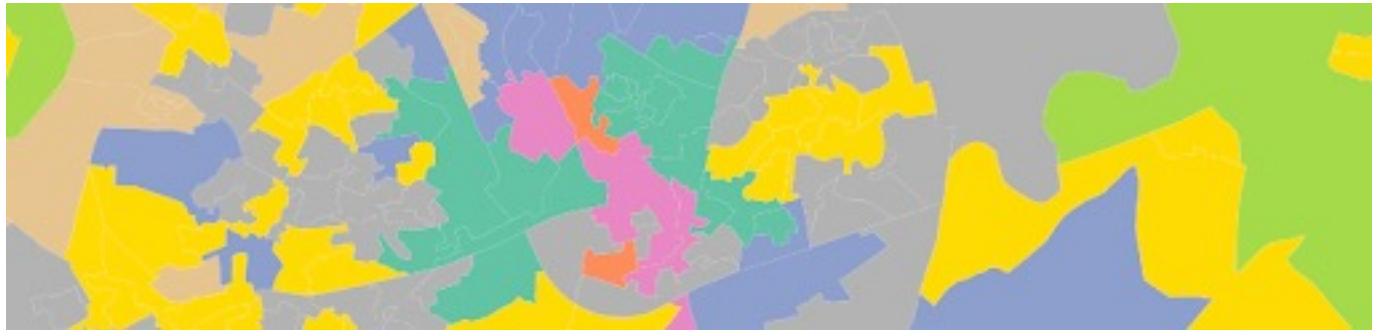
Principles of Spatial Analysis

WEEK 07: POINT PATTERN ANALYSIS

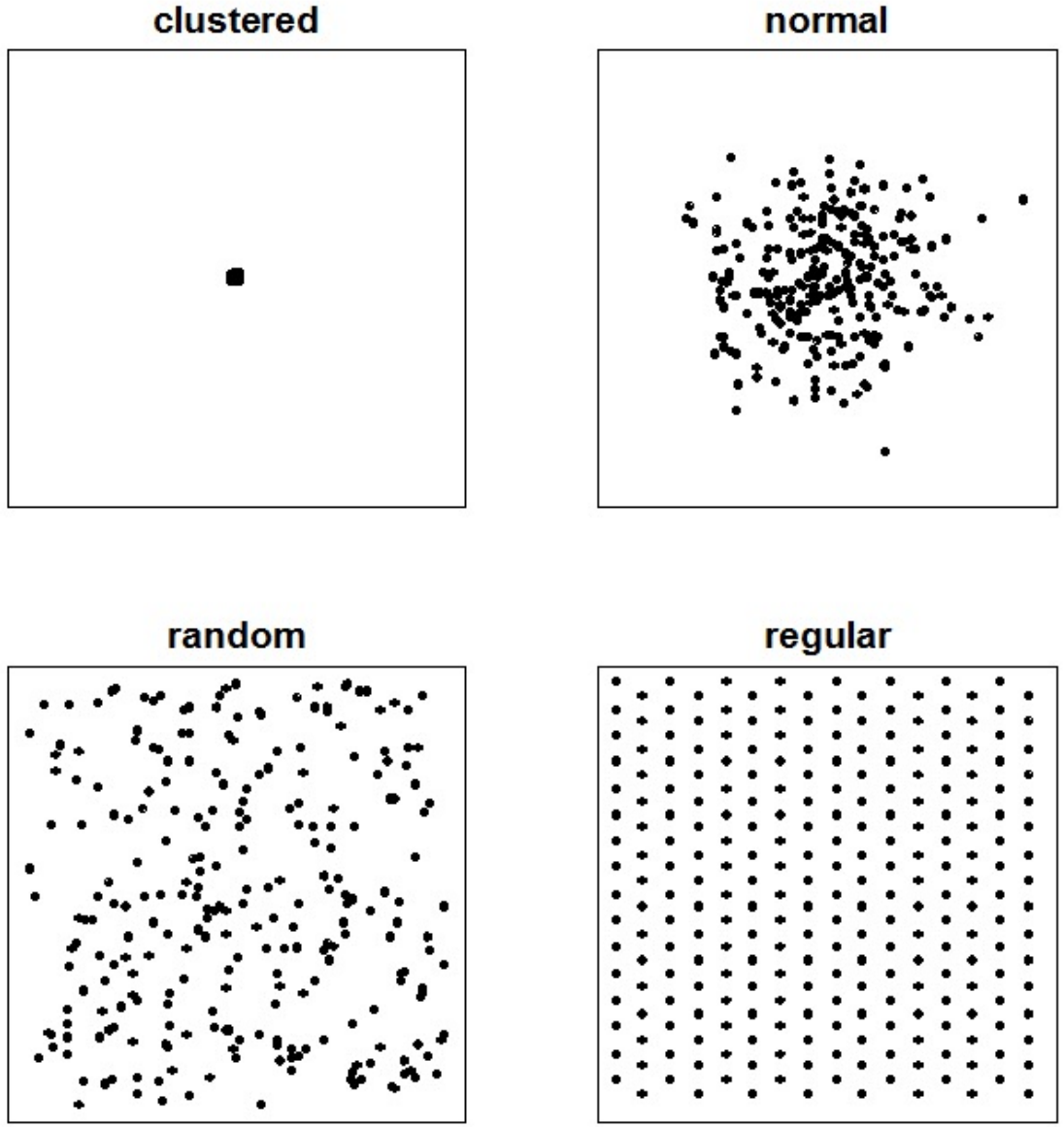


UCL

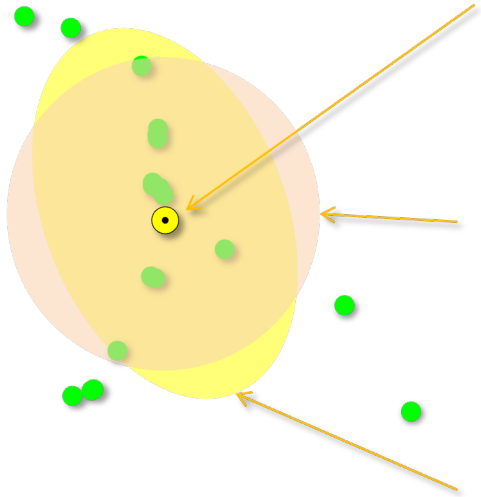
This week

- Focus analysis directly on point events rather than aggregation to some type of administrative geography.
- Roughly three ways to describe or characterise point processes: descriptive (summary) statistics, distance-based methods (average nearest neighbour, Ripley's K), density-based methods (DBSCAN, kernel density estimation).
- Some practical examples.

Point pattern analysis



Descriptive statistics I



Mean center is the computed average X and Y coordinate values.

$$\bar{s} = \left(\frac{\sum_{i=1}^{n} x_i}{n}, \frac{\sum_{i=1}^{n} y_i}{n} \right)$$

Standard distance is a measure of the variance between the average distance of the features to the mean center.

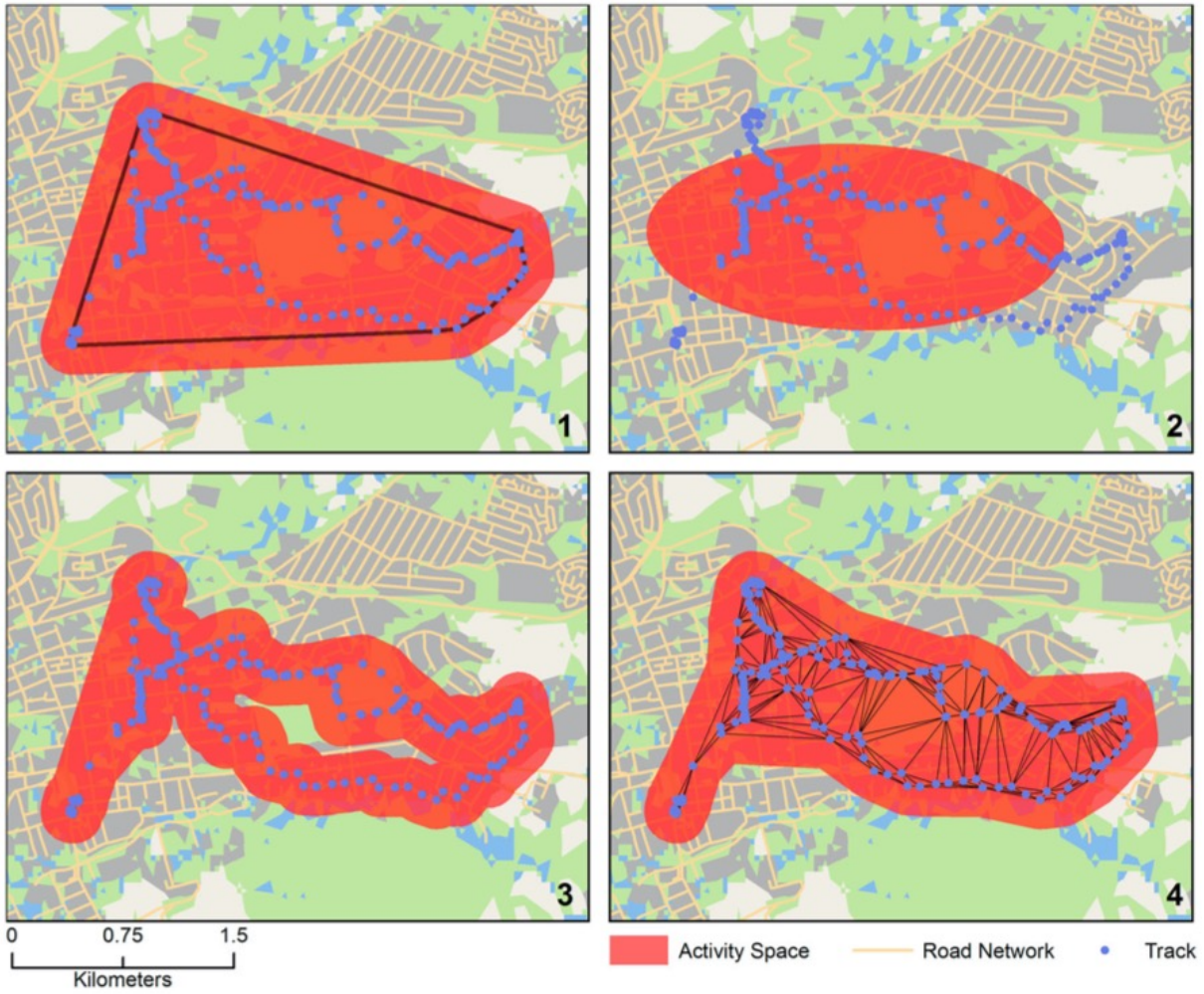
$$d = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu_x)^2 + (y_i - \mu_y)^2}{n}}$$

Standard deviational ellipse computes separate standard distances for each axis.

