A comparison of wagering mechanisms by simulation

1 Mechanism

- Competitive scoring rule with Brier score (CSR)
- No-arbitrage wagering mechanism (NAW)
- Proxy pari-mutuel mechanism (PPM)

2 Metric and Evaluation Method

- Money exchange - evaluated as the total amount of money lost by all losers after the future event is realized. When comparing, we normalize the money exchange according to the total amount of wagers.
  - For CSR: We assign each forecaster S_i as its wager. The net payoff of a forecaster is given by the competitive scoring rule

    $$f_i(p_i, x) = B(p_i, x) - \frac{\sum_{j \neq i} B(p_j, x)}{n-1}.$$

    The Brier score is defined as $B(p, x) = 2 - 2(1-p)^2$. We regard a point of score as one unit of money. The money exchange is independent of S_i for all i, but in order to guarantee that no forecaster will lose money more than its wager, we shall have $S_i \geq 2$. Therefore, we set $S_i = 2$ for all i.
  - For NAW: The total money lost by forecasters may exceed the total money won by forecasters. Therefore, we set the money exchange as the total amount of money lost by all losers.
  - For PPM: The total money lost by forecasters equals the total money won by forecasters.

3 Simulation Parameter and Result

3.1 Max gain and lose of a single forecaster

In this section, we examine the max gain of a completely right forecaster,i.e., someone who forecasts 1 and the event eventually happens, and max lose of a completely wrong forecaster, i.e., someone who forecasts 1 and eventually the event doesn't happen. We ranged the number of forecasters for 5 to 30 with a step 5. For each number of forecasters, we ran 1000 experiments. Among these forecasters, we select one to forecast 1 for event occurrence, while the rest forecasts are i.i.d. drawn from certain distributions. The wagers of all forecasters are all set to one unit of money.

1. The other forecasts follow uniform distribution.

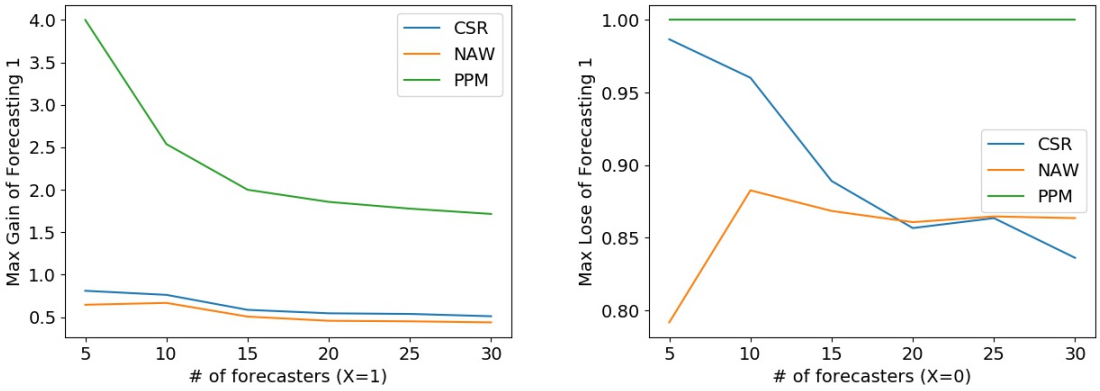

Figure 1: The extreme payoffs of forecasting 1 when others' are drawn from **uniform distribution**

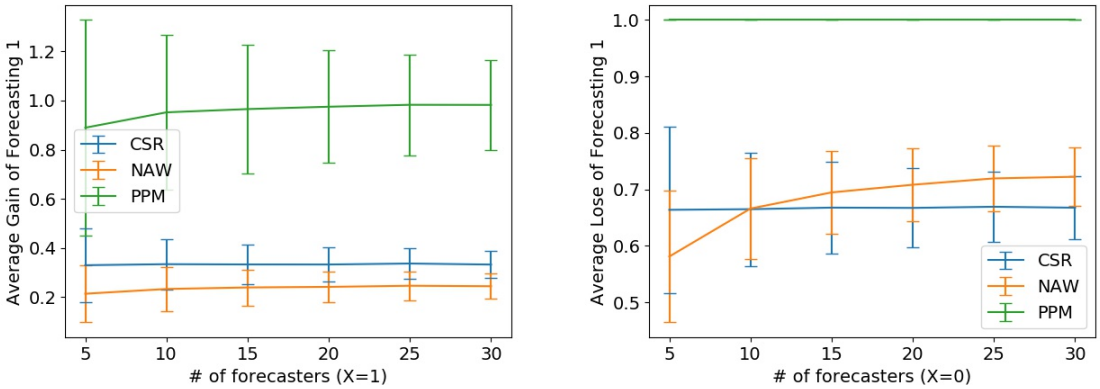

Figure 2: The average payoff of forecasting 1 when others' are drawn from **uniform distribution**

