

Cribs: Regression modeling for home price

Technical Summary Report

Jun Tae Son | Omer Saif Cheema | Yusheng Zhu

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Abstract:

The dataset we used in the analysis had historical data on house sales in King County, Washington from May 2014 to May 2015. The goal of the analysis is to generate the best model for predicting home sale price in King county.

We included the following features in the analysis; date of house sales, location of the property, the number of bedrooms and bathrooms, the size of the property, floors, waterfront, view index, conditions of house, levels of construction and design, renovation history and basement. We analyzed how influential these features are on house price and conducted prediction test using our final model. We assumed that the prices of the houses are only affected by the variables used in the data set and factors like interest rates, property taxes other political and economic factors stay constant.

Our final model indicates that house prices in King County are most influenced by interior area of a house, the location of a house and whether a house has a basement or not. Finally, we are excited to declare that the predictors in our final model can explain over 75 percent of variation in house prices in King County.

Introduction:

In this report, we built a multivariate regression model of house prices using dataset composed of 21,613 houses in King County, Washington. The goal of the analysis is to generate the best fitted regression model for predicting home sale price in King county.

Before starting the analysis, we hypothesized that location of the house, number of bedrooms and the quality of construction of a house are the biggest predictors of house prices. Also, during the analysis we assume that the prices of the houses are only affected by the variables used in the data set and factors like interest rates, property taxes other political and economic factors stay constant.

Finally, you can see the name and the description of the variables from the data set in the appendix (see table 1). Below is the methodology we used to find the best regression model for predicting house prices in King County.

Methodology:

The data set used in the analysis was obtained from Kaggle.com and you can download it from the following link: <https://www.kaggle.com/harlfoxem/housesalesprediction>. The data set is composed of 21 attributes and 21,613 rows of observations. The data was imported by using the Proc Import function in SAS and then, 1500 rows were randomly taken out of the original data set using different seed values by all three members.

Preprocessing:

- Recode Qualitative Covariates and Create Dummy Variables:

- A. **DATE:** Every instance under the date features in the original dataset had 15 digits. We reduced the number of digits for every instance from 15 to 8 to make YYYYMMDD format, and then we assigned each date to one of the four quarters in a year; first, second, third and fourth quarter. Finally, by making Q1 as the base level, we created three dummy variables: Q2, Q3 and Q4 (see table 2 in the appendix).
- B. **Location:** The location of each house purchased in the dataset was given in terms of its coordinates. Firstly, we calculated the medians for longitude and latitude of all the instances. Based on the median values for longitude and latitude, we divided the section from north(N) to south(S), and west(W) to east(E). We assigned N and W as the base level (where S=0 and E=0), and generated three location dummy variables NE, SW, and SE. Please see table 3 in the appendix for more detail.
- C. **Grade:** The variable grade in the dataset could have a value from 1 to 13. We divided the grade index into four sections: 1-3 as short of building construction and design, 4-6 as below average level of construction and design, 7-10 as above average level of construction and design, and 11-13 as high-quality level of construction and design. (see table 4 for more details in the appendix).
- D. **View:** The view index in the dataset had five grades from 0 to 4. So, we created binary dummy variable of view_good which had a value of 1 when the view index was 3 or 4, and a value of 0 otherwise.
- E. **Floor:** The range of floor variable in the dataset was between 1 and 3.5. We calculated the mean value of the floor (1.5) and created a dummy variable, floor_h. If the floor>1.5 then floor_h=1.
- F. **Condition:** The condition index in the dataset had five grades from 1 to 5. So, we created the dummy variable condition_good which had a value of 1 when the condition feature had a value greater than 4, and a value of 0 otherwise.
- G. **Renovated:** The original dataset had yr_renovated variable which contained renovation history for the specific property. We created binary dummy variable 'renovated' which had a value of 1 when the property has a renovation record.

- H. **Basement:** The original dataset had `sqft_basement`, the size of basement. We created a binary dummy variable 'basement' which had a value of 1 when the property had a basement.
- **Transformation of Variables:**
We applied transformation method on dependent and independent variables that violated one of regression assumptions: linearity, constant variance, independence, and normality. We transformed dependent variable 'price' into 'ln_price', and independent variables 'sqft_living', 'sqft_lot' and 'sqft_above' into 'ln_sqft_living', 'ln_sqft_lot' and 'ln_sqft_above' using log transformation. Please see the next section for more details.

Model Approach:

- A. **Multicollinearity:** After the preprocessing, we created the Pearson correlation coefficient table to see the relationship among predictors, and ran the regression model with VIF option to check whether there was multicollinearity between independent variables. We removed predictors with multicollinearity problem if we found any.
- B. **Interactive Variables:** We created the interactive variables by combining two different predictors that seemed to have a joint effect. If there was multicollinearity between the interactive variables and its parts, we centered the quantitative part(s) of the interactive variables and created new centered quantitative variable(s) and new centered interactive variables.
- C. **Outliers and Influential Points:** We ran the regression model with option "r" and "influence" to check whether there were any outliers and influence points or not. If there were outliers and influence points, we kept on removing them until the R-square and Adj R-square stopped increasing.
- D. **Data splitting:** After removing outliers and influence points, we split the sample into a training and testing set using 75/25 ratio and created a separate training set and a testing set.
- E. **Model selections:** we ran two different selection methods on our full regression model to find the best fitted model. If two different models were given by the selection methods, we choose the one with the least number of predictors or/and with the highest adj R-square.
- F. **Prediction test using two observations:** We copied the first two rows of predictors from the training set and merge them with the testing set and created a new dataset called predict. Then we ran the regression model with that dataset and calculated the predicted price, confidence interval, and prediction interval of all the data.

Validation Approach:

- A. **Model validity test:** Based on the model selection results on training set, each team member generated their own regression model. And we conducted goodness of fit test and compared the number of predictors, RMSE, and adjusted R-square values.
- B. **Predictive Performance Test:** Each team member used their train/test split dataset created previously and added a new response variable column called new_y. Then we added the observed values from ln_price values from the training set in the new_y and left the new_y values for test data empty. Then we ran the regression model for training set and predicted values on the test set. Finally, we calculated the R-square, adj. R-square, RMSE, MAE, cross validated R-square for all three models and selected the final model that showed the best performance.

Bonus Analysis (Partial Correlation) (Omer):

The Pearson correlations shows that some of the predictors are highly correlated with the response. However, as we have seen, some predictors are also highly correlated with other predictors, making it difficult to determine which predictors are actually important.

Partial correlations allow us to see correlations between each predictor and the response, after adjusting for the other predictors. The partial methodology was copied from the report called Housing Prices Multiple Regression – Multicollinearity and Model Building written by M. Smith. You can access the report from the link given in the references and you can see that Omer used partial correlation by looking at his code.

The Pearson correlation showed that the correlation of ln_price against ln_sqft_above, ln_sqft_lot and ln_sqft_above is 0.654, 0.1549 and 0.5632. That shows that ln_price has positive moderate relationship with ln_sqft_above; positive weak relationship with ln_sqft_lot and positive moderate relationship with ln_sqft_above. However, partial correlation for ln_price against ln_sqft_above, ln_sqft_lot and ln_sqft_above is 0.4026, -0.063 and -0.01033. This means that the true relationship between ln_sqft_living is still positive and moderate but the relation between ln_sqft_lot and ln_sqft_above is actually weak and negative (see figure 1 for more detail)

Analysis, Results and Findings:

Regression Assumption

The response variable in the initial regression model was price. The independent variables in the model were bathrooms, bedrooms, sqft_living, sqft_lot, sqft_above, floor_h, waterfront, view_good, condition_good, grade_b, grade_a, grade_h, renovated, basement, NE, SW, SE, Q2, Q3, Q4.

```
PROC REG;  
model price =bathrooms bedrooms sqft_living sqft_lot sqft_above  
          floor_h waterfront view_good condition_good grade_b grade_a  
          grade_h renovated basement NE SW SE Q2 Q3 Q4;  
run;
```

However, distribution of the dependent variable in the initial model was positively skewed as you can see in the figure 2 in the appendix. Since the normality assumption was violated (see figure 3), we decided to create a new dependent variable, ln_price, by taking the natural log of the variable price. By doing that the dependent variable in the model became normally distributed (see figure 4 and 5). According to the scatterplot matrix of dependent variables and continuous predictors (see figure 6), sqft_living and sqft_above appear to have moderate positive linear relationship with ln_price. And we couldn't find linearity relationship between dependent variable and sqft_lot. Residual plots of all three continuous predictors showed a discernible pattern (see figure 7). To solve the constant variance and independence problem, we applied log transformation and created new predictors: ln_sqft_living, ln_sqft_lot, and ln_sqft_abov. These new variables didn't violate constant variance and independence assumption and improved linearity with ln_price (see figure 8).

After the transformation of the dependent variable and the three continuous variables the initial regression model looked like below:

```
PROC REG;  
model ln_price =bathrooms bedrooms ln_sqft_living ln_sqft_lot  
ln_sqft_above floor_h waterfrontview_good condition_good grade_b  
grade_a grade_h renovated basement NE SW SE Q2 Q3 Q4;  
run;
```

Multicollinearity

There are two ways of determining multicollinearity among the independent variables.

- If correlation coefficient value of two independent variables has more than 0.9, we may conclude that there is multicollinearity problem.
- If two independent variables have higher than 10 variance influence, or lower than 0.1 tolerance value, then we may conclude that there is multicollinearity problem.

