

1. Linked queue

Microsoft Visual Studio 偵錯主控台

```
Do what???
```

1. Insert
2. Pop
3. Front number
4. Back number
5. Size
6. Print queue
7. Quit

```
1
Insert value: 2
Queue: 2

1
Insert value: 22
Queue: 2 22

1
Insert value: 222
Queue: 2 22 222

3
the first element is 2

4
the last element is 222

5
the size of the queue is 3

6
Queue: 2 22 222

2
Number 2 has been popped.
Queue: 22 222

2
Number 22 has been popped.
Queue: 222

2
Number 222 has been popped.
Queue Is Empty!!!!

2
Queue Is Empty!!!!
```

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```
3
the first element is 2

4
the last element is 222

5
the size of the queue is 3

6
Queue: 2 22 222

2
Number 2 has been popped.
Queue: 22 222

2
Number 22 has been popped.
Queue: 222

2
Number 222 has been popped.
Queue Is Empty!!!!

2
Queue Is Empty!!!!

3
Queue Is Empty!!!!

4
Queue Is Empty!!!!

5
the size of the queue is 0

6
Queue Is Empty!!!!

7
Bye bye!!!

C:\Users\User\vs\hw2\Debug\hw2.exe (處理序 29332)
若要在偵錯停止時自動關閉主控台，請啟用「工具」->
```

```
1  #include<iostream>
2  using namespace std;
3  template<typename T> struct Node
4  {
5      Node<T>* nxt;
6      T val;
7      Node() {
8          nxt = NULL;
9      }
10 };
11 template<typename T> class Queue {
12 public:
13     int size = 0;
14     Node<T>* head, * tail;
15     Queue() {
16         head = new Node<T>;
17         head->nxt = NULL;
18         tail = head;
19     }
20     void push(T val) {
21         size++;
22         Node<T>* add;
23         add = new Node<T>;
24         add->val = val;
25         add->nxt = NULL;
26         tail->nxt = add;
27         tail = tail->nxt;
28     }
29     void pop() {
30         if (size == 0) cout << "Queue Is Empty!!!!\n";
```

```

28
29 void pop() {
30     if (size == 0) cout << "Queue Is Empty!!!!\n";
31     else
32     {
33         cout << "Number " << (head->nxt)->val << " has been popped.\n";
34         size--;
35         head = head->nxt;
36         print();
37     }
38
39 }
40 void front() {
41     if (size == 0) cout << "Queue Is Empty!!!!\n";
42     else cout << "the first element is " << (head->nxt)->val << "\n";
43 }
44 void back() {
45     if (size == 0) cout << "Queue Is Empty!!!!\n";
46     else cout << "the last element is " << tail->val << "\n";
47 }
48 void size_q() {
49     cout << "the size of the queue is " << size << "\n";
50 }
51 void print() {
52     Node<T>* tmp = head;
53     if (size == 0) cout << "Queue Is Empty!!!!\n";
54     else
55     {
56         cout << "Queue: ";
57         while (tmp != tail) {
58             cout << (tmp->nxt)->val << " ";

```

```

59             tmp = tmp->nxt;
60         }
61         cout << "\n";
62     }
63 }
64 };
65
66 int main() {
67     Queue<int> q;
68     cout << "Do what???\n";
69     cout << "1. Insert\n2. Pop\n3. Front number\n4. Back number\n5. Size\n6. Print queue\n7. Quit\n\n";
70     int flag = 1;
71     while (flag) {
72         int n;
73         cin >> n;
74         switch (n)
75         {
76             case 1:
77                 int m;
78                 cout << "Insert value: ";
79                 cin >> m;
80                 q.push(m);
81                 q.print();
82                 break;
83             case 2:
84                 q.pop();
85                 break;

```

