 1.41) and $\sqrt{3}$ (or approximately 1.73).

2.2 2-(b)

According to the above sub-question, the closest point to $(X_1, X_2, X_3) = (0, 0, 0)$ is the fifth observation, $(X_1, X_2, X_3) = (-1, 0, 1)$. Therefore, the KNN prediction with $K = 1$ would be **green**.

2.3 2-(c)

From before, we can see that the closest three points to $(X_1, X_2, X_3) = (0, 0, 0)$ are the second, fifth, and sixth observation, two of which are red and one of which are green. Therefore, the KNN prediction with $K = 3$ would be **red**.

2.4 2-(d)

As in the `LogitKNN` notebook that we looked at in class, if the Bayes decision boundary is highly nonlinear, an extremely large value for K would be underfitting and/or ignoring information. On the other hand, if very small, KNN classifier would be overfitting the data. However, it would be better to have a moderately smaller (than larger) value of K than to have something that is extreme on either side.

2.5 2-(e)

Let us first set up the training data and the target values. In the below code, note that `R` refers to red and `G` refers to green.

```
In [124]: obs_array = np.array(obs_lst)
          print(obs_array)
          y_array = ['R','R','R','G','G','R']
          print()
          print(y_array)
```

```
[[ 0  3  0]
 [ 2  0  0]
 [ 0  1  3]
 [ 0  1  2]
 [-1  0  1]
 [ 1  1  1]]

['R', 'R', 'R', 'G', 'G', 'R']
```

Now let us run the KNN classifier provided from `scikit-learn`, and predict the label for $(X_1, X_2, X_3) = (1, 1, 1)$.

```
In [125]: knn_2 = neighbors.KNeighborsClassifier(n_neighbors=2)
          knn_fit = knn_2.fit(obs_array, y_array)
```

```
In [127]: test_array = np.array([[1, 1, 1]])
          knn_fit.predict(test_array)

Out[127]: array(['G'],
                dtype='<U1')
```

Surprisingly, it is seen that despite $(X_1, X_2, X_3) = (1, 1, 1)$ being the sixth observation labelled with **red**, the classification is actually show to be **green**. To dig deeper, let us examine what the closest observations to $(X_1, X_2, X_3) = (1, 1, 1)$ are.

```
In [114]: for obs in obs_lst:
              print()
              print(EuclideanDist(np.array([1, 1, 1]), obs))


2.44948974278

1.73205080757

2.2360679775

1.41421356237

2.2360679775

0.0
```

It is seen that the sixth (which is the said point itself) and fourth points are the closest ones to $(X_1, X_2, X_3) = (1, 1, 1)$. With one observation being labelled green and the other red, the KNN classifier may not be able to make the decision without some sort of tie-breaking rule. The third observation closest to $(X_1, X_2, X_3) = (1, 1, 1)$ is the second observation, labelled red. Therefore, it must not be that the tie-breaking rule is incorporating one more closest point to the classifier.

```
In [132]: alphabets_a_to_l_without_g = ['A', 'B', 'C', 'D', 'E', 'F', 'H', 'I', 'J', 'K', 'L']
          for alphabet in alphabets_a_to_l_without_g:
              y_array_try = [alphabet, alphabet, alphabet, 'G', 'G', alphabet]
              knn_fit_try = knn_2.fit(obs_array, y_array_try)
              pred = knn_fit_try.predict(test_array)
              print(pred)

['A']
['B']
['C']
['D']
['E']
['F']
['G']
['G']
```

```
['G']
['G']
['G']
```

But notice the code chunk above. It seems that the tie-breaking rule is simply that if a letter (or string variable) precedes another, that letter is chosen as a tie-breaker. Because `R` succeeds `G` in Python and in the actual alphabet, it was not chosen (or at least this is my conjecture). Therefore, while the classifier says it is **green**, we should keep in mind that both are possible answers (or because the point is the sixth observation, perhaps more towards **red** than green).

