Prof. M. A. Abdou
TA: Tyler Rhodes

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MAE 237D

Fusion Engineering and Design

FINAL EXAM

Take Home Exam

Due: Thursday, March 17, 2016
at 4:00pm
(Submit in 44-114 Eng IV to Emily or Jesse)

Attempt Only Six Problems

Name: David Li

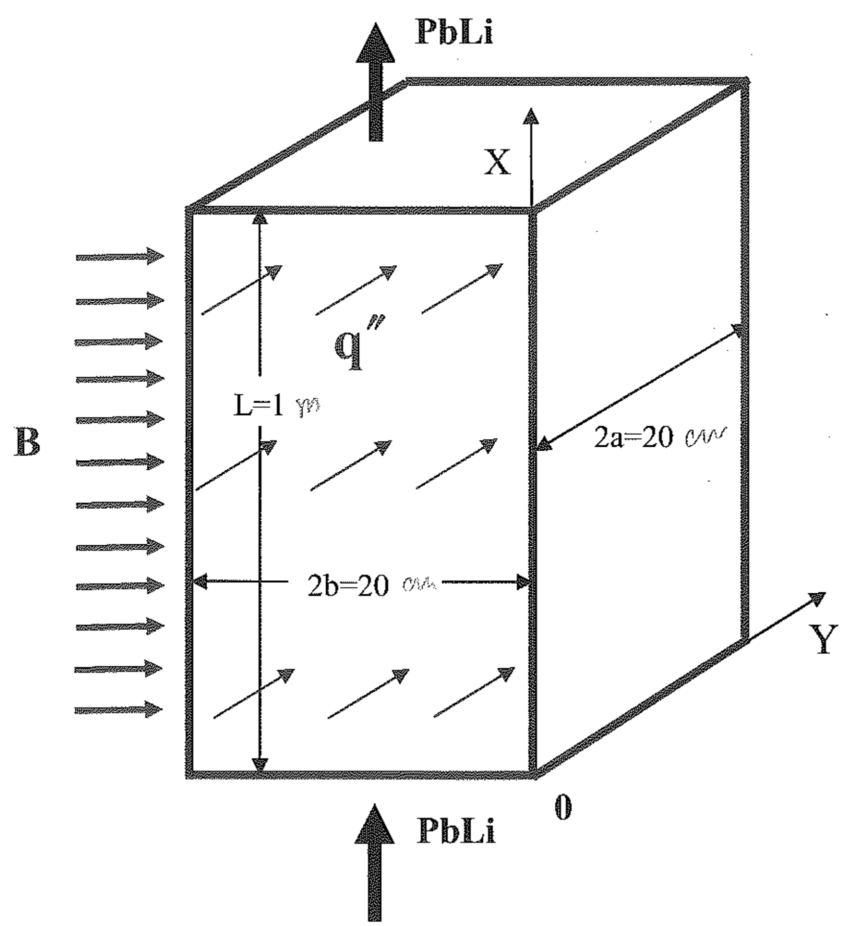
Student ID#: 304589834

- **Include the details of your solutions**
- **Provide informal citations for any sources used**
- **Make, indicate, and justify any significant assumptions**
- **Please work independently**

Problem 1

In a self-cooled poloidal PbLi blanket, the liquid metal flows through rectangular ducts made of RAFM steel. The wall thickness of the duct is 2 mm. Consider one of the front ducts (facing the plasma), assuming idealized conditions when the duct is fully decoupled electrically from the rest of the blanket and also neglect heat exchange with all other ducts. The flow velocity is 0.5 m/s. The toroidal magnetic field is 5 T. The PbLi flow is exposed to volumetric heating that varies with the radial distance y as $q'''(y) = 30\times10^6 \exp\{-y/a\}$, W/m^3. The surface heat flux is 0.5 MW/m^2. The inlet temperature in the PbLi is 400°C. The internal duct cross-sectional dimensions $2a$ and $2b$ and the length L are shown in the figure.

a) Calculate basic dimensionless parameters: the Hartmann number Ha, Reynolds number Re, magnetic Reynolds number Re_m, interaction parameter N, and the wall conductance ratio c.
b) Estimate the MHD pressure drop without and with electrical insulation (assuming ideal electrical insulation).
c) What can you say about the shape of the velocity profile in the two cases: (1) if the duct is perfectly insulated; and (2) if there is no any electrical insulation?
d) What flow regime (laminar or turbulent) will likely occur?
e) Estimate temperature increase in PbLi: Tout-Tin.