2. The other forecasts follow Beta(1, 0.2), where the mode is forecasting 1.

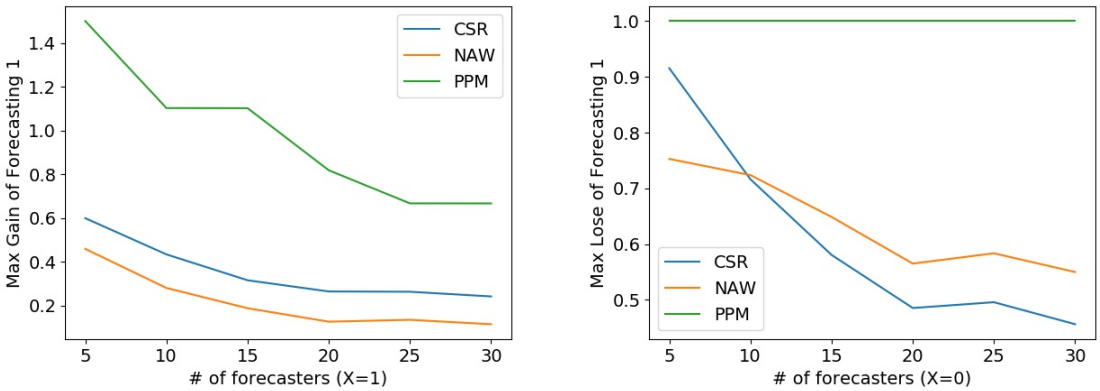

Figure 3: The extreme payoffs of forecasting 1 when others' are drawn from **Beta(1,0.2)**

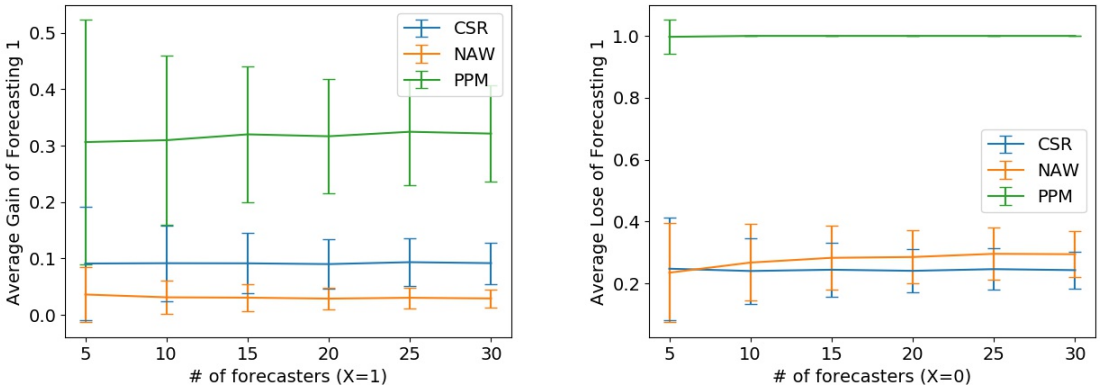

Figure 4: The average payoff of forecasting 1 when others' are drawn from **Beta(1,0.2)**

3. The other forecasts follow Beta(0.2, 1), where the mode is forecasting 0.

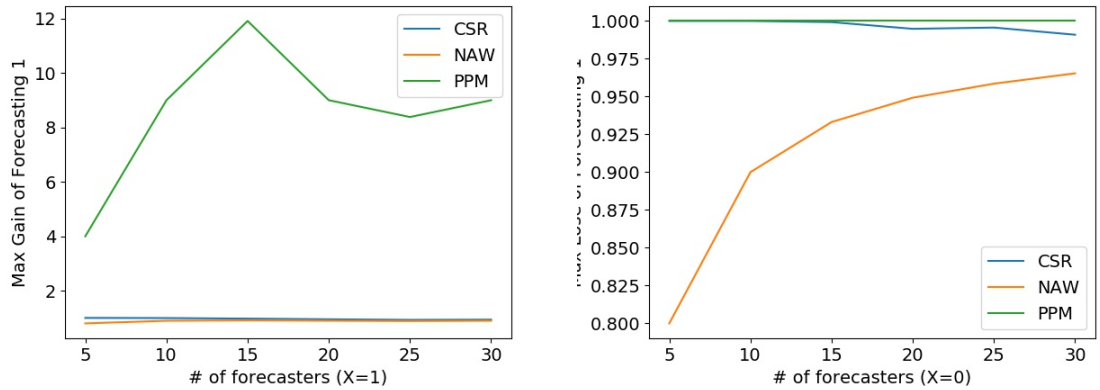

Figure 5: The extreme payoffs of forecasting 1 when others' are drawn from **Beta(0.2,1)**