3 Problem 3

For this problem, let us use the data without missing values once more. I had named it `data_nomiss` above.

3.1 3-(a)

Firstly, let us try to find the median for the variable mpg in `data_nomiss`. It turns out that the said median is 22.75.

```
In [13]: median_mpg = data_nomiss['mpg'].median()
         print(median_mpg)
```

```
22.75
```

Now let us create the variable `mpg_high` as directed from the question.

```
In [14]: data_nomiss = pd.DataFrame(data_nomiss) ## needs to be on a new dataframe
         data_nomiss['mpg_high'] = 0              ## initializing things to 0
         rowcnd = data_nomiss.mpg >= median_mpg   ## row condition for mpg_high
         data_nomiss.loc[rowcnd, 'mpg_high'] = 1 ## above or equal to median = 1
         data_nomiss.tail(10)
```

```
Out[14]:       mpg  cylinders  displacement  horsepower  weight  acceleration  year  \
         387  26.0          4         156.0        92.0    2585          14.5    82
         388  22.0          6         232.0       112.0    2835          14.7    82
         389  32.0          4         144.0        96.0    2665          13.9    82
         390  36.0          4         135.0        84.0    2370          13.0    82
         391  27.0          4         151.0        90.0    2950          17.3    82
         392  27.0          4         140.0        86.0    2790          15.6    82
         393  44.0          4          97.0        52.0    2130          24.6    82
         394  32.0          4         135.0        84.0    2295          11.6    82
         395  28.0          4         120.0        79.0    2625          18.6    82
         396  31.0          4         119.0        82.0    2720          19.4    82

              origin                         name  mpg_high
```

```
387        1  chrysler lebaron medallion         1
388        1             ford granada l          0
389        3            toyota celica gt         1
390        1          dodge charger 2.2          1
391        1            chevrolet camaro         1
392        1             ford mustang gl         1
393        2                   vw pickup         1
394        1               dodge rampage         1
395        1                 ford ranger         1
396        1                  chevy s-10         1
```

Let us prepare for the dependent variable and the regressors.

```
In [15]: y_logit = data_nomiss['mpg_high'].values
         x_logit = data_nomiss[['cylinders', 'displacement', 'horsepower', 'weight',
                                'acceleration', 'year', 'origin']].values

         ## h-stacking the constant terms
         const_nm = np.ones(num_nm).reshape((num_nm, 1))
         x_logit_with_const = np.hstack((const_nm, x_logit))
         x_logit_with_const
```

```
Out[15]: array([[   1. ,    8. ,  307. , ...,   12. ,   70. ,    1. ],
                [   1. ,    8. ,  350. , ...,   11.5,   70. ,    1. ],
                [   1. ,    8. ,  318. , ...,   11. ,   70. ,    1. ],
                ...,
                [   1. ,    4. ,  135. , ...,   11.6,   82. ,    1. ],
                [   1. ,    4. ,  120. , ...,   18.6,   82. ,    1. ],
                [   1. ,    4. ,  119. , ...,   19.4,   82. ,    1. ]])
```

Let us run the logistic regression.

```
In [17]: logit = sm.Logit(y_logit, x_logit_with_const)
         logit_stats = logit.fit()
         print(logit_stats.summary())
```

```
Optimization terminated successfully.
         Current function value: 0.200944
         Iterations 9
                           Logit Regression Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  392
Model:                          Logit   Df Residuals:                      384
Method:                           MLE   Df Model:                            7
Date:                Tue, 19 Feb 2019   Pseudo R-squ.:                  0.7101
Time:                        16:58:23   Log-Likelihood:                -78.770
converged:                       True   LL-Null:                       -271.71
                                        LLR p-value:                 2.531e-79
==============================================================================
```

```
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const        -17.1549      5.764     -2.976      0.003     -28.452      -5.858
x1            -0.1626      0.423     -0.384      0.701      -0.992       0.667
x2             0.0021      0.012      0.174      0.862      -0.021       0.026
x3            -0.0410      0.024     -1.718      0.086      -0.088       0.006
x4            -0.0043      0.001     -3.784      0.000      -0.007      -0.002
x5             0.0161      0.141      0.114      0.910      -0.261       0.293
x6             0.4295      0.075      5.709      0.000       0.282       0.577
x7             0.4773      0.362      1.319      0.187      -0.232       1.187
==============================================================================

Possibly complete quasi-separation: A fraction 0.14 of observations can be
perfectly predicted. This might indicate that there is complete
quasi-separation. In this case some parameters will not be identified.
```

