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MAE 237 D Final

Problem 3

a) $\vec{B} = B\,\hat{e}_z \qquad \vec{u}(r) = 2U_{avg}\left(1-\frac{r^2}{R^2}\right)\hat{e}_x \qquad y^2+z^2=r^2$ along pipe wall

$$\nabla^2\phi = \nabla\cdot(\vec{u}\times\vec{B})$$

Right idea, but wrong coordinate system. Should have used cylindrical coordinates to compute div(uxB) then applied separation of variables wrt r and theta so that the BC can be applied at r=R.

$$\vec{u}\times\vec{B} = \begin{vmatrix} i & j & k \\ 2u_{avg}\left(1-\frac{r^2}{R^2}\right) & 0 & 0 \\ 0 & 0 & B \end{vmatrix} = -2BU_{avg}\left(1-\frac{r^2}{R^2}\right) j$$

$$\nabla\cdot(\vec{u}\times\vec{B}) = -2BU_{avg}\cdot\frac{\partial}{\partial y}\left(1-\frac{r^2}{R^2}\right)$$

along the wall, $r^2 = y^2+z^2$ so $\frac{\partial}{\partial y}\left(1-\frac{r^2}{R^2}\right) = \frac{\partial}{\partial y}\left(1-\frac{y^2+z^2}{R^2}\right) = \frac{-2y}{R^2}$

$\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2} = 2BU_{avg}\frac{2y}{R^2}$ assume $\phi = Y(y)Z(z)$

$Z\frac{d^2Y}{dy^2}+Y\frac{d^2Z}{dz^2}=0$ for homogeneous solution ; $\phi(0,R)=\phi(0,-R)=\phi(R,0)=\phi(-R,0)=0$

$Y(R)=Y(-R)=Z(R)=Z(-R)=0$

$\frac{1}{Y}\frac{d^2Y}{dy^2} = \frac{-1}{Z}\frac{d^2Z}{dz^2} = A \qquad Y''=AY \qquad Z''=-AZ$

$\Rightarrow$

$Z = C_1\sin\left(\frac{\pi z}{R}\right)+ \qquad Z' = \frac{C_1R}{\pi}\cos\left(\frac{\pi z}{R}\right) \qquad Z'' = \frac{-C_1R^2}{\pi^2}\sin\left(\frac{\pi z}{R}\right) = -AZ = -AC_1\sin\left(\frac{\pi z}{R}\right)$

$A = \frac{R^2}{\pi^2}$

$Y = C_2e^{Y\frac{R}{\pi}} \qquad Y' = \frac{R}{\pi}C_2e^{YR/\pi} \qquad Y'' = \frac{R^2}{\pi^2}C_2e^{YR/\pi} = AY$

$Y(R)=0 \Rightarrow C_2e^{R^2/\pi}=0 \Rightarrow C_2=0$

$\phi = \frac{2BU_{avg}}{R^2}yz^2 \qquad \phi_{yy}=0 \qquad \phi_{zz} = \frac{4BU_{avg}}{R^2}y \qquad \phi_{yy}+\phi_{zz} = 4\frac{BU_{avg}}{R^2}y$

Right idea, but incorrect formula to start

b) $\phi(R,0) = 4\frac{BU_{avg}}{R} \qquad \phi(-R,0) = -4B\frac{U_{avg}}{R}$

potential difference between A and B is $8BU_{avg}\cdot\frac{1}{R}$

$= 8\cdot 1T\cdot 0.1m/s\cdot 0.1m = \boxed{0.08V}$