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MAE 237D Final

Problem 1

Assume kinematic viscosity of PbLi $\nu = 1.4 \cdot 10^{-7} \frac{m^2}{s}$ [from handout 17]

a) $Ha = Bb\left(\frac{\sigma}{\nu\rho}\right)^{0.5} = 5T \cdot 0.1m \left(\frac{0.7 \cdot 10^6 \frac{1}{Ohm \cdot m}}{1.4 \cdot 10^{-7} \frac{m^2}{s} \cdot 9300 \frac{kg}{m^3}}\right)^{0.5} = 1.16 \cdot 10^4$

$Re = \frac{U_0 L}{\nu} = \frac{0.5 m/s \cdot 1m}{1.4 \cdot 10^{-7} \frac{m^2}{s}} = 3.57 \cdot 10^6$

$Re_m = \frac{U_0 L}{\eta} = U_0 L \cdot \sigma \mu_0 = 0.5 m/s \cdot 1m \cdot 0.7 \cdot 10^6 \frac{1}{Ohm \cdot m} \cdot 0.001\ Pa \cdot s = 350 = 3.5 \cdot 10^2$

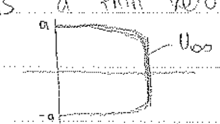
$N = \sigma B^2 \frac{L}{\rho U_0} = 0.7 \cdot 10^6 \frac{1}{Ohm \cdot m} \cdot (5T)^2 \cdot \frac{1m}{9300 \frac{kg}{m^3} \cdot 0.5 m/s} = 3.76 \cdot 10^3$

$c = \frac{\sigma_w t_w}{\sigma a} = \frac{1.4 \cdot 10^6 \frac{1}{Ohm \cdot m} \cdot 2mm}{0.7 \cdot 10^6 \frac{1}{Ohm \cdot m} \cdot 10 cm} \cdot \frac{1 cm}{10 mm} = 0.04$

b) without insulation: $\Delta p_{MHD} = LJB \approx L\sigma U_0 B^2 c = 1m \cdot 0.7 \cdot 10^6 \frac{1}{Ohm \cdot m} \cdot 0.5 m/s (5T)^2 \cdot 0.04$
$= 0.35\ MPa$

with ideal electrical insulation, the MHD pressure drop is zero

c) perfectly insulated duct: The velocity profile has a thin velocity boundary layer with steep gradients:



no electrical insulation: The velocity profile is similar to the insulated case, except there are side layers where the local velocity is actually higher than the core velocity.

d) $\frac{Re}{Ha} = \frac{3.57 \cdot 10^6}{1.16 \cdot 10^4} \approx 308$

The critical Reynolds number is ~380 experimentally and 48,000 theoretically so this flow would most likely be laminar.

e) surface heat flux: $q_1 = 0.5\ \frac{MW}{m^2} \cdot 1m \cdot 0.2m = 0.1\ MW$

volumetric heating $q_2 = \int_0^{2a} 30 \cdot 10^6\, e^{(-y/a)}\, dy = 30 \cdot 10^6 \cdot \left[-a e^{-y/a}\right]_0^{2a}$
$= 30 \cdot 10^6 \cdot [-.1e^{-2} + .1e^{0}] = 2.59 \cdot 10^6\ W = 2.59\ MW$

The surface heat flux can be neglected

$\rho C_p U \frac{dT}{dx} = q_2 \qquad \frac{dT}{dx} = 2.59\ MW \cdot \frac{1}{9300\ \frac{kg}{m^3} \cdot 190\ \frac{J}{kg\,K} \cdot 0.5\ m/s} = 2.93\ K/m$

$dT = 2.93\,dx = 2.93\ K = T_{out} - T_{in} \qquad \boxed{T_{out} = 402.93°C}$

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Problem 3

a) $\vec{B} = B\,\hat{e}_z$ $\quad \vec{u}(r) = 2U_{avg}\left(1-\frac{r^2}{R^2}\right)\hat{e}_x$ $\quad y^2+z^2=r^2$ along pipe wall

$$\nabla^2\phi = \nabla\cdot(\vec{u}\times\vec{B})$$

$$\vec{u}\times\vec{B} = \begin{vmatrix} i & j & k \\ 2u_{avg}\left(1-\frac{r^2}{R^2}\right) & 0 & 0 \\ 0 & 0 & B \end{vmatrix} = -2B\,U_{avg}\left(1-\frac{r^2}{R^2}\right)\hat{j}$$

$$\nabla\cdot(\vec{U}\times\vec{B}) = -2B\,u_{avg}\cdot\frac{\partial}{\partial y}\left(1-\frac{r^2}{R^2}\right)$$

along the wall, $r^2 = y^2+z^2$ so $\frac{\partial}{\partial y}\left(1-\frac{r^2}{R^2}\right) = \frac{\partial}{\partial y}\left(1-\frac{y^2+z^2}{R^2}\right) = \frac{-2y}{R^2}$

$$\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2} = 2B\,U_{avg}\frac{2y}{R^2}$$ assume $\phi = Y(y)\,Z(z)$