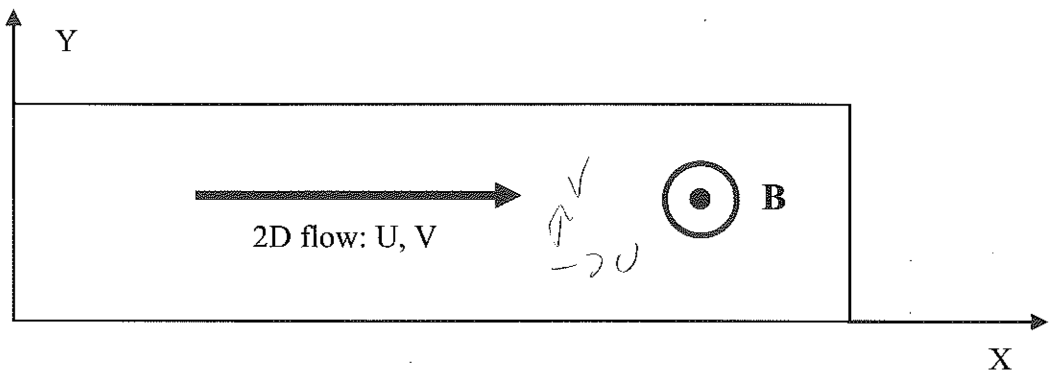
Physical properties

Fe: σ=1.4×10^6 1/Ohm-m, k=33 W/m-K, ρ=7800 kg/m3, C_p=750 J/kg-K

PbLi: σ=0.7×10^6 1/Ohm-m, k=15 W/m-K, ρ=9300 kg/m^3, C_p=190 J/kg-K, μ=0.001 Pa-s

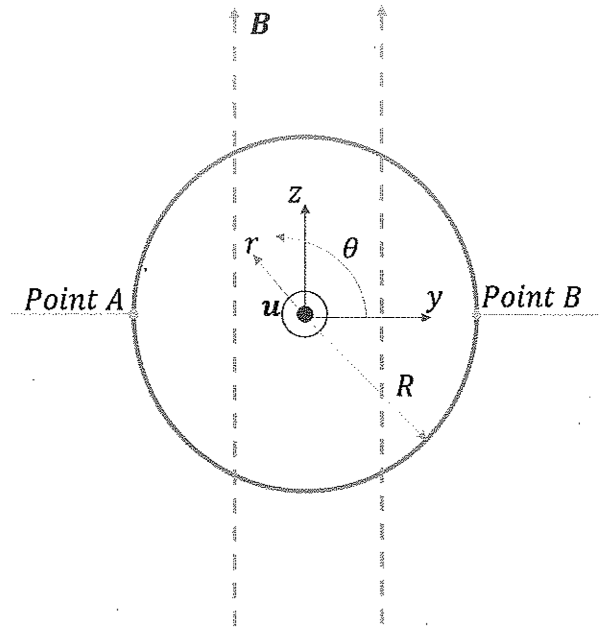
Problem 2

Derive the vorticity equation ($\omega = \frac{\partial U}{\partial y} - \frac{\partial V}{\partial x}$) for a 2D MHD flow (in the x-y plane) of electrically conducting fluid in a constant spanwise magnetic field (the field is in z direction). Based on this equation conclude what kind of MHD effect will be experienced by the flow.



Problem 3

Consider a fully developed MHD flow in a non-conducting circular pipe with radius R in the presence of a uniform magnetic field in the z-direction ($\boldsymbol{B} = B\widehat{e_z}$) as shown in the figure.



For such a configuration evaluate the following:

a) The distribution of electric potential along the wall (r = R) of the pipe for a given axisymmetric velocity profile $\boldsymbol{u}(r) = 2u_{avg}\left(1 - \frac{r^2}{R^2}\right)\widehat{e_x}$ (here u_{avg} is the average fluid velocity) by solving 2D Poisson equation for electric potential in the y-z plane with the assumption that the velocity profile is not affected by the magnetic field. *[HINT: Use the method of separation of variables.]*

b) Potential difference between points A and B for magnetic field strength B of 1 Tesla, average velocity u_{avg} of 10 cm/sec and pipe radius R of 10 cm.

Problem 4

a) Draw a schematic of a vertical cross-section of a tokamak reactor showing all major reactor components.

b) Describe concisely the functions of all components in (a) above.

c) What is the main difference between a tokamak and other toroidal confinement plasma devices?

d) Draw a unit cell of a DCLL blanket illustrating the primary geometric regions and materials.

e) Compare the features, advantages and disadvantages, of DCLL blanket to separately cooled PbLi blanket.

f) Discuss how tritium is extracted from ceramic breeder blankets.

