Prof. M. A. Abdou
TA: Tyler Rhodes

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MAE 237D

Fusion Engineering and Design

FINAL EXAM

Take Home Exam

Due: Thursday, March 17, 2016
at 4:00pm
(Submit in 44-114 Eng IV to Emily or Jesse)

Attempt Only Six Problems

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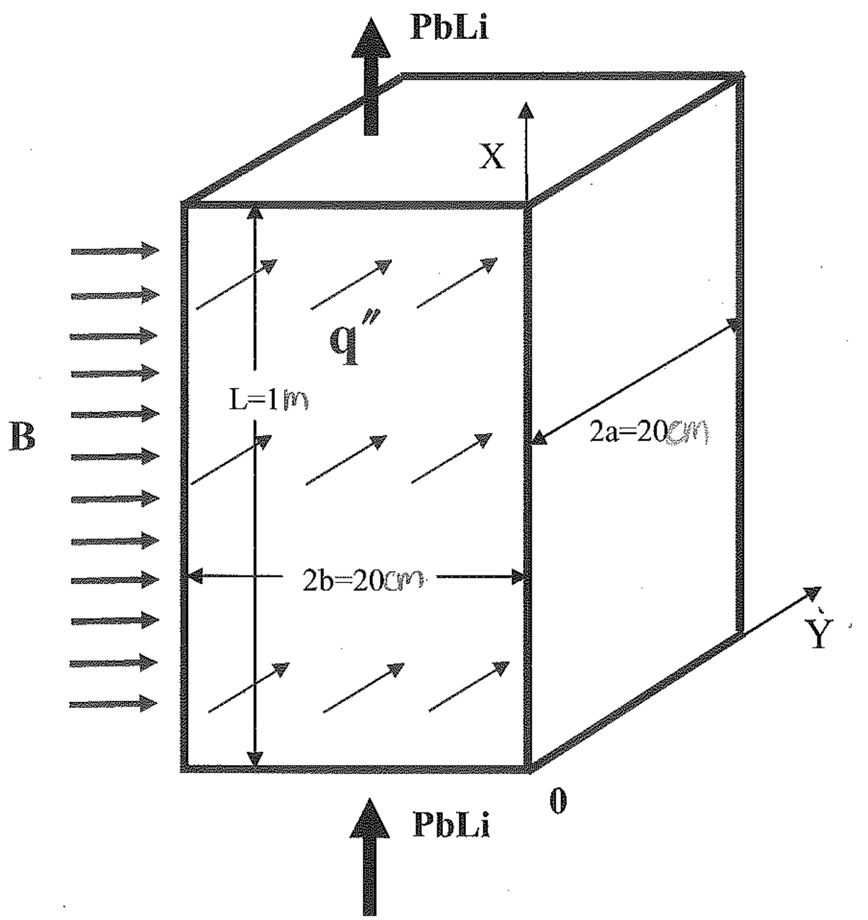
Student ID#: 804037581

- **Include the details of your solutions**
- **Provide informal citations for any sources used**
- **Make, indicate, and justify any significant assumptions**
- **Please work independently**

Problem 1

In a self-cooled poloidal PbLi blanket, the liquid metal flows through rectangular ducts made of RAFM steel. The wall thickness of the duct is 2 mm. Consider one of the front ducts (facing the plasma), assuming idealized conditions when the duct is fully decoupled electrically from the rest of the blanket and also neglect heat exchange with all other ducts. The flow velocity is 0.5 m/s. The toroidal magnetic field is 5 T. The PbLi flow is exposed to volumetric heating that varies with the radial distance y as $q'''(y) = 30\times10^6 \exp\{-y/a\}$, W/m^3. The surface heat flux is 0.5 MW/m^2. The inlet temperature in the PbLi is 400°C. The internal duct cross-sectional dimensions $2a$ and $2b$ and the length L are shown in the figure.

a) Calculate basic dimensionless parameters: the Hartmann number Ha, Reynolds number Re, magnetic Reynolds number Re_m, interaction parameter N, and the wall conductance ratio c.
b) Estimate the MHD pressure drop without and with electrical insulation (assuming ideal electrical insulation).
c) What can you say about the shape of the velocity profile in the two cases: (1) if the duct is perfectly insulated; and (2) if there is no any electrical insulation?
d) What flow regime (laminar or turbulent) will likely occur?
e) Estimate temperature increase in PbLi: Tout-Tin.



Physical properties

Fe: σ=1.4×10^6 1/Ohm-m, k=33 W/m-K, ρ=7800 kg/m3, C_p=750 J/kg-K

PbLi: σ=0.7×10^6 1/Ohm-m, k=15 W/m-K, ρ=9300 kg/m^3, C_p=190 J/kg-K, μ=0.001 Pa-s

Problem I:

(a): $Ha = BL\sqrt{\frac{\sigma}{\mu}}$ $B = 5T$, $L = \frac{2b}{2} = b = 0.1m$, $\sigma_{PbLi} = 0.7\times10^6\ S/m$, $\mu = 0.001\ kg/m.s$

$\Rightarrow Ha = 5\times 0.1\cdot\sqrt{\frac{0.7\times10^6}{0.001}} = \boxed{13229}$

$Re = \frac{\rho U_0 L}{\mu}$ (the definition of Reynold number is consisted in lecture note)

$= \frac{9300\cdot 0.5\cdot 0.1}{0.001} = \boxed{465000}$

$Re_m = \frac{U_0 L}{\eta} = U_0 L \mu_0 \sigma = 0.5\cdot 0.1\times 4\pi\times10^{-7}\times 0.7\times10^6 = \boxed{0.043982}$

$N = \frac{Ha^2}{Re} = \frac{(13229)^2}{465000} = \boxed{376.36}$

$C_w = \frac{\sigma_w t_w}{\sigma_e L} = \frac{1.4\times10^6\cdot 2\times10^{-3}}{0.7\times10^6\cdot 0.1} = \boxed{0.04}$

(b): Pressure Drop in the Hartmann flow

$\Delta P = \lambda\frac{l}{2b}\frac{\rho U_0^2}{2}$ where $l = 1m$, $2b = 0.2m$, $\Rightarrow \frac{l}{2b} = 5$

$P_d = \frac{\rho U_0^2}{2} = \frac{9300\cdot 0.5\cdot 0.5}{2} = 1162.5\ Pa$

λ is the pressure drop coefficient.

and $\lambda = \frac{4}{Re}\frac{Ha^2}{C_w+1}\frac{C_w Ha + \tanh Ha}{Ha - \tanh Ha} = \frac{4N}{C_w+1}\frac{C_w Ha + \tanh Ha}{Ha - \tanh Ha}$

(i): without electrical insulation: where $C_w = 0.04$

$\Rightarrow \lambda = \frac{4\times 376.36}{0.04+1}\cdot\frac{0.04\cdot 13229 + \tanh(13229)}{13229 - \tanh(13229)} = \underline{58.015}$