In this analysis, we generated Pearson correlation coefficient table and calculated variance influence to detect multicollinearity among the predictors. The correlation coefficient value between ln_sqft_living and ln_sqft_above was about 0.9 and VIF values for ln_sqft_living and ln_sqft_above were also higher than 10. We also found multicollinearity problems on grade dummy variables. Grade_b and grade_a had a

high correlation value close to 0.9 and VIF values for all three grade dummy variables were larger than 10 (see figure 9). To solve the multicollinearity from our model, we took out one of the variables with multicollinearity and rerun the model to see how it changed (see table 5)

Interaction variables

In Jun's model, he created the interactive variables between the location of the house and the size of the living area (minus the basement). In the model, he created three new interactive variables, `above_ne`, `above_sw` and `above_se`, by multiply `ln_sqft_above` with NE, SW and SE. However, when he rerun the regression model it showed that there was multicollinearity (VIF was greater than 10) between the interactive variables and location variables (NE, SW and SE) (See figure 8). To fix the multicollinearity problem, we centered `ln_sqft_above` and created a new variable called `ln_sqft_above_c`. Afterwards, we created new interactive variables by multiplying NE, SW and SE with `ln_sqft_above_c` and the new interactive variables were not multicollinear with NE, SW and SE anymore (see figure 10 & 11).

Omer also made the interactive variables between location of the house but he used the complete interior area of the house (`ln_sqft_living`). In his model, he created three new interactive variables, `ln_sqft_living_NE`, `ln_sqft_living_SW` and `ln_sqft_living_SE`, with NE, SW and SE. However, just like Jun's interactive variable, Omer's variables were also multicollinear with location variables because their VIF value was greater than 10. He fixed the multicollinearity problem by centering `ln_sqft_living` (`ln_sqft_living_c`) and created centered interactive variables: `ln_sqft_living_NE_c`, `ln_sqft_living_SW_c` and `ln_sqft_living_SE_c` (look at figure 12 and 13 for more detail).

Yusheng create 7 interaction variables in her model. 3 interaction variables were made between `ln_sqft_living` and the 3 location dummy variables---by multiplying `ln_sqft_living` with NE, SW and SE respectively, based on the assumption that the Square footage of the apartments interior living space and the house location have a joint effect on the house value. Same as Jun and Omer, it turned out that the VIF value of the 3 interaction variables appear to be much higher than 10 in her model. To fix this multicollinearity problem, she centered the `ln_sqft_living` into `ln_sqft_living_c` by subtracting the mean from the original data and recomputed the interaction variables--`ln_sqft_living_NE_c`, `ln_sqft_living_SW_c` and `ln_sqft_living_SE_c` in the new model, and the VIF of these interaction terms have all dropped to a value below 10 afterwards (see figure 14 & 15).

Apart from the joint effect assumption , Yusheng made another assumption that interaction terms should be added between predictors whose correlation coefficients is greater than 0.5, From the Pearson correlation coefficient table (Figure 16 & 17), 4 pairs : $\text{Cov}(\text{bathrooms}, \text{bedrooms}) = 0.53987$, $\text{Cov}(\text{bedrooms}, \text{ln_sqft_living}) = 0.64290$, $\text{Cov}(\text{bathrooms}, \text{ln_sqft_living}) = 0.74995$, $\text{Cov}(\text{bathrooms}, \text{floor_h}) = 0.6007$ can be found to satisfy the assumption .Thus, creating another 4 interaction variables accordingly, and then adding each interaction variable to the model one by one checking the P-value to test if each interaction variable is significant enough to stay in the model. Next, using the same mean centering method on the continuous variables to fix the multicollinearity problem.

Outliers and Influential points

An outlier is an observation that lies an abnormal distance from other values in a random sample from a population. There is an outlier in a dataset when its studentized residual values is greater than or less than +3 or -3.

Influence point is an observation which has excessive influence on the fit of the regression model. We used Cook's Distance to identify influential points in this dataset. In this model an observation is an influential point if it's Cook's Distance was greater than 0.0003. See table 6 to look at how many observations each member took out.

Splitting the data to training set and testing:

Before running the selection methods to find the best model for each member, we split our sample dataset to a training and testing set. 75% of the sample data was randomly assigned to a training set and the rest of the 25% data was stored in a testing set. Then each teammate ran the selection methods on the training set to produce the final model for each member.

Model Selection Methods

Jun: (see table 7)

Jun used Backward and Stepwise method to get the best set of predictors in the final regression model. After applying two selection methods, he removed 5 predictors but Adj-R-square and RMSE stayed the same (0.7307 and 0.2357 respectively). The possible reason for this is because he already got the model with low variance by eliminating 153 outliers and influential points in the previous section.

But when Jun ran the testing model using the set of 16 predictors driven from backward and stepwise selection methods, 'renovated' and 'NW' variables turned out to be not significant. Thus, he decided to remove those two insignificant predictors and finalized his model.

```
proc reg data=house_train;  
model ln_price =bedrooms ln_sqft_above_c waterfront view_good condition_good  
grade_b grade_h basement SW SE Q4 above_NE_c above_SW_c above_SE_c/stb  
run;
```

Omer:

Before running the two different selection methods, Omer had 21 predictors in his regression model. Running the Stepwise method gave Omer 14 predictors on training set and running the ADJR SQ on training set gave omer 16 predictors (see Table 8 for more detail about the selection methods and the final model). The final model Omer selected had 14 predictors given by the Stepwise method. However, when Omer ran his final model with the testing set 5 predictors became insignificant (as you can see in Figure 16). He had to take out variables, bathrooms, bedrooms, waterfront, NE and Q2. After taking out the variables Omer's real final model had only 9 predictors, as you can see below:

```

*Testing Final model after taking out bathrooms, bedrooms, waterfront, NE and Q2;
TITLE 'checking final model with testing set after taking out Q2';
PROC REG data=houseP_test;
model ln_price=ln_sqft_living_c view_good condition_good
      grade_b grade_a renovated SW SE ln_sqft_living_SE_c/stb vif;
run;
title;

```

Yusheng:

Before the selection method, Yusheng had 25 predictors in total. Running the forward selection method gave Yusheng 15 predictors on training set and running the Cp method on training set gave Yusheng 17 predictors. (See table 9 for more detail about the selection methods and the final model), Yusheng chose the model of Cp method as her final model because it has a better R square than another model. Then, she removed bathrooms_c and condition_good from her final model as they are not significant variables on training set.

However, when Yusheng ran her final model on testing set, 6 predictors became insignificant, She had to take out 6 insignificant variables, bedrooms_c, grade_b, grade_a, renovated, NE and Q2. So Yusheng had only 9 predictors remaining in her final model.

```

* remove insignificant predictors in testing set;
* remove bedrooms_c,grade_b,grade_a,renovated,NE,Q2;
title Final Model on testing;
proc reg data=house_test;
model ln_price = ln_sqft_living_c waterfront view_good SW SE bb_c bathliving_c bedliving_c living_SE_c/vif stb;
run;

```

Validation and Performance Diagnostics on the Regression Model

Based on the model selection analysis, each team member generated the following regression models:

Model 1 (Jun): $\ln_price = 13.3336 - 0.0746 \cdot \text{bedrooms} - 0.6983 \cdot \ln_sqft_above_c + 0.7534 \cdot \text{waterfront} + 0.3226 \cdot \text{view_good} + 0.1536 \cdot \text{condition_good} - 0.1123 \cdot \text{grade_b} + 0.4532 \cdot \text{grade_h} + 0.3768 \cdot \text{basement} - 0.5468 \cdot \text{SW} - 0.3844 \cdot \text{SE} - 0.1054 \cdot \text{Q4} - 0.2190 \cdot \text{above_NE_c} + 0.2811 \cdot \text{above_SW_c} - 0.2688 \cdot \text{above_SE_c}$

Model 2 (Omer): $\ln_price = 13.6272 - 0.6689 \cdot \ln_sqft_living_c + 0.3597 \cdot \text{view_good} + 0.15775 \cdot \text{condition_good} - 0.5265 \cdot \text{grade_b} - 0.4731 \cdot \text{grade_a} + 0.1918 \cdot \text{renovated} - 0.4127 \cdot \text{SW} - 0.3761 \cdot \text{SE} - 0.2520 \cdot \ln_sqft_living_SE_c$

Model 3 (Yusheng): $\ln_price = 13.16315 - 0.6982 \cdot \ln_sqft_living_c + 0.6044 \cdot \text{waterfront} + 0.4120 \cdot \text{view_good} - 0.4693 \cdot \text{SW} - 0.4133 \cdot \text{SE} - 0.0906 \cdot \text{bb_c} + 0.2633 \cdot \text{bathliving_c} + 0.1038 \cdot \text{bedliving_c} - 0.2716 \cdot \text{living_SE_c}$

To compare model validity and predictive performance, we compared the above fitted models as follows.

<Model validity test on training set>

	<i>M1</i>	<i>M2</i>	<i>M3</i>
<i>The number of predictors</i>	14	9	9
<i>Goodness of Fit</i>	p<0.001	p<0.001	p<0.001
<i>RMSE</i>	0.23755	0.29616	0.30424
<i>R square</i>	0.7301	0.6762	0.6637
<i>Adjusted R square</i>	0.7263	0.6743	0.6610

Based on Goodness of Fit test, we may conclude that all three models fitted the set of observations well enough. Even though the number of predictors on model 1 was higher than the rests, model 1 had the lowest RMSE and the highest adjusted R square value.