```

79         cin >> m;
80         q.push(m);
81         q.print();
82         break;
83     case 2:
84         q.pop();
85         break;
86     case 3:
87         q.front();
88         break;
89     case 4:
90         q.back();
91         break;
92     case 5:
93         q.size_q();
94         break;
95     case 6:
96         q.print();
97         break;
98     case 7:
99         flag = 0;
100        cout << "Bye bye!!!";
101        break;
102    default:
103        cout << "Please input number 1 to 7 !!!\n";
104        break;
105    }
106    cout << "\n";
107 }
108

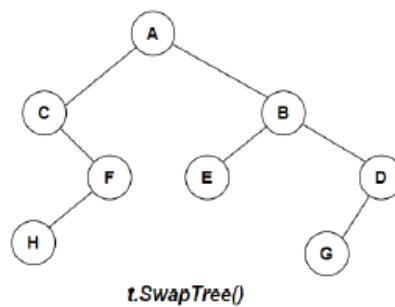
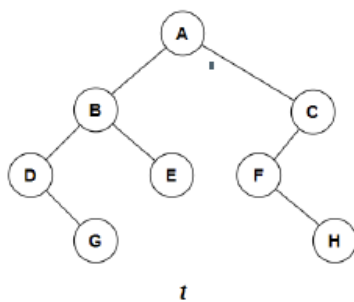
```

2. Swaptree()

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Traverse the tree: D G B E A F H C

Traverse the swaptree: C H F A E B G D



```

1  #include<iostream>
2  using namespace std;
3  struct Node {
4      char data;
5      struct Node* left;
6      struct Node* right;
7  };
8  struct Node* insert_node(char data) {
9      struct Node* node = (struct Node*)malloc(sizeof(struct Node));
10     node->data = data;
11     node->left = NULL;
12     node->right = NULL;
13     return node;
14 }
15 void swaptree(struct Node* node) {
16     if (node == NULL) {
17         return;
18     }
19     else {
20         swaptree(node->left);
21         swaptree(node->right);
22         struct Node* tmp;
23         tmp = node->left;
24         node->left = node->right;
25         node->right = tmp;
26     }
27 }
28 void traversal(struct Node* node) {           //inorder traversal
29     if (node == NULL) return;
30     traversal(node->left);
31     cout << node->data << " ";
32 }

```

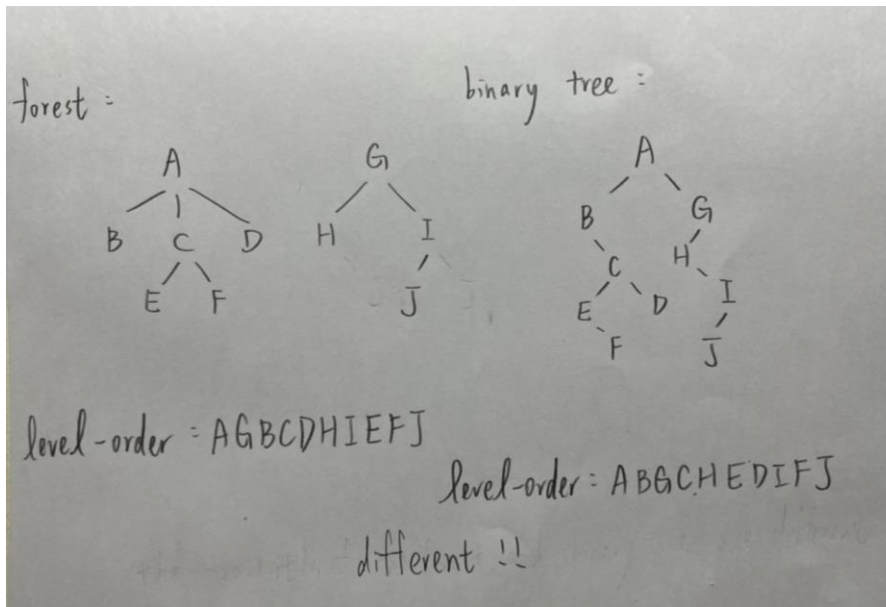
找不到任何問題