$\Rightarrow \Delta P = 58.015\times 5\times 1162.5 = 337212\ Pa = \boxed{0.33721\ MPa}$

(ii) with electrical insulation, where $C_w = 0$

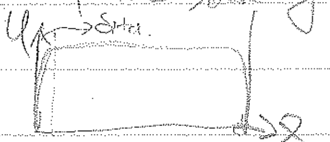
$\Rightarrow \lambda = \frac{4N}{1}\cdot\frac{\tanh(Ha)}{Ha - \tanh(Ha)} = \frac{4\times 376.36}{1}\cdot\frac{\tanh(13229)}{13229 - \tanh(13229)} = \boxed{0.11381}$

$\Rightarrow \Delta P = 0.11381\times 5\times 1162.5 = \boxed{661.52\ Pa}$

(c): (i) if the duct is perfectly insulated,

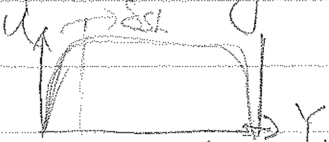
all four walls will form very thin boundary layer.

near two walls perpendicular to B field, the thickness of the boundary layer (Hartmann boundary layer)



$$\delta_{Ha} = b\,\frac{\ln Ha}{Ha} \approx \frac{b}{Ha} = 7.5592\times10^{-6}\,m$$

near two walls parallel to B field, the thickness of the boundary layer (Side boundary layer or Shercliff B.L)

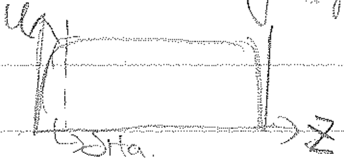


$$\delta_{SL} \approx \frac{b}{Ha^{1/2}} = 869.43\times10^{-6}\,m.$$

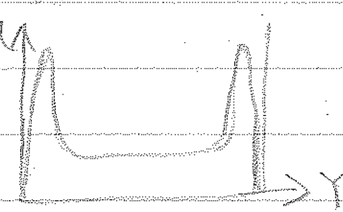
Away from the boundary layer, the flow will become flat in the bulk zone

(ii): if the duct is finite conducting,

the velocity near the Hartmann boundary layer is similar to the case of perfect insulating way



However, the flow near the side layer will form two jets and the velocity profile is like a "M" shape.



(d): To determine the flow regime

Hartmann layer transition to a turbulent state generally occurs at a critical value of "R" where $R_{cr} = \frac{Re}{Ha} < 150$.

In this problem, $R = \frac{465000}{13229} = 35.15$

Even for the side layer transition, $R_{cr} < 100$.

All in all, $R < R_{cr}$ of Hartmann layer and Re of Shercliff layer.

Therefore, the flow is laminar regime

(e): With the assumption of thermal insulation on all other three walls except for the heating wall,

therefore, from energy conservation,

$$\dot{m}c_p(T_{out}-T_{in}) = Q_{tot}$$

$$Q_{tot} = q''A + \int_V \dot{q}'''dV$$

$$= q''(2b\cdot l) + \int_0^{2a} \dot{q}'''dy\cdot(2b\cdot l)$$

$$= [q'' + 30\times10^6\int_0^{2a}\exp(-\tfrac{y}{a})dy]\cdot(2b\cdot l)$$

$$= [0.5\times10^6 + 30\times10^6\cdot[-a\exp(-\tfrac{y}{a})]\Big|_0^{2a}]\,(2b\cdot l)$$

$$= \{0.5\times10^6 + 30\times10^6[-0.1[\exp(-2)-1]]\}(2b\cdot l).$$

$$= [0.5\times10^6 + 30\times8.64665\times10^4]\cdot(0.2\times1)$$

$$= 3.0940\times0.2\times10^6 = \underline{0.61879\times10^6\,W}.$$

$$\Rightarrow T_{out}-T_{in} = \frac{Q_{tot}}{\dot{m}c_p} = \frac{Q_{tot}}{\rho U A_c\cdot c_p} = \frac{61879\ [W]}{9300\times0.5\times0.2\times0.2\times190\ [W/K]}$$

$$\boxed{= 17.51\ [K]}$$

Problem 4

a) Draw a schematic of a vertical cross-section of a tokamak reactor showing all major reactor components.

b) Describe concisely the functions of all components in (a) above.

c) What is the main difference between a tokamak and other toroidal confinement plasma devices?

d) Draw a unit cell of a DCLL blanket illustrating the primary geometric regions and materials.

e) Compare the features, advantages and disadvantages, of DCLL blanket to separately cooled PbLi blanket.

f) Discuss how tritium is extracted from ceramic breeder blankets.

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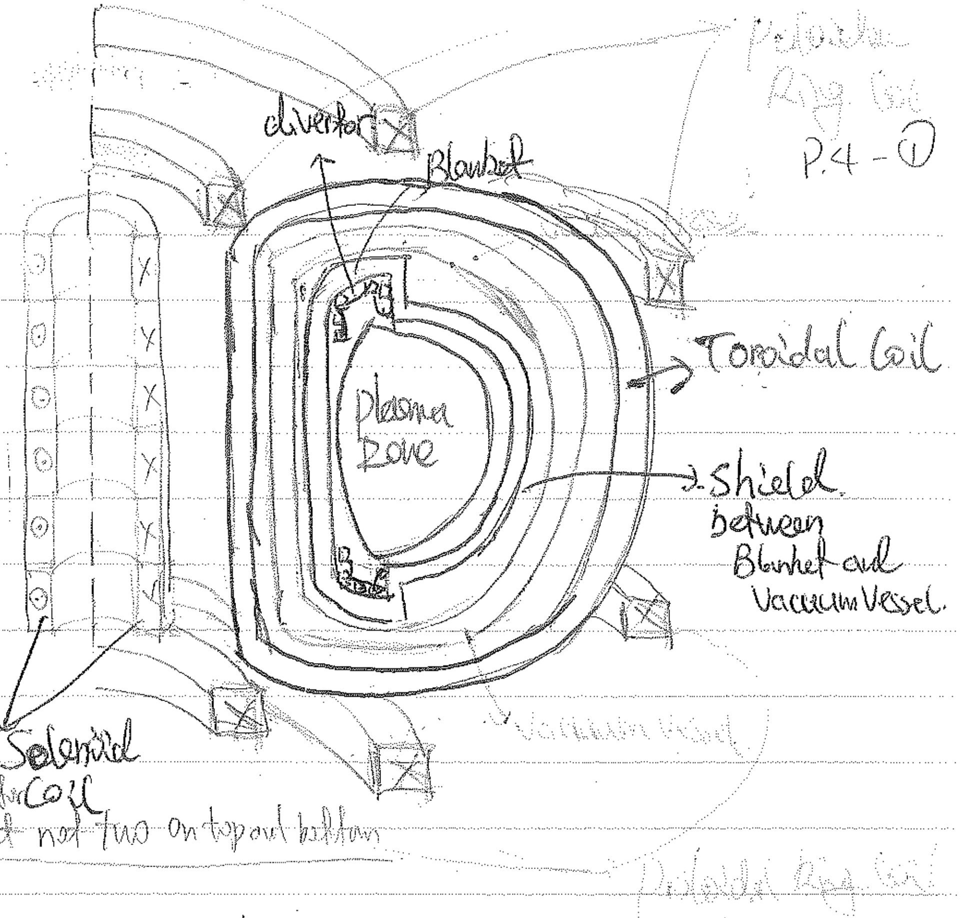
Problem 4:

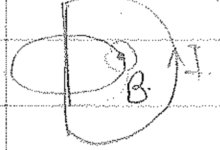
(a): Axisymmetric to the left figure

Apologize for using color pens.