Problem 5

A tokamak reactor with superconducting TF coils has a major radius of 6.8m, an aspect ratio of 3, and a neutron wall load of 3.6 MW/m^2. It has a breeding blanket that attenuates the neutrons by two orders of magnitude followed by 90 cm of 85% Pb+15% B_4C.

a) Calculate the reactor fusion power.

b) Calculate the total heat load into the cryogenic system.

c) Calculate the total power required to remove the nuclear heating deposited in the magnet.

d) Calculate the radiation-induced resistivity in the copper stabilizer at the point of maximum magnetic field after 4 years of continuous reactor operation.

e) If the tritium breading ratio is 1.15, calculate the rate of tritium production in the blanket in kg/s.

Problem 6

a) State and explain cryogenic stabilization criterion for superconducting magnet.

b) Discuss concisely radiation effects on components of superconducting magnets.

c) Compare the functions of bulk shielding, penetration shielding, and biological shielding in a tokamak fusion power plant.

d) What is the most promising structural material for a fusion DEMO? Why?

Problem 7

a) Calculate Q values for Li^6 (n, t) and Li^7 (n, n't), and specify if they are exothermic or endothermic.

b) If a 1 MeV neutron undergoes elastic scattering at 45 degrees with a Li^6 target in the blanket what is the heat deposited in the material per interaction?

c) An (n,α) reaction in a particular nuclide has a Q-value of -5 MeV calculate the neutron kerma factor for 14 MeV neutrons.

d) A particular shield composition has a total energy attenuation coefficient of 0.138 cm^{-1}, what is the shield thickness required to achieve energy attenuation of four orders of magnitude?

e) Write down the Neutron Transport Equation and describe the physical meaning of each term. Which term is the one that requires a more difficult mathematical treatment?

f) Neutronics calculations for a fusion blanket show the following reaction rates per fusion neutron:

REACTION	**REACTION RATE** Per fusion neutron	**|Q| - VALUE MeV**
V(n,2n)	0.1	13
V(n,γ)	0.05	8
^{6}Li(n,α)	0.80	4.8
^{7}Li(n,γ)	0.02	5
^{7}Li(n,n',α)	0.4	2.4

f1) Calculate the tritium breeding ratio.

f2) Calculate the energy multiplication factor

f3) If a tokamak reactor using the above blanket produces 3000 MW of fusion power and has a thermal conversion efficiency of 35%, calculate the reactor electric power output.

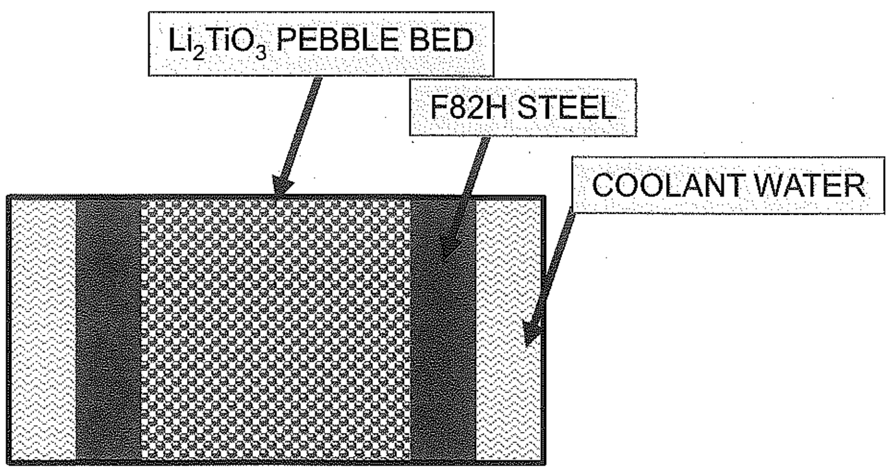
Problem 8

Consider a 1D, pebble bed-type blanket configuration with a 2-cm wide (along the tokamak's radial direction) breeder volume cooled on both sides by water at a bulk temperature of $T_f = 300$ °C. Water is flowing at 5 m/s through an equivalent hydraulic coolant channel of 1 cm with a structural wall thickness of 3 mm. (See the sketch below)

a) Calculate the temperature distribution across the pebble breeder element, structure, and water, considering the following:
   - Single size pebble bed of lithium Li_2TiO_3 pebbles of 1 mm diameter.
   - Constant volumetric heat generation rate in the breeder region of 8 MW/m^3
   - A temperature jump of 25 °C exists at the interface of pebble bed and steel
   - Use thermal properties of stainless steel for F82H

b) Calculate the purge gas pressure drop across a 1 meter tall pebble bed as a function of superficial purge gas velocity of 1, 5, and 10 cm/s for a single size bed of 1 mm pebble. Assume an average purge gas temperature of 600 °C and random packing of spheres.

c) How much tritium will permeate to the coolant from the pebble bed region through the F82H wall, if the superficial purge gas velocity is 1, 5, and 10 cm/s?
   - Assume diffusion limited control.
   - Average tritium generation rate in the breeder region= 1.21e-7 g/s.
   - Use bed average temperature for tritium partial pressure estimation.