$$d_x = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu_x)^2}{n}}$$

$$d_y = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \mu_y)^2}{n}}$$

Descriptive statistics II



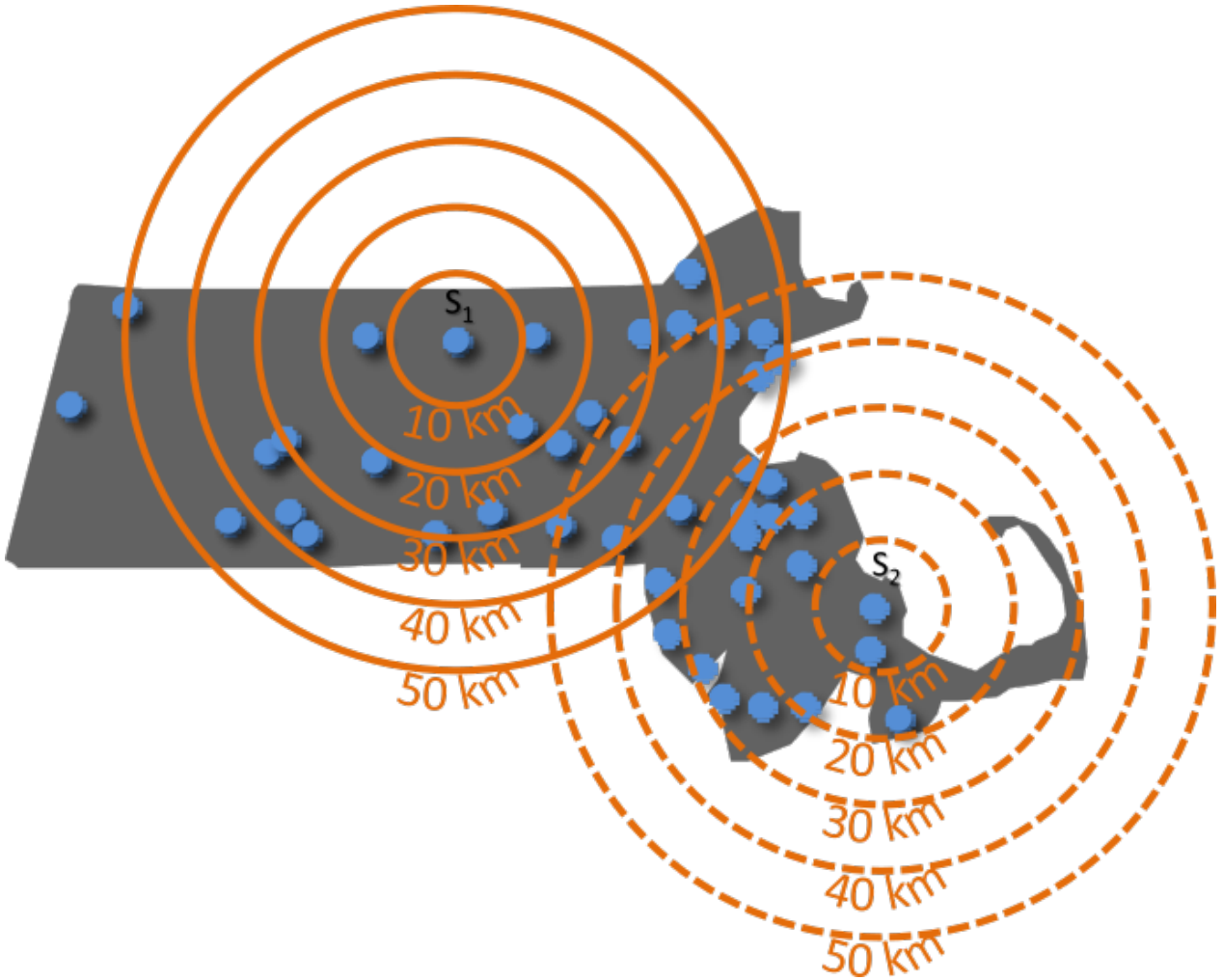
Distance-based measures I

- Using distance as a summary measure to describe a point process.
- Measure of spread of the distribution.
- Well-known example is average nearest neighbour (ANN).
- ANN measures the distance between each feature and its nearest neighbor's location. It then averages all these nearest neighbor distances.
- If the average distance is less than the average for a hypothetical random distribution, the distribution of the features being analysed is considered clustered. If the average distance is greater than a hypothetical random distribution, the features are considered dispersed.

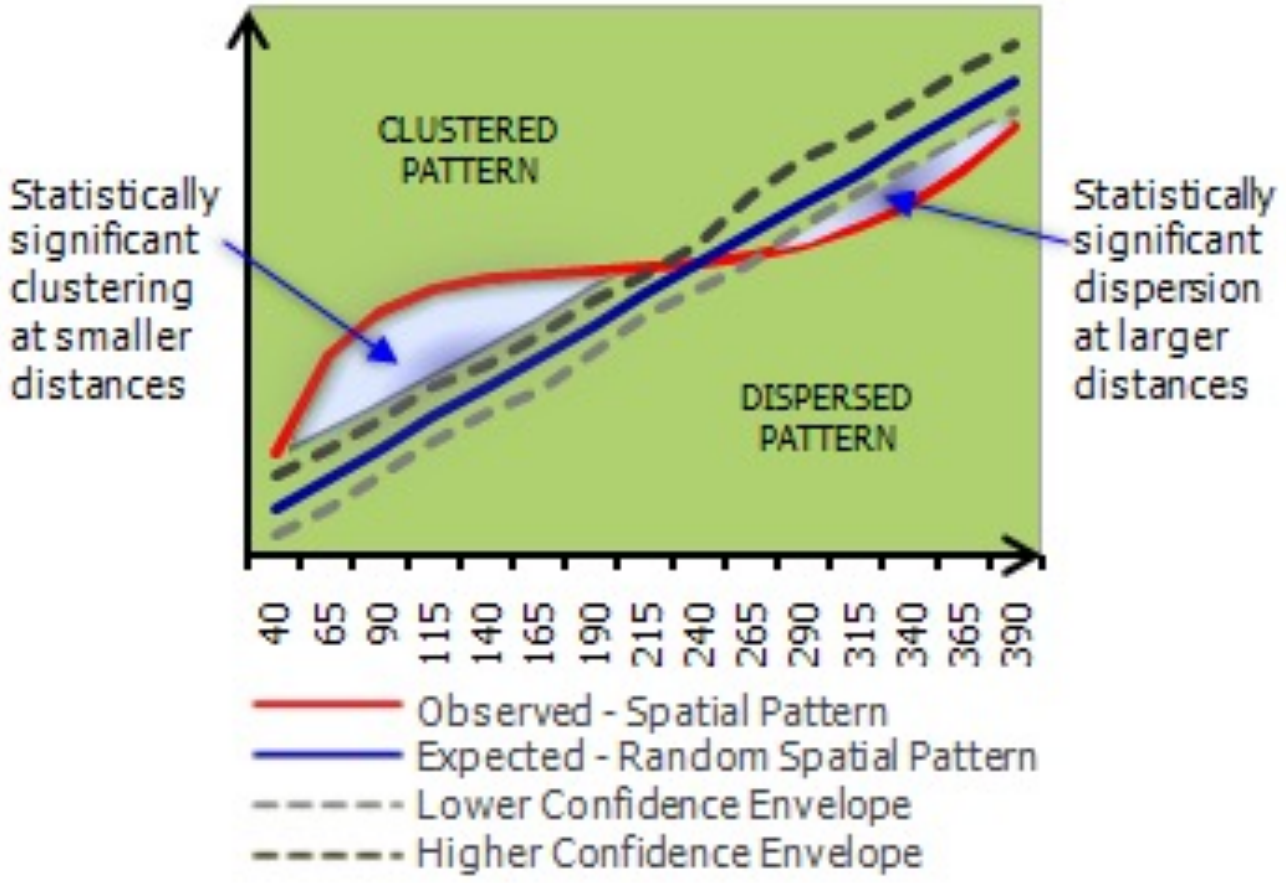
Distance-based measures II

- Another example is Ripley's K.
- Ripley's K counts the number of observations within a user defined set of distances and compares this to to a hypothetical (random) pattern of observations.
- Ripley's K function is generally calculated at multiple distances allowing you to see how point pattern distributions can change with scale. For example, at near distances, the points could cluster, while at farther distances, points could be dispersed.
- Distance-based measure of dispersion across scales

Distance-based measures III



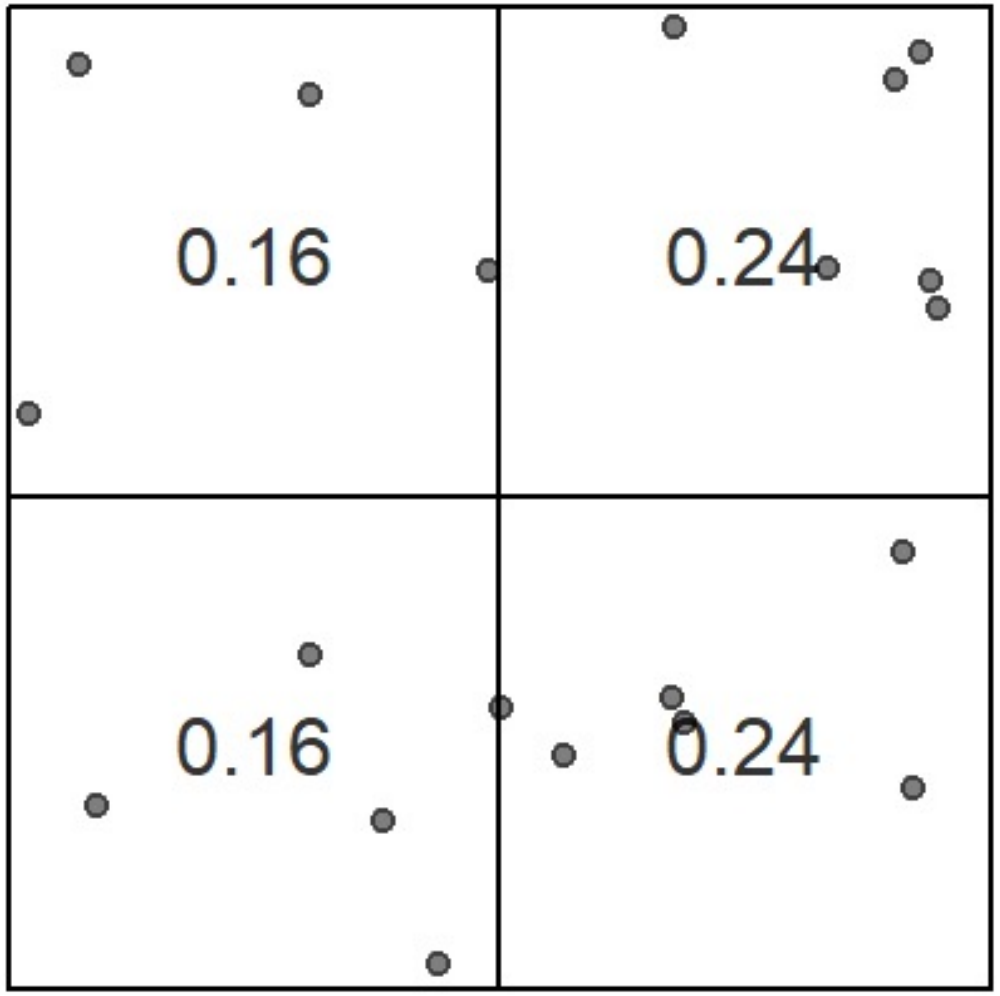
Distance-based measures IV



Density-based measures I

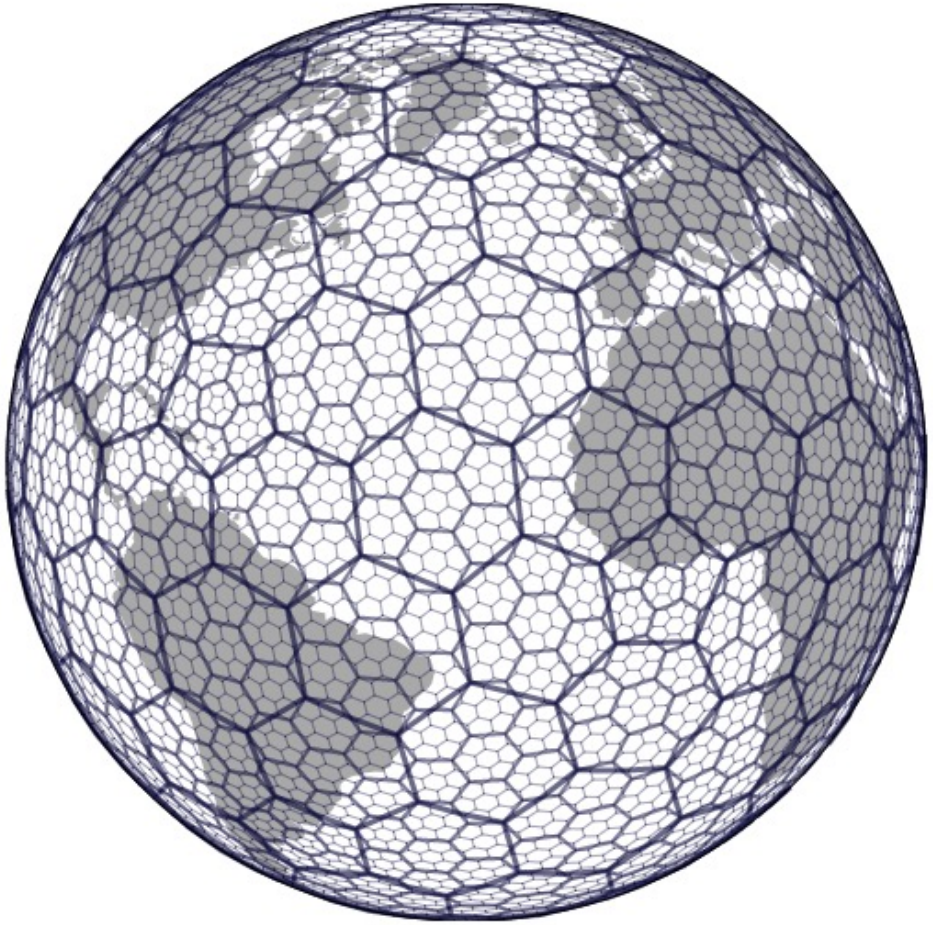
- Measure of the intensity of a point process.
- Density-Based Spatial Clustering of Applications with Noise (DBSCAN) to find high-density clusters.
- Kernel Density Estimation (KDE)