```

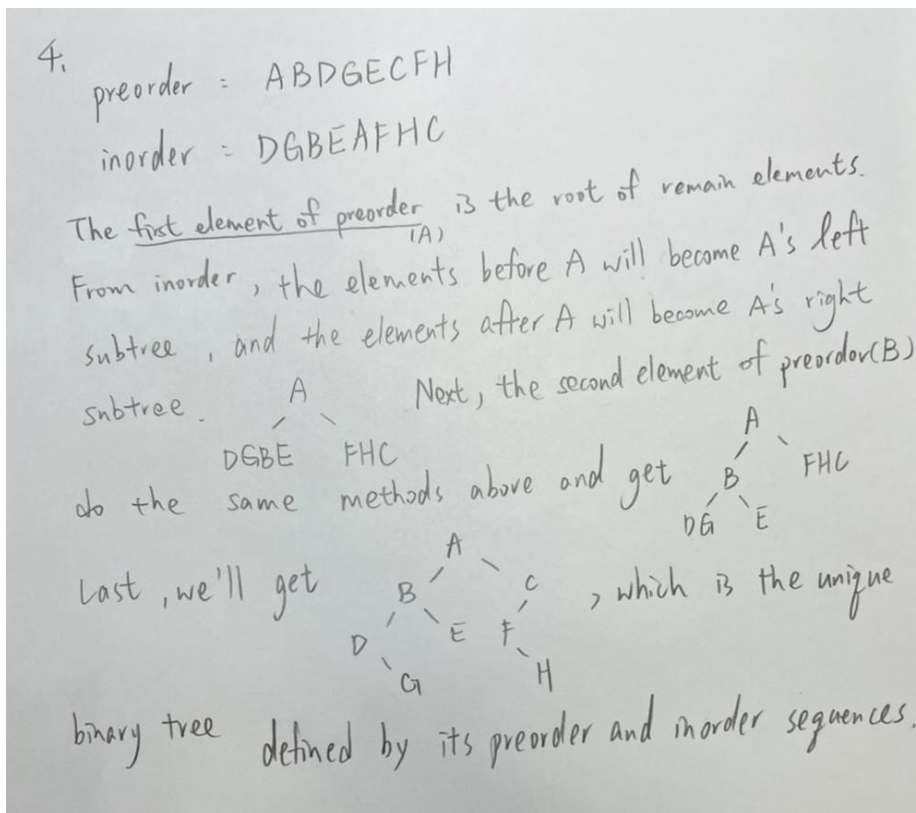
27 }
28 void traversal(struct Node* node) {           //inorder traversal
29     if (node == NULL) return;
30     traversal(node->left);
31     cout << node->data << " ";
32     traversal(node->right);
33 }
34 int main() {
35     struct Node* root = insert_node('A');
36     root->left = insert_node('B');
37     root->left->left = insert_node('D');
38     root->left->left->right = insert_node('G');
39     root->left->right = insert_node('E');
40     root->right = insert_node('C');
41     root->right->left = insert_node('F');
42     root->right->left->right = insert_node('H');
43
44     cout << "Traverse the tree: ";
45     traversal(root);
46     cout << "\n\n";
47     swaptree(root);
48     cout << "Traverse the swaptree: ";
49     traversal(root);
50     cout << "\n\n";
51 }

```

3. Level-order traversal

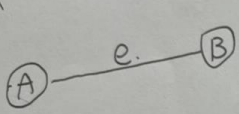


4. Unique tree from preorder and inorder



5. Sum of degrees

5.




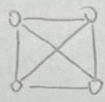
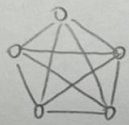


e is an edge of the graph which has 2 ends (A, B)
 we count twice when we sum the vertex degrees (deg(A) and deg(B))
 Hence, every edge counts twice in the graph.

$\Rightarrow \text{Sum of degrees} = 2E$
 \uparrow number of edges.

6. $n(n-1)/2$

6.

		edges
one :		0
two :		1
three :		3
four :		6
five :		10

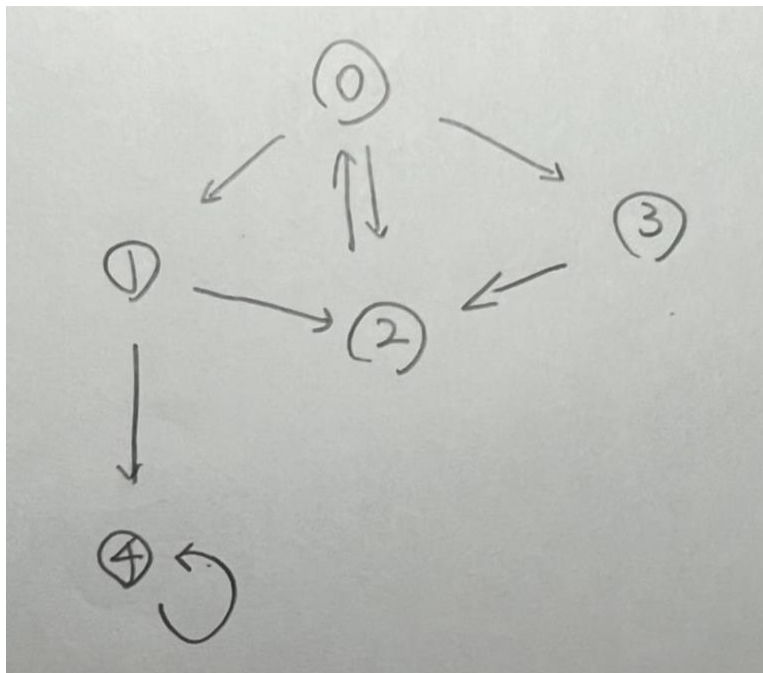
each vertex has $n-1$ edges times n vertex,
 every edge will count twice for two vertex, so.
 the number of edges in an n -vertex complete graph is

$$\frac{n(n-1)}{2} *$$

7. bfs

