# Example 1

Genus 4 superelliptic curve $y^3 = x^5 - 2x^4 - 2x^3 - 2x^2 - 3x$.

Using work of Poonen and Schaefer, Magma can show that the rank of the Jacobian of this curve is equal to 1:

```
> Qx<x>:=PolynomialRing(RationalField());
> RankBounds(x^5 - 2*x^4 - 2*x^3 - 2*x^2 - 3*x,3);
1 1
```

We take $p = 7$ and initial precision $N = 20$:

```
> load "coleman.m";
> Q:=y^3 - (x^5 - 2*x^4 - 2*x^3 - 2*x^2 - 3*x);
> p:=7;
> N:=20;
> data:=coleman_data(Q,p,N);
```

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There are 5 obvious rational points on the curve:

```
P1:=set_point(1,-2,data);
P2:=set_point(0,0,data);
P3:=set_point(-1,0,data);
P4:=set_point(3,0,data);
P5:=set_bad_point(0,[1,0,0],true,data);
```

Where the last point is the point at infinity.

```
IP1P2,N2:=coleman_integrals_on_basis(P1,P2,data:e:=50);
> IP1P2;
(12586493*7 + O(7^10)    19221514*7 + O(7^10)
-19207436*7 + O(7^10)   -10636635*7 + O(7^10)
128831118 + O(7^10)        67444962 + O(7^10)
-23020322 + O(7^10)  401602170*7^-1 + O(7^10))
> N2;
10
```

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We can now solve for the 1-forms $\omega$ such that $\int_{P_1}^{P_2} \omega = 0$:

```
> K:=pAdicField(p,N2);
> M:=Matrix(4,1,Vector(K,[IP1P2[i]: i in [1..4]]));
> W:= Kernel(M); w1:=W.1; w2:=W.2; w3:=W.3;
```

We find $3 = g - r$ independent $\omega$, so we can set the integrals $\int_{P_1}^{P} \omega$ to zero on all residue disks to find a finite subset $S \subset X(\mathbf{Q}_p)$ which contains $X(\mathbf{Q})$.

We compute that $|S| = 5$, so the list of points that we found is complete.

This has all been automated, we could also have run:

```
> Qpoints:=Q_points(data,1000); // search height <= 1000
> #effective_chabauty(data:Qpoints:=Qpoints,e:=50),#Qpoints;
5 5
```

# Some references

paper: 'Explicit Coleman integration for curves' at:

`https://arxiv.org/abs/1710.01673`

Magma code: 'Coleman' at:

`https://github.com/jtuitman/Coleman/`

(`examples.pdf` contains lots of examples)