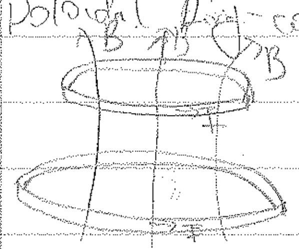


Note: ITER use only one set of divertor on the bottom of the blanket not two on top and bottom

(b): - Toroidal magnet coils: will provide the toroidal B field so that the plasma could be confined along with the toroidal field direction.



- Poloidal Ring coils: primary function of poloidal coils is to shape the plasma and contribute to its stability by "pinching" it away from the walls.



- Central solenoid coils: when the tokamak starts to operate, the powerful currents will be induced in the plasma zone and maintained during long plasma pulses. Those induced currents will also induce poloidal B field from Ampere's Law so as to confine the plasma.

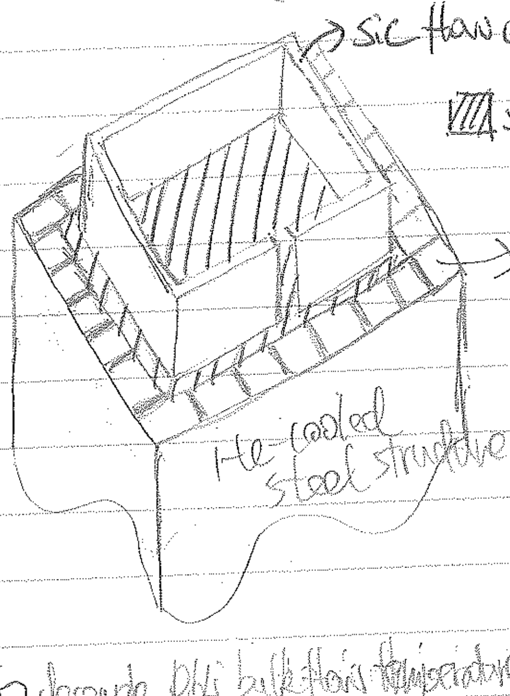
- Divertor: the divertor is to extract heat (α particle) and ash produced by the fusion reaction, to minimize plasma contamination and to protect the surrounding walls from thermal and neutronic loads.

— Blanket + sheild :, Blanket + sheild modules that completely covers the inner walls of the vacuum vessel protect the steel structure and the superconducting toroidal coils from the heat and high-energy neutrons produced by the fusion reactions. for some fusion reactions like D-T, the blanket will also produce the tritium for sustainable fusion reaction.

— Vacuum vessel: the plasma particles need a high-vacuum environment which is provided by the vacuum vessel. The vacuum vessel can also improve the radiation shielding and plasma stability. In other words, the larger the vacuum chamber volume, the easier it is to confine the plasma.

(c) The transient poloidal field lines in tokamak are manipulated by driving a current through the plasma itself, while in other toroidal confinement plasma devices, the poloidal B field lines are produced by external currents.
Also, the drift of the plasma in the tokamak will be corrected by external poloidal coil while in some other devices like stellarator, the drift will be cancelled out by the special shape of toroidal magnet coils.

(d)



- First wall and ferritic steel structure is cooled by helium
- Breeding zone is self-cooled.
- Structure and Breeding zone are separated by SiCf/SiC composite flow channel insert that provide thermal insulation to decouple PbLi bulk flow temperature from ferritic steel wall and provide electrical insulation to reduce MHD pressure drop in the flowing breeding zone.

$\Delta P_{MHD} \approx C_0 L \sigma_e V B^2$

(e): For DCLL Blanket, the structure is cooled by He, then high speed of liquid coolant is not required so that the MHD pressure drop can be lower.

FCI provides the thermal insulating of breeding zone to structure and the electric insulting of the liquid metal then the MHD pressure drop can be further decreases

No thermal stress issues on the inner surface of structure because of the FCI

Disadvantage: the corrosion of FCI, will result in a higher MHD pressure drop instead.

For Separately-cooled Lead Lithium Blanket

Advantages: Almost all energy will be removed by separate He stream, so that the speed of liquid metal coolant can be very low to reduce the MHD pressure drop.

No issues from FCI

Disadvantages: Low velocity of PbLi leads to high tritium partial pressure, which leads to tritium permeation (very serious problem)

T: Outlet temperature is limited by PbLi compatibility with RAFM steel structure ~500°C

(f): In ceramic breeder blankets, high power neutrons will hit the lithium in the ceramic pebbles such as Li_2O, Li_4SiO_4, Li_2TiO_3, Li_2ZrO_3 and the nuclear reaction between neutrons and lithium will produce tritium which will diffuse out from the pebble. Then the tritium will be removed with a purge gas (primarily helium + little hydrogen) flowing through the open porosity of packed beds of lithium ceramics. The whole tritium transport consists of five mechanisms: 1) Bulk diffusion, 2) grain boundary diffusion, 3) surface adsorption/desorption 4) pore diffusion and 5) purge flow convection. Also, tritium is not released just as T_2 but also as a composition of HT, T_2O, HTO which requires further process e.g. tritium extraction

For the tritium extraction system, the purge gas with H_2, HT, HTO H_2O will first flow through the RTMS (room temperature molecular sieve) for HTO and H_2O absorption where the tritium in HTO composite has been extracted.
Then, the remaining gas will flow through the CMS (Cryogenic molecular sieve) bed for HT and H_2 absorption, which the tritium in HT will be extracted.
At last, purge gas of He with small amount of H_2 (0.1%) will flow back to Blanket Modules.

Problem 5

A tokamak reactor with superconducting TF coils has a major radius of 6.8m, an aspect ratio of 3, and a neutron wall load of 3.6 MW/m^2. It has a breeding blanket that attenuates the neutrons by two orders of magnitude followed by 90 cm of 85% Pb+15% B_4C.

a) Calculate the reactor fusion power.

b) Calculate the total heat load into the cryogenic system.