$Z\frac{d^2Y}{dy^2} + Y\frac{d^2Z}{dz^2} = 0$ for homogeneous solution ; $\phi(0,R)=\phi(0,-R)=\phi(R,0)=\phi(-R,0)=0$

$Y(R)=Y(-R)=Z(R)=Z(-R)=0$

$\frac{1}{Y}\frac{d^2Y}{dy^2} = \frac{-1}{Z}\frac{d^2Z}{dz^2} = A \qquad Y''=AY \qquad Z''=-AZ$

$\Rightarrow$

$Z = C_1\sin\left(\frac{\pi z}{R}\right) + \quad Z' = \frac{C_1R}{\pi}\cos\left(\frac{\pi z}{R}\right) \qquad Z'' = \frac{-C_1R^2}{\pi^2}\sin\left(\frac{\pi z}{R}\right) = -AZ = -AC_1\sin\left(\frac{\pi z}{R}\right)$

$A=\frac{R^2}{\pi^2}$

$Y = C_2e^{Y\frac{R}{\pi}} \qquad Y' = \frac{R}{\pi}C_2e^{YR/\pi} \qquad Y'' = \frac{R^2}{\pi^2}C_2e^{YR/\pi} = AY$

$Y(R)=0 \Rightarrow C_2e^{R^2/\pi} = 0 \Rightarrow C_2=0$

$\phi = \frac{2B\,U_{avg}}{R^2}yz^2 \qquad \phi_{yy}=0 \qquad \phi_{zz} = \frac{4B\,U_{avg}}{R^2}y \qquad \phi_{yy}+\phi_{zz} = 4\frac{B\,U_{avg}}{R^2}y$

b) $\phi(R,0) = 4\frac{B\,U_{avg}}{R} \qquad \phi(-R,0) = -4B\frac{U_{avg}}{R}$

potential difference between A and B is $8B\,U_{avg}\cdot\frac{1}{R}$

$= 8\cdot 1T\cdot 0.1m/s\cdot 0.1m = \boxed{0.08V}$

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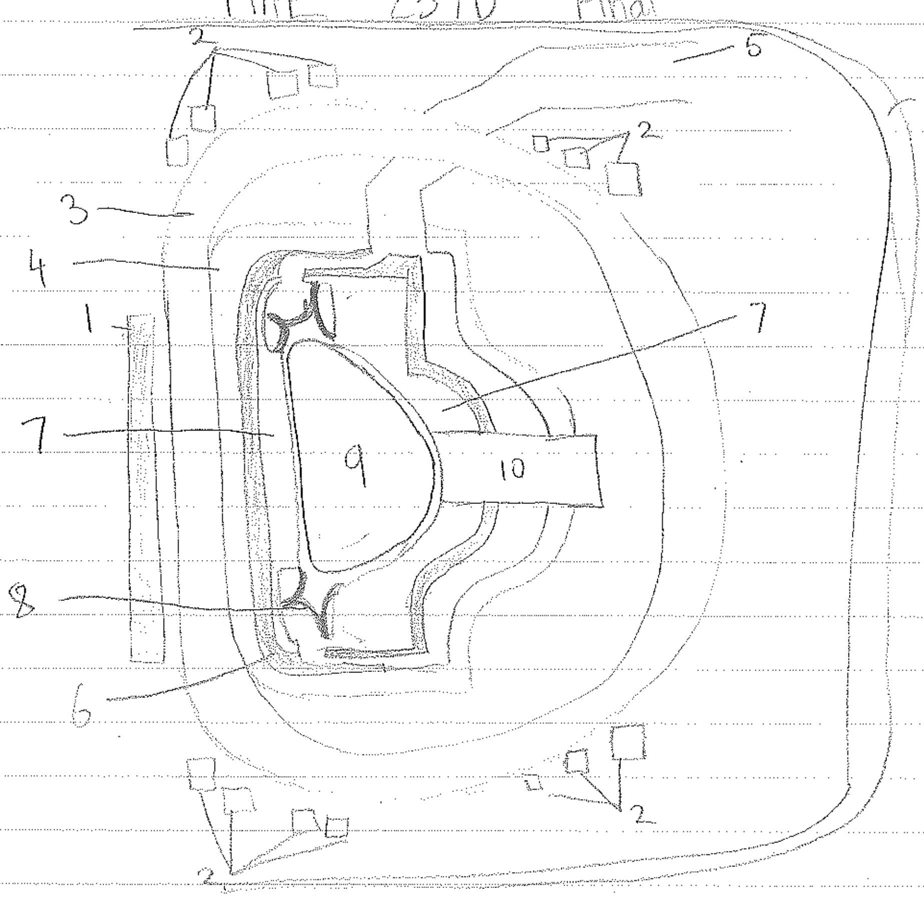
Problem 4