<Predictive performance test on testing set>

	<i>M1</i>	<i>M2</i>	<i>M3</i>
<i>RMSE</i>	0.2410	0.2993	0.2722
<i>MAE</i>	0.1962	0.2436	0.2209
<i>R square</i>	0.7665	0.6933	0.7179
<i>Adjusted R square</i>	0.7563	0.6857	0.7109
<i>Cross-validated R square</i>	0.0262 (<0.3)	0.0153 (<0.3)	0.0494 (<0.3)

Cross-validating statistics for all three models were less than 0.3, indicating that all of them were good for prediction. However, M1 had the lowest root mean squared error and mean absolute error. Adjusted R square for M1 was the highest among the regression models: 75.63% of the variability in home sale price can be explained by M1. Since M1 performed better on prediction, we decided to use M1 as our final regression model.

Interpretation of influential parameters

In this section, we are going to interpret regression coefficients of three predictors in the final model that had the strongest influence on home sale price (see Figure 19). When we determine the strongest predictor on the response variable, we need to compare standard estimates among predictors. In the final model, $\ln_sqft_above_c$, SW, and basement variables were determined as the most influential predictors on the response because they had the highest standard estimates.

Sqft_above: Interpretation of parameter estimate for $\ln_sqft_above_c$ was complicated for the following reasons

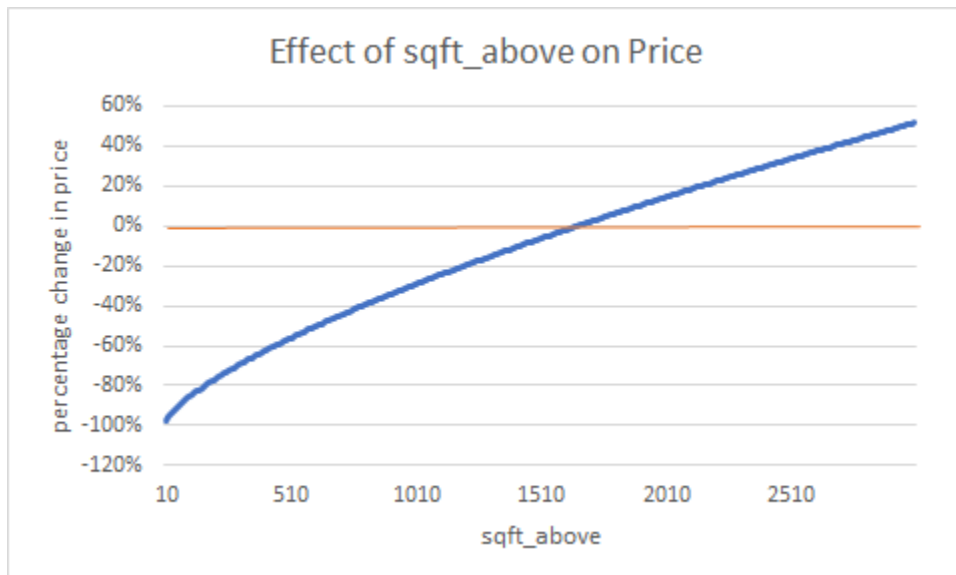
1. We applied log transformation on both home sale price and $sqft_above$ because distribution of home sale price was not normally distributed, and residual plot for $sqft_above$ was not randomly scattered.
2. Price and $sqft_above$ does not have linear relationship.
3. We centered \ln_sqft_above to solve multicollinearity problem with interaction variables.

When we assigned $sqft_above$ as 1, home sale price decreased by 99.43%. But whenever we input higher $sqft_above$ value, the percentage of price significantly increased. And the percentage of price change became positive when $sqft_above$ reached 1,654. From this analysis, we could determine that home sale price starts to increase if the size of living space without basement is larger than 1,654 sqft and the property is located in north west part of King county (see figure 20).

$$100 * (e^{(-0.6983 * (7.4107 - \ln(1)))} - 1) = -99.43\%$$

$$100 * (e^{(-0.6983 * (7.4107 - \ln(1654)))} - 1) = 0.01\%$$

Figure 20 (Effect of $sqft_above$ on price):

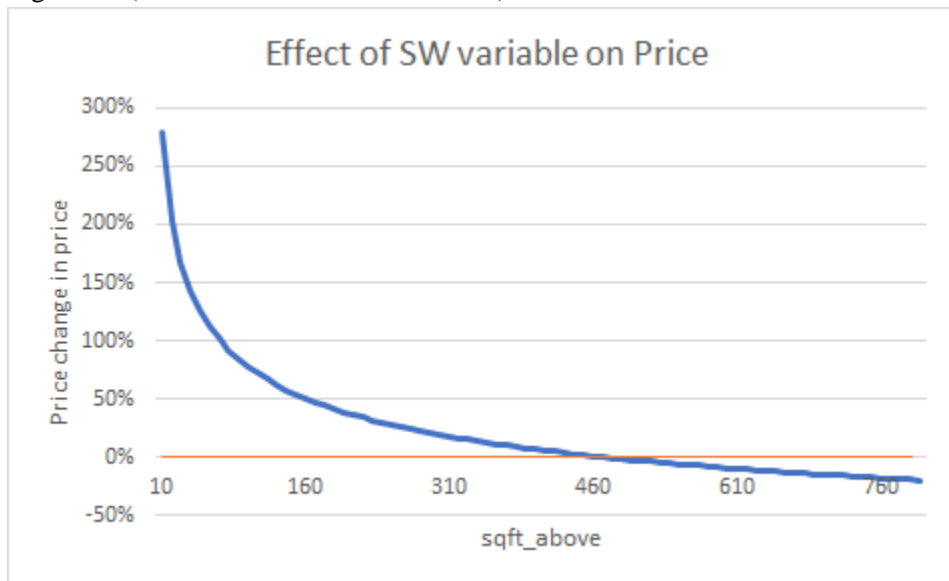


SW: We cannot interpret SW variable without considering the interaction variable above_SW because above_SW is the interactive variable of SW. When we increase the sqft_above in the SW region by 1, the home sale price increased by 660.68%. And the percentage of price change became negative when sqft_above reached 473. Therefore, we may conclude that the effect on SW variable on home sale price starts to decrease if the size of living space without basement is larger than 473 sqft and the property is located in south west part of King county (see figure 21).

$$100*(e^{(-0.5468*1)}-1) + 100*(e^{(0.2811*(7.4107-\ln(1))*1}-1) = -42.12\% + 702.80\% = 660.68\%$$

$$100*(e^{(-0.5468*1)}-1) + 100*(e^{(0.2811*(7.4107-\ln(473))*1}-1) = -42.12\% + 42.08\% = -0.04\%$$

Figure 21 (Effect of SW variable on Price):



Basement: basement is positively associated with home sale price. Assuming all other variables constant, home sale price increases by 45.76% if the property has a basement.

$$100*(e^{0.3768} - 1) = 100*(1.4576 - 1) = 45.76\%$$

Prediction results of two observations

We copied first two observations from training set and created new dataset called “pred”. And we combined pred data and testing set to predict the home sale price for the two instances. The result is shown as below.

Obs	Dependent Variable	Predicted Value	Std Error Mean Predict	95% CL Mean		95% CL Predict		Residual
1	12.1495 .	12.5380	0.0522	12.4353	12.6407	12.0624	13.0136	-0.3885 .
2	12.7038 .	13.1537	0.0454	13.0643	13.2430	12.6808	13.6266	-0.4499 .

Since the actual values of \ln_price fall in prediction interval, we may determine that the predictions were correct for both instances. Now we want to see the predicted value of price, thus we converted \ln_price to price by applying inverse function of $\log()$.

Obs	Dependent Variable	Predicted Value	Std Error Mean Predict	95% CL Mean		95% CL Predict		Residual
1	\$188,999.57	\$278,730.32				\$173,234.25	\$448,471.31	.
2	\$328,995.71	\$515,916.39				\$321,515.17	\$827,860.55	.

- Predicted value of the first observation is \$278,730.32 and prediction interval is in range between \$173,234.25 and \$448,471.31.
- Predicted value of the second observation is \$328,995.71 and prediction interval is in range between \$321,515.17 and \$827,860.55.

Although actual \ln_price and predicted \ln_price appeared to be similar, we noticed that actual prices in dollar deviated significantly from the predicted values. Therefore, we need further studies on real estate industry such as effects on global economy, regulatory from government, education institutions in the region, etc.

Future work:

Since the log transformation has caused the inaccuracy on model predictions, we may consider trying more transformation methods to improve the model's accuracy in the future. Unavoidably, there were some odd revelations about the influence of bedrooms on house prices in our model, which is violating our hypothesis mentioned in the introduction of this report. We want to investigate on a whole dataset or other large housing dataset to confirm this finding. Besides, latitude and longitude analysis is currently beyond the scope of this class, we may need to use third party software like Tableau to analyze latitude and longitude in the future.