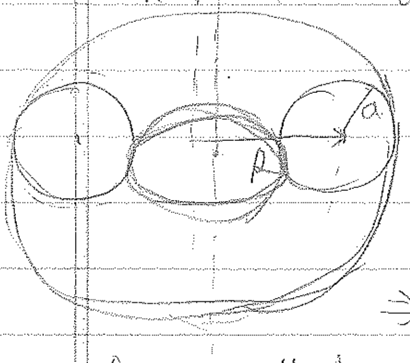
c) Calculate the total power required to remove the nuclear heating deposited in the magnet.

d) Calculate the radiation-induced resistivity in the copper stabilizer at the point of maximum magnetic field after 4 years of continuous reactor operation.

e) If the tritium breading ratio is 1.15, calculate the rate of tritium production in the blanket in kg/s.

Problem 5: $R = 6.8m$, $AR = \frac{R}{a} = 3 \Rightarrow$ minor radius $a = \frac{R}{AR} = \frac{6.8m}{3} = 2.267\,m$

(a): then in an ideal donut shape (circular cross-section)



therefore the total surface area of the plasma region

$S_{plasma} = 2\pi R \cdot 2\pi a = 4\pi^2 Ra = \underline{715.88\,m^2} \approx S_{first\ wall}$

Therefore, the total neutron power is equal to

neutron wall loads $\times$ first wall surface area

$\Rightarrow P_{neutron} = 3.6\,MW/m^2 \times 715.88\,m^2 = \underline{2577.168\,MW}$

Assume all the neutrons produced by the fusion reaction, so that energy multiplication factor $\mathcal{E} = 1$. and assume the fusion reaction is D-T reaction

Therefore, the total fusion power

$$\frac{P_{fusion}}{P_{neutron}} = \frac{17.58}{14.06} \Rightarrow P_{fusion} = \frac{17.58}{14.06} \cdot P_{neutron} = \boxed{3222.376\,MW}$$

(b): With 90cm of 85% Pb + 15% B_4C shielding, with neutron energy attenuation coefficient $\underline{\mu_n = 0.0977\,cm^{-1}}$, then the neutron intensity is attenuated by $\underline{e^{-0.0977 \times 90} = 1.5179 \times 10^{-4}}$

plus two order of magnitude attenuated by breeding blanket, then total attenuation is $\underline{1.5179 \times 10^{-6}}$

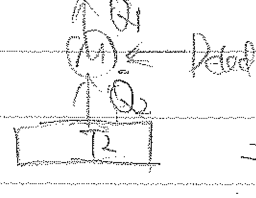
Therefore the neutron wall load into the cryogenic system

$$P_n = 1.5179 \times 10^{-6} \times 3.6\,MW/m^2 = 5.46444\,W/m^2.$$

Let's assume the heat load into the cryogenic system all come from the kinetic energy of neutron, then, the total heat load $P_{heat} = P_n = \boxed{5.46444\,W/m^2}$

(c): Assume an ideal thermodynamic cryogenic system is employed to remove heat from the superconducting magnets operating at $\underline{4K}$, and the heat is rejected at room temperature ($\underline{300K}$)

For a reversed Carnot cycle the thermal efficiency $\eta = \frac{T_2}{T_1 - T_2} = \frac{4K}{300K - 4K} = \frac{\dot{Q}_2}{P_{electric}}$



To calculate the surface area of cryogenic system, we assume the thickness of breeding blanket is 70cm, and thickness of shielding is 90cm

$\Rightarrow S_{cryogenic} = 2\pi R \cdot 2\pi (a + 1.6) = 1038.019\,m^2$.

$\Rightarrow \boxed{\dot{Q}_2} = P_{heat} \cdot S_{cryogenic} = 5.672\,kW$ and $\boxed{P_{electric} = \frac{\dot{Q}_2}{\eta} = 419.728\,kW}$

total power required to remove heat

(d). from the paper "Radiation considerations for superconducting fusion magnets",
we have the expression of radiation induced resistivity in copper
as $\rho_r = 3\times10^{-7}[1-\exp(-563\,dpa)]\,\Omega\,cm$.
we use the displacement energy Ed of 40eV.
then the neutron flux density of Ed

$$\phi(Ed) = 5.4644 \tfrac{J}{sm^2} \cdot \frac{1\ eV}{1.60218\times10^{-19}J} \cdot \frac{\#}{40eV}$$

$$= 8.5265\times10^{17}\ \#/m^2\cdot s = 8.5265\times10^{21}\ \#/cm^2\cdot s$$

and the displacement cross-section

$$\sigma_d(Ed) = 5.08\times10^{-2}\,b = 5.08\times10^{-2}\times10^{-24}\,cm^2 \text{ (from atom.kaeri.re.kr)}$$

With an exposure of 4 years. (Instead of doing the integrate, we estimate the average)
so that $dpa = \phi(Ed)\,\sigma_d(Ed)\cdot t$

$$= 8.5265\times10^{21}\times5.08\times10^{-26}\times4\times365\times24\times3600$$

$$= 54639 \text{ (too large to be correct, Ed or } \sigma_d \text{ could be wrong).}$$

$\Rightarrow \rho_r = 3\times10^{-7}\,\Omega\,cm$ which is the saturation resistivity of copper.

Alternatively, the ρ_r can also be found by looking at the figure of "maximum radiation induced resistivity in copper stabilizer as a function of inner blanket/shield thickness" in the paper where the expression of ρ_r being found.

(e): For a given tritium breeding ratio TBR = 1.15,

the rate of tritium production $\dot{N}^{+}$ in the blanket can be calculated from calculating the tritium consumption in the plasma $\dot{N}^{-}$

and $\frac{\dot{N}^{+}}{\dot{N}^{-}} = TBR = 1.15$.

In 1 kg of tritium, the number of tritium atoms

$$N_t = \frac{1\,kg}{3.016042\,amu} \cdot \frac{1\,amu}{1.66053878 \times 10^{-27}\,kg/atom}$$

$$= 1.9966988 \times 10^{26}\ atoms/kg$$

Since one tritium atom is consumed in one D-T reaction,

then $\dot{N}^{-} = \frac{P_{fusion}}{E_{DT}} \cdot \frac{1}{N_t}$

$$= \frac{3222.376 \times 10^{6}\,J/s \cdot atoms}{17.58 \times 10^{6}\,eV} \cdot \frac{1\,eV}{1.602 \times 10^{-19}\,J} \cdot \frac{1\,kg}{1.9966988 \times 10^{26}\,atoms}$$

$$= 5.7304 \times 10^{-6}\ kg/s.$$

therefore, tritium production rate in blanket

$$\dot{N}^{+} = \dot{N}^{-} \times TBR = 5.7304 \times 10^{-6} \times 1.15 = \boxed{6.590 \times 10^{-6}\ kg/s}$$

Problem 6

a) State and explain cryogenic stabilization criterion for superconducting magnet.

b) Discuss concisely radiation effects on components of superconducting magnets.

c) Compare the functions of bulk shielding, penetration shielding, and biological shielding in a tokamak fusion power plant.

d) What is the most promising structural material for a fusion DEMO? Why?