a)

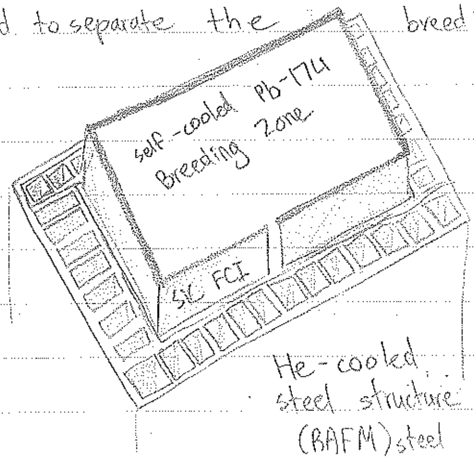


b)

1. Solenoid coil: this magnet is used to induce a poloidal field to move the plasma
2. Poloidal magnets: These magnets are used to shape the plasma in the D shape.
3. Toroidal field coil: These magnets create the toroidal magnetic field to confine the plasma
4/5. The vacuum vessel (4) along with the vacuum pumping duct (5) are used to remove the organic molecules that would be broken up in the hot plasma and to create low density inside the reactor. Keeps background pressure low.
6. Shield: the shield protects the magnets, structures, and personell from neutrons that were not absorbed by the blanket.
7. First wall/blanket: Used to convert the energy from the neutrons into thermal energy, breeds tritium. The plasma heat load is removed by the 1st wall.
8. Divertor plates: Used to remove the α particles from the D-T reaction
9. Plasma: Mix of deuterium and tritium where the fusion reaction takes place
10. RF Antenna: radio frequency waves are used to heat the plasma
11. Cryostat: Used to maintain low temperatures for the magnets and other systems.

d) A tokamak uses magnetic confinement, there are other reactors that use inertial confinement to keep the plasma in the toroidal shape. Another magnetic confinement reactor is the stellerator which differs from the tokamak with a much more complicated shape for its magnet. Theoretically, a stellerator can have the magnet on continuously while the tokamak needs constant magnetic field flux so the center solenoid cannot run continuously.

d/e) Features of DCLL: first wall and ferritic steel structure cooled with helium. The breeding zone is self cooled. Silicon carbide (SiC) flow channel inserts (FCIs) are used to separate the breeding zone from the structure.



Features of separately cooled PbLi blanket: All energy is removed by a separate helium coolant.

Advantages of DCLL: Higher temperature PbLi can be used. Higher efficiency by enabling use of the Brayton cycle. Reduced MHD pressure drop.

Disadvantages of DCLL: More difficult to fabricate the FCIs. Neutron irradiation of the SiC could also be a problem. Buoyancy could affect the interface temperature, heat losses, and tritium transport.

f) In a ceramic breeder, tritium that is created travels to the grain boundaries of the pebble bed through bulk diffusion, then moves along the grain boundaries. Tritium undergoes surface desorption with the oxides in the ceramic to T_2O. Low pressure gas (helium purge) removes tritium through the interconnected porosity of the ceramic breeder. Finally, convective mass transfer removes the tritium out of the blanket through the helium purge channels.