Conclusion:

In this project, we presented a process of building a multivariate regression model for a simplified problem of estimating housing prices in King County, Washington. From the above model diagnostics, we could conclude that all the predictors in our final model are significant and there is no problem from the residual examinations. So, it is a valid model. Except for the intercept predictor, there are 14 predictors in the model. The model can explain 75.63% variations of the housing prices. Also, Our model has been proved to have good prediction performance with its cross validated R square being 0.0262 below to 0.3 . Especially, the predictor number of bedrooms with a very small coefficient and standardized estimates doesn't seem to have significant influence on home value. To conclude, the housing prices are closely related to the size of the house, the condition of construction and design of the house, house location and with or without basement. The number of bedrooms also affects the home prices but to a very limited degree.

However, we should also be aware that this is a simplified model and we only considered the information provided in this dataset alone. According to the hedonic pricing model suggested in our references, more reliable analysis should include both internal factors such as number of stories, heating/AC system, and age of the house, and external factors such as property taxes, school district, and air quality.

Appendix:

All relevant output should be included here and cross referenced in your Analysis, Results & Findings section.

Table 1:

Dependent variable	Description
Price	Price of each home sold

Independent variable	Description
ID	Unique ID for each home sold
Date	Date of the home sale
Bedrooms	Number of bedrooms
Bathrooms	Number of bathrooms, where 0.5 counts for a room with a toilet but no shower
Sqft_living	Square footage of the apartments interior living space
Sqft_lot	Square footage of the land space
Floors	Number of floors
Waterfront	A dummy variable for whether the apartment was overlooking the waterfront or not
View	An index from 0 to 4 of how good the view of the property was
Condition	An index from 1 to 5 on the condition of the apartment
Grade	An index from 1 to 13, where 1-3 falls short of building construction and design, 7 has an average level of construction and design, and 11-13 have a high-quality level of construction and design
Sqft_above	The square footage of the interior housing space that is above ground level
Sqft_basement	The square footage of the interior housing space that is below ground level
yr_built	The year the house was initially built

yr_renovation	The year of the house's last renovation
zipcode	What zipcode are the house is in
lat	Latitude
long	Longitude
Sqft_living15	The square footage of interior housing living space for the nearest 15 neighbors
Sqft_lot15	The square footage of the land lots of the nearest 15 neighbors

Table 2:

	Q2	Q3	Q4
First Quarter	0	0	0
Second Quarter	1	0	0
Third Quarter	0	1	0
Fourth Quarter	0	0	1

Table 3:

	NE	SW	SE
S =0 and E=0	0	0	0
S =0 and E=1	1	0	0
S=1 and E=0	0	1	0
S=1 and E=1	0	0	1

Table 4:

	grade_b	grade_a	grade_h
grade (1 to 3)	0	0	0
grade (4 to 6)	1	0	0
grade (7 to 10)	0	1	0
Grade (11 to 13)	0	0	1

Figure 1 (Omer's Partial Correlation)

Pearson Correlation Coefficients, N = 1500 Prob > r under H0: Rho=0				
	ln_price	ln_sqft_living	ln_sqft_lot	ln_sqft_above
ln_price	1.00000	0.65433 <.0001	0.15499 <.0001	0.56329 <.0001
ln_sqft_living	0.65433 <.0001	1.00000	0.30850 <.0001	0.87096 <.0001
ln_sqft_lot	0.15499 <.0001	0.30850 <.0001	1.00000	0.32287 <.0001
ln_sqft_above	0.56329 <.0001	0.87096 <.0001	0.32287 <.0001	1.00000

Pearson Partial Correlation Coefficients, N = 1500 Prob > r under H0: Partial Rho=0		
	ln_price	ln_sqft_living
ln_price	1.00000	0.40626 <.0001
ln_sqft_living	0.40626 <.0001	1.00000

Pearson Partial Correlation Coefficients, N = 1500 Prob > r under H0: Partial Rho=0		
	ln_price	ln_sqft_lot
ln_price	1.00000	-0.06354 0.0139
ln_sqft_lot	-0.06354 0.0139	1.00000

Pearson Partial Correlation Coefficients, N = 1500 Prob > r under H0: Partial Rho=0		
	ln_price	ln_sqft_above
ln_price	1.00000	-0.01033 0.6895
ln_sqft_above	-0.01033 0.6895	1.00000

Figure 2 (Histogram) (Before transformation of price):

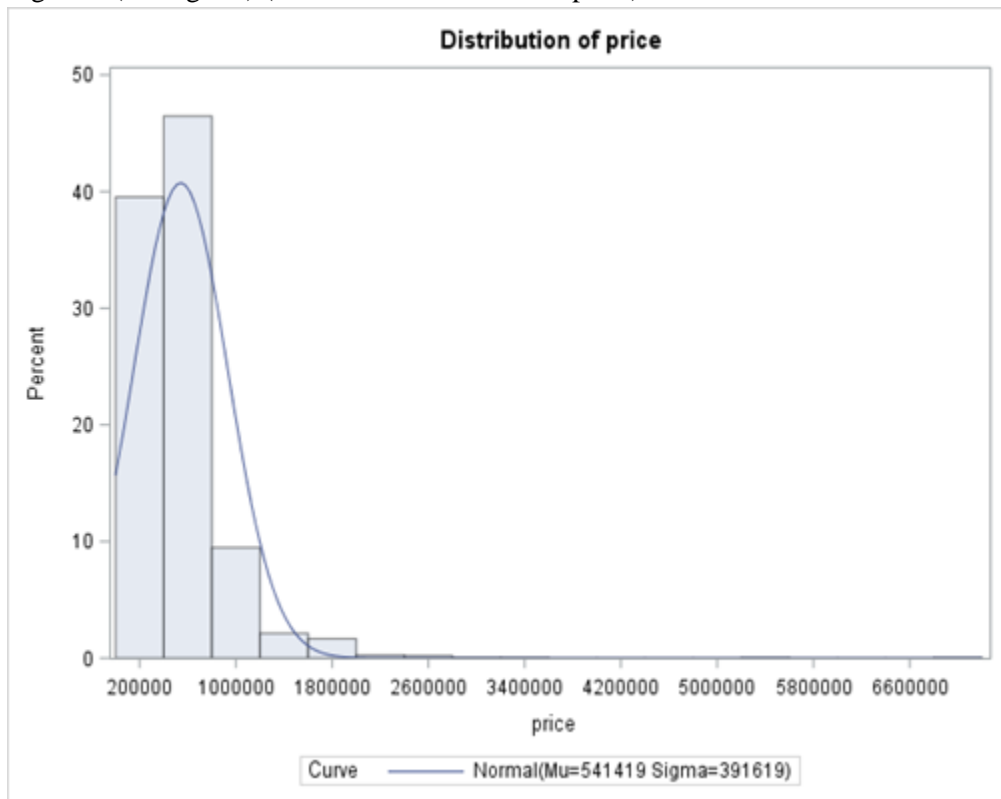


Figure 3 (NNP) (Before transformation of price):

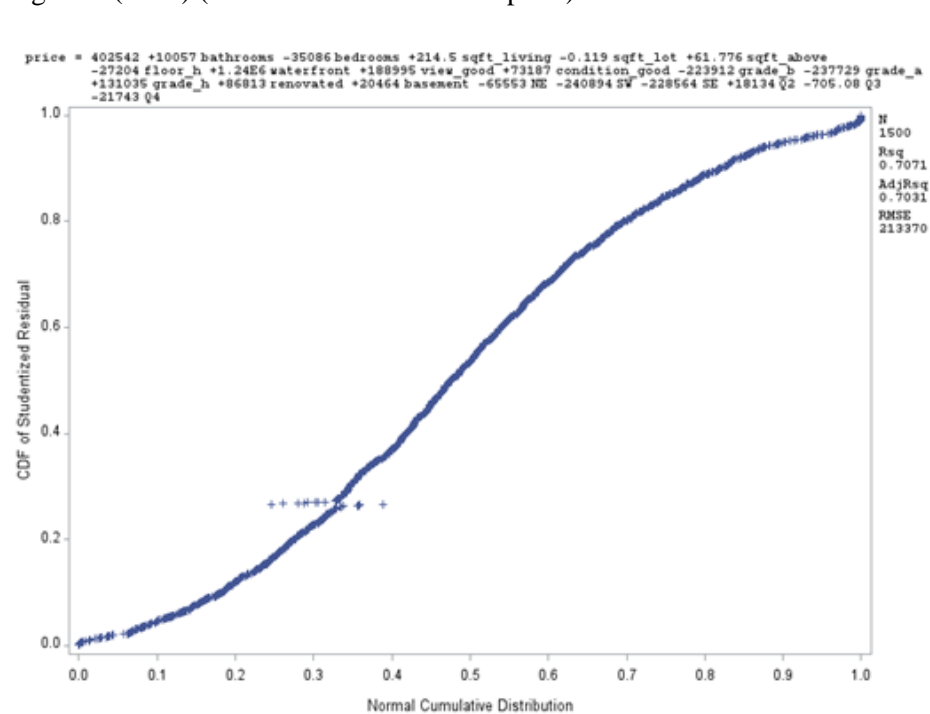


Figure 4 (Histogram) (After transformation of price to ln_price):