Problem 6:

(a) Cryogenic stabilization criterion for superconducting magnet is

$$I^2R \leq q''Pl$$

where I is the applied current in superconducting magnet.

R is the resistance of the superconducting coil

q'' is the surface cooling flux.

P is the perimeter of coolant module.

l is the length of the superconducting magnet toroidally.

The cryogenic stabilization criterion says the thermal energy by Ohmic heating from superconducting coils must be all removed by the cooling system.

since $R = \frac{\rho l}{a}$, where a is the cross section perpendicular to applied current of superconducting coils.

$$\Rightarrow \boxed{I^2\rho \leq q''Pa}$$

(b): - Structure of S.C magnet has very small radiation damage because of very small neutron reaction cross-section.

- For stabilizer, the neutron irradiation at cryogenic temperatures will produce immobile point defects which results in a radiation induced resistivity followed by a large Ohmic heating power. This defects or damage needs to be recovered by annealing.

- For insulators, neutron radiation-induced microstructural variation will result in the change of mechanical and dielectric properties, especially for the mechanical strength, dielectric strength, electric resistivity.

- For superconducting filaments, the effect of neutron radiation will diminish the superconducting region of current density — temperature — magnetic field phase space.

Specifically, the critical current density will be reduced with increase of neutron fluence. the critical temperature will also be lower with introduction of disorder from neutron radiation.

(c): - Bulk shield, surrounding the blanket modules is to protect vacuum vessel and toroidal superconducting magnets. (thickness ~ 1m)
- Penetration shield, around penetrations will prevent radiation streaming through these penetrations to the vacuum vessel or superconducting magnets. (thickness ~ 20cm – 1m)
- Biological shield, the forest shielding, typically concrete, is to prevent tritium going outside to protect personnel in the operating rooms and outside (thickness ~ 2m)

(d): Reduced Activation Ferritic Martensitic (RAFM) steel could be the promising structural material for a fusion DEMO.
Because in RAFM steel, highly activating alloying elements (Mo, Nb) were replaced by those (W, Ta) offering lower activation.
Specifically, 8~10% Cr for good fracture properties and corrosion resistance.
1~2% W for good mechanical properties (ductility, strength, ...)
0.07% Ta: for stabilization of grain size and improves strengths.

Problem 7

a) Calculate Q values for Li^6 (n, t) and Li^7 (n, n't), and specify if they are exothermic or endothermic.

b) If a 1 MeV neutron undergoes elastic scattering at 45 degrees with a Li^6 target in the blanket what is the heat deposited in the material per interaction?

not enough info.

c) An (n,α) reaction in a particular nuclide has a Q-value of -5 MeV calculate the neutron kerma factor for 14 MeV neutrons.

d) A particular shield composition has a total energy attenuation coefficient of 0.138 cm^{-1}, what is the shield thickness required to achieve energy attenuation of four orders of magnitude?

e) Write down the Neutron Transport Equation and describe the physical meaning of each term. Which term is the one that requires a more difficult mathematical treatment?

f) Neutronics calculations for a fusion blanket show the following reaction rates per fusion neutron:

$^{50}V(n,2n)^{49}V$ $\quad$ $^{51}V(n,2n) \to {}^{50}V$

REACTION	REACTION RATE Per fusion neutron	|Q| - VALUE MeV
V(n,2n)	0.1	13 -13.
V(n,γ) $^{51}V(n,\gamma)^{52}V$	0.05	8 +8.
$^6Li(n,\alpha)$	0.80	4.8 +
$^7Li(n,\gamma)$	0.02	5 +
$^7Li(n,n',\alpha)$	0.4	2.4 –

f1) Calculate the tritium breeding ratio.

f2) Calculate the energy multiplication factor

f3) If a tokamak reactor using the above blanket produces 3000 MW of fusion power and has a thermal conversion efficiency of 35%, calculate the reactor electric power output.

Problem 7.

(a): ${}^6_3Li + n \longrightarrow He + T$ from Shultis & Faw, Table B.1: Atomic Mass Tables

$\Rightarrow Q = [M({}^6_3Li) + m_n - M({}^4_2He) - M({}^3_1H)]c^2$ $M({}^6_3Li) = 6.01512\,u$, $uc^2 = 931.494\,MeV$

$= (6.01512 + 1.00866 - 4.00260 - 3.01605)uc^2$ $m_n = 1.00866\,u$

$= 5.13 \times 10^{-3} \times 931.494\,MeV$ $M({}^4He) = 4.00260\,u$

$= 4.78\,MeV$ (exothermic) $M(T) = 3.01605\,u$

${}^7_3Li + n \longrightarrow n + He + T$ $M({}^7_3Li) = 7.01600\,u$.

$\Rightarrow Q = [M({}^7_3Li) + m_n - m_n - M({}^4_2He) - M({}^3_1H)]c^2$

$= (7.01600 - 4.00260 - 3.01605)uc^2$

$= -2.65 \times 10^{-3} \times 931.494\,MeV$

$= -2.47\,MeV$ (endothermic)

(b): For an elastic scattering with incident neutron (E) and still target nucleus the scattered neutron energy $E' = \frac{E}{(A+1)^2}[\cos\theta + \sqrt{A^2 - \sin^2\theta}]^2$.

for ${}^6Li$ (n,n) reaction with $E = 1\,MeV$, $\theta = 45°$ ... $A = 6$.

then $E' = \frac{1\,MeV}{(6+1)^2}[\cos(45°) + \sqrt{6^2 - \sin^2(45°)}]^2$

$= \boxed{0.9067\,MeV}$

then the recoiling nucleus ${}^6Li$ will has an energy $E_A = (E - E') = \boxed{0.0933\,MeV}$

(c): total energy deposited $E_H = E_r + E_c = E + Q = 14 - 5 = 9\,MeV$

then we need to find out the cross section of this reaction

Assume the nuclide reactant is ${}^A_ZX$, the product is ${}^{A-3}_{Z-2}Y$ with $Q = -5\,MeV$

$\Rightarrow {}^A_ZX + n \rightarrow \alpha + {}^{A-3}_{Z-2}Y \Rightarrow m(X) + m_n - m(He) - m(Y) = \frac{-5\,MeV}{931.494\,MeV}u = \underline{-5.3677 \times 10^{-3}}$

$\Rightarrow m(X) - m(Y) = -5.3677 \times 10^{-3} + 4.002603 - 1.008665 = 2.98857\,u$

looking through the Atomic mass Table in Shultis & Faw, we could find ${}^A_ZX$ and ${}^{A-3}_{Z-2}Y$ based on this mass difference. After that we can also find the cross section $\sigma_{X(n,\alpha)Y}(E_n = 14\,MeV)$

macroscopic Kerma factor $= \frac{\rho_X N_A}{M_X} \cdot \sigma_{X(n,\alpha)Y} E_H$ or microscopic Kerma factor $= \sigma_{X(n,\alpha)Y} E_H$