Density-based measures II



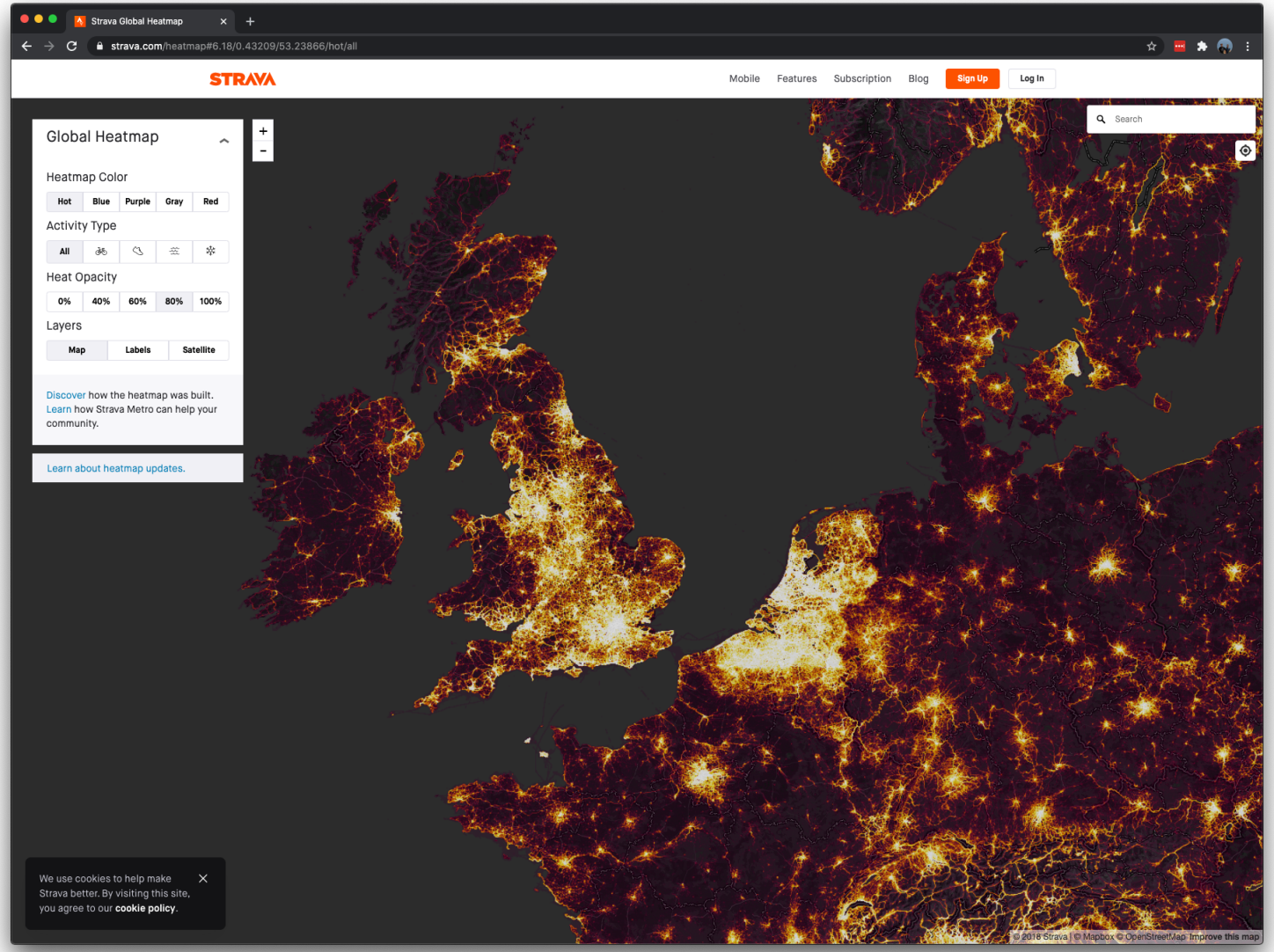
Gimond, M. 2020. *Geodesic geometry*. [online] https://mgimond.github.io/Spatial/index.html

Density-based measures III



Uber. 2018. *H3: Uber's Hexagonal Hierarchical Spatial Index*. [online] https://eng.uber.com/h3/

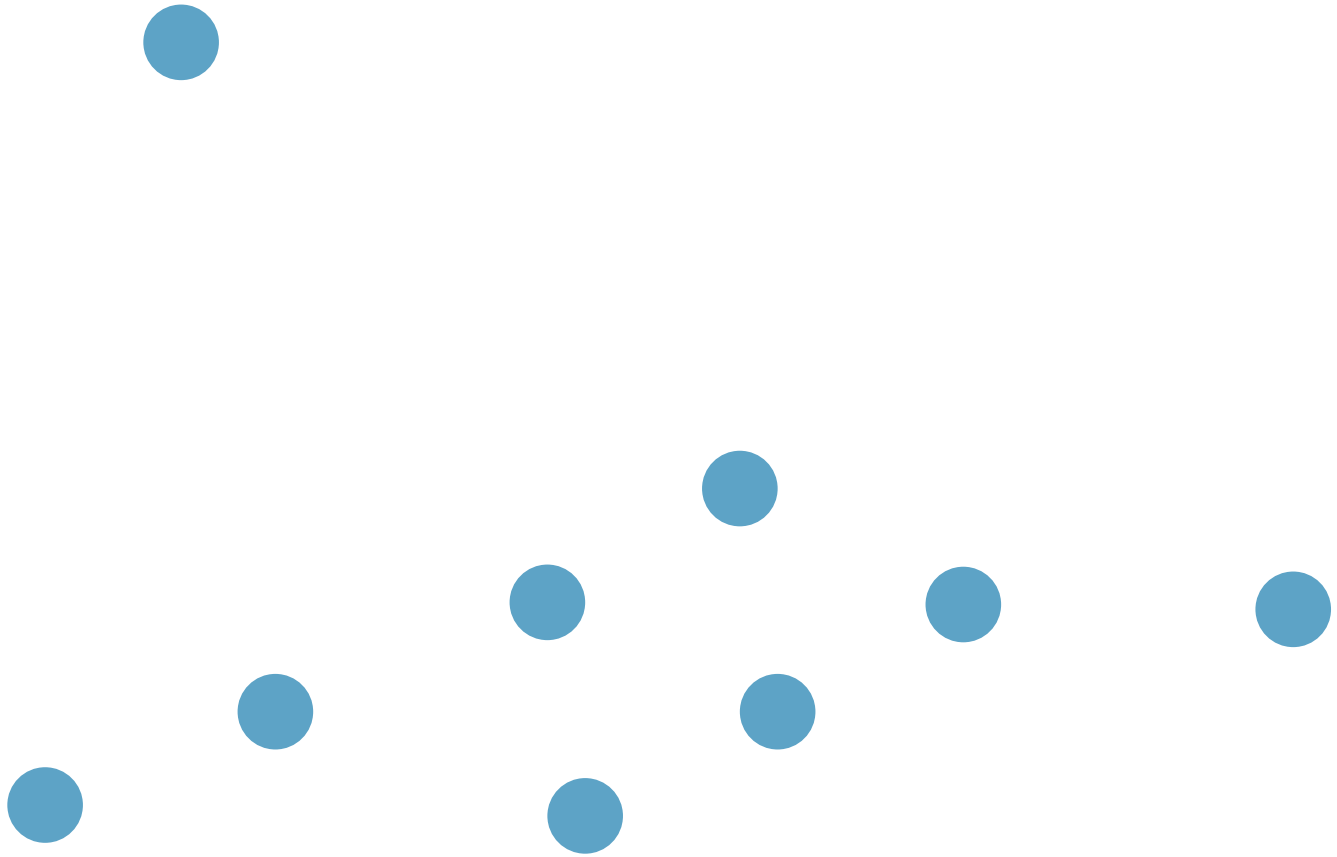
Density-based measures IV



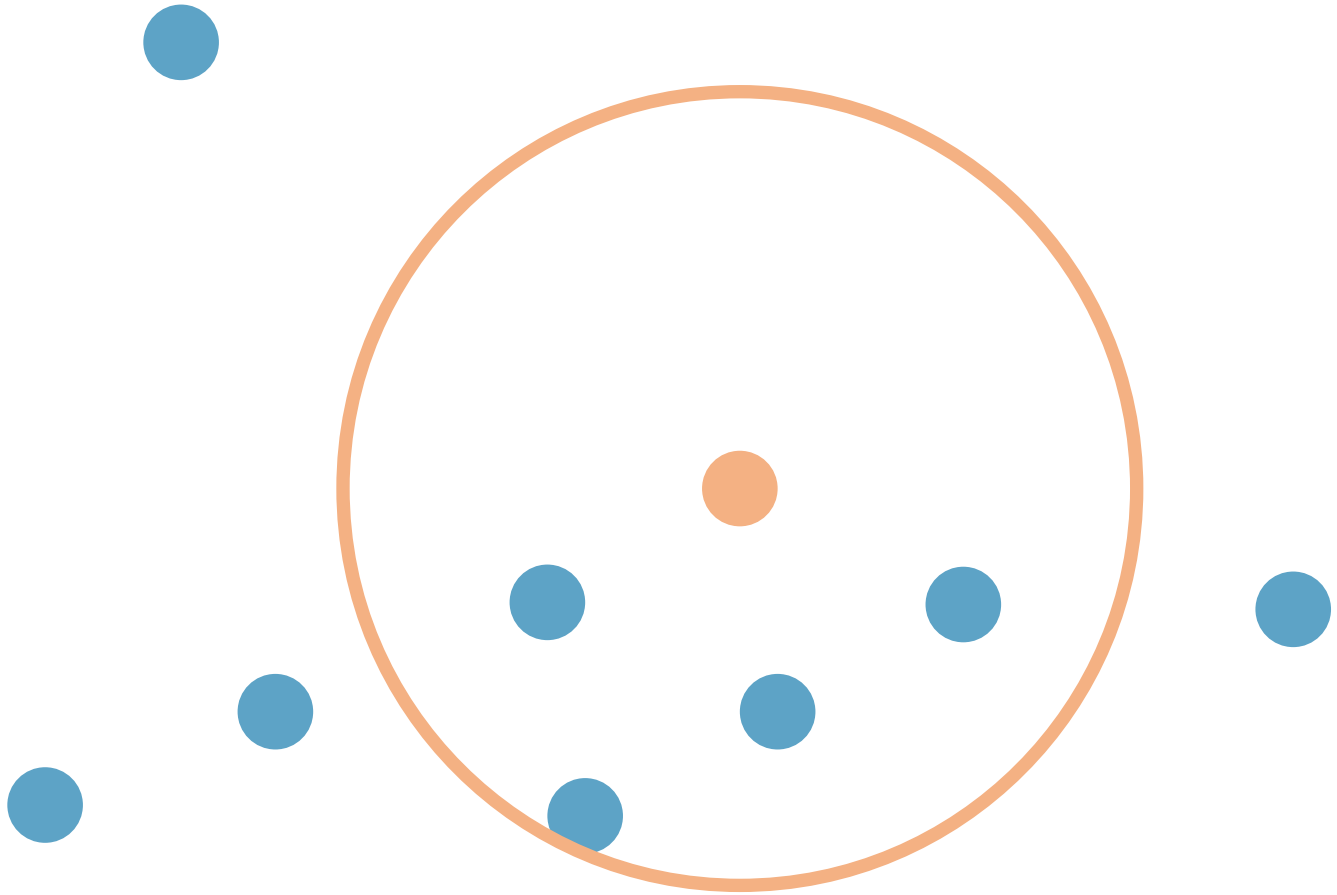
DBSCAN I

- Density-Based Spatial Clustering of Applications with Noise (DBSCAN)
- DBSCAN is probably one of the most common clustering algorithms and also most cited in scientific literature
- Functionally DBSCAN detects clusters of points, with noise, that reach the minimum density threshold based on two inputs: `minpts` and `epsilon` neighbourhood radius (`eps`), where `eps` is the maximum distance, or search radius, between each point in a cluster and `minpts` is the minimum number of points to be assigned to a cluster