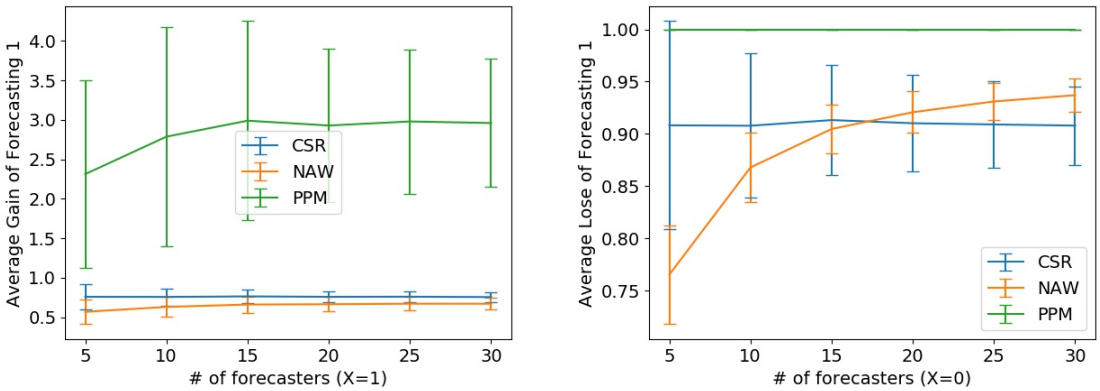

Figure 6: The average payoff of forecasting 1 when others' are drawn from **Beta(0.2,1)**

4. The other forecasts follow Beta(100, 100), where the mode is forecasting 0.5.

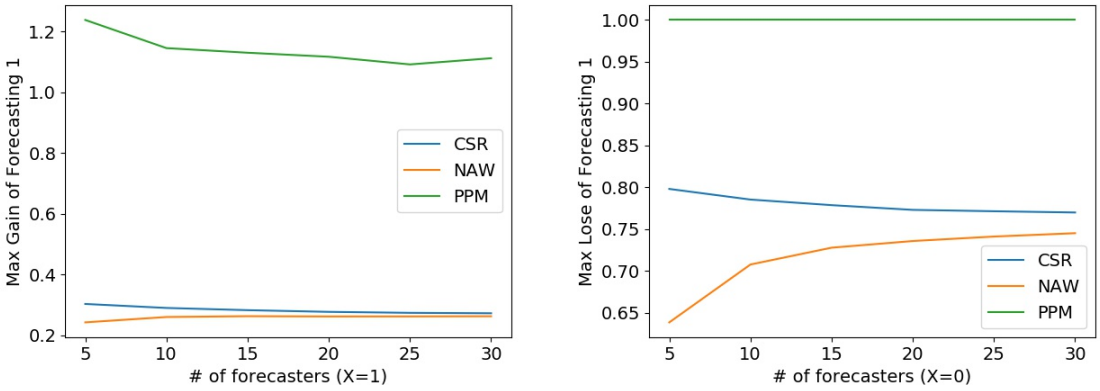

Figure 7: The extreme payoffs of forecasting 1 when others' are drawn from **Beta(100,100)**

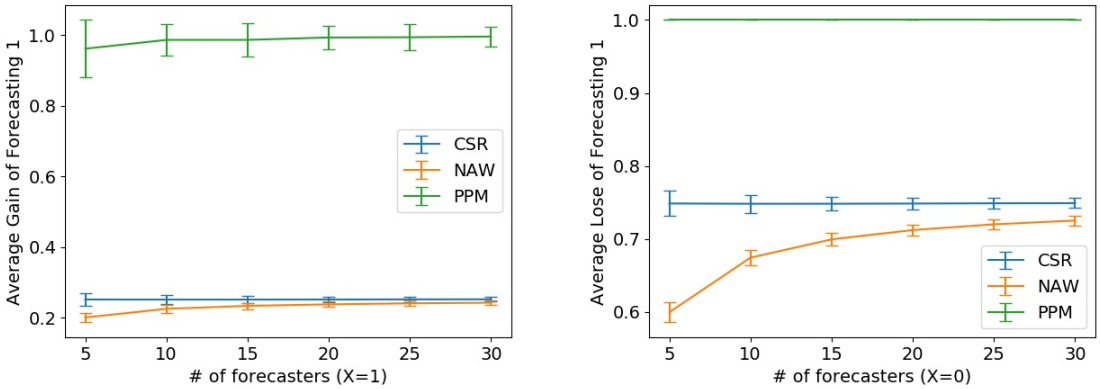

Figure 8: The average payoff of forecasting 1 when others' are drawn from **Beta(100,100)**