(d): Neutron beam attenuation satisfies the following eqn as

$$I(x) = I_0 e^{-\Sigma_R x} \qquad \Sigma_R = 0.138\,cm^{-1}$$

to achieve energy attenuation at four orders of magnitude

then $10^{-4} = e^{-\Sigma_R x_s} \Rightarrow x_s = \ln(10^{-4})\cdot\left(-\frac{1}{\Sigma_R}\right) = \boxed{66.742\,cm}$

(e): $$\underbrace{\frac{\partial n}{\partial t}}_{①} + \underbrace{v\hat{\Omega}\cdot\nabla n}_{②} + \underbrace{v\Sigma_t n(\vec{r},E,\hat{\Omega},t)}_{③} = \underbrace{\int_{4\pi} d\hat{\Omega}' \int_0^\infty dE'\, v'\Sigma_s(E'\to E, \hat{\Omega}'\to\hat{\Omega})\, n(\vec{r},E',\hat{\Omega}',t)}_{④} + \underbrace{S(\vec{r},E,\hat{\Omega},t)}_{⑤}$$

①. the rate change of neutron density.

②: neutron density flux or loss due to leakage.

③: neutron density loss due to collision.

④: neutron density gain due to inscattering into dE about E and direction $\hat{\Omega}$ in $d\hat{\Omega}$.

⑤: neutron density gain due to the source.

Term ④, the gain by inscattering is the most difficult to treat.

(f) We assume only D-T reaction is interested in this problem.

(1) for each D + T ⟶ He + n + 17.58 MeV

There is one tritium being consumed and one fusion neutron generated.

Among with these five reactions, only $^6Li(n,\alpha)$ and $^7Li(n,n'\alpha)$ could produce tritium. Therefore, Tritium Breeding Ratio

$$TBR = \frac{0.8 + 0.4}{1.0} = \boxed{1.2}$$

(2): The total energy produced in the system per fusion reaction

$$E = \underbrace{17.58\,MeV}_{DT} + \underbrace{0.1(-13)\,MeV}_{V(n,2n)} + \underbrace{0.05\cdot 8\,MeV}_{V(n,\gamma)} + \underbrace{0.8\cdot 4.8\,MeV}_{^6Li(n,\alpha)} + \underbrace{0.02\cdot 5\,MeV}_{^7Li(n,\gamma)} + \underbrace{0.4\cdot(-25}_{^7Li(n,n'\alpha)}$$

$= 19.66\,MeV. \quad = 14.06\varepsilon + 3.52\,MeV$ where ε = energy multiplication factor.

$$\Rightarrow \varepsilon = \frac{19.66 - 3.52}{14.06} = \boxed{1.1479}$$ which is in the reasonable range

α particle neutrons fusion power

f. (3): thermal power $P_{th} = P_\alpha + \epsilon P_n = \left(\frac{3.52}{17.58} + \epsilon \frac{14.06}{17.58}\right) P_f = 1.1183\, P_f$

then $\frac{\text{the electrical power out}}{\text{thermal power in}} = \frac{P_{electric}}{P_{th}} = \frac{P_{electric}}{1.1183 P_f} = \eta_{th} = 0.35$

$\Rightarrow P_{electric} = 0.35 \times 1.1183 \times 3000\ MW$

$= \boxed{1174\ MW}$

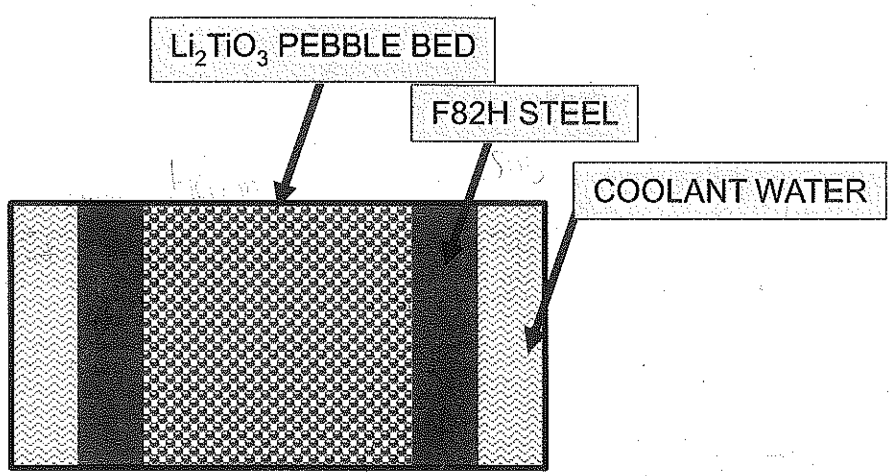
Problem 8

Consider a 1D, pebble bed-type blanket configuration with a 2-cm wide (along the tokamak's radial direction) breeder volume cooled on both sides by water at a bulk temperature of $T_f = 300$ °C. Water is flowing at 5 m/s through an equivalent hydraulic coolant channel of 1 cm with a structural wall thickness of 3 mm. (See the sketch below)

a) Calculate the temperature distribution across the pebble breeder element, structure, and water, considering the following:
   - Single size pebble bed of lithium Li_2TiO_3 pebbles of 1 mm diameter.
   - Constant volumetric heat generation rate in the breeder region of 8 MW/m^3
   - A temperature jump of 25 °C exists at the interface of pebble bed and steel
   - Use thermal properties of stainless steel for F82H

b) Calculate the purge gas pressure drop across a 1 meter tall pebble bed as a function of superficial purge gas velocity of 1, 5, and 10 cm/s for a single size bed of 1 mm pebble. Assume an average purge gas temperature of 600 °C and random packing of spheres.

c) How much tritium will permeate to the coolant from the pebble bed region through the F82H wall, if the superficial purge gas velocity is 1, 5, and 10 cm/s?
   - Assume diffusion limited control.
   - Average tritium generation rate in the breeder region= 1.21e-7 g/s.
   - Use bed average temperature for tritium partial pressure estimation.