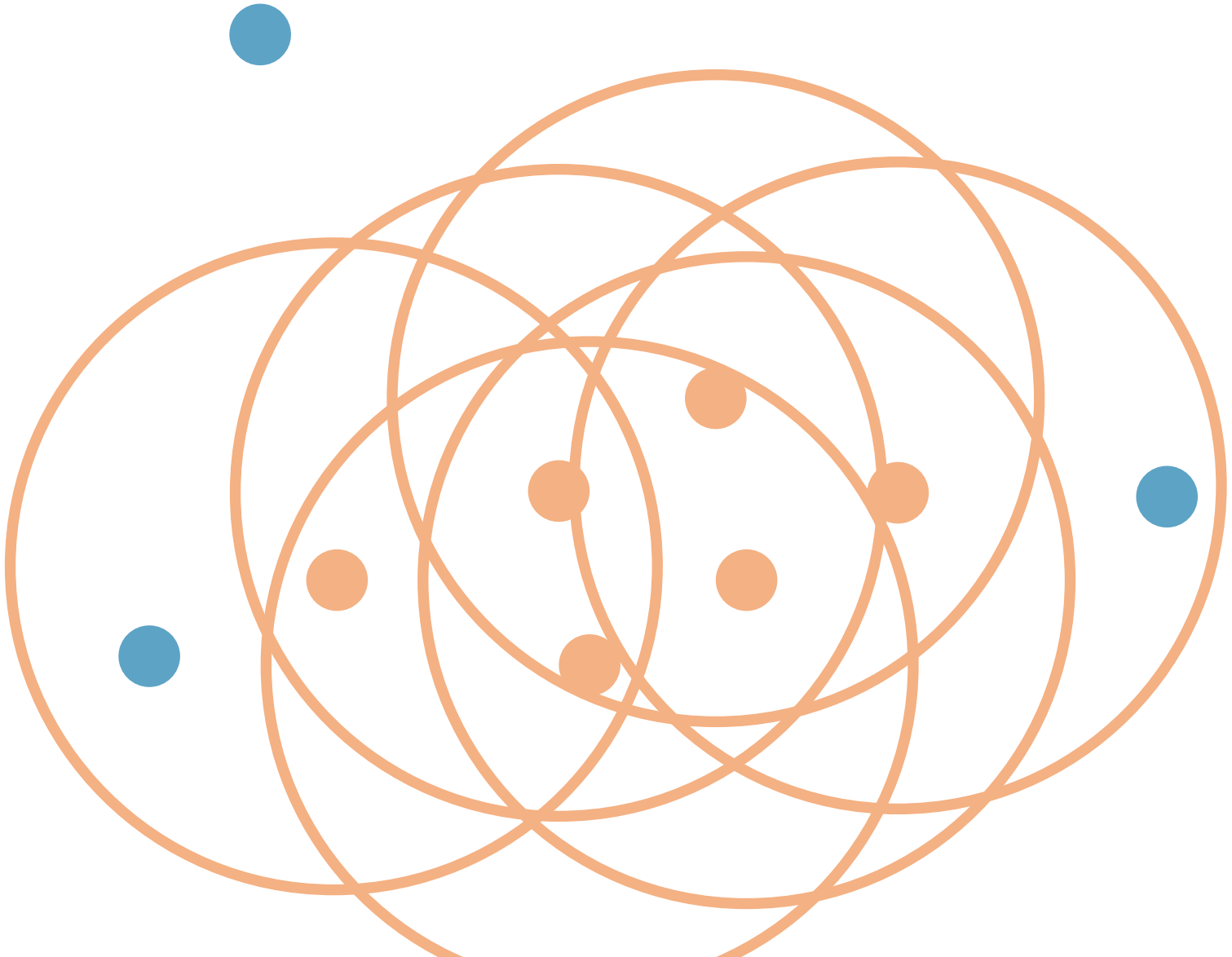
DBSCAN II



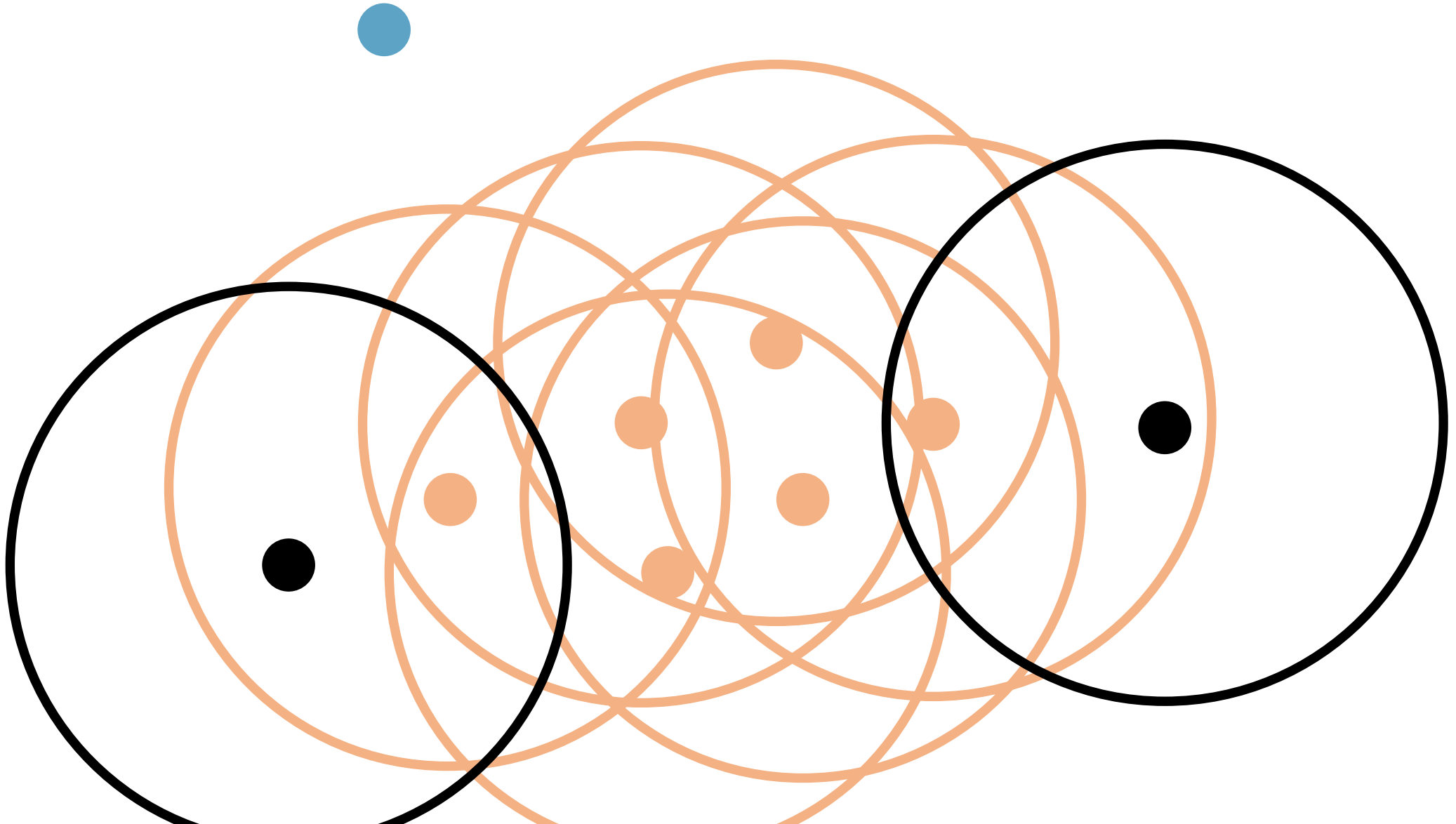
DBSCAN III



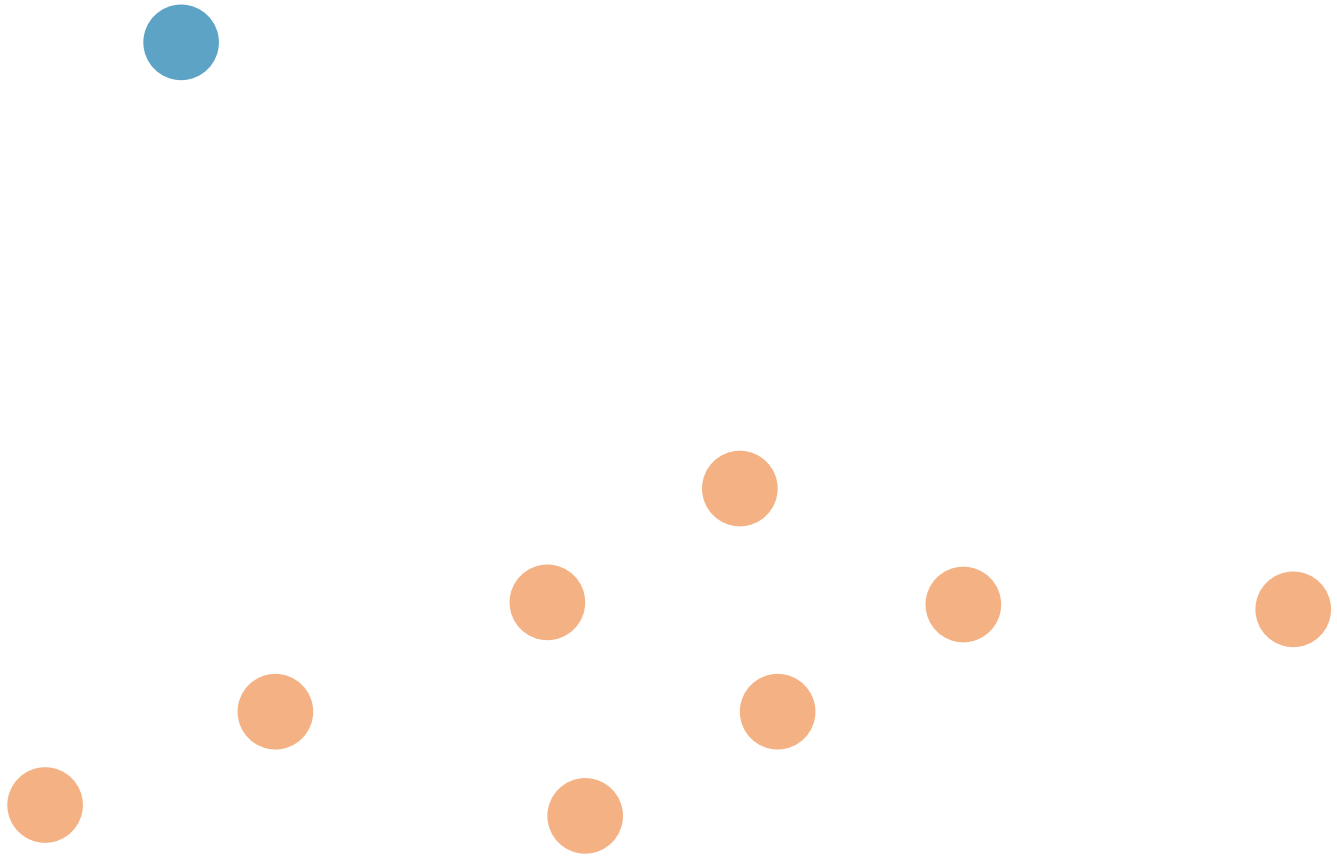
DBSCAN IV



DBSCAN V



DBSCAN V



DBSCAN VI

- No need to specify number of clusters a priori.
- DBSCAN can find non-linearly separable clusters (i.e. arbitrarily shaped).
- However: highly dependent on the distance measure and not robust to data sets with large differences in densities as the parameters cannot be chosen to cover both situations.

Kernel density estimation I

- Kernel density estimation (KDE) is a non-parametric way to estimate the probability density function of a random variable. It uses local information defined by windows (called kernels) to estimate densities of specified features at given locations.
- In essence it is a smoothing function where a continuous curve is created, based on a finite data sample.