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Problem 5

a) $R = 6.8\,m$ $\quad A = 3 = \frac{R}{a} \Rightarrow a = 2.267\,m$ $\quad P_{nw} = 3.6\ \frac{MW}{m^2}$

Surface Area $= 4\pi^2 a R = 608.5\ m^2$ $\quad P_n = P_{nw} \cdot SA = 2190.6\ MW$

$$P_f = P_n \cdot \frac{17.58}{14.06} = \boxed{2739\,MW}$$

b) $P_n = 2190.6\,MW$ $\quad P_{nb}$ after blanket attenuation $= 21.9\,MW = L_{TE}(r_0)$

$$L_{TE}(r) = L_{TE}(r_0)\, e^{-\mu_t (r - r_0)} = 21.9\,MW\ e^{-0.0976 \cdot 90} = 0.3355\ MW$$

c) Assume room temperature T_r at $293\,K$ and magnet operating temperature $T_0 = 4\,K$

$$P = \dot{Q} \cdot \frac{T_r - T_0}{T_0} = 335.5\,kW \cdot \frac{293K - 4K}{4K} = 24{,}241\,kW = \boxed{24.24\,MW}$$

d) From "Radiation Damage in the Copper Stabilizer in a Superconducting Magnet" by R. E. Nygren

$$\rho_R = 400\left[1 - \exp(-I_w \cdot 10^{-19})\right]\ n\Omega\,cm$$

$$I_w = P_{nw} t_0 = 3.6\ \frac{MW}{m^2} \cdot 4\,years = 14.4\ \frac{MW\text{-}yr}{m^2} \cdot \frac{4.43 \cdot 10^{17}\ n/m^2}{1\,MW/m^2} = 6.38 \cdot 10^{18}$$

$$\rho_R = 188.6\ n\Omega\,cm = \boxed{1.886 \cdot 10^{-7}\ \Omega\,cm}$$

e) $P_{nw} = J E_0$ $\quad J = \frac{P_{nw}}{E_0}$ $\quad J \cdot SA = \frac{P_n}{E_0} = \frac{2190.6\,MW}{14.06\,MeV} = \frac{2190.6\,W}{14.06\,eV} \cdot \frac{1\,eV}{1.602 \cdot 10^{-19}\,J}$

$J \cdot SA = 9.7256 \cdot 10^{20}\ n/s$

$$T = \frac{\#\text{ of tritium atoms produced in the blanket}}{\#\text{ of fusion neutron}} = 1.15 \qquad \#\text{ in blanket} = 1.118 \cdot 10^{21}\ \frac{\text{tritium}}{s}$$

$$t_{\text{in blanket}} = 1.118 \cdot 10^{21}\ \frac{\text{tritium}}{s} \cdot \underbrace{\frac{1\,kg}{1.9966988 \cdot 10^{26}\ \text{atoms}}}_{\text{from HW 1}} = \boxed{5.6 \cdot 10^{-6}\ \frac{kg}{s}}$$

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Problem 6

a) cryogenic stablization criterion : $I^2 \rho \leq q'' p a$ a: normal conductor cross sectional area
I: operating current ρ: total resistivity of the stabilizer q'': heat flux p: cooled perimeter

This criterion requires that the heat transfer from the stabilized superconducting magnet must be sufficient to transfer the I^2R heat that is generated in the stabilizing material when a flux jump occurs.

b) Neutron radiation on the superconductor diminishes the superconducting region of current density - temperature - magnetic field phase which makes it more difficult to operate the magnet while it is superconducting. The normal conductor used for cryogenic stability in the superconductor experiences a dramatic increase in resistance due to radiation. Radiation also causes the organic insulators in the superconducting magnets to physically deteriorate, though inorganic insulators fare better but are brittle.

c) The bulk shield surrounds the blanket to protect the vacuum vessel and the superconducting magnets.
The penetration shield is used around penetrations for the beams, ducts, access ports ...
The biological shield protects personnel in central rooms and outside of the building. Typically concrete.

[EFDA presentation]

d) Reduced Activation Ferritic Martensitic (RAFM) steel is the most promising structural material for a fusion DEMO. RAFM steel can be optimized for good fracture properties, corrosion resistance, ductility, strength grain size, and strength. RAFM has no nickle which is radioactive due to its 6 isotopes. Oxide dispersion strengthened RAFM steel can also be used at slightly higher temperatures.