```
t main() {  
    Graph g(5);  
    g.add(0, 1);  
    g.add(0, 2);  
    g.add(0, 3);  
    g.add(1, 2);  
    g.add(1, 4);  
    g.add(2, 0);  
    g.add(3, 2);  
    g.add(4, 4);  
    cout << "BFS from vertex 2: ";  
    g.bfs(2);  
}
```

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BFS from vertex 2: 2 0 1 3 4
C:\Users\User\vs\hw2\Debug\hw2..
若要在偵錯停止時自動關閉主控台，
按任意鍵關閉此視窗...



```

1  #include<iostream>
2  #include<list>
3  using namespace std;
4
5  class Graph {
6      int vertex;
7      list<int>* adj;
8  public:
9      Graph(int vertex) {
10         this->vertex = vertex;
11         adj = new list<int>[vertex];
12     }
13     void add(int value, int index) {
14         adj[value].push_back(index);
15     }
16     void bfs(int s) {
17         bool* visited = new bool[vertex];
18         for (int i = 0; i < vertex; i++) {
19             visited[i] = false;
20         }
21
22         list<int> queue;
23         visited[s] = true;
24         queue.push_back(s);
25         list<int>::iterator i;
26         while (!queue.empty()) {
27             s = queue.front();
28             cout << s << " ";
29             queue.pop_front();
30             for (i = adj[s].begin(); i != adj[s].end(); i++) {
31                 if (!visited[*i]) {

```

找不到任何問題