Kernel density estimation II

Tycho — python — 85×25

```
>>> def make_data(N, f=0.3, rseed=1):
...     rand = np.random.RandomState(rseed)
...     x = rand.randn(N)
...     x[int(f * N):] += 5
...     return x
...
>>> x = make_data(1000)
>>> x
array([ 1.62434536e+00, -6.11756414e-01, -5.28171752e-01, -1.07296862e+00,
        8.65407629e-01, -2.30153870e+00,  1.74481176e+00, -7.61206901e-01,
        3.19039096e-01, -2.49370375e-01,  1.46210794e+00, -2.06014071e+00,
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        4.88518147e-01, -7.55717130e-02,  1.13162939e+00,  1.51981682e+00,
```
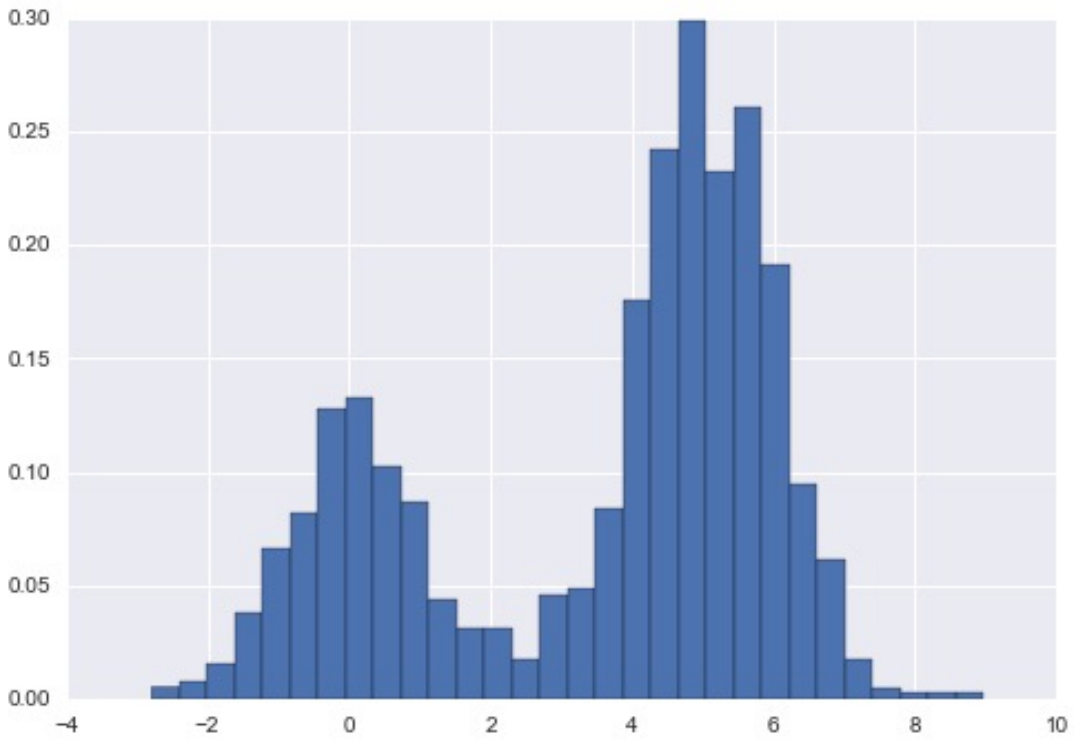
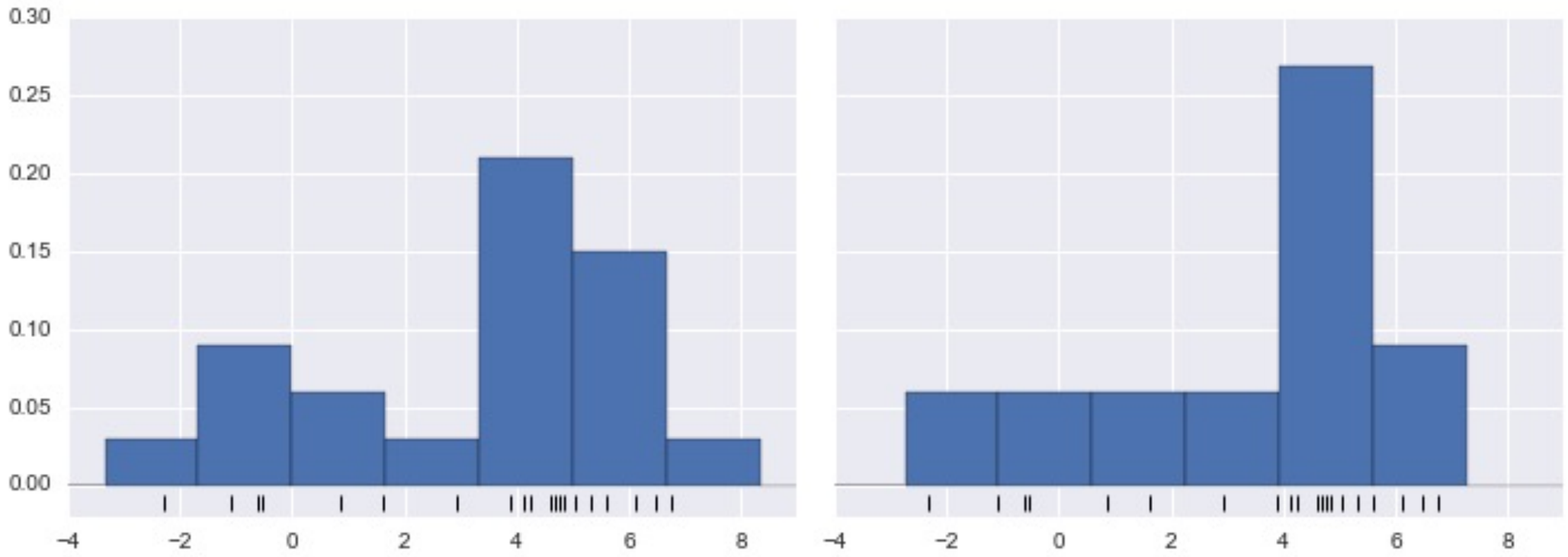

Kernel density estimation III



VanderPlas, J. 2016. *Python data science handbook: essential tools for working with data*. O'Reilly Media, Inc.

Kernel density estimation IV



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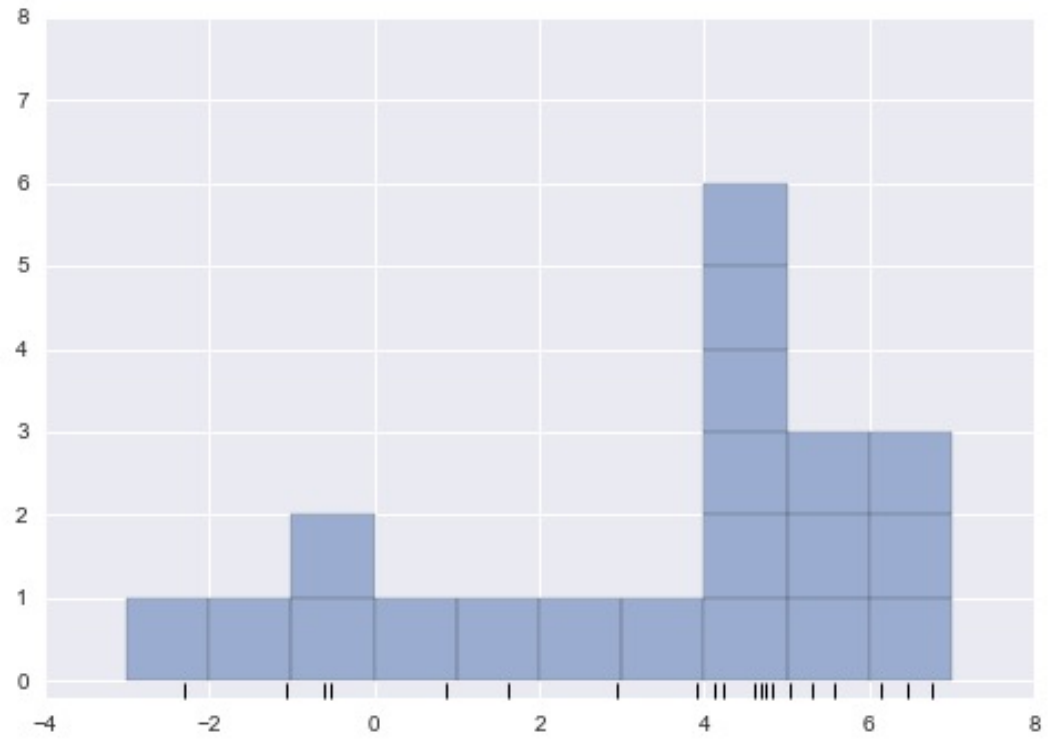
Kernel density estimation V



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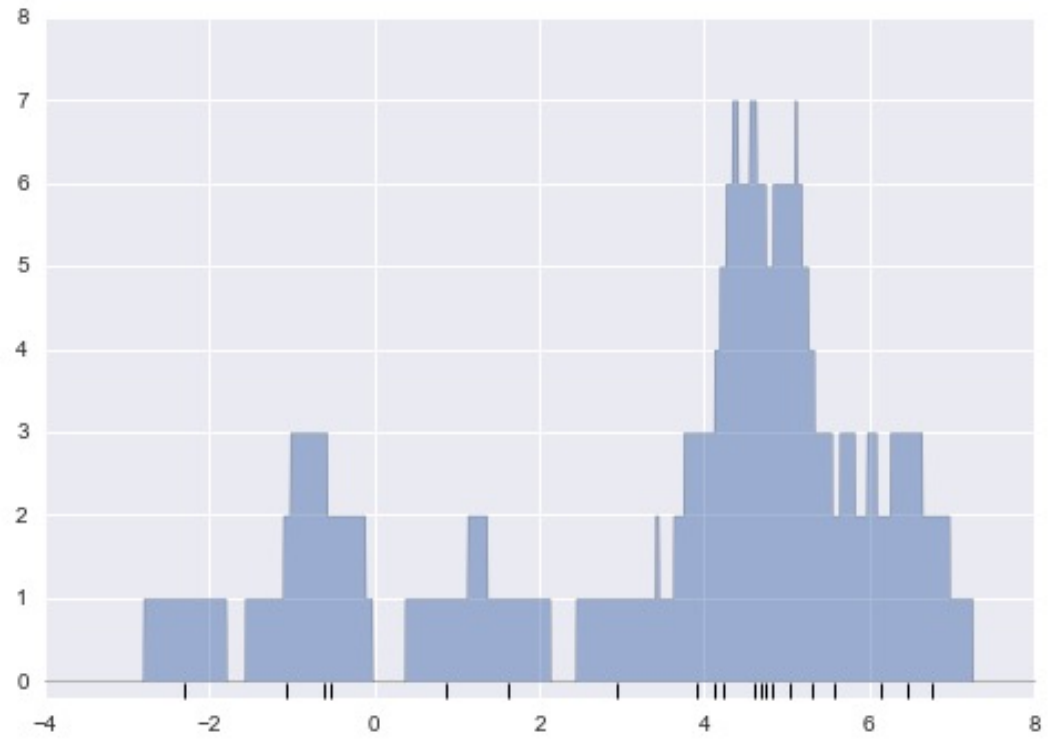
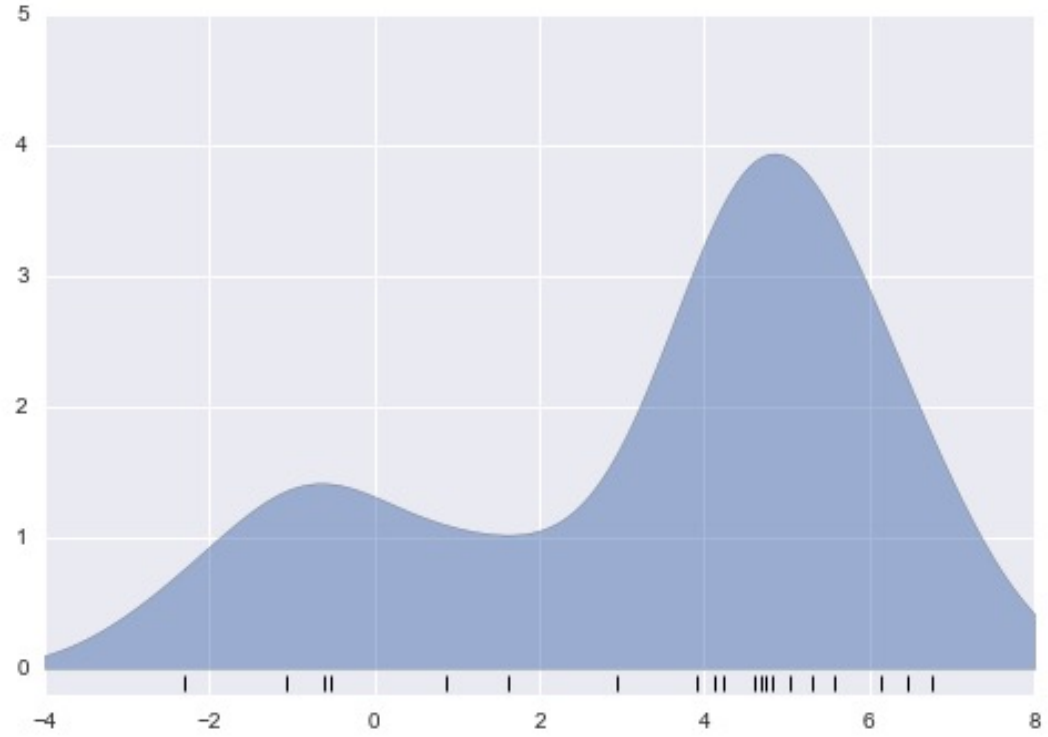
Kernel density estimation VI



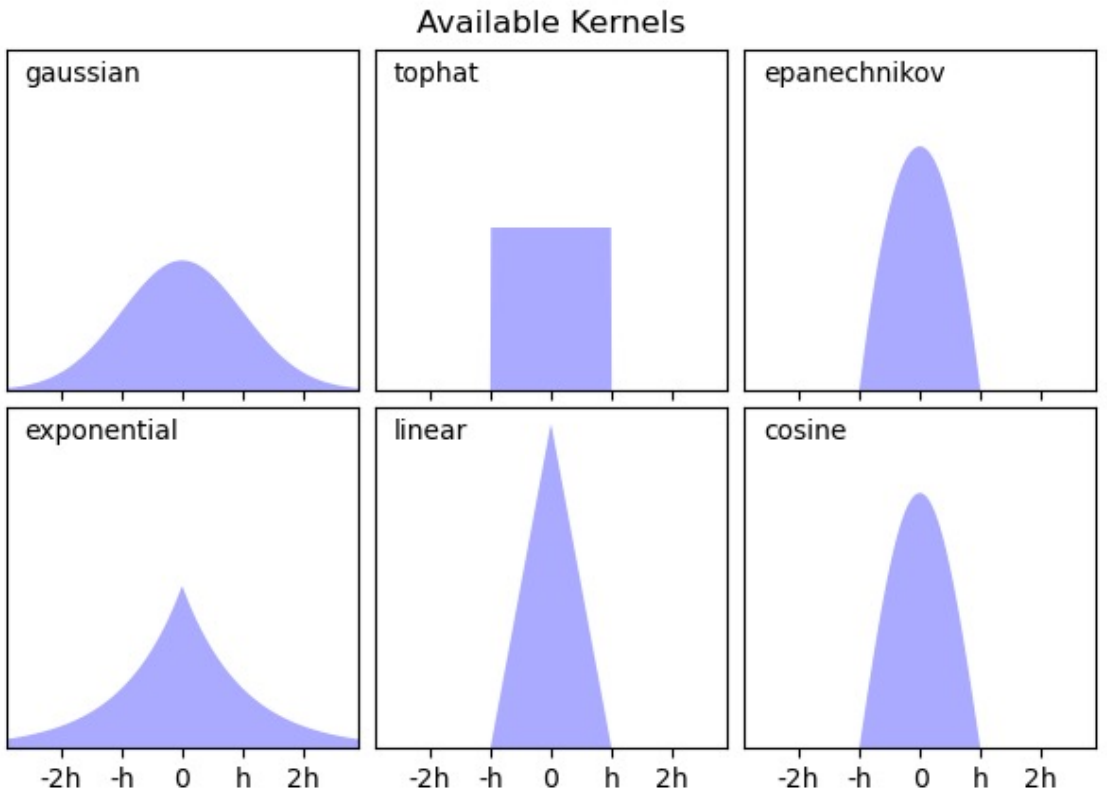
VanderPlas, J. 2016. *Python data science handbook: essential tools for working with data*. O'Reilly Media, Inc.