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Problem 7

a) $Li^6(n,t)\,{}^4_2He$ $\quad Q = \{m({}^6_3Li) + m({}^1_0n) - m({}^3_1t) - m({}^4_2He)\}c^2$

$Q = \{6.015122u + 1.008665u - 3.016049u - 4.002603u\} \cdot 931.5 \frac{MeV}{u} = 4.783 MeV$

$+Q \Rightarrow$ exothermic

$Li^7(n,n't)\,{}^4_2He$ $\quad Q = \{m({}^7_3Li) + m({}^1_0n) - m({}^1_0n) - m({}^3_1t) - m({}^4_2He)\}c^2$

$Q = \{7.016004u - 3.016049u - 4.002603u\} \cdot 931.5 \frac{MeV}{u} = -2.467 MeV$

$-Q \Rightarrow$ endothermic

b) 1 MeV neutron elastic scattering at 45° with Li^6

The neutron leaves energy with the ^6Li which briefly becomes a charged particle before depositing this energy as heat.

$$E_{Li} = \left[\sqrt{\frac{m_n m_n \cdot E_n}{(m_n + m_{Li})^2}}\cos\theta_y \pm \sqrt{\frac{m_n m_n \cdot E_n}{(m_n+m_{Li})^2}\cos^2\theta_y + \left[\frac{m_{Li}-m_n}{m_n+m_{Li}}E_n + \frac{m_{Li}\cdot Q}{m_n+m_{Li}}\right]}^{\,.712736}\right]^2$$

$m_n = 1.008665\,u \quad m_{Li} = 6.015122\,u \quad E_n = 1\,MeV \quad \theta_y = 45° \quad Q = 0$

$$E_{Li} = \left[0.1015455 + 0.85035\right]^2 = \boxed{0.906108 \text{ MeV/ interaction}}$$

c) Using nndc.bnl.gov, the closest (n,α) reaction with $Q = -5$ MeV is $^{26}Mg(n,\alpha)^{23}Ne$ where $Q = -5.4166$ MeV

at 14 MeV, $\sigma = 0.0840343$ b $\qquad$ assume α and ^{23}Ne deposit energy locally

$E_H = 14MeV - 5.4166MeV = 8.5834 MeV$

$k = \sigma E_H = 0.0840343b \cdot \frac{10^{-24}cm^2}{b} \cdot 8.5834 MeV = \boxed{7.213 \cdot 10^{-25}\ MeV \cdot cm^2}$

d) $\mu_t = 0.138\ cm^{-1} \qquad L_{TE}(r) = L_{TE}(r_0)\,e^{-0.138(\Delta r)} \qquad \frac{L_{TE}(r)}{L_{TE}(r_0)} = 1 \cdot 10^{-4} = e^{-0.138(\Delta r)}$

$\ln(1 \cdot 10^{-4}) = -0.138\,\Delta r \Rightarrow \boxed{\Delta r = 66.74\ cm}$

e) $$\underbrace{\frac{\partial n}{\partial t}}_{5} + \underbrace{v\hat{\Omega}\cdot\nabla n}_{4} + \underbrace{v\Sigma_t n(r,E,\hat{\Omega},t)}_{2} = \underbrace{\int_{4\pi} d\hat{\Omega}' \int_0^\infty dE' v'\Sigma_s(E'\to E, \hat{\Omega}'\to\hat{\Omega})\,n(r,E',\hat{\Omega}',t)}_{3} + \underbrace{s(r,E,\hat{\Omega},t)}_{1}$$

① is the rate of source neutrons that appear in the volume

② is neutrons that are lost due to collisions in the volume

③ This is the inscattering term where neutrons are gained due to E' and Ω' neutrons scattering into E and Ω neutrons

④ these are the neutrons that leak through the surface of the volume

⑤ This is the rate of change of neutrons.

The inscattering term ③ requires a more difficult mathematical treatment.

f) $V(n, 2n)^{4}V$ $\quad V(n, \gamma)^{4}V^{*}$ $\quad {}^{6}Li(n, \alpha)T$ $\quad {}^{7}Li(n, \gamma)^{8}Li^{*}$

${}^{7}Li(n, n', \alpha)T$ only ${}^{6}Li(n, \alpha)$ and ${}^{7}Li(n, n', \alpha)$ produce tritium

f1) $$TBR = \frac{\text{number produced}}{\text{fusion neutron}} = \frac{\text{reaction rates of } {}^{6}Li(n,\alpha) + {}^{7}Li(n,n',\alpha)}{1} = 0.80 + 0.4 = 1.2$$

f2) $$\epsilon = \frac{0.1(-13\text{MeV}) + 0.05(8) + 0.80(4.8) + 0.02(5) + 0.4(-2.4)}{14.06\text{MeV}} = 0.148$$

f3) $$P_e = \eta_{th} (P_\alpha + \epsilon P_n) = 0.35\left[\frac{3.52}{17.58} + 0.148\,\frac{14.06}{17.58}\right](3000\text{MW}) = 334.47\text{MW}$$