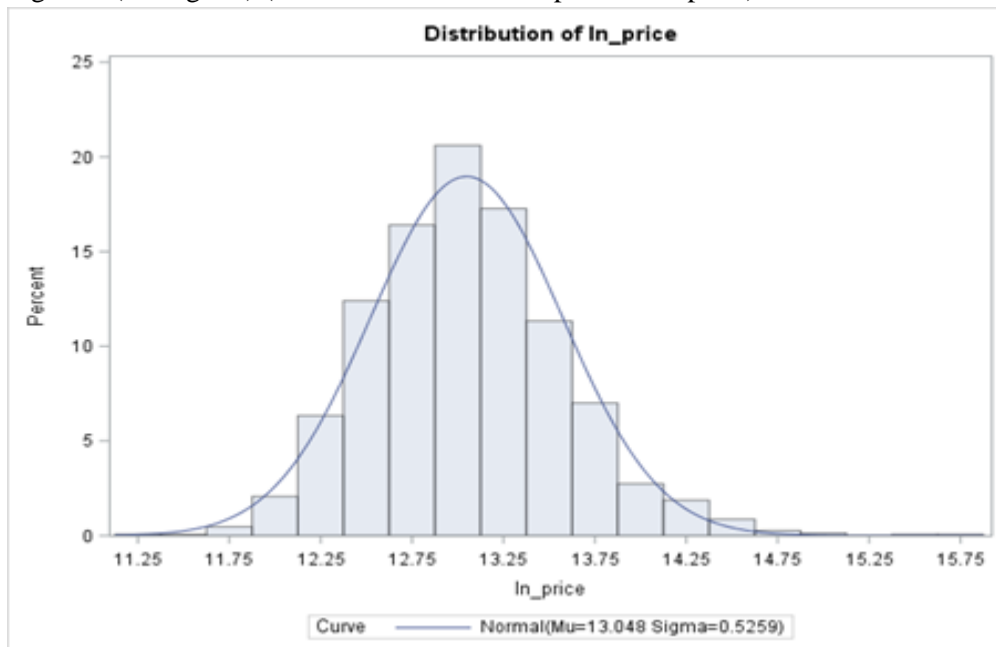


Figure 5 (NNP) (After transformation of price to ln_price):

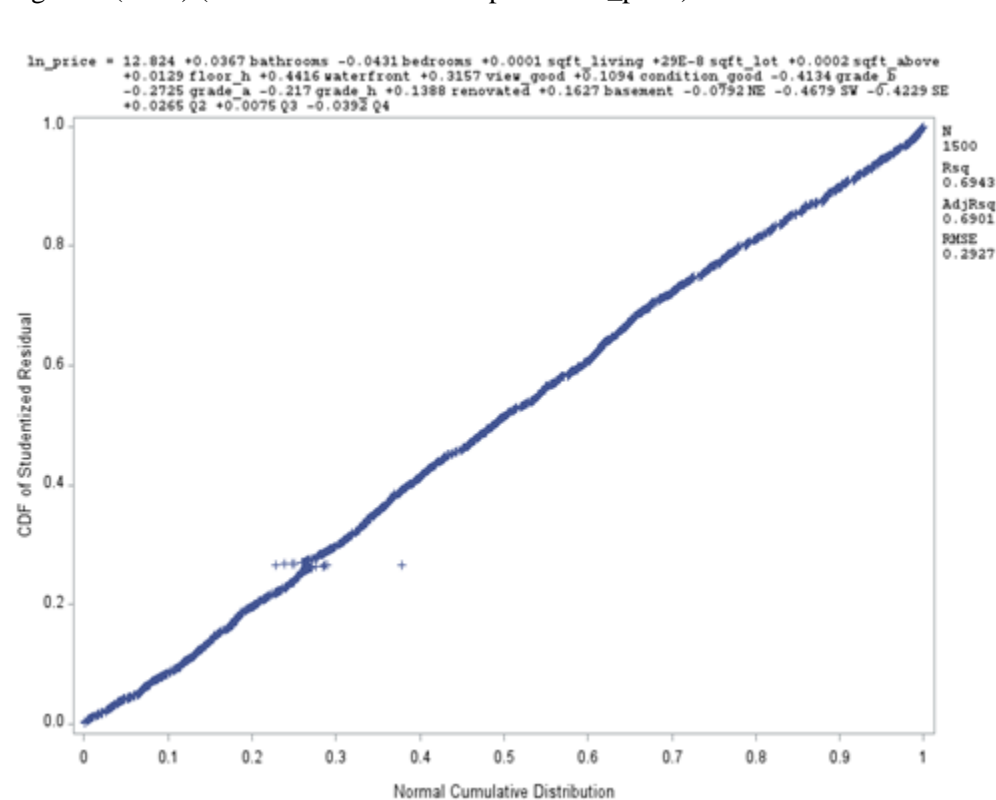


Figure 6 (Scatter plot matrix of ln_price against sqft_living, ln_sqft_living, sqft_lot, ln_sqft_lot, sqft_above, ln_sqft_above):

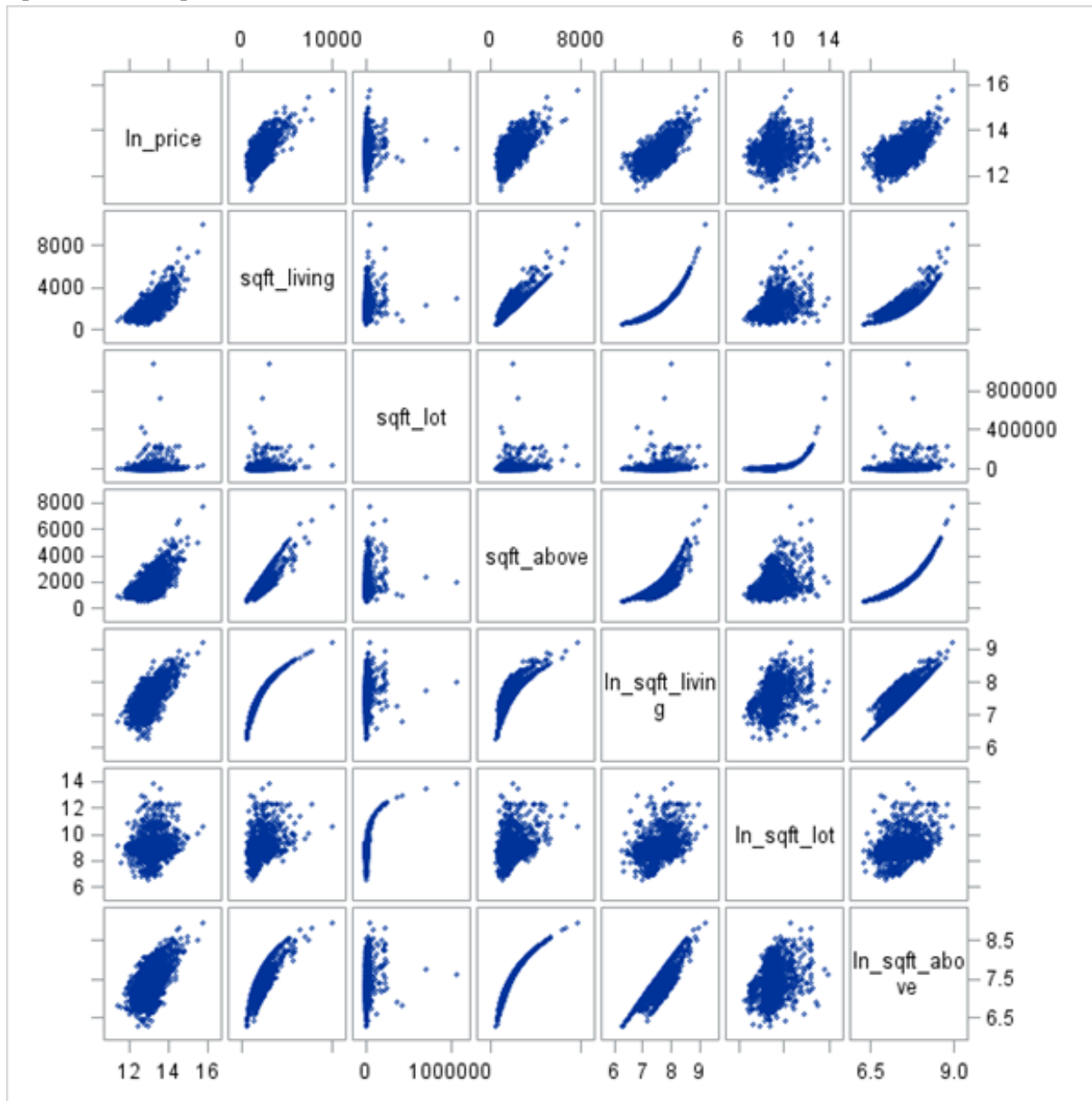


Figure 7 (Studentized Residual of sqft_lot, sqft_above and sqft_living):