Kernel density estimation VII



VanderPlas, J. 2016. *Python data science handbook: essential tools for working with data*. O'Reilly Media, Inc.

Kernel density estimation VIII



Scikit-learn. 2020. *Density estimation.* [online] https://scikit-learn.org/stable/modules/density.html

Kernel density estimation IX

- A two-dimensional KDE can be mathematically represented as follows:

$$\hat{f}(x,y) = \frac{1}{nh^2} \sum_{i=1}^{n} K\left(\frac{d_{i,(x,y)}}{h}\right)$$

where $\hat{f}(x,y)$ is the estimated density value at location (x,y), n is the total number of event points under concern h is a measure of window width (i.e. kernel bandwidth), $d_{i,(x,y)}$ is the distance between event point location i and j, and K is a density function characterising how the contribution of point i varies as a function of $d_{i,(x,y)}$.

Two examples

Point pattern analysis ‘in action’ in some actual research:

- Not al point data sets can be properly analysed with summary measures, e.g. analysis of GPS trajectory data (‘movement data’).
- Some problems do not per se look like a ‘point pattern analysis’ problem, but actually can be conceived as such, e.g. surname profiling.

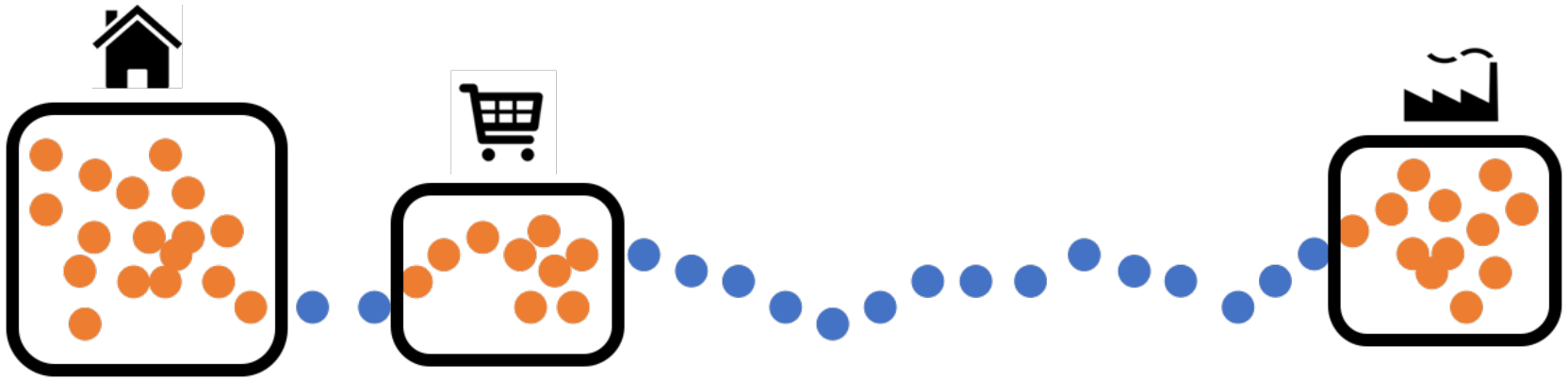
Movement data I

Van Dijk, J. T. 2018. Identifying activity-travel points from GPS-data with multiple moving windows. *Computers, Environment and Urban System* 70: 84-101

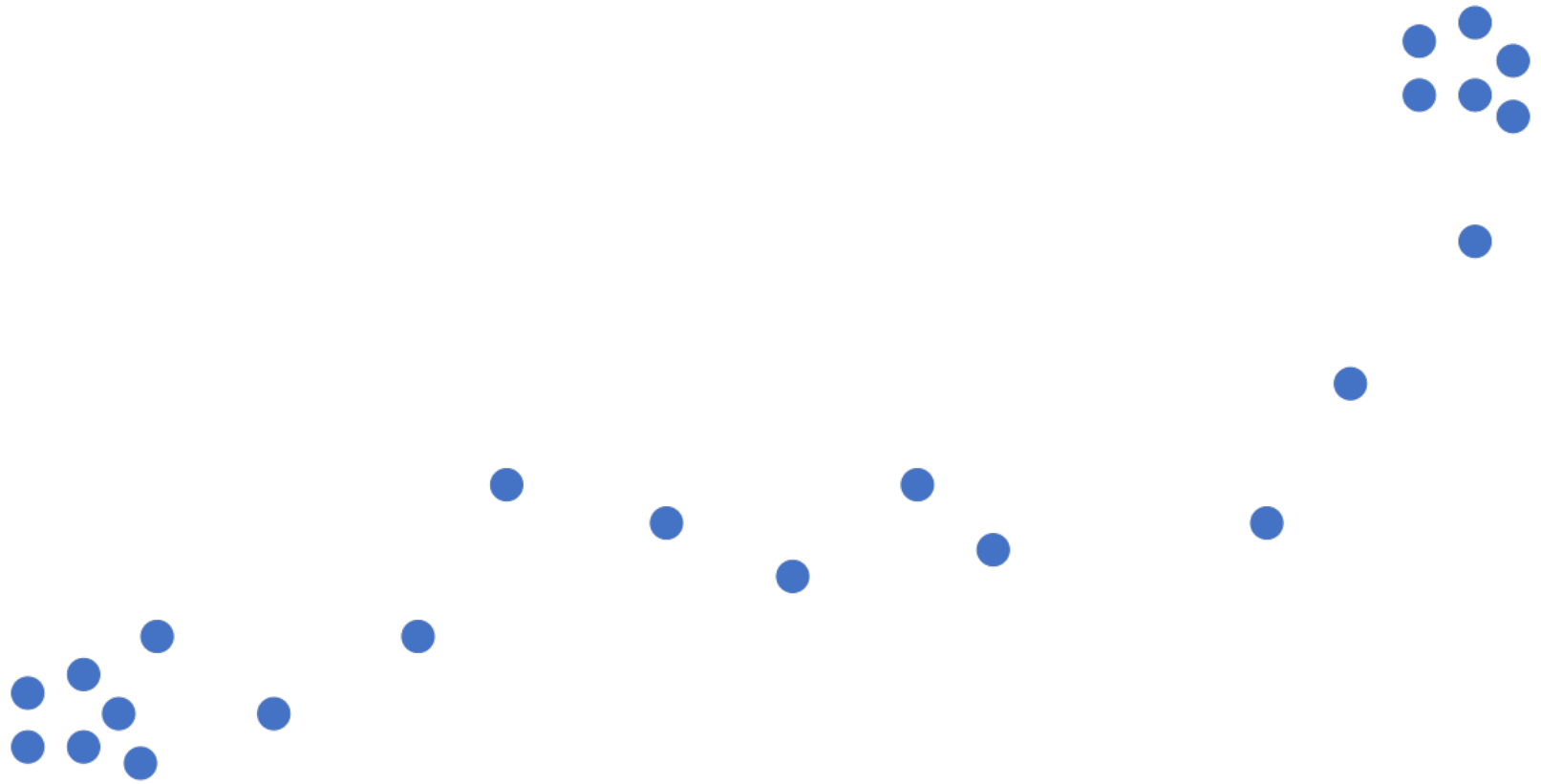
Movement data II

Van Dijk, J. T. 2018. Identifying activity-travel points from GPS-data with multiple moving windows. *Computers, Environment and Urban System* 70: 84-101