Note that $x1$, $x2$, $x3$, $x4$, $x5$, $x6$, and $x7$ in the table above refer to cylinders, displacement, horsepower, weight, acceleration, year, and origin respectively (and *const* referring to the constant term). We see that among the regressors (excluding the constant term), those that have statistically significant coefficients at the 5% (or p-value of 0.05) level are **weight** and **year**.

3.2 3-(b)

I have split the data into training and testing datasets as directed by the question using below code chunk.

```
In [29]: X_train, X_test, y_train, y_test = \
             train_test_split(x_logit_with_const, y_logit, test_size = 0.5, random_state=10)
```

3.3 3-(c)

```
In [27]: LogReg = LogisticRegression()
         fitted_train_LogReg = LogReg.fit(X_train, y_train)
         fitted_train_LogReg.coef_

Out[27]: array([[-0.07022621, -0.67604786,  0.00608728, -0.03802261, -0.00505466,
                 -0.13489425,  0.29986833, -0.15403736]])
```

And as seen from the above code chunk's output, we can see that:

$$(\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7) \approx (-0.0702, -0.6760, 0.0061, -0.0380, -0.0051, -0.1349, 0.2999, -0.1540)$$

in which the numbers have been rounded up to the nearest ten-thousandth.

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0.1 3-(d)

After producing the predicted dependent variables using the code chunk directly below, I have also produced the confusion matrix as follows.

```
In [33]: y_pred = LogReg.predict(X_test)
```

```
In [34]: confusion_matrix_ = confusion_matrix(y_test, y_pred)
         print("Confusion matrix:")
         print(confusion_matrix_)
```

```
Confusion matrix:
[[86 13]
 [12 85]]
```

As seen from below, in the test data for the dependent variable, there were 99 observations with `mpg_high = 0` and 97 observations with `mpg_high = 1`. Therefore, we can see that out of the 99 with `mpg_high = 0` in the test data, the logistic regression classifier has correctly classified 86 of them, which is approximately 86.87%. On the other hand, out of the 97 with `mpg_high = 1` in the test data, the logistic regression classifier has correctly classified 85 of them, which is approximately 87.63%. This would be comparison via "recall" or "true positive rate."

```
In [35]: print("# of test y-data = 0:", sum(y_test == 0), "; # of test y-data = 1:", sum(y_tes
```

```
# of test y-data = 0: 99 ; # of test y-data = 1: 97
```

```
In [36]: print("Recall for low mpg")
         print(confusion_matrix_[0][0] / sum(y_test == 0) * 100)
         print()
         print("Recall for high mpg")
         print(confusion_matrix_[1][1] / sum(y_test == 1) * 100)
```

```
Recall for low mpg
86.8686868687

Recall for high mpg
87.6288659794
```

On the other hand, one can also calculate the "precision," in which the model has predicted 98 observations as having low mpg, but only 86 are actually so (approximately 87.75%). Similarly, 98 observations are classified as having high mpg, but only 85 are actually so (approximately 86.73%). So in terms of recall, the model is very slightly better at predicting high mpg, but with precision it is slightly better at predicting low mpg.

However, if one were to calculate the $F1$ score (i.e. the harmonic mean of precision and recall), we would have 87.31% for low mpg and 87.18% for high mpg (approximate values). This, again, is very close to one another. Therefore, I conclude that the model is approximately equally good at prediction of low and high mpg.