Problem 8:

(a): We can do two different assumptions on the arrangement of ceramic pebble in the pebble bed.

case(a) The simplest one is to assume in 1D, ceramic pebbles are attaching to each other in the pebble bed so that the effective thermal conductivity is the thermal conductivity of Li_2TiO_3 which has comparable conductivities with Li_4SiO_4

In figure 1.8 from Jin's lecture notes

$$\underline{k_{Li_4SiO_4}(T=600°C) = 2.6\ W/m\cdot K \text{ at } 80\%\ TD}$$

case(b) The second case when there is a fair void fraction, generally $\varepsilon = 0.355$ for the ceramic pebbles sitting in Helium gas, $(T=600°C)$, $k_f = 0.2525\ W/m\cdot K$.

$$\Rightarrow K = \frac{k_s}{k_f} = \frac{2.2}{0.2525} = 8.713. \text{ and } \varepsilon = 0.355 \quad B = 1.25\left(\frac{1-\varepsilon}{\varepsilon}\right)^{1.11} = 2.4253.$$

from Zehner–Schlunder correlation is

$$\frac{k_e}{k_f} = (1-\sqrt{1-\varepsilon}) + \frac{2\sqrt{1-\varepsilon}}{1-B/K}\left[\frac{(1-\frac{1}{K})B}{(1-B/K)^2}\ln\left(\frac{K}{B}\right) - \frac{B+1}{2} - \frac{B-1}{1-B/K}\right]$$

$$= 0.19688 + 2.225798\cdot[5.27225 - 1.71265 - 1.975069]$$

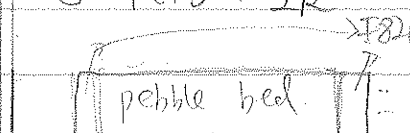
$$= 3.7237$$

$$\Rightarrow \underline{k_e} = 3.7237 \times 0.2525 = \underline{0.9402\ W/m\cdot K}$$

— For 1D steady thermal energy equation with no convection in pebble bed

$$\text{then } +k\frac{\partial^2 T}{\partial x^2} + q''' = 0 \Rightarrow \frac{\partial^2 T}{\partial x^2} = -\frac{q'''}{k}$$

$$\Rightarrow T(x) = \frac{-q'''}{2k}x^2 + C_1 x + C_2.$$



with B.C. $T(x=\pm 0.1(cm)) = T_{pw}$ → Temperature at pebble wall

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0 \Rightarrow C_1 = 0.$$

$$\Rightarrow T(x) - T_{pw} = \frac{q'''}{2k_e}(0.1^2 - x^2)$$

— for 1D steady thermal energy equation for the structure we have pure conduction there so that

$T_{s,w}$ = temperature at $x = \pm 1$ cm
$T_{w,w}$ = temperature at $x = \pm 1.3$ cm

$$\frac{k_s(T_{s,w} - T_{w,w})}{\delta_s} = q_w$$

$k_s = 33\ W/m\cdot K$ for F82H (evaluated at $T_{(1cm)} = 500°C$)

$\delta_T = 0.3$ cm, thickness

q_w is the heat flux between water coolant to structure

— At last, for the water coolant,

Since we have Dittus-Boelter equation for Nusselt number

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.3}$$ all the properties of coolant (water) are used at temperature of $T_f = 300K$ (not $300°C$)

$$Re_D = \frac{\rho U D}{\mu} = \frac{997.1 \times 5 \times 0.01}{0.798 \times 10^{-3}} \approx \underline{62475}$$

and the equation of Nu_D above valid for $Re_D \geq 10000$ (I think there's a typo for the $T_f = 300°C$ in problem statement. water will be boiling at that temperature ~~at the general temperature of water~~ coolant in solid breeder blanket is about $70°C$, but I use 300K here

so we can use this expression.

and $Pr = 6.2$

$$\Rightarrow Nu_D = 0.023 \times (62475)^{4/5} (6.2)^{0.3} = \underline{272.9}$$

from the definition of Nusselt number

$$Nu_D = \frac{hD}{k_{water}} = \frac{q_w D}{k_w (T_w - T_m)}$$ and $k_{water} = 0.609\, W/m \cdot K$

$D = 0.01m$, hydraulic coolant channel

Assume the temperature and wall heat flux are the same on the interface of water coolant and the F82H steel. then we have two equations and two unknowns q_w, T_w

$$\Rightarrow \begin{cases} \dfrac{q_w D}{k_w (T_{ww} - T_f)} = Nu_D = 272.9 \\ \dfrac{k_s (T_{s,w} - T_{ww})}{\delta_s} = q_w \end{cases} \Rightarrow \frac{k_s}{k_w} \frac{(T_{sw} - T_{ww})}{(T_{ww} - T_f)} \cdot \frac{D}{\delta_s} = Nu_D$$

$$\Rightarrow \frac{T_{sw} - T_{ww}}{T_{ww} - T_f} = Nu_D \cdot \frac{\delta_s}{D} \cdot \frac{k_w}{k_s} = 272.9 \cdot \frac{0.3}{1} \cdot \frac{0.609}{33} = 1.511$$

$\because$ since there's a $25°C$ temperature jump at the interface of pebble bed and steel

then $T_{pw} - T_{sw} = 25$

so now, we have two equations

$$\begin{cases} T(x) - (T_{sw} + 25) = \dfrac{q'''}{2k_e}(0.1^2 - x^2), \quad q''' = 8 \times 10^6\, W/m^3, \quad k_e = \begin{cases} 2.0\, W/mK & \text{case (a)} \\ 0.9402\, W/mK & \text{case (b)} \end{cases} \\ \dfrac{T_{sw} - T_{ww}}{T_{ww} - T_f} = 1.511, \quad T_f = 300K. \end{cases}$$

$$\Rightarrow \boxed{T(x) = T_{ww} + 1.511(T_{ww} - T_f) + 25 + \frac{q'''}{2k_e}(0.1^2 - x^2)}$$

b): from the equation 2.1 in Jon's lecture notes

$$\frac{\Delta p}{L} = \frac{180 U \mu}{d_p^2} \frac{(1-\epsilon)^2}{\epsilon^3}$$

with random packing of spheres, the packing fraction is assume to be $\phi = 0.6$ which is the reasonable assumption between 0.52 of simple cubic structure and 0.68 of body centered cube.

$\Rightarrow$ $\epsilon = 1-\phi = 0.4$ for helium at $T = 600°C$, $\mu = 4.18 \times 10^{-5}$ Pa·s. from EngineeringToolbox.com

$$\Rightarrow \Delta P = \frac{180 \cdot \bar{U} \cdot \mu L}{d_p^2} \left(\frac{(1-\epsilon)^2}{\epsilon^3}\right) = \frac{180 \times 4.18 \times 10^{-5} \cdot 1}{(1 \times 10^{-3})^2} \left(\frac{0.6^2}{0.4^3}\right) \cdot \bar{U} = 42322.5 \bar{U}$$

$$= \begin{cases} 423.225 \text{ Pa} & \text{with } \bar{U} = 1 \text{cm/s} \\ 2116.125 \text{ Pa} & \text{with } \bar{U} = 5 \text{cm/s} \\ 4232.25 \text{ Pa} & \text{with } \bar{U} = 10 \text{cm/s} \end{cases}$$

(c): First, calculate the average temperature of solid breeding blanket

$$\bar{T} = \frac{1}{0.02m} \int_{-0.01}^{0.01} T(x) \cdot dx = \frac{1}{0.02} \int_{-0.01}^{0.01} (450.55 + 1.5385 \times 10^6 (0.01^2 - x^2)) dx$$

$$= \frac{11.4623}{0.02} = 573 (K)$$

and then the partial pressure of tritium can be calculated by the ideal gas law

$$P_T = \frac{n_T}{V} RT = C_{TP} RT. \quad C_{TP} \text{ is the tritium concentration in the pebble bed}$$