```

28             cout << s << " ";
29             queue.pop_front();
30             for (i = adj[s].begin(); i != adj[s].end(); i++) {
31                 if (!visited[*i]) {
32                     visited[*i] = true;
33                     queue.push_back(*i);
34                 }
35             }
36         }
37     }
38 };
39
40 int main() {
41     Graph g(5);
42     g.add(0, 1);
43     g.add(0, 2);
44     g.add(0, 3);
45     g.add(1, 2);
46     g.add(1, 4);
47     g.add(2, 0);
48     g.add(3, 2);
49     g.add(4, 4);
50     cout << "BFS from vertex 2: ";
51     g.bfs(2);
52 }

```


8. number of spanning tree

8.

$$n=1, \quad 0, \quad 1 \text{ 種} \geq 2^{1-1} - 1 = 0$$

$$n=2, \quad \text{---}, \quad 1 \text{ 種} \geq 2^{2-1} - 1 = 1$$

$$n=3, \quad \begin{array}{c} \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} \quad \begin{array}{c} \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} \quad \begin{array}{c} \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array}, \quad 3 \text{ 種} \geq 2^{3-1} - 1 = 3$$

$$n=4, \quad \begin{array}{ccccccc} \begin{array}{c} \circ \quad \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} & \begin{array}{c} \circ \quad \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} & \begin{array}{c} \circ \quad \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} & \begin{array}{c} \circ \quad \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} & \begin{array}{c} \circ \quad \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} & \begin{array}{c} \circ \quad \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} & \begin{array}{c} \circ \quad \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} \\ \begin{array}{c} \circ \quad \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} & \begin{array}{c} \circ \quad \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} & \begin{array}{c} \circ \quad \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} & \begin{array}{c} \circ \quad \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} & \begin{array}{c} \circ \quad \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} & \begin{array}{c} \circ \quad \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} & \begin{array}{c} \circ \quad \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array} \end{array}, \quad 14 \text{ 種} \geq 2^{4-1} - 1 = 7$$

$$n \geq 5$$



n^{th}

連接 n^{th} 至前 $n-1$ 個至少 $n-1$ 種.

$f(n-1)$ 種.

$$\Rightarrow f(n) \geq f(n-1) \cdot (n-1)$$

$$\text{同理, } f(n-1) \geq f(n-2) \cdot (n-2)$$

$$\text{因此, } f(n) \geq f(n-1) \cdot (n-1) \geq f(n-2) \cdot (n-2) \cdot (n-1) \geq f(n-3) \cdot (n-3) \cdot (n-2) \cdot (n-1) \geq (n-1)!$$

$$\Rightarrow \text{prove } (n-1)! \geq 2^{n-1} - 1 \quad \forall n \geq 5$$

$$n=5, \quad 4! = 24 \geq 2^{4-1} - 1 = 7. \quad \text{true.}$$

$$\text{假設 } n=k \text{ 時 } (k-1)! \geq 2^{k-1} - 1 \text{ 成立.}$$

$$\Rightarrow k! = k \cdot (k-1)! \geq k \cdot 2^{k-1} - 1 > 2^k - 1 \quad (\because k \geq 5) \text{ 成立.}$$

$$\text{由數學歸納法得證 } (n-1)! \geq 2^{n-1} - 1 \quad \forall n \geq 5$$

$$\Rightarrow \# \text{ of spanning trees in a complete graph with } n \text{ vertices} \geq 2^{n-1} - 1 \quad *$$

9. Topolterator

```
vector<int> adj_list[105]; //input the graph with adjancy list
int in_degree[105]; // calculate the indegree in advance
int n; // n nodes in the DAG

class TopoIterator {
public:
    void topo_sort() {
        queue<int> q;
        vector<int> order;
        for (int i = 0; i < n; i++) {
            if (in_degree[i] == 0) q.push(i);
        }
        while (q.size()) {
            int tmp = q.front();
            q.pop();
            order.push_back(tmp);
            for (int i = 0; i < adj_list[tmp].size(); i++) {
                int next = adj_list[tmp][i];
                in_degree[next]--;
                if (in_degree[next] == 0) {
                    q.push(next);
                }
            }
        }
        for (int i = 0; i < order.size(); i++) {
            cout << order[i] << " ";
        }
        cout << "\n";
    }
};
```

10. ShortestPath

- 1) If we want to get ShortestPath(4), we can only get 4 to 6 and can't get the others shortest path. Similarly, we can't get any ShortestPath of 6 to other vertices because of the directed relationship between the vertices.
- 2) 0 -> 2 -> 1 -> 3 -> 4 -> 6 or 0 -> 2 -> 1 -> 3 -> 5 -> 6 distance = 8