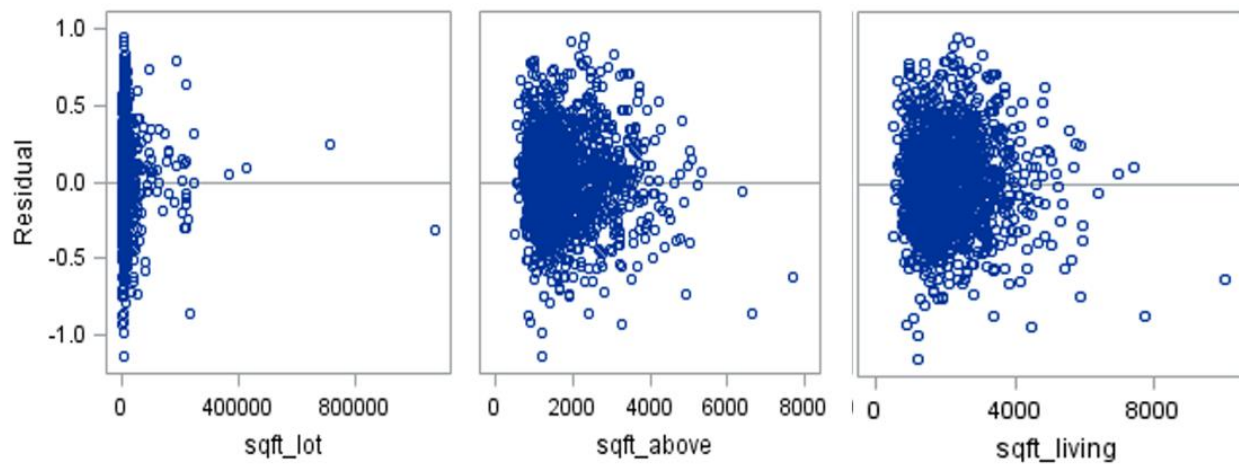


Figure 8 (Studentized Residual of ln_sqft_lot, ln_sqft_above and ln_sqft_living):

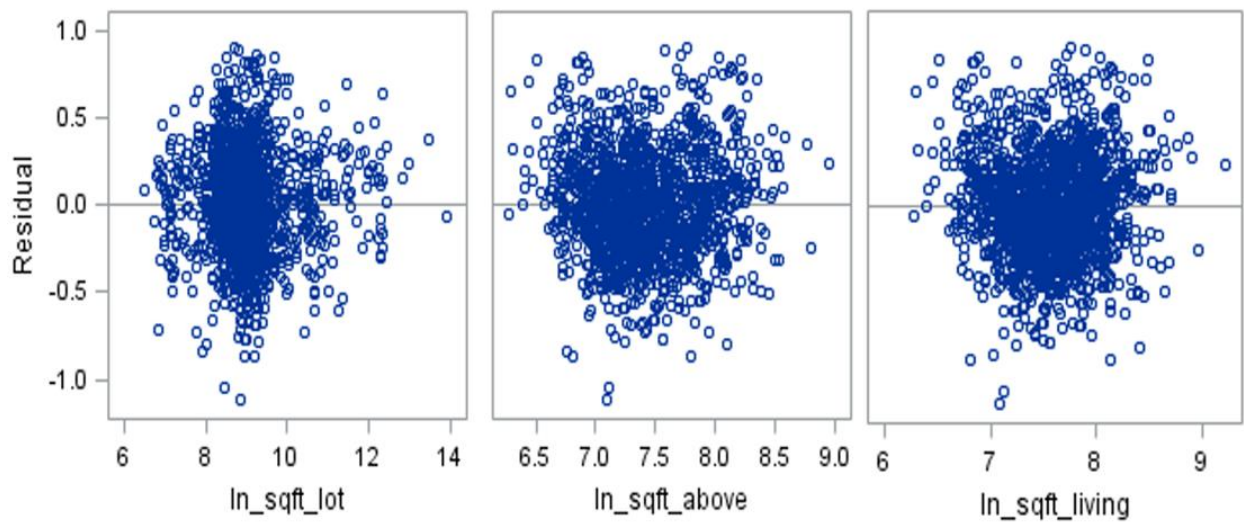


Figure 9 (Multicollinear variables):

Parameter Estimates							
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Standardized Estimate	Variance Inflation
Intercept	1	8.39111	0.37662	22.28	<.0001	0	0
bathrooms	1	0.03855	0.01901	2.03	0.0427	0.05494	3.39315
bedrooms	1	-0.05085	0.01131	-4.50	<.0001	-0.08779	1.76245
ln_sqft_living	1	0.34843	0.07950	4.38	<.0001	0.27898	18.73892
ln_sqft_lot	1	0.00825	0.01104	0.75	0.4552	0.01403	1.63199
ln_sqft_above	1	0.37033	0.07790	4.75	<.0001	0.29872	18.26104
floor_h	1	0.02289	0.02481	0.92	0.3563	0.02135	2.47633
waterfront	1	0.57510	0.11616	4.95	<.0001	0.07968	1.19819
view_good	1	0.31097	0.04611	6.74	<.0001	0.11020	1.23478
condition_good	1	0.10497	0.01751	5.99	<.0001	0.09581	1.18166
grade_b	1	-0.64175	0.30235	-2.12	0.0340	-0.37891	147.40875
grade_a	1	-0.57728	0.30238	-1.91	0.0564	-0.37012	173.84572
grade_h	1	-0.22742	0.30962	-0.73	0.4627	-0.06346	34.52347
renovated	1	0.13142	0.04026	3.26	0.0011	0.04899	1.04177
basement	1	0.15484	0.03409	4.54	<.0001	0.14257	4.55776
NE	1	-0.08258	0.02525	-3.27	0.0011	-0.06479	1.81451
SW	1	-0.48057	0.02321	-20.70	<.0001	-0.37494	1.51725
SE	1	-0.43954	0.02306	-19.06	<.0001	-0.38602	1.89748
Q2	1	0.03738	0.02279	1.64	0.1011	0.03283	1.85217
Q3	1	0.00366	0.02339	0.16	0.8755	0.00309	1.79921
Q4	1	-0.03938	0.02405	-1.64	0.1018	-0.03187	1.75290

Table 5 (Multicollinear variables removed):

	Jun	Omer	Yusheng
The first variable each member removed	grade_a: It has the highest VIF value (173.84572)	Grade_h: It didn't had the biggest VIF (19.12) out of all grade dummy variables but it was the only one that was insignificant.	ln_sqft_above: It has the highest coefficient with ln_sqft_living (0.87) in the Pearson correlation coefficient table(see at figure 13).
Rerun the model			
The second variable each member removed	ln_sqft_living: It has the highest VIF value (18.73723)	ln_sqft_above: it has the highest VIF value (19.59)	Didn't remove second variable that has multicollinearity, but created interaction terms between pairs whose correlation coefficient is greater than 0.5.

Figure 10 (Multicollinearity between locations and interactive variables - Jun's model):

NE	1	-0.98076	0.42302	-2.32	0.0206	-0.76947	526.01499
SW	1	1.50962	0.44492	3.39	0.0007	1.17778	575.41293
SE	1	-2.02487	0.39141	-5.17	<.0001	-1.77830	564.27863
Q2	1	0.03128	0.02243	1.39	0.1634	0.02747	1.85275
Q3	1	-0.00015690	0.02303	-0.01	0.9946	-0.00013230	1.80039
Q4	1	-0.03450	0.02367	-1.46	0.1451	-0.02792	1.75190
above_NE	1	0.12069	0.05680	2.12	0.0338	0.72373	554.07246
above_SW	1	-0.27541	0.06135	-4.49	<.0001	-1.55578	573.49987
above_SE	1	0.21326	0.05303	4.02	<.0001	1.40986	586.92954

Figure 11 (No Multicollinearity after centering ln_sqft_above_c -Jun's model):

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Standardized Estimate	Variance Inflation
Intercept	1	13.00881	0.11171	116.45	<.0001	0	0
bathrooms	1	0.05282	0.01832	2.88	0.0040	0.07528	3.25660
bedrooms	1	-0.03928	0.01092	-3.60	0.0003	-0.06782	1.69892
ln_sqft_lot	1	0.01634	0.01075	1.52	0.1289	0.02779	1.59764
ln_sqft_above_c	1	-0.61638	0.05004	-12.32	<.0001	-0.49719	7.78024
floor_h	1	-0.00047173	0.02434	-0.02	0.9845	-0.00043987	2.46048
waterfront	1	0.62725	0.11502	5.45	<.0001	0.08691	1.21284
view_good	1	0.36256	0.04580	7.92	<.0001	0.12848	1.25805
condition_good	1	0.11594	0.01728	6.71	<.0001	0.10582	1.18773
grade_b	1	-0.10559	0.02813	-3.75	0.0002	-0.06234	1.31704
grade_h	1	0.28316	0.05946	4.76	<.0001	0.07901	1.31463
renovated	1	0.15590	0.03970	3.93	<.0001	0.05812	1.04591
basement	1	0.26620	0.02055	12.96	<.0001	0.24511	1.70912
NE	1	-0.08633	0.02578	-3.35	0.0008	-0.06773	1.95312
SW	1	-0.53139	0.02507	-21.20	<.0001	-0.41459	1.82621
SE	1	-0.44447	0.02329	-19.08	<.0001	-0.39034	1.99807
Q2	1	0.03128	0.02243	1.39	0.1634	0.02747	1.85275
Q3	1	-0.00015690	0.02303	-0.01	0.9946	-0.00013230	1.80039
Q4	1	-0.03450	0.02367	-1.46	0.1451	-0.02792	1.75190
above_NE_c	1	-0.12069	0.05680	-2.12	0.0338	-0.04982	2.62554
above_SW_c	1	0.27541	0.06135	4.49	<.0001	0.09448	2.11505
above_SE_c	1	-0.21326	0.05303	-4.02	<.0001	-0.09167	2.48124

Figure 12 (multicollinearity between location and interactive variables - Omer's model):