Movement data III

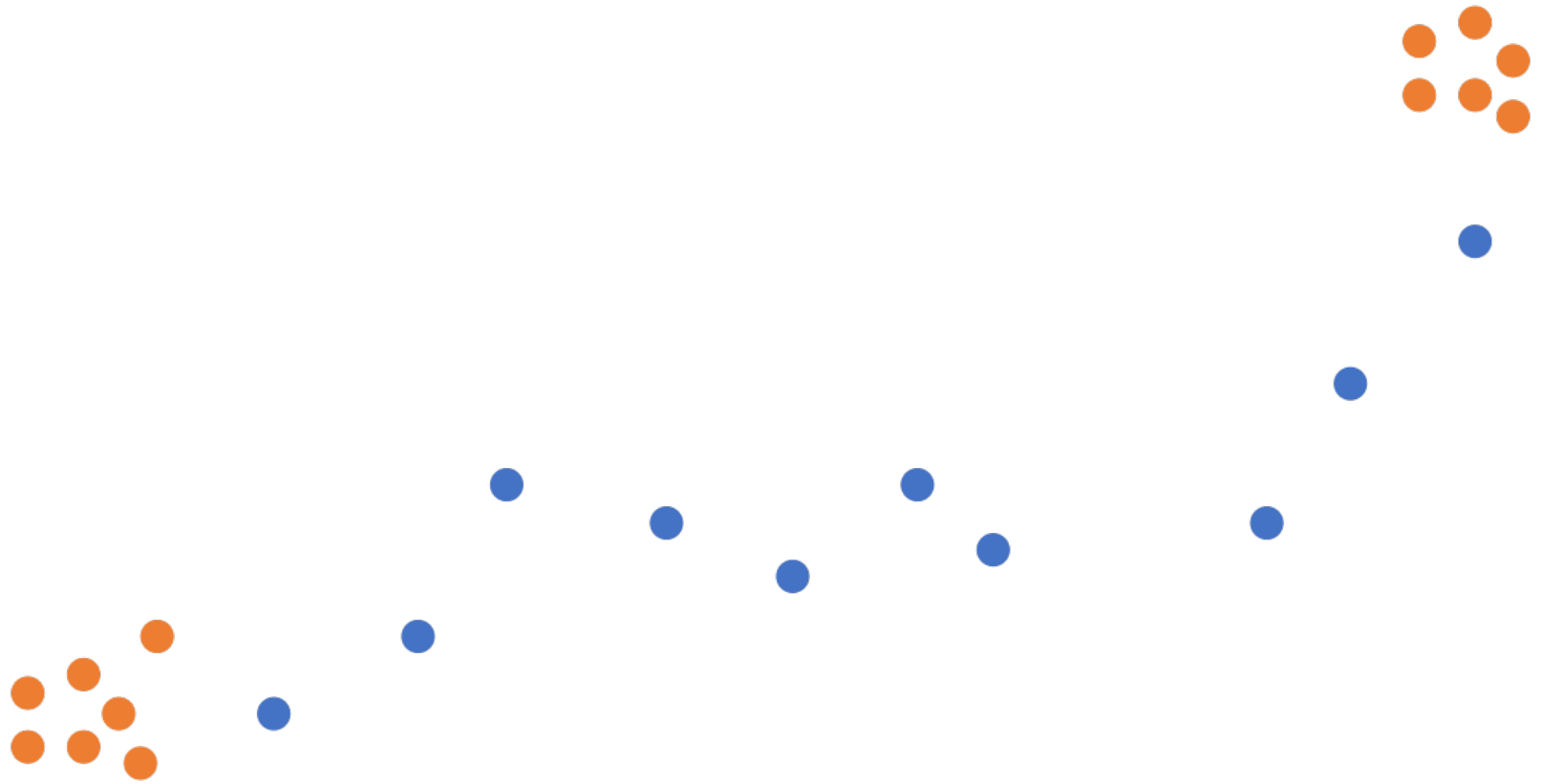


Movement data IV



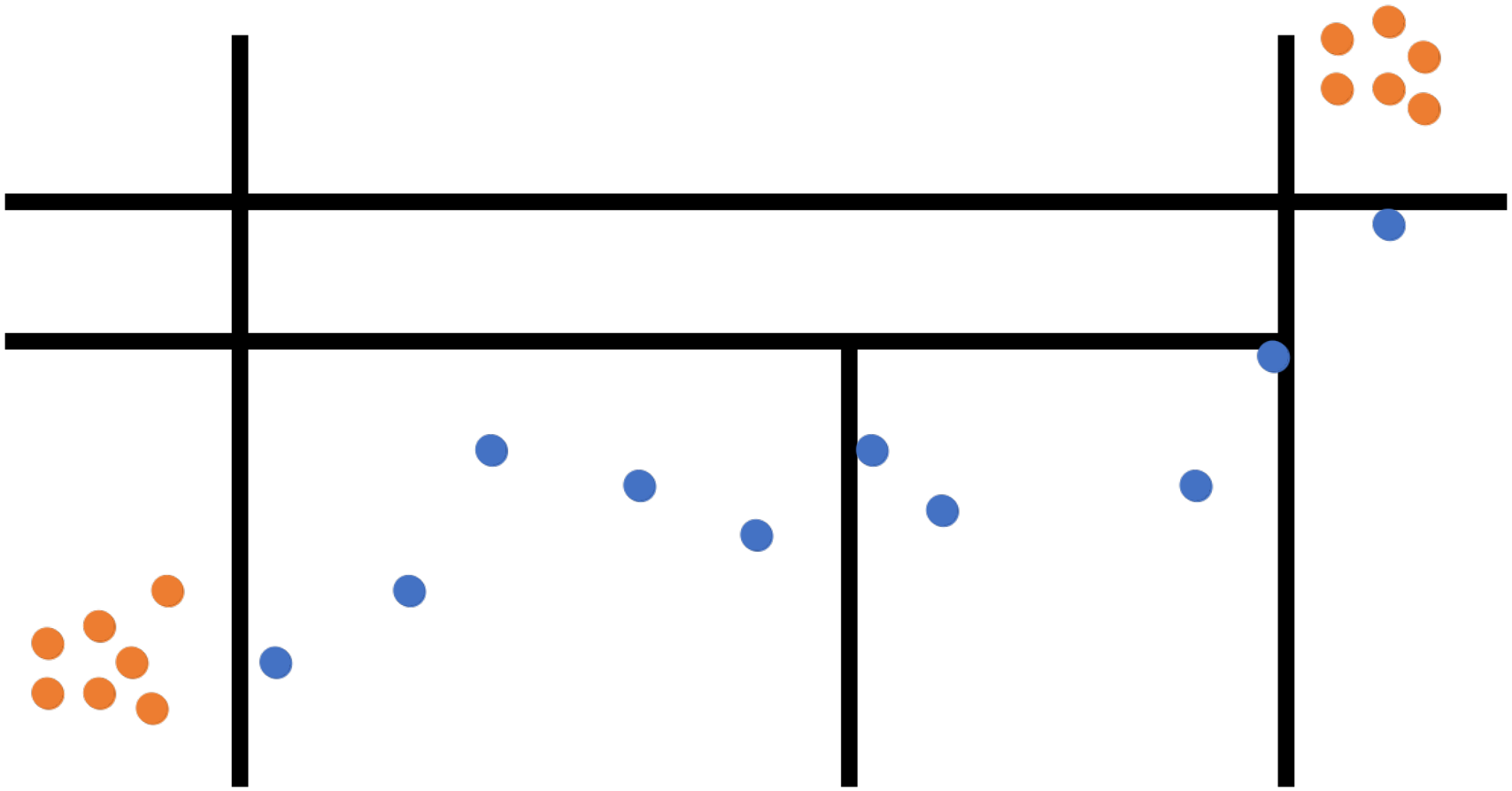
Van Dijk, J. T. & T. de Jong. 2017. Post-processing GPS tracks in reconstructing travelled routes in a GIS-environment: Network subset selection and attribute adjustment. *Annals of GIS* 23(3): 203-217

Movement data V



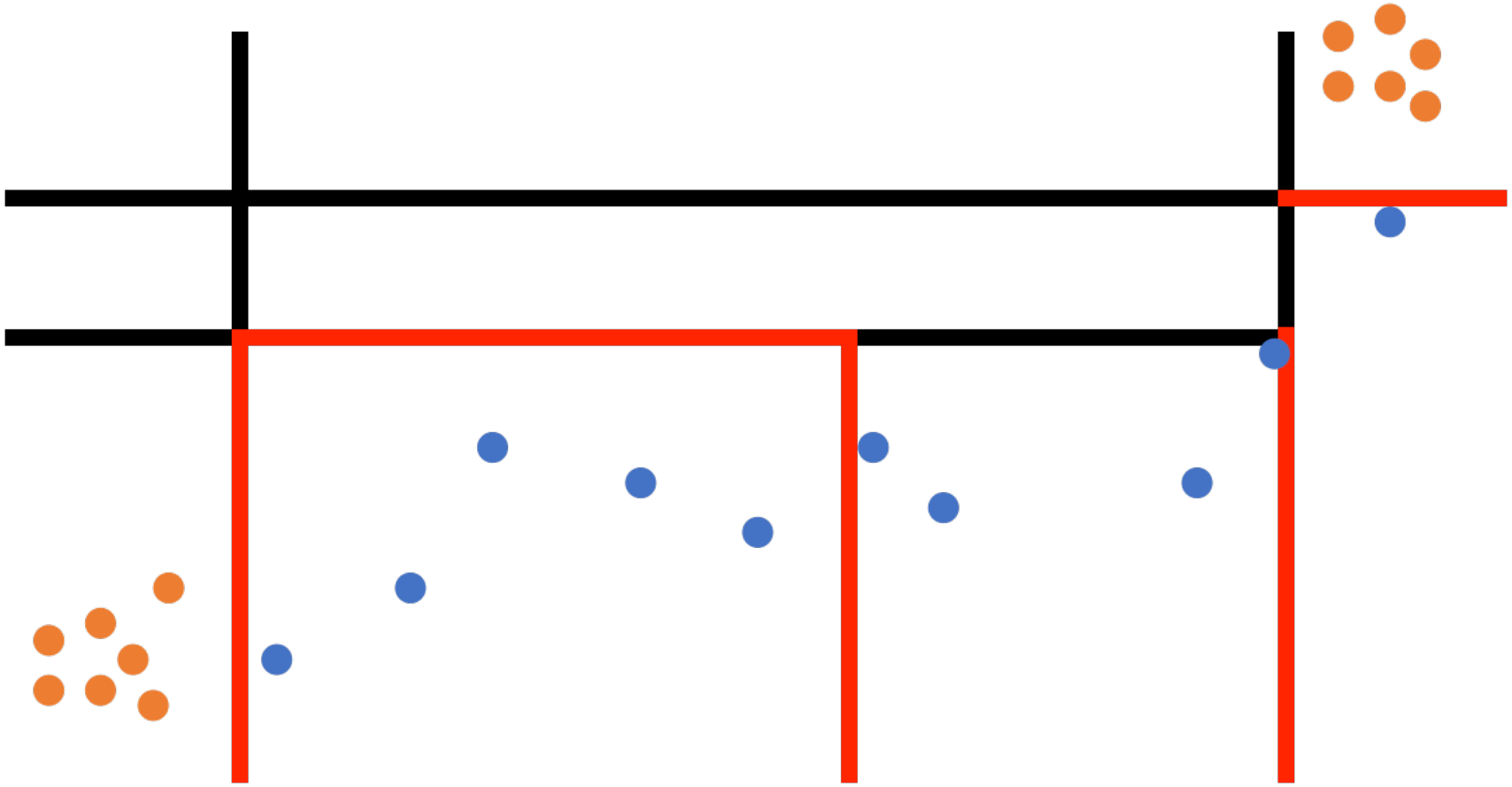
Van Dijk, J. T. & T. de Jong. 2017. Post-processing GPS tracks in reconstructing travelled routes in a GIS-environment: Network subset selection and attribute adjustment. *Annals of GIS* 23(3): 203-217

Movement data VI



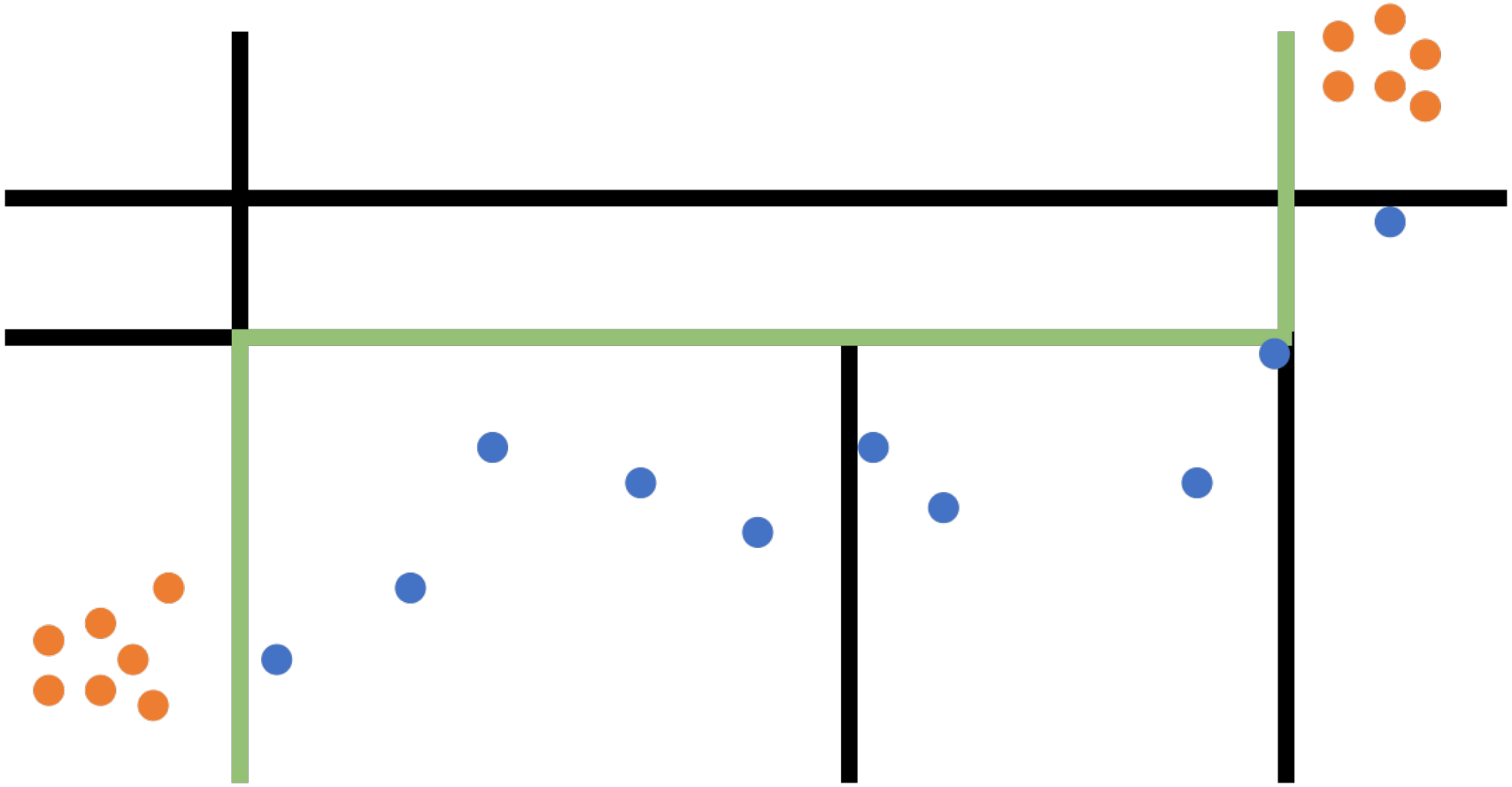
Van Dijk, J. T. & T. de Jong. 2017. Post-processing GPS tracks in reconstructing travelled routes in a GIS-environment: Network subset selection and attribute adjustment. *Annals of GIS* 23(3): 203-217

Movement data VII



Van Dijk, J. T. & T. de Jong. 2017. Post-processing GPS tracks in reconstructing travelled routes in a GIS-environment: Network subset selection and attribute adjustment. *Annals of GIS* 23(3): 203-217

Movement data VIII



Van Dijk, J. T. & T. de Jong. 2017. Post-processing GPS tracks in reconstructing travelled routes in a GIS-environment: Network subset selection and attribute adjustment. *Annals of GIS* 23(3): 203-217

Surname profiling I

- Personal names contain socio-demographic information.
- But also: many surnames contain spatial information at a variety of scales relating to the origins of many of their bearers.
- Data: Historic Census of Population 1851-1911, Consumer Registers 1997– 2016.
- 1.2 million surnames with locations (Historic Parishes, unit postcodes and geo-coded addresses for several years of data).

