Some computational details

$$f(x,y) = y^4 + 5x^4 - 6x^2y^2 + 6x^3 + 26x^2y + 10xy^2 - 10y^3 - 32x^2 - 40xy + 24y^2 + 32x - 16y = 0.$$

$$\vec{\omega} := \begin{pmatrix} 1 \\ x \\ y \\ -160x^4/3 + 736x^3/3 - 16x^2y/3 + 436x^2/3 - 440xy/3 + 68y^2/3 \\ -80x^3/3 + 44x^2 - 40xy/3 + 68y^2/3 - 32 \\ -16x^2y + 28x^2 + 72xy - 4y^2 - 160x/3 + 272/3 \end{pmatrix} \frac{dx}{\left(\frac{\partial f}{\partial y}\right)}$$

We take $p = 17$ and use the correspondences Z with $\xi_Z = 6a_q - \mathrm{Tr}(a_q)Id$ for $q = 7, 11$:

$$Z_1 = \begin{pmatrix} 0 & 112 & -656 & -6 & 6 & 6 \\ -112 & 0 & -2576 & 15 & 9 & 27 \\ 656 & 2576 & 0 & 3 & 3 & -3 \\ 6 & -15 & -3 & 0 & 0 & 0 \\ -6 & -9 & -3 & 0 & 0 & 0 \\ -6 & -27 & 3 & 0 & 0 & 0 \end{pmatrix} \quad Z_2 = \begin{pmatrix} 0 & -976 & -1104 & 10 & -6 & 18 \\ 976 & 0 & -816 & -3 & 1 & 3 \\ 1104 & 816 & 0 & -3 & 3 & -11 \\ -10 & 3 & 3 & 0 & 0 & 0 \\ 6 & -1 & -3 & 0 & 0 & 0 \\ -18 & -3 & 11 & 0 & 0 & 0 \end{pmatrix}$$

Some references

paper: 'Explicit Chabauty-Kim for the Split Cartan Modular Curve of Level 13'

https://arxiv.org/abs/1711.05846

Popular accounts at:

https://www.quantamagazine.org/mathematicians-crack-the-cursed-curve-20171207/

https://www.nemokennislink.nl/publicaties/de-zeven-oplossingen-van-de-vervloekte-kromme/

Magma code: 'Cartan 13' (in progress) at:

https://github.com/jtuitman/Cartan13/