David Li
30458983?

Problem 1

a) $Ha = BL\sqrt{\frac{\sigma}{\rho\nu}} = BL\sqrt{\frac{\sigma}{\mu}}$

$Ha = 5 \times \frac{20}{100} \times \sqrt{\frac{0.7\times10^6}{0.001}}$ $\boxed{Ha = 26457.513}$

$Re = \frac{U_0 L}{\nu} = \frac{U_0 L \rho}{\mu}$

$= \frac{0.5 \times 1 \times 9300}{0.001}$ $\boxed{Re = 4650000}$

$R_m = \frac{U_0 L}{\eta} = U_0 L \mu_0 \sigma$

$= 0.5 \times 1 \times 4\pi \times 10^{-7} \times 0.7 \times 10^6$ $\boxed{R_m = 0.440}$

$N = \frac{\sigma B^2 L}{\rho U_0} = \frac{0.7\times10^6 \times 5^2 \times \frac{20}{100}}{9300 \times 0.5}$ $\boxed{N = 752.688}$

$c = \frac{\sigma_w t_w}{\sigma L} = \frac{1.4\times10^6 \times {}^2/_{1000}}{0.7\times10^6 \times {}^{10}/_{100}}$ $\boxed{c = 0.04}$

b) ΔP_{MHD} without electrical insulation:

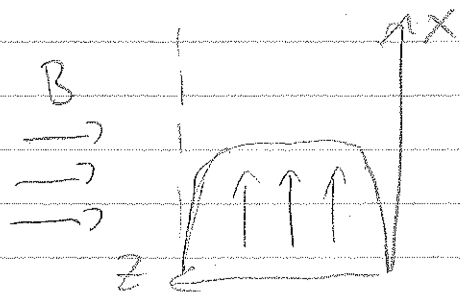
$\Delta P_{MHD} = L\sigma V B^2 \frac{\sigma_w t_w}{\sigma a}$

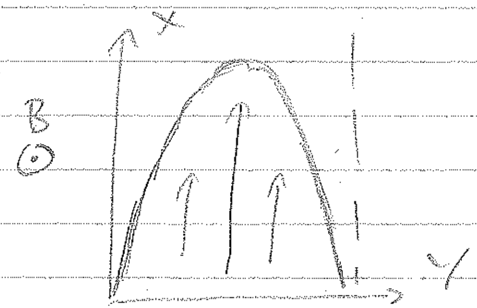
$= 1 \times 0.7\times10^6 \times 0.5 \times 5^2 \times \frac{1.4\times10^6 \times {}^2/_{1000}}{0.7\times10^6 \times \frac{20}{100} \times \frac{20}{100}}$

$\boxed{\Delta P_{MHD} = 875000 \text{ Pa}}$

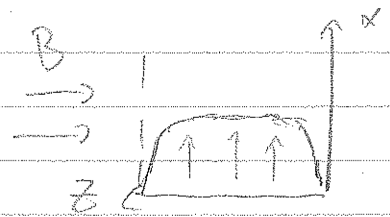
ΔP_{MHD} with ideal insulation $= \boxed{0 \text{ Pa}}$

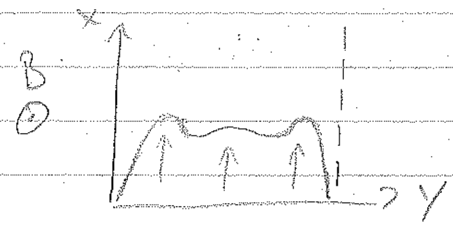
c) 1) In the Hartmann walls, there are thin velocity boundary layers with steep gradients. Basic MHD velocity profile. In the side wall orientation, it is essentially a hydrodynamic profile





2) Significant pressure drop.
In the Hartmann walls, thin Hartmann layers, similar to insulating walls.
In the side walls, side layer local velocities exceed core velocity in a narrow domain.





d) Turbulent flow likely, Re is extremely high.

David Li
30458483?