“Van Dijk”

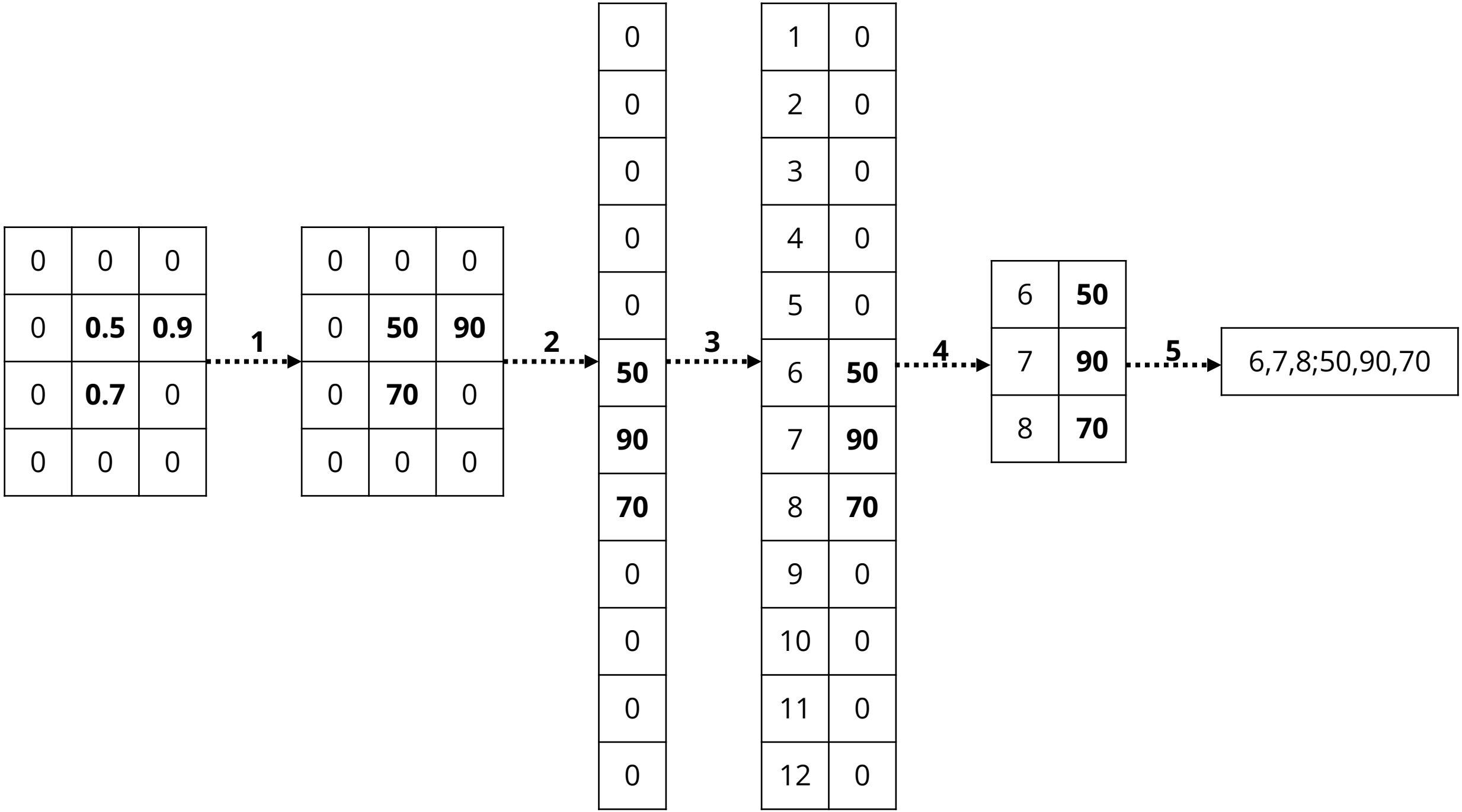
“Lansley”

“Rossall”

Surname profiling II

Combination on various point pattern analysis techniques:

- kernel densities to map the surname concentrations of names found in Great Britain executed over a 1000m x 1000m grid.
- deconstruction of grids as sparse matrices to optimise storage and database retrieval (storing 1.2 million KDEs is challenging), followed by DBSCAN to create contours of highest relative density (vectorisation).


0 0 0
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0 70 0
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50
90
70
0
0
0
0
3
1 0
2 0
3 0
4 0
5 0
6 50
7 90
8 70
9 0
10 0
11 0
12 0
4
6 50
7 90
8 70
5
6,7,8;50,90,70

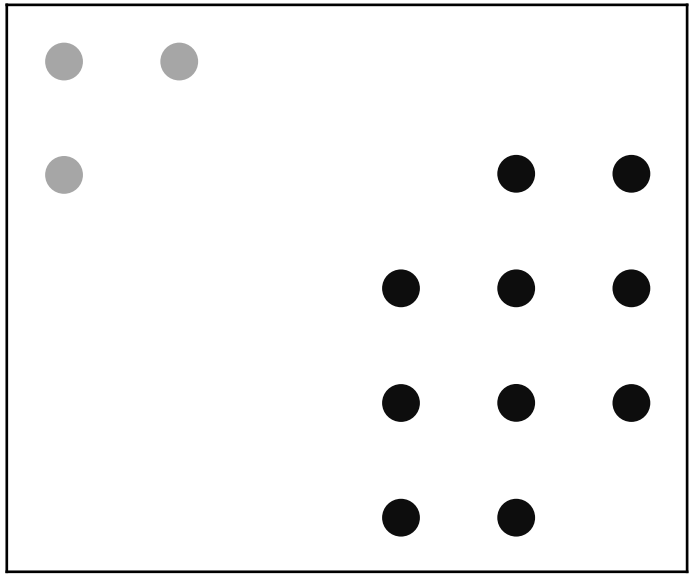
1

50	**90**	20	0	10	10
80	10	0	20	**40**	**50**
30	0	10	**40**	**60**	**80**
0	0	20	**50**	**70**	**50**
0	0	30	**50**	**90**	30

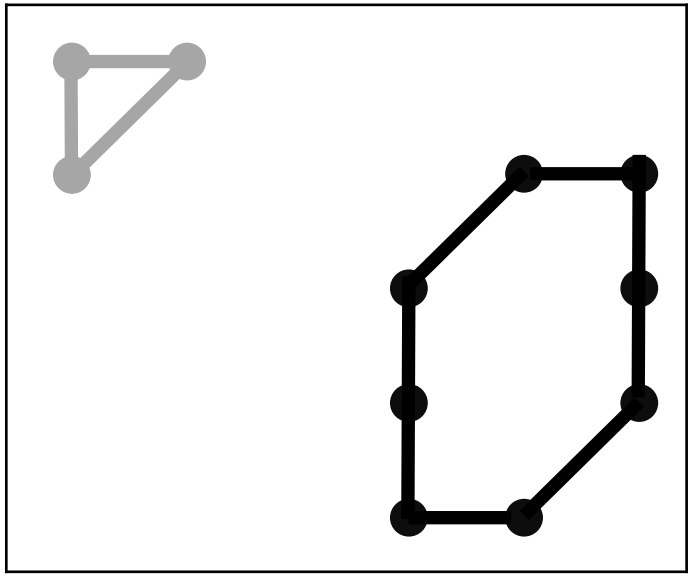
2

50	**90**			
80			**40**	**50**
		40	**60**	**80**
		50	**70**	**50**
		50	**90**	

3



4

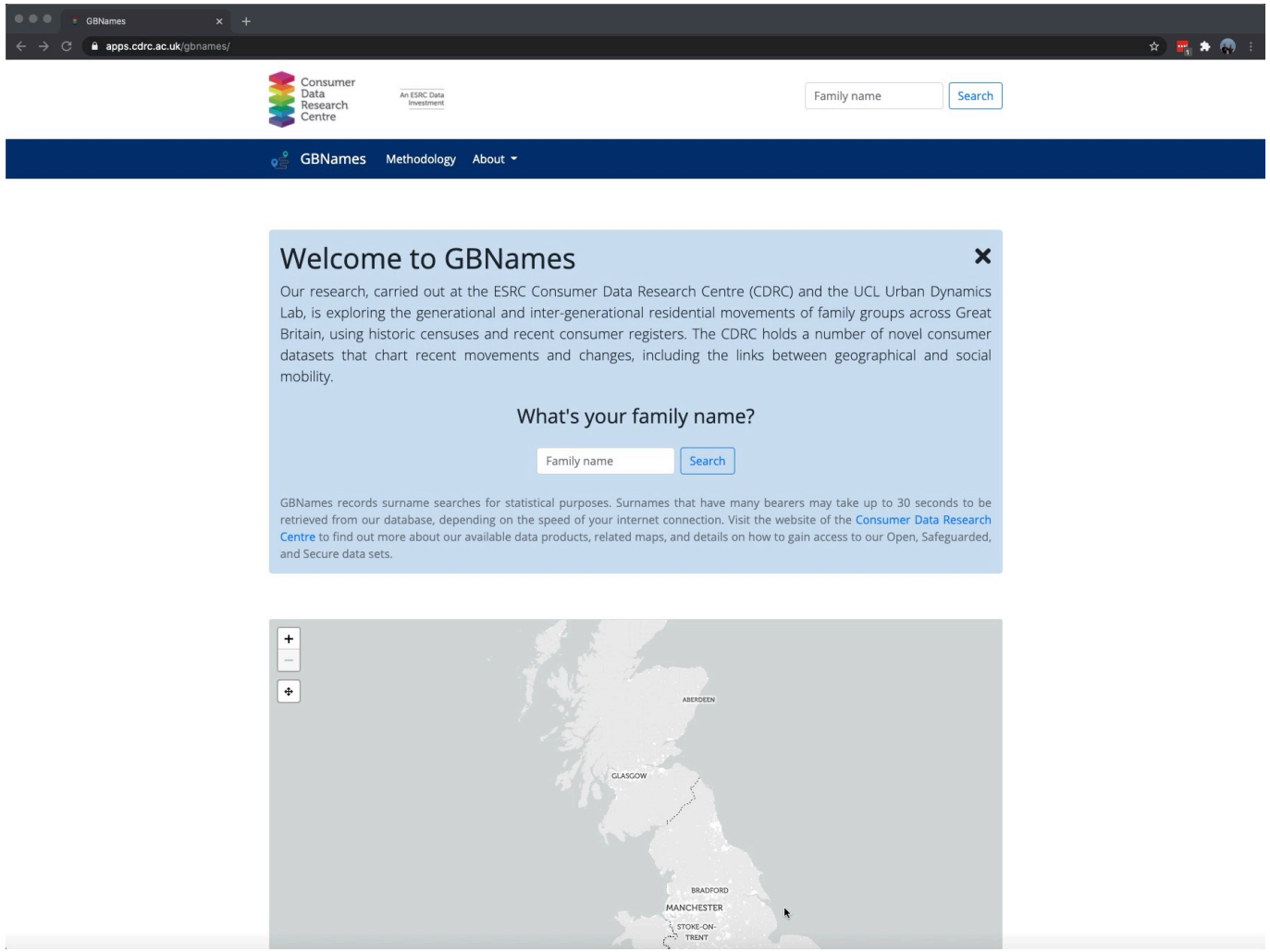


Surname profiling III

Why does the DBSCAN work?

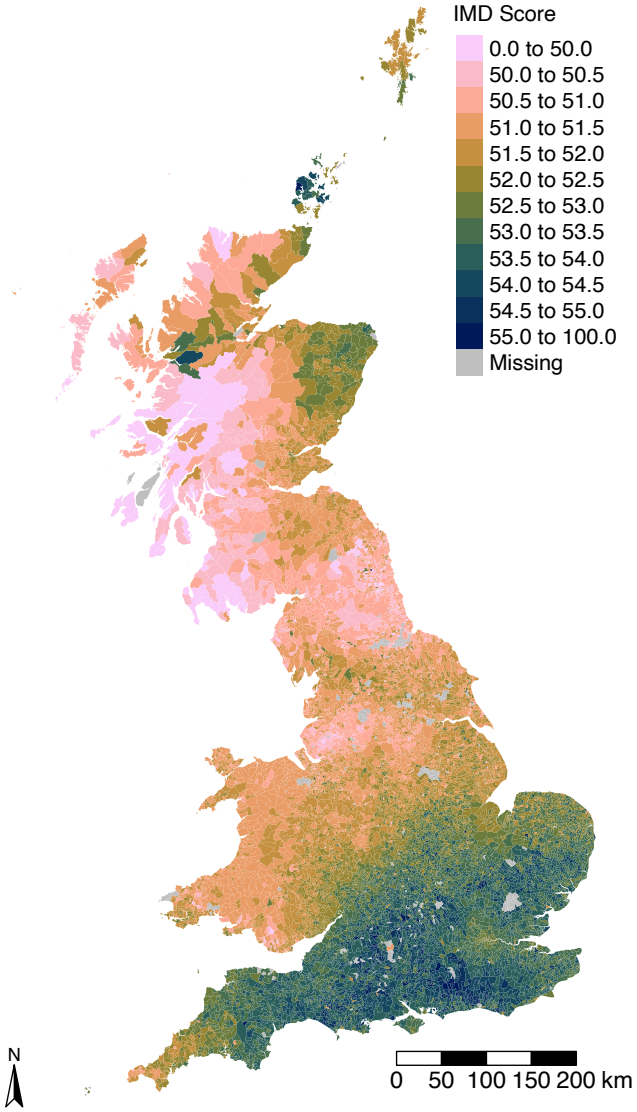
- Because the 1000 ×1000 m grid resolution never changes, we know that two adjacent grid centre points are always within a distance of approximately 1415 metres and we can thus use a distance constrained neighbourhood search.
- It is fast.

Surname profiling IV



Van Dijk, J. T. & P. A. Longley. 2020. Interactive display of surname distributions in historic and contemporary Great Britain. *Journal of Maps* 16(1): 68-76

Surname profiling V



Longley, P. A. Van Dijk, J. T., & Lan, T. 2021. The geography of intergenerational social mobility in Great Britain. *Nature Communications* 12: 6050.

Conclusion

- Point pattern analysis rather than aggregation to some type of administrative geography – we talked through several techniques today.
- Different approaches possible depending on the type of question you try to answer (e.g. characterisation of a point process versus cluster identification).
- Not all data may present themselves as clear candidates for these types of analyses.

Questions

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