NE	1	-0.58897	0.42929	-1.37	0.1703	-0.45894	514.26479
SW	1	0.02222	0.38423	0.06	0.9539	0.01742	416.74522
SE	1	-1.96243	0.38322	-5.12	<.0001	-1.67702	492.88135
Q2	1	0.02684	0.02305	1.16	0.2444	0.02403	1.95671
Q3	1	-0.02309	0.02382	-0.97	0.3326	-0.01963	1.88480
Q4	1	-0.01307	0.02462	-0.53	0.5956	-0.01052	1.80565
ln_sqft_living_NE	1	0.06730	0.05648	1.19	0.2336	0.40292	525.52383
ln_sqft_living_SW	1	-0.06547	0.05161	-1.27	0.2047	-0.38046	413.26043
ln_sqft_living_SE	1	0.19953	0.05074	3.93	<.0001	1.30175	503.68349

Figure 13 (No Multicollinearity after centering ln_sqft_living_c -Omer's model):

Parameter Estimates							
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Standardized Estimate	Variance Inflation
Intercept	1	13.61905	0.13592	100.20	<.0001	0	0
bathrooms	1	0.07111	0.01804	3.94	<.0001	0.10436	3.22209
bedrooms	1	-0.05537	0.01146	-4.83	<.0001	-0.09664	1.83823
ln_sqft_lot	1	0.01005	0.01137	0.88	0.3769	0.01689	1.67801
ln_sqft_living_c	1	-0.63372	0.04804	-13.19	<.0001	-0.51630	7.04092
floor_h	1	0.01637	0.02390	0.68	0.4936	0.01552	2.35983
waterfront	1	0.36661	0.09178	3.99	<.0001	0.06744	1.31001
view_good	1	0.29661	0.04679	6.34	<.0001	0.10941	1.36863
condition_good	1	0.10677	0.01794	5.95	<.0001	0.09523	1.17665
grade_b	1	-0.48549	0.06364	-7.63	<.0001	-0.28191	6.27573
grade_a	1	-0.47320	0.05412	-8.74	<.0001	-0.30110	5.45058
renovated	1	0.16780	0.04247	3.95	<.0001	0.06033	1.07181
basement	1	0.00191	0.01960	0.10	0.9225	0.00178	1.53030
NE	1	-0.08161	0.02482	-3.29	0.0010	-0.06359	1.71882
SW	1	-0.47137	0.02326	-20.27	<.0001	-0.36943	1.52677
SE	1	-0.45824	0.02286	-20.05	<.0001	-0.39159	1.75322
Q2	1	0.02684	0.02305	1.16	0.2444	0.02403	1.95671
Q3	1	-0.02309	0.02382	-0.97	0.3326	-0.01963	1.88480
Q4	1	-0.01307	0.02462	-0.53	0.5956	-0.01052	1.80565
ln_sqft_living_NE_c	1	-0.06730	0.05648	-1.19	0.2336	-0.02342	1.77568
ln_sqft_living_SW_c	1	0.06547	0.05161	1.27	0.2047	0.02640	1.99039
ln_sqft_living_SE_c	1	-0.19953	0.05074	-3.93	<.0001	-0.08024	1.91364

Figure 14 (Before centering for multicollinearity between interactive variables - Yusheng's model):

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Standardized Estimate	Variance Inflation
Intercept	1	13.16448	0.69064	19.06	<.0001	0	0
bathrooms	1	-1.01280	0.24475	-4.14	<.0001	-1.49810	636.01222
bedrooms	1	-0.82097	0.20086	-4.09	<.0001	-1.44012	602.43419
ln_sqft_living	1	-0.03989	0.10769	-0.37	0.7111	-0.03269	37.78730
ln_sqft_lot	1	-0.00110	0.01093	-0.10	0.9199	-0.00185	1.64002
floor_h	1	-0.08420	0.08039	-1.05	0.2951	-0.08045	28.62760
waterfront	1	0.32937	0.09411	3.50	0.0005	0.05672	1.27456
view_good	1	0.30469	0.04529	6.73	<.0001	0.11165	1.33658
condition_good	1	0.12714	0.07920	1.61	0.1086	0.02525	1.20019
grade_b	1	-0.20254	0.24239	-0.84	0.4035	-0.11846	97.52511
grade_a	1	-0.10346	0.24717	-0.42	0.6756	-0.06634	121.89738
grade_h	1	0.14020	0.24900	0.56	0.5735	0.04034	24.90157
renovated	1	0.14389	0.04093	3.52	0.0005	0.05227	1.07288
NE	1	-0.06815	0.41286	-0.17	0.8689	-0.05356	510.83235
SW	1	-0.07257	0.38022	-0.19	0.8487	-0.05710	434.26487
SE	1	-1.61683	0.37653	-4.29	<.0001	-1.40284	517.92543
Q2	1	0.03033	0.02232	1.36	0.1744	0.02740	1.97319
Q3	1	-0.01777	0.02307	-0.77	0.4413	-0.01520	1.88897
Q4	1	-0.00433	0.02382	-0.18	0.8559	-0.00351	1.81614
bb	1	-0.06464	0.01635	-3.95	<.0001	-0.51979	83.89590
bathliving	1	0.16931	0.03439	4.92	<.0001	2.13653	913.90692
bedliving	1	0.11834	0.02875	4.12	<.0001	1.79770	925.95889
bathfloor	1	0.01913	0.03365	0.57	0.5698	0.05034	38.05805
living_NE	1	0.00143	0.05425	0.03	0.9789	0.00866	521.32727
living_SW	1	-0.05279	0.05108	-1.03	0.3016	-0.30786	430.61732
living_SE	1	0.15748	0.04976	3.17	0.0016	1.04303	527.01608

Figure 15 (After centering for multicollinearity between interactive variables - Yusheng's model):

Parameter Estimates							
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Standardized Estimate	Variance Inflation
Intercept	1	13.20197	0.26051	50.68	<.0001	0	0
bathrooms_c	1	-0.04746	0.02179	-2.18	0.0296	-0.07020	5.04071
bedrooms_c	1	0.06513	0.01140	5.71	<.0001	0.11425	1.94178
ln_sqft_living_c	1	-0.71305	0.04559	-15.64	<.0001	-0.58430	6.77398
ln_sqft_lot	1	-0.00110	0.01093	-0.10	0.9199	-0.00185	1.64002
floor_h	1	-0.04386	0.02174	-2.02	0.0439	-0.04190	2.09420
waterfront	1	0.32937	0.09411	3.50	0.0005	0.05672	1.27456
view_good	1	0.30469	0.04529	6.73	<.0001	0.11165	1.33658
condition_good	1	0.12714	0.07920	1.61	0.1086	0.02525	1.20019
grade_b	1	-0.20254	0.24239	-0.84	0.4035	-0.11846	97.52511
grade_a	1	-0.10346	0.24717	-0.42	0.6756	-0.06634	121.89738
grade_h	1	0.14020	0.24900	0.56	0.5735	0.04034	24.90157
renovated	1	0.14389	0.04093	3.52	0.0005	0.05227	1.07288
NE	1	-0.05735	0.02398	-2.39	0.0169	-0.04507	1.72402
SW	1	-0.47057	0.02269	-20.74	<.0001	-0.37024	1.54706
SE	1	-0.42951	0.02189	-19.63	<.0001	-0.37266	1.74972
Q2	1	0.03033	0.02232	1.36	0.1744	0.02740	1.97319
Q3	1	-0.01777	0.02307	-0.77	0.4413	-0.01520	1.88897
Q4	1	-0.00433	0.02382	-0.18	0.8559	-0.00351	1.81614
bb_c	1	-0.06464	0.01635	-3.95	<.0001	-0.10841	3.64963
bathliving_c	1	0.16931	0.03439	4.92	<.0001	0.14299	4.09338
bedliving_c	1	0.11834	0.02875	4.12	<.0001	0.11048	3.49750
bathfloor_c	1	-0.01913	0.03365	-0.57	0.5698	-0.01753	4.61446
living_NE_c	1	-0.00143	0.05425	-0.03	0.9789	-0.00051103	1.81731
living_SW_c	1	0.05279	0.05108	1.03	0.3016	0.02095	1.99355
living_SE_c	1	-0.15748	0.04976	-3.17	0.0016	-0.06517	2.05722

Figure 16 ($R(\ln_sqft_living, \ln_sqft_above) = 0.87$ Close to 0.9 -Yusheng's model):

	bathrooms	bedrooms	<u>\ln_sqft_living</u>	<u>\ln_sqft_lot</u>	<u>\ln_sqft_above</u>
bathrooms	1.00000	0.53987 <.0001	0.74995 <.0001	0.04890 0.0590	0.69527 <.0001
bedrooms	0.53987 <.0001	1.00000	0.64290 <.0001	0.19279 <.0001	0.54862 <.0001
\ln_sqft_living	0.74995 <.0001	0.64290 <.0001	1.00000	0.31093 <.0001	0.87012 <.0001
\ln_sqft_lot	0.04890 0.0590	0.19279 <.0001	0.31093 <.0001	1.00000	0.32291 <.0001
<u>\ln_sqft_above</u>	0.69527 <.0001	0.54862 <.0001	0.87012 <.0001	0.32291 <.0001	1.00000
floor_h	0.60007 <.0001	0.22396 <.0001	0.43967 <.0001	-0.16111 <.0001	0.59946 <.0001
waterfront	0.07033 0.0066	-0.01767 0.4953	0.07347 0.0045	0.10215 <.0001	0.07122 0.0059

