STAT8008: Time Series & Multi-Variate Analysis

Department of Mathematics
Cork Institute of Technology

Lecture 03: Seasonal Index & Forecasting

Seasonal Variation (S)

- Seasonal variations are measured in Index Form.
- For example, let us suppose sales figures for the shop at C.I.T. are recorded quarterly over a number of years and an index of sales calculated. The resulting index is set out as follows:

Quarter	Index (%)
1	115
2	122
3	46
4	117

If the sales in the shop were exactly the same for every quarter, then the index of sales would be 100% for each quarter of the year, amounting to 400% in total.

Seasonal Variation (S)

In reality, sales are not the same every quarter- some periods are busier than others. The index above reflects this.

- The 1st quarter figure suggests that sales are 115% of normal.
- The 2nd quarter figure suggests that sales are 122% of normal
- The 3rd quarter figure suggests that sales are 46% of normal
- The 4th quarter figure suggests that sales are 117% of normal

When quantifying the Seasonal Variation, we calculate a **seasonal index**.

Cyclical Variation (C)

- Cyclical variations only become apparent when data has been recorded over long periods of time, i.e. 25 or 30 years or even more.
- The duration of these cycles can be measured in terms of their turning points, or in other words, from trough to trough or peak to peak. These cycles are recurrent rather than strictly periodic.
- No highly accurate method of forecasting this type of activity has been devised.
- Unless you have data available over a long period of time, cyclic patterns are not usually fit by forecasting models.

Residual Variation (R)

- Residual variation is variation due to the occurrence of unpredictable events and therefore by its nature may exist to a lesser or greater extent within the time series.
- For example, a quarterly seasonal index is being calculated (as mentioned above) the sum of the 4 quarterly index values should equal 400%. If the sum fails to equal 400% then this is an indication that some residual variation exists.
- Consequently an adjustment to each of the figures is made, so that the sum will equal 400%. This adjustment eliminates the residual variation component.

Time Series Components

We have already stated that a time series is composed of 4 components:

1. Secular Trend (T)
2. Seasonal Variation (S)
3. Cyclical Variation (C)
4. Residual Variation (R)

The overall time series is denoted by A.

Methods

In carrying out an analysis of the time series, we may wish to isolate or depict the seasonal component (S). In order to do this we must eliminate the trend (T), the cyclical component (C) and the residual component (R) from the overall time series (A). This can be done in either of two ways, thus bringing us to two models which have been established for analysing a time series:

1. The Additive Model:

$$A = T + S + C + R$$

2. The Multiplicative Model:

$$A = T * S * C * R$$

Additive vs Multiplicative

- The method of seasonal decomposition used depends upon the relationship between the values of the series and the seasonal fluctuations.
- If seasonal **fluctuations are greater/smaller** as a series increases/decreases, then the multiplicative method should be used.
- If however the seasonal **fluctuations remain fairly constant** as a series increases, then the additive method should be used.

Example

A local auctioneer examines his income from the sale of private housing during the years 2007 to 2010 inclusive. The income (000s) is presented on a quarterly basis as follows:

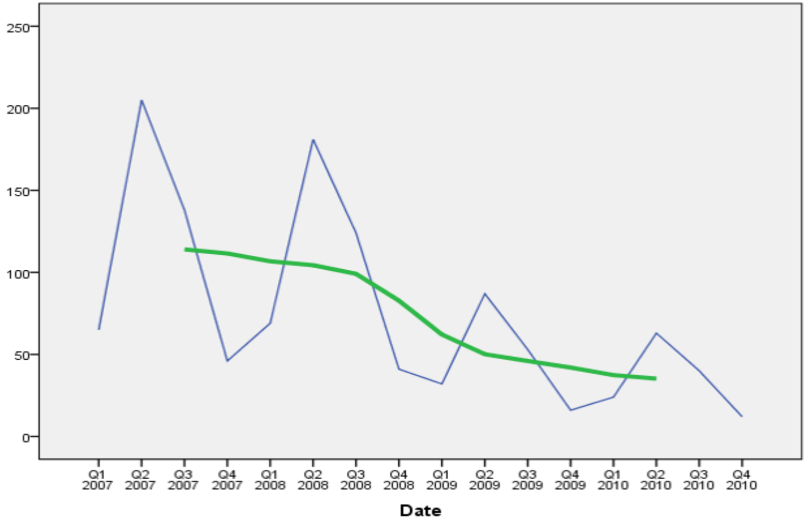
Year	Q1	Q2	Q3	Q4
2007	65	205	138	46
2008	69	181	124	41
2009	32	87	53	16
2010	24	63	40	12