Problem 1 cont.

e) $q'''(y) = 30\times10^6 \times e^{-\frac{y}{10cm}}$

$$\int_0^{20} q'''(y)\,dy = 30\times10^6 \times \left(-10e^{-\frac{y}{10}}\right]_0^{20} + C$$

$$q'' = 30\times10^6 \times (10 - 10e^{-2}) + 0.5\times10^6 \ \frac{W}{m^2}$$

$$q'' = 259899415 \ \frac{W}{m^2} \quad a = 1m \times \frac{20}{100}m = 0.2\ m^2$$

$$q = 51979883.01\ W$$

$$q = \dot{m}c_p(T_{out} - T_{in})$$
$$\dot{m} = \rho U_o a$$

$$T_{out} - T_{in} = \frac{q}{\rho U_o a c_p}$$
$$= \frac{51979883.01}{9500 \times 0.5 \times \frac{20}{100} \times \frac{20}{100} \times 140}$$

$$\boxed{T_{out} - T_{in} = 1470.851°C}$$

David Li
304589834

Problem 3)

a) $E = u \times B$

$= \left(2u_{avg}\left(1-\frac{r^2}{R^2}\right) \times B\right)\hat{e}_y$

$V = -\int E\,dy$

for $\theta = 0°$

$V = 2u_{avg}B\int\left(1-\frac{y^2}{R^2}\right)dy$

$= 2u_{avg}B\left(y - \frac{y^3}{3R^2}\right) + C$

for $y = R\cos\theta$

$V = 2u_{avg}B\left(R\cos\theta - \frac{1}{3}R\cos^3\theta\right)$

$$\boxed{V = 2u_{avg}BR\left(\cos\theta - \frac{1}{3}\cos^3\theta\right) + C}$$

b) for A: $\theta = 180°$

$V_A = 2 \times \frac{10}{100} \times 1 \times \frac{10}{100}\left(\cos 180° - \frac{1}{3}\cos^3 180°\right) + C$

$V_A = -\frac{1}{75} + C$ V

for B: $\theta = 0°$

$V_B = 2 \times \frac{10}{100} \times 1 \times \frac{10}{100} \times \left(\cos 0° - \frac{1}{3}\cos^3 0°\right) + C$

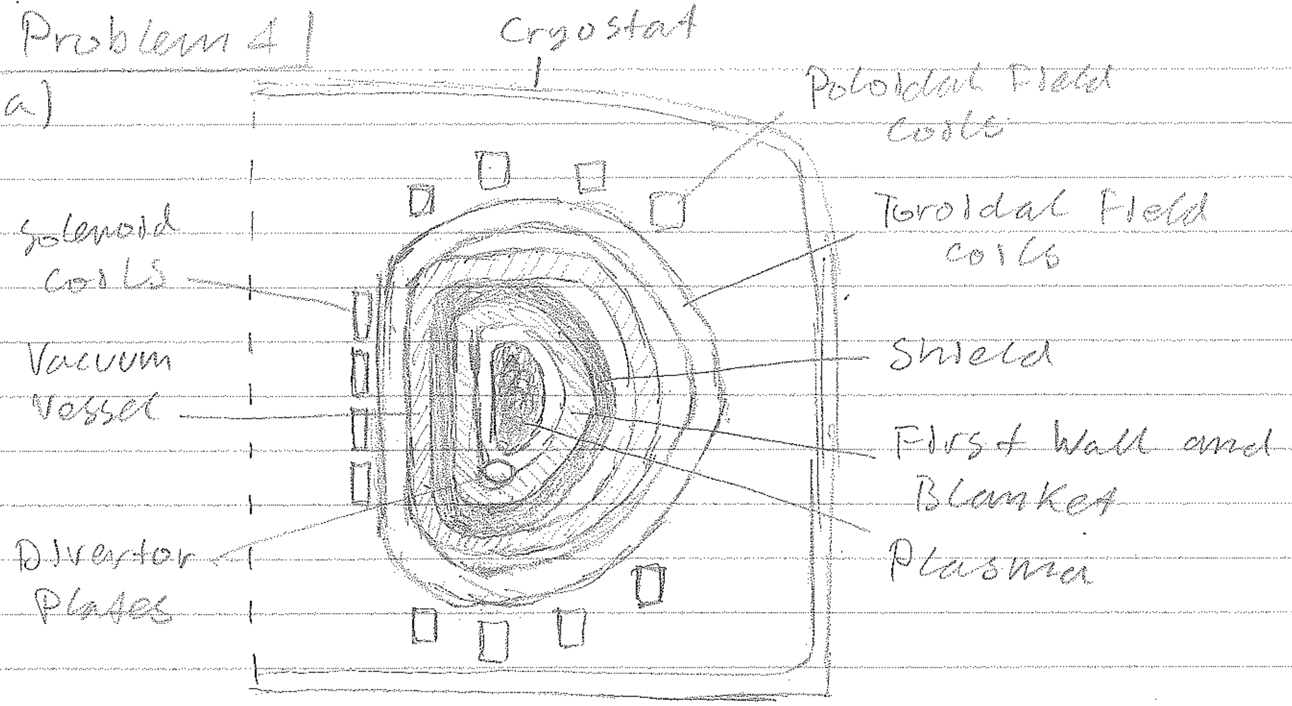
$V_B = \frac{1}{75} + C$ V

$$\boxed{V_B - V_A = 0.02\overline{6}\ V}$$

David Li
304589834

Problem 4.1

a)