Figure 17 (Yusheng)

	\ln_price	bathrooms	bedrooms	\ln_sqft_living	\ln_sqft_lot	floor_h	waterfront
\ln_price	1.00000	0.54016 <.0001	0.33448 <.0001	0.65700 <.0001	0.15203 <.0001	0.28463 <.0001	0.16353 <.0001
bathrooms	0.54016 <.0001	1.00000	0.53987 <.0001	0.74995 <.0001	0.04890 0.0590	0.60007 <.0001	0.07033 0.0066
bedrooms	0.33448 <.0001	0.53987 <.0001	1.00000	0.64290 <.0001	0.19279 <.0001	0.22396 <.0001	-0.01767 0.4953
\ln_sqft_living	0.65700 <.0001	0.74995 <.0001	0.64290 <.0001	1.00000	0.31093 <.0001	0.43967 <.0001	0.07347 0.0045
\ln_sqft_lot	0.15203 <.0001	0.04890 0.0590	0.19279 <.0001	0.31093 <.0001	1.00000	-0.16111 <.0001	0.10215 <.0001
floor_h	0.28463 <.0001	0.60007 <.0001	0.22396 <.0001	0.43967 <.0001	-0.16111 <.0001	1.00000	0.05748 0.0264
waterfront	0.16353 <.0001	0.07033 0.0066	-0.01767 0.4953	0.07347 0.0045	0.10215 <.0001	0.05748 0.0264	1.00000

Table 6 (Outliers and Influential Points):

Model	Outliers removed	Influential points removed	Explanation
M1	153		I took out all possible outliers ($>+3$) and influential points based on studentized residual values and Cook's Distance (>0.0003).
M2	8	0	When I removed the 8 outliers the RMSE decreased; R-square increased and Adj R-square increased. However, when I removed 20 or more influential points my R-square and Adj-R-square decreased instead of increasing. Therefore, I only took out the biggest outliers from my dataset. Also, since King County has one of the richest residents in United States, most of the outliers given by the model might not be outliers. It is also one of the richest county in the country.
M3	8		I remove 8 observations that are both outliers and influential points. My Adj-R-square increased from 0.66 to 0.67 after this step.

Table 7 (Jun's Selection Methods):

Independent variables in the regression model before selection method	bathrooms bedrooms ln_sqft_lot ln_sqft_above_c floor_h waterfront view_good condition_good grade_b grade_h renovated basement NE SW SE Q2 Q3 Q4 above_ne_c above_sw_c above_se_c	# of predictors= 21 Adj R-square = 0.7307 RMSE = 0.23566 The above values given by running the training set.
Independent variables given by both Backward and Stepwise method	bedrooms ln_sqft_above_c waterfront view_good condition_good grade_b grade_h renovated basement NE SW SE Q4 above_NE_c above_SW_c above_SE_c	# of predictors= 16 Adj R-square = 0.7307 RMSE = 0.23566 The above values given by running the training set.
Final model variables after taking out two insignificant predictors	bedrooms ln_sqft_above_c waterfront view_good condition_good grade_b grade_h renovated basement NE SW SE Q4 above_NE_c above_SW_c above_SE_c	# of predictors= 14 Adj R-square = 0.7263 RMSE = 0.23755 The above values given by running the training set.

Table 8 (Omer's Selection Methods):

Independent variables in the regression model before selection method	bathrooms, bedrooms, ln_sqft_lot, ln_sqft_living_c, floor_h, waterfront ,view_good, condition_good, grade_b, grade_a, renovated, basement, NE, SW, SE, Q2, Q3, Q4, ln_sqft_living_NE_c, ln_sqft_living_SW_c, ln_sqft_living_SE_c.	# of predictors= 21
Independent variables given by Stepwise method	bathrooms, bedrooms, ln_sqft_living_c, waterfront ,view_good, condition_good, grade_b, grade_a, renovated, NE, SW, SE, Q2, ln_sqft_living_SE_c.	# of predictors=14 R-square = 0.6932 Adj R-square = 0.6894 RMSE = 0.28511 The above values given by running the training set.
Independent variables given by ADJR SQ method	bathrooms, bedrooms, ln_sqft_living_c, floor_h, waterfront, view_good, condition_good, grade_b, grade_a, renovated, NE, SW, SE, Q2, ln_sqft_living_SW_c, ln_sqft_living_SE_c.	# of predictors=16 R-square=0.6943 Adj R-square=0.6898 RMSE=0.28488 Variables floor_h, ln_sqft_living_SW_c are insignificant. The above values given by running the training set
Final Model Variables	bathrooms, bedrooms, ln_sqft_living_c, waterfront ,view_good, condition_good, grade_b, grade_a, renovated, NE, SW, SE, Q2, ln_sqft_living_SE_c.	# of predictors=14 R-square = 0.6932 Adj R-square = 0.6894 RMSE = 0.28511 The above values given by running the training set.

Figure 18 (Showing Omer's model after selection method on testing set):

Parameter Estimates							
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Standardized Estimate	Variance Inflation
Intercept	1	13.61568	0.17496	77.82	<.0001	0	0
bathrooms	1	0.04730	0.03465	1.37	0.1731	0.06696	2.90327
bedrooms	1	-0.03794	0.02399	-1.58	0.1147	-0.06443	2.00353
ln_sqft_living_c	1	-0.68681	0.07311	-9.39	<.0001	-0.55047	4.14393
waterfront	1	0.07685	0.18282	0.42	0.6745	0.01467	1.46930
view_good	1	0.32628	0.08133	4.01	<.0001	0.13619	1.39087
condition_good	1	0.17464	0.03613	4.83	<.0001	0.14908	1.14786
grade_b	1	-0.46167	0.14371	-3.21	0.0014	-0.27594	8.90479
grade_a	1	-0.42327	0.12127	-3.49	0.0005	-0.27166	7.31114
renovated	1	0.17625	0.08199	2.15	0.0323	0.07181	1.34670
NE	1	-0.03112	0.04513	-0.69	0.4909	-0.02562	1.66618
SW	1	-0.42550	0.04715	-9.02	<.0001	-0.31601	1.47978
SE	1	-0.39518	0.04477	-8.83	<.0001	-0.32437	1.62992
Q2	1	0.00238	0.03399	0.07	0.9441	0.00204	1.02074
ln_sqft_living_SE_c	1	-0.22287	0.08605	-2.59	0.0100	-0.08931	1.43468

Table 9 (Yusheng's Selection Methods) :

Independent variables in the regression model before selection method	Bathrooms,bedrooms, ln_sqft_living_c, ln_sqft_lot , floor_h,waterfront,view_good, condition_good, grade_b, grade_a ,grade_h, renovated, NE, SW, SE, Q2 ,Q3, Q4, bb_c,bathliving_c,bedliving_c , bathfloor_c , living_NE_c ,living_SW_c, living_SE_c	# of predictors= 25 The above values given by running the training set.
Independent variables given by Forward Selection method	ln_sqft_living, SE, SW ,bathliving_c, view_good ,grade_h, bedrooms ,living_SE_c, renovated ,grade_b, waterfront, Q2 ,bedliving_c, bb_c, NE	# of predictors=15 R-square=0.6889 Adj R-square=0.6847 RMSE=0.29341

		The above values given by running the training set
Independent variables given by Mallows' Cp Selection method	bathrooms, bedrooms ,ln_sqft_living_c, waterfront ,view_good, condition_good, grade_b, grade_a, renovated ,NE, SW, SE, Q2, bb_c, bathliving_c, bedliving_c, living_SE_c,	# of predictors=17 R-square=0.6904 Adj R-square=0.6857 RMSE=0.29297 Variables bathrooms_c and condition_good are insignificant.
Final Model Variables	Ln_sqft_living_c, waterfront view_good , SW, SE , bb_c, bathliving_c, bedliving_c, living_SE_c	# of predictors=9 R-square=0.6637 Adj R-square=0.6610 RMSE=0.30424 The above values given by running the training set.

Figure 19 (strongest parameters):

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Standardized Estimate
Intercept	1	13.33356	0.06481	205.74	<.0001	0
bedrooms	1	-0.07462	0.01863	-4.00	<.0001	-0.13250
ln_sqft_above_c	1	-0.69832	0.07163	-9.75	<.0001	-0.58981
waterfront	1	0.75344	0.25665	2.94	0.0036	0.08290
view_good	1	0.32257	0.09350	3.45	0.0006	0.10520
condition_good	1	0.15364	0.02909	5.28	<.0001	0.14629
grade_b	1	-0.11225	0.04717	-2.38	0.0179	-0.07263
grade_h	1	0.45324	0.15117	3.00	0.0029	0.08612
basement	1	0.37681	0.03111	12.11	<.0001	0.37483
SW	1	-0.54677	0.03925	-13.93	<.0001	-0.45987
SE	1	-0.38443	0.03249	-11.83	<.0001	-0.36842
Q4	1	-0.10542	0.03069	-3.43	0.0007	-0.09220
above_NE_c	1	-0.21899	0.10001	-2.19	0.0293	-0.08224
above_SW_c	1	0.28107	0.10550	2.66	0.0081	0.10071
above_SE_c	1	-0.26882	0.08775	-3.06	0.0024	-0.13145

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