1. Plot the data on a time series graph.
2. Calculate a four quarter centered moving average for the data and plot this on your graph in 1.
3. Using a multiplicative model, determine seasonal index values for the data.
4. The decline in sales can be described by the linear trend equation $y = 138 - 7.5x$ [x-units: one quarter: $x = 1$ gives 1st quarter 2007]. Use the linear trend equation to calculate trend values for each quarter of 2011.
5. Seasonalise the values from part 4 to obtain a forecast of sales for 2011.

Example - seen in previous lecture

Year	Quarter	(A) Income (€000's)	4 Quarter moving total	2 Step total	(T) 4 Quarter centred Moving Average
2007	I	65		*	*
	II	205		*	*
			454		
	III	138		912	114.0
			458		
	IV	46		892	111.5
			434		
2008	I	69		854	106.8
			420		
	II	181		835	104.4
			415		
	III	124		793	99.1
			378		
	IV	41		662	82.8
			284		
2009	I	32		497	62.1
			213		
	II	87		401	50.1
			188		
	III	53		368	46.0
			180		
	IV	16		336	42.0
			156		
2010	I	24		299	37.4
			143		
	II	63		282	35.3
			139		
	III	40		*	*
	IV	12		*	*

Example - seen in previous lecture



Example: Seasonal Variation

Year	Quarter	Income (A)		
2007	I	65		
	II	205		
	III	138		
	IV	46		
2008	I	69		
	II	181		
	III	124		
	IV	41		
2009	I	32		
	II	87		
	III	53		
	IV	16		
2010	I	24		
	II	63		
	III	40		
	IV	12		

Example: Seasonal Variation

Year	Quarter	Income (A)	4-Q centred MA (T)	
2007	I	65		
	II	205		
	III	138	114.0	
	IV	46	111.5	
2008	I	69	106.8	
	II	181	104.4	
	III	124	99.1	
	IV	41	82.8	
2009	I	32	62.1	
	II	87	50.1	
	III	53	46.0	
	IV	16	42.0	
2010	I	24	37.4	
	II	63	35.3	
	III	40		
	IV	12		

Example: Seasonal Variation

Year	Quarter	Income (A)	4-Q centred MA (T)	$\frac{A}{T} \times 100$ (%)
2007	I	65		
	II	205		
	III	138	114.0	121.1
	IV	46	111.5	41.3
2008	I	69	106.8	64.6
	II	181	104.4	173.4
	III	124	99.1	125.1
	IV	41	82.8	49.5
2009	I	32	62.1	51.5
	II	87	50.1	173.7
	III	53	46.0	115.2
	IV	16	42.0	38.1
2010	I	24	37.4	64.2
	II	63	35.3	178.5
	III	40		
	IV	12		

Example: Seasonal Indexes (%)

	Quarter			
Year	I	II	III	IV
2007	—	—	121.1	41.3
2008	64.6	173.4	125.1	49.5
2009	51.5	173.7	115.2	38.1
2010	64.2	178.5	—	—
Mean	60.1	175.2	120.5	43.0

$$\text{Residual Variation} = \frac{400}{60.1 + 175.2 + 120.5 + 43.0} = 1.003$$

Example: Adjusted Seasonal Indexes

Quarter	Adjusted Seasonal Indexes (%)
I	$60.1 \times 1.003 = 60.3$
II	$175.2 \times 1.003 = 175.7$
III	$120.5 \times 1.003 = 120.9$
IV	$43.0 \times 1.003 = 43.1$