3.2 Average performances of different mechanisms under different wager distributions

1. Forecasts are drawn from uniform distribution, wagers are drawn from Beta(100,100) (normal-distribution-like with mean 0.5 in range [0,1])

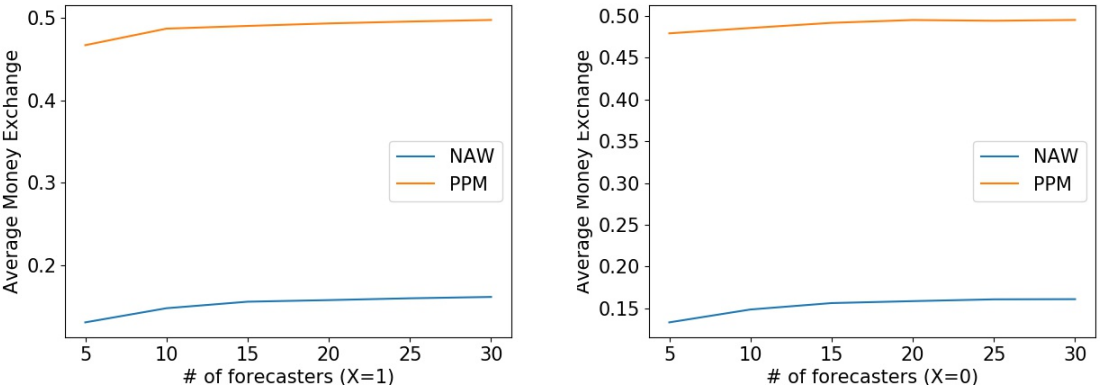

Figure 9: Average money exchange (Forecasts: Uniform, Wagers: Beta(100,100))

2. Forecasts are drawn from uniform distribution, wagers are drawn from Pareto(1.16,1) (in DCA paper)

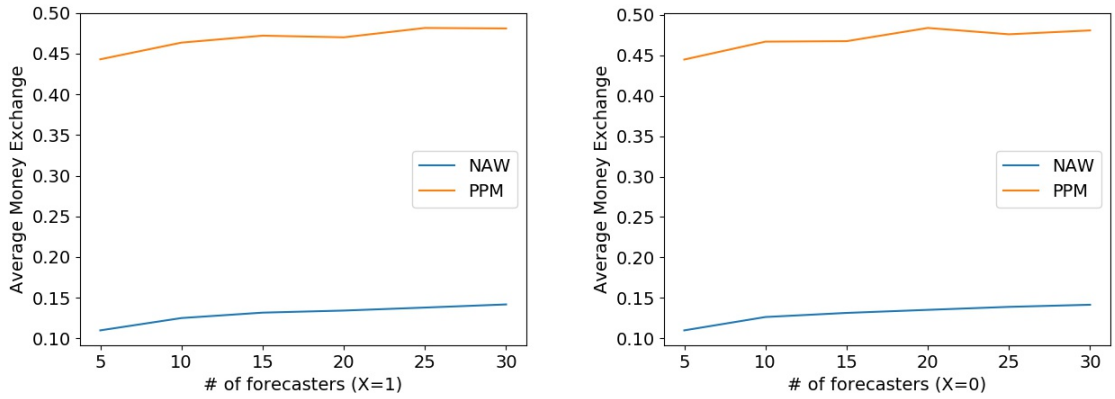

Figure 10: Average money exchange (Forecasts: **Uniform**, Wagers: **Pareto(1.16,1)**)