To calculate the mole concentration of tritium, let's assume the flowing rate of tritium by purge gas is much larger than the tritium permeation rate from F82H.

so that the gen rate of tritium is approximately equal to the flow rate

Therefore $\dot{G} = \rho U X_T A = U m_{TP} A$

with A square cross section of A (2cm × 2cm).

then $m_{TP} = \frac{\dot{G}}{UA}$

for $U = 1$ cm/s, $m_{TP} = \frac{1.21 \times 10^{-7} g/s}{0.01 \times 0.02^2 \, m^3/s} = 0.03025 \frac{g}{m^3} = 3.025 \times 10^{-5} \frac{kg}{m^3}$

therefore, $C_{TP} = \frac{m_{TP}}{A_T} = \frac{3.025 \times 10^{-5} \, kg/m^3}{3.016049 \, kg/mole} = 1.003 \frac{mole}{m^3}$

$\Rightarrow P_T = 1.003 \frac{mole}{m^3} \times 8.314 \, J/mole \cdot K \cdot 573 (K) = 4778$ Pa $\rightarrow$ seems to be too high

we need one more condition of T_{wc}, the temperature at the interface between water coolant and F82H steel to close this problem

Let's assume $T_{wc} = 350K$, which is a reasonable assumption

pebble bed

for case (a): $T(x) = 350 + 1.511(350-300) + 25 + \frac{8\times10^6}{2\times2.6}(0.01^2 - x^2)$

$= 450.55 + 1.5385\times10^6(0.01^2 - x^2)$ [K]

for case (b): $T(x) = \underbrace{450.55}_{T_{pw}} + 4.2544\times10^6(0.01^2 - x^2)$ for $-0.01 \le x \le 0.01$

and in the region of structure,

F82H steel

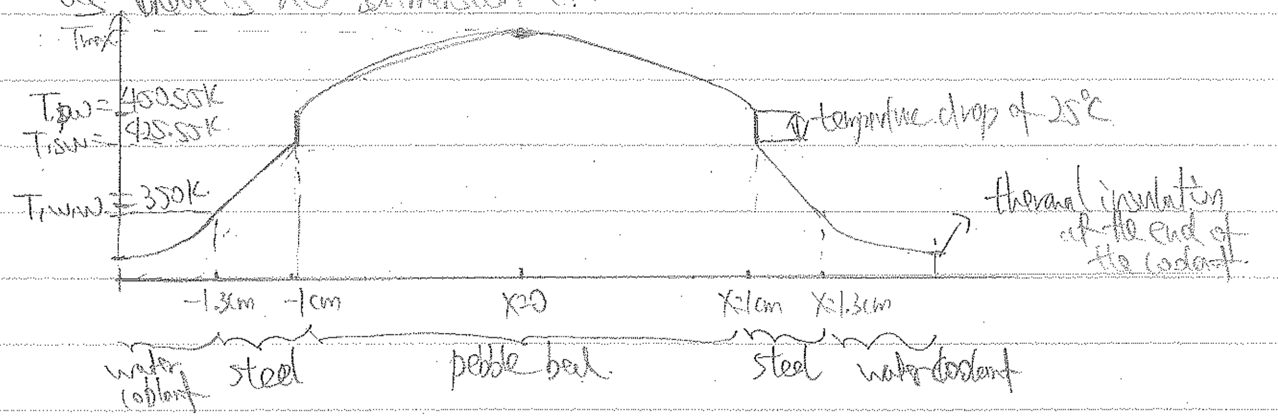
$$k\frac{\partial T}{\partial x} = q \Rightarrow T - (T_{sw} - 25) = \frac{q_w}{k_s}(|x| - 0.01)$$

$$\Rightarrow T(x) = (T_{pw} - 25) + \frac{T_{sw} - T_{wc}}{\delta_s}(|x| - 0.01)$$

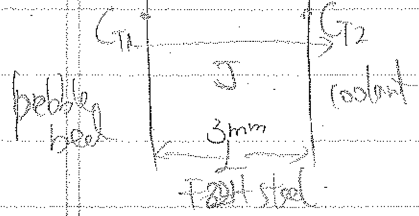
$$= 425.55 + 75.55\left(\frac{|x|}{0.003} - \frac{10}{3}\right) \quad x\,[m] \quad (|x| \in [0.01, 0.013])$$

for the water, we can only get the quantitative profile as there is no dimension ...

$\Rightarrow$



Now, consider the permeation process, with diffusion limited control,



then the tritium permeation flux $J = \dfrac{D_S(C_{T1} - C_{T2})}{L}$

where $C_{T1} = K_S\sqrt{P_{TP}}$ P_{TP} is the partial pressure of tritium gas (T_2)

and K_S the solubility of tritium on F82H steel.

from the experimental data of tritium permeation through F82H plate, by Chikada, Suzuki, Matsuzaki, Terai and Muroga in Journal of Nuclear Material. 417(2011), 1241-1244, the permeability of tritium through F82H plate at $T = 400°C$; the "permeability flux

$$J = 6.7741 \times 10^{-9}\ \frac{mole}{m^2 S}$$

the diffusivity and solubility of tritium – F82H are unknown. If we know those properties, then we can do our calculation.

However, in the same paper written by Chikada, the permeability of tritium through F82H plate at $T = 573K$ is given as $9.8\times10^{-11}\ mol/m.S\sqrt{Pa}$

$\Rightarrow$ the permeation through 3mm of F82H is equal to $9.8\times10^{-11} \times 0.003 \times \sqrt{4598}$
$= 2\times10^{-11}$ mole/S.

therefore, the total mass of tritium that will permeate to the coolant is

$M_{T_2 loss} = 2\times10^{-11}\ mole.S \cdot 3.01604\ kg/mole = \boxed{6.0321\times10^{-11}\ kg/s}$ with $U = 1cm/s$

with $U = 5cm/s$ $\Rightarrow C_{T_2 Pebble} = 0.2006\ \frac{mole}{m^3} \Rightarrow P_{T_2 pebble} = 955.6\ Pa$

$\Rightarrow M_{T_2 loss} = 9.8\times10^{-11} \times 0.003 \times \sqrt{955.6} \times 3.01604 = \boxed{2.74108\times10^{-11}\ kg/s}$

with $U = 10cm/s$, do the same calculation, $M_{T_2 loss} = \boxed{1.90752\times10^{-11}\ kg/s}$

$\Rightarrow$

velocity U	mass of tritium permeate to the coolant $M_{T_2 loss}$
1 cm/s	6.0321×10^{-11} kg/s
5 cm/s	2.74108×10^{-11} kg/s
10 cm/s	1.90752×10^{-11} kg/s