Note that the adjusted indexes now sum to 400%

Example: Forecasting

- 16 quarterly figures were given, spanning 2007 to 2010 inclusive. So $x = 17$, $x = 18$, $x = 19$ and $x = 20$ will represent each quarter of 2011. We substitute $x = 17$ into the linear trend equation ($y = 138 - 7.5x$) to calculate a **trend** value for the first quarter of 2013.
- This value is then **seasonalised** to give a more realistic **forecast** of income for the first quarter of 2013. The process is repeated for $x = 18$, $x = 19$ and $x = 20$.

$$\text{Forecast of Income} = T * S$$

Example: Forecasting

Filling the x values into the trend equation and seasonalising gives:

$$x = 17 \Rightarrow y = 10.5 \qquad \times 60.3\% = 6.33 \text{ (000's)}$$
$$x = 18 \Rightarrow y = 3.0 \qquad \times 175.7\% = 5.271 \text{ (000's)}$$
$$x = 19 \Rightarrow y = -4.5 \qquad \times 120.9\% = -5.441 \text{ (000's)}$$
$$x = 20 \Rightarrow y = -12 \qquad \times 43.1\% = -5.172 \text{ (000's)}$$

- When **trend** values are multiplied by a seasonal index we refer to this as **seasonalisation of data**.

Interpretation:

- The downward trend since 2009 seems to be continuing into 2013. The forecasted figures are lower again than those of 2012. The seasonal variations are apparent again in 2013, i.e. 1st and 4th quarter figures are low while income is higher for the 3rd quarter and peaks in the 2nd quarter as in previous years.

Exercise 1

The table below shows passenger movements by air for a small airport. Figures are in thousands.

	Quarter			
Year	I	II	III	IV
2008	62	98	144	86
2009	51	84	128	68
2010	41	67	104	52

1. Plot the data on a time series graph.
2. Calculate a four quarter centered moving average for the data and plot this on your graph in part 1.
3. Calculate a seasonal index for each quarter, using a multiplicative model.
4. Deseasonalise the figures for the year 2010 and comment on the resulting trend.

Exercise 2

The manager of a local Cooperative shop which sells fuel, along with many other products, suspects that sales of domestic coal have been declining in recent years. Thus the quarterly sales figures (in 000's) for the last four years were reported for analysis:

	Quarter			
Year	I	II	III	IV
2008	72	38	16	58
2009	64	35	14	49
2010	55	32	10	40
2011	43	29	8	35

1. Plot the data on a time series graph.
2. Establish a four quarter centered moving average for the data and plot this on your graph in part 1.
3. Use an **Additive** model to establish a seasonal index for the data.
4. The decline in sales can be described by the linear trend equation $y = 52.0 - 1.72x$ [x-units: one quarter: $x = 1$ gives 1st quarter 2008]. Use the linear trend equation to calculate trend values for each quarter of 2011 and then seasonalise these values to obtain a forecast of sales for 2012.

Exercise 3

A company manufacturing components for personal computers is planning its production and thus uses quarterly sales figures over the past three years to forecast future demand. The sales figures (in 000's) are as follows:

	Quarter			
Year	I	II	III	IV
2009	658	601	547	672
2010	648	618	524	656
2011	550	528	415	534

1. Plot the data on a time series graph.
2. Calculate a four quarter centered moving average for the data and plot this on your graph in part 1.
3. Using a multiplicative model, determine seasonal index values for the data.
4. The trend in the data can be depicted by the linear equation $y = 667 - 13.5x$ [x-units: 1 quarter, $x = 1$ gives 1st quarter 2009] . Use the linear trend equation and the seasonal index to forecast figures for each quarter of 2012.
5. Comment on the trend for 2012. How does it compare with the trend in previous years?