b) - plasma - needs to be contained and kept at extremely high temperatures to drive the Deuterium-Tritium fusion cycle.
- toroidal field coils - provides the toroidal magnetic field which holds the plasma in place in a ring
- poloidal field coils - help shape the plasma so that the cross section is in the desired shape
- central solenoid coils - creates a toroidal electric current that flows inside the plasma, which creates a poloidal magnetic field as well as heating the plasma via ohmic heating
- vacuum vessel - keeps the shield, blanket, and plasma in a vacuum, so that neutrons dispelled by the plasma are not stopped by the air

b cont) - shield - protects the vacuum vessel, magnets, cryostat, and other components and personnel from the radiation and neutrons from the plasma.

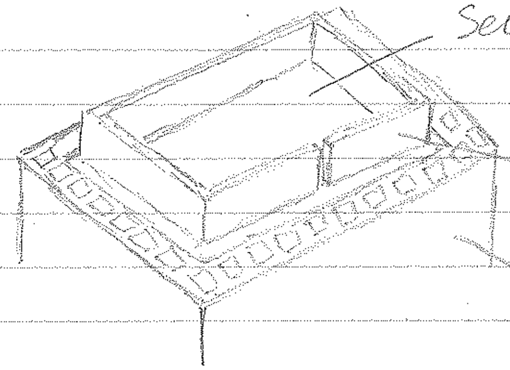
- First wall / Blanket - provides a physical boundary for the plasma; converts the kinetic energy of neutrons and gamma rays and radiation into heat and extracts it into usable power; breeds tritium with a lithium breeder, neutron multiplier, and an extraction method (purge gas for solid ceramic breeder, etc.), helps with radiation shielding
- divertor - removes α-particles in the form of "He ash" from the plasma, as well as other impurities
- cryostat - keeps the magnets at the extremely low temperatures necessary for superconducting

c) The main difference between a tokamak and other toroidal confinement plasma devices is the creation of a toroidal current to create a poloidal magnetic field and to provide ohmic heating.

David Li
304589834

Problem 4 cont.

d)



e) DCLL: the First Wall and structure are cooled with He while the liquid breeder is self-cooled. The structure and breeder are separated by SiC FCIs that provide thermal and electrical insulation to decouple the temperatures and reduce MHD pressure drop in the breeder. The difference in temperature between the breeder and the structure leads to higher thermal efficiency. However, care must be taken to keep the steel structure below 550°C and the interface temperature below 480°C.
PbLi: PbLi is circulated slowly to extract tritium while a seperate He stream removes the heat. This aids with avoiding the MHD pressure drop. However, the low velocity of the breeder leads to high tritium partial pressure, which could cause tritium permeation. The compatibility of PbLi with RAFM steel structure also necessitates a smaller temperature difference, limiting thermal efficiency of the cooling. These issues are improved on by DCLL.

f) The ceramic breeder is purged by a low pressure Helium gas to remove tritium through the "interconnected porosity" in the breeder.

David Li
304589834

Problem 51

a) $P_{nw} = \frac{P_n}{S} = \frac{E_n}{E_{DT}} \frac{P_{fusion}}{S}$

$E_n = 14.06$ MeV, $E_{DT} = 17.58$ MeV, $P_{nw} = 3.6\ MW/m^2$

$A = \frac{R}{a}$ $3 = \frac{6.8}{a}$ $a = \frac{34}{15}$ m

$S = 4\pi^2 a R = 4\pi^2\left(\frac{34}{15}\right)(6.8) = 608.49401\ m^2$

$P_{fusion} = \left(\frac{E_{DT}}{E_n}\right) S P_{nw} = \left(\frac{17.58}{14.06}\right)(608.49401)(3.6)$

$\boxed{P_{fusion} = 2739.002\ MW}$

b) $P_n = S P_{nw} = (608.494)(3.6) = 2190.578$ MW

$\frac{1}{100} P_n = 21.906$ MW

for 85% Pb and 15% B_4C, total energy attenuation coefficient $\mu_t = 0.0976\ cm^{-1}$

$\mu_t \times t = 0.0976\ cm^{-1} \times 90\ cm = 8.784$

$\frac{L_{TE}(r)}{L_{TE}(r_0)} = e^{-8.784} = 1.532 \times 10^{-4}$

$L_{TE}(r) = 21.906 \times 1.532 \times 10^{-4} = 3.355 \times 10^{-3}$ MW

$= \boxed{3.355\ kW}$

c) Assume ideal thermodynamic cryogenic system based on reversed Carnot cycle. Assume the superconducting magnets are operating at 4 K and the heat is rejected at room temperature (300 K)

$\eta_R = \frac{Q_L}{W_{,in}} = \frac{T_L}{T_H - T_L}$ $W_{,in} = \frac{T_H - T_L}{T_L} Q_L$

$W_{,in} = \frac{300-4}{4} \times 3.355$ $\boxed{W_{,in} = 248.283\ kW}$

d) $I_w = P_w \cdot t_0 \cdot F$

Assume $F = 1$

$$= \frac{3.6\ MW}{m^2} \cdot 4\,yr \cdot 1 = 14.4\ \frac{MW \cdot yr}{m^2}$$

→ Assume $\Delta^0_{BS} = 1.1\ m$

using Fig. 10 from Handout 12

$$\boxed{\rho = 3 \times 10^{-8}\ \Omega \cdot cm = 3 \times 10^{-10}\ \Omega \cdot m}$$

* thickness of blanket not given

e) $\dot{m}_t = \frac{P_f}{E_{DT}} M_t$

$$= \left(\frac{2739.002\ MW}{17.58\ MeV}\right)\left(\frac{1\ eV}{1.6022 \times 10^{-19}\ W \cdot s}\right)\left(\frac{1\ kg}{1.9966988 \times 10^{26}\ T\ atom}\right)$$

$$\dot{m}_t = 4.870 \times 10^{-6}\ \frac{kg}{s}$$

$$\dot{m}_t^+ = 1.15\,\dot{m}_t = 1.15 \times 4.870 \times 10^{-6}$$

$$\boxed{\dot{m}_t^+ = 5.601 \times 10^{-6}\ \frac{kg}{s}}$$

David Lo
304589834

Problem 6

a) The cryogenic stabilization criterion is: $\boxed{I^2 \rho_{cu} \leq q P a}$

It basically states that for cryogenic stabilization to be provided, the power added by ohmic heating in the copper must be equal to or less than the power we are able to remove. Thus the following equation: $I^2 R_{cu} \leq q P l$

where $R_{cu} = \frac{\rho_{cu} l}{a}$, q is the heat we can remove, P is the cooled parameter, a is the area of the stabilizer

$$I^2\left(\frac{\rho_{cu} l}{a}\right) \leq q P l \Rightarrow \underline{I^2 \rho_{cu} \leq q P a}$$

b) The superconducting magnet assembly is composed of a superconductor, a stabilizer, the structure, and an electric and thermal insulator.

Radiation causes atomic displacements and transmutations that cause changes in the physical and mechanical properties of materials.

- For superconductors, neutron radiation decreases the superconducting region of current density-temperature-magnetic field phase space. This, in effect, causes the critical current in the superconductor to decrease as fluence increases.

b cont.) - In stabilizers, which are commonly copper or aluminum, radiation induces an increase in resistivity as the displacements per atom (dpa) of conductor. These defects can be annealed and removed.

- In insulators, radiation cause physical deterioration from neutron damage. Organic insulators are more susceptible to this; inorganic retain their physical properties better but suffer from brittleness. With enough radiation damage, insulators lose their resistances and become conductors.
- In the structures, radiation damage can cause swelling, loss of ductility, changes in creep rate, changes in fatigue life, and loss of fracture toughness, amongst other effects.

c) The <u>bulk shield</u> surrounds the blanket to protect the vacuum vessel and superconducting magnets.
The <u>penetration shield</u> is specifically for components that penetrate the bulk shield and the structure in general, such as natural beams, vacuum ducts, and fuel ducts.
The <u>biological shield</u> comprises the reactor building walls and is typically made of concrete. It is designed to protect the reactor personnel and sensitive equipment in the central rooms and outside.

David Li
30458983?

Problem 6 cont.

d) the most promising structural material for a fusion DEMO is ferritic steel. This is because stainless steel is weak against radioactivity, and prone to swelling from radiation damage. It also has poor thermal properties, and has higher thermal stresses for the same temperature difference.

David Li
304589834

Problem 7

a) $Q = (m_i - m_f)c^2$

$Li^6 + n \rightarrow T + \alpha$

$m_{Li6} = 6.015122795$ amu

$m_n = 1.008644904$ amu

$m_T = 3.0160492$ amu

$m_\alpha = 4.001506466$ amu

$1 \text{ amu} = 1.660539040 \times 10^{-27}$ kg

$c = 299792458$ m/s

$$Q_{Li6} = (6.015122795 + 1.008701 - 3.0160492 - 4.001506466)(1.660539040 \times 10^{-27})(299792458)^2$$

$$Q_{Li6} = 7.658 \times 10^{-13} \text{ J} \times \frac{1 \text{ MeV}}{1.6021766208 \times 10^{-13} \text{ J}}$$

$$\boxed{Q_{Li6} = 4.785 \text{ MeV}}$$

$Q_{Li6} > 0$, $Li^6(n,t)$ is exothermic

$Li^7 + n \rightarrow T + \alpha + n$

$m_{Li7} = 7.01600455$ amu

$$Q_{Li7} = (7.01600455 - 3.0160492 - 4.001506466)(1.660539040 \times 10^{-27})(299792458)^2$$

$$Q_{Li7} = -3.957 \times 10^{-13} \text{ J} \times \frac{1 \text{ MeV}}{1.6021766208 \times 10^{-13} \text{ J}}$$

$$\boxed{Q_{Li7} = -2.467 \text{ MeV}}$$

$Q_{Li7} < 0$, $Li^7(n,n't)$ is endothermic

b) $E' = \frac{E}{(A+1)^2}\left[\cos\alpha + \sqrt{A^2 - \sin^2\alpha}\right]^2$

$A_{Li^6} = 6.015122795$ amu

$E' = \frac{1}{(6.015+1)^2}\left[\cos 45^\circ + \sqrt{6.015^2 - \sin^2 45^\circ}\right]^2$

$E' = 0.906881165$ MeV

$\boxed{E - E' = 0.093118 \text{ MeV}}$

c) for 14 MeV neutrons, $Q = -5$ MeV

$E_H = E_n - E_n' + Q = 14 - 0 + (-5) = 11$ MeV

$k = \sigma E_H \qquad \boxed{k = 11\sigma \text{ [MeV}\cdot\text{barn]}}$

* σ not given, reaction unknown

d) $e^{-\mu_t \cdot t} = \frac{1}{10^4} \qquad \mu_t = 0.138 \text{ cm}^{-1}$

$-\mu_t \cdot t = \ln \frac{1}{10^4} \qquad t = -\frac{1}{0.138} \ln \frac{1}{10^4}$

$\boxed{t = 66.742 \text{ cm}}$

e) $\frac{\partial n}{\partial t} + v\hat{\Omega}\cdot\nabla n + v\Sigma_t n(r, E, \hat{\Omega}, t)$

$= \int_{4\pi} d\hat{\Omega}' \int_0^\infty dE' v' \Sigma_s(E' \to E, \hat{\Omega}' \to \Omega) n(r, E', \hat{\Omega}', t)$

$+ s(r, E, \hat{\Omega}, t)$

①. $s(r, E, \hat{\Omega}, t)$: Any neutron sources in control volume

David Li
304589834

Problem 7 cont.

② $\int_{4\pi} d\hat{\Omega}' \int_0^{\infty} dE' v' \Sigma_s(E' \to E, \hat{\Omega}' \to \hat{\Omega})\, n(r, E', \hat{\Omega}', t)$:
Gain from scattering inside of the control volume, changing neutrons of a different energy and direction E' and Ω' into the energy and direction of interest, E and Ω. This is known as the inscattering term.

③ $v\Sigma_t n(r, E, \Omega, t)$:
The opposite of ②; neutrons in the control volume whose energy and direction are changed by a collision.

④ $\frac{\partial n}{\partial t} + v\hat{\Omega}\cdot\nabla n$: The neutrons going out of the surface of the control volume minus the neutrons going into the surface.

④ Number ②, the inscattering term, is the most difficult term to treat mathematically.

f) 1) TBR = 0.8 + 0.02 + 0.4

$\boxed{TBR = 1.22}$

2) $E_n = 14.06$ MeV

$(0.1 \times 13) + (0.05 \times 8) + (0.8 \times 4.8) + (0.02 \times 5) + (0.4 \times -2.4) = 4.68$ MeV

$\frac{14.06 + 4.68}{14.06} = \boxed{1.333}$

3) $3000\text{ MW} \times 1.333 \times 0.35$

$= \boxed{1399.502\text{ MW}}$