3. Forecasts are drawn from Beta(0.3,0.3), wagers are drawn from Beta(100,100) (in DCA paper)

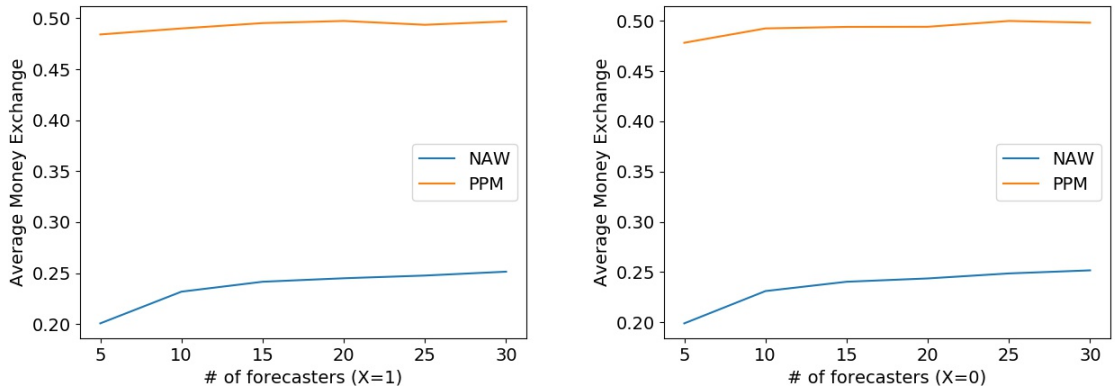

Figure 11: Average money exchange (Forecasts: **Beta(0.3,0.3)**, Wagers: **Beta(100,100)**)

4. Forecasts are drawn from Beta(0.3,0.3), wagers are drawn from Pareto(1.16,1) (in DCA paper)

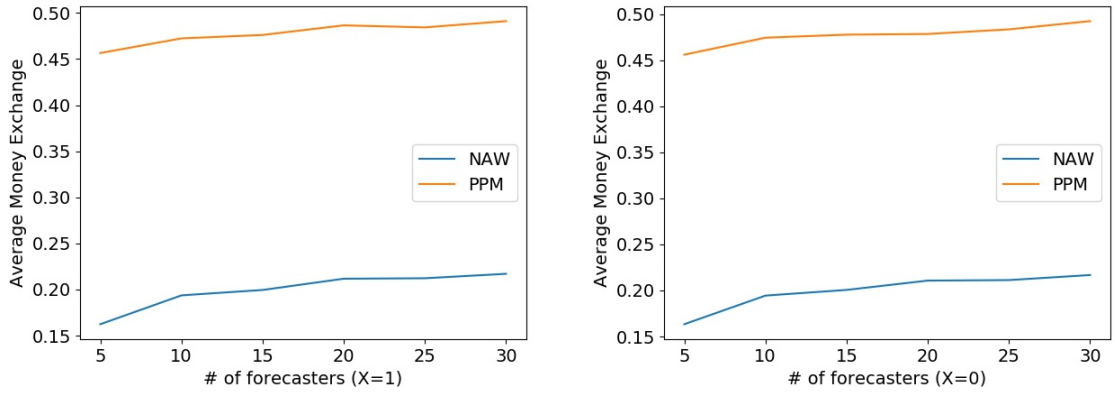

Figure 12: Average money exchange (Forecasts: **Beta(0.3,0.3)**, Wagers: **Pareto(1.16,1)**)

5. Forecasts are drawn from Beta(1,0.2), wagers are drawn from Beta(100,100) (normal-distribution-like with mean 0.5 in range [0,1])

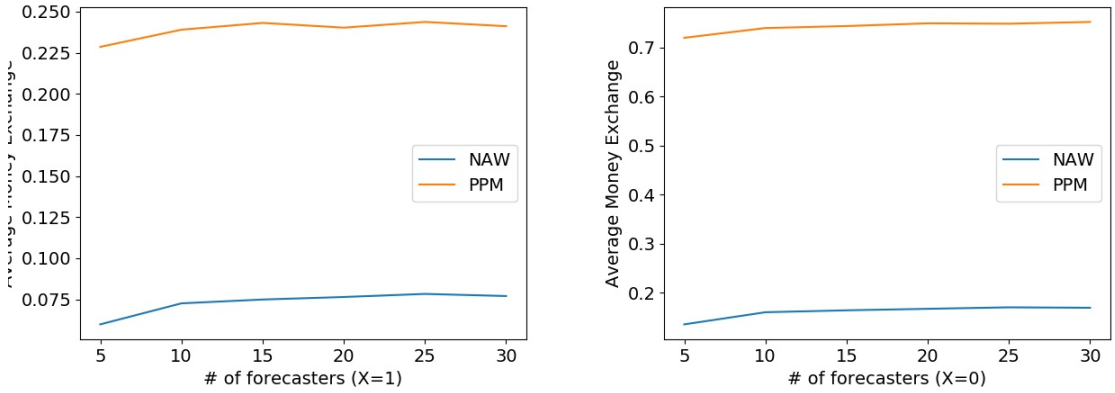

Figure 13: Average money exchange (Forecasts: **Beta(1,0.2)**, Wagers: **Beta(100,100)**)

6. Forecasts are drawn from Beta(1,0.2), wagers are drawn from Pareto(1.16,1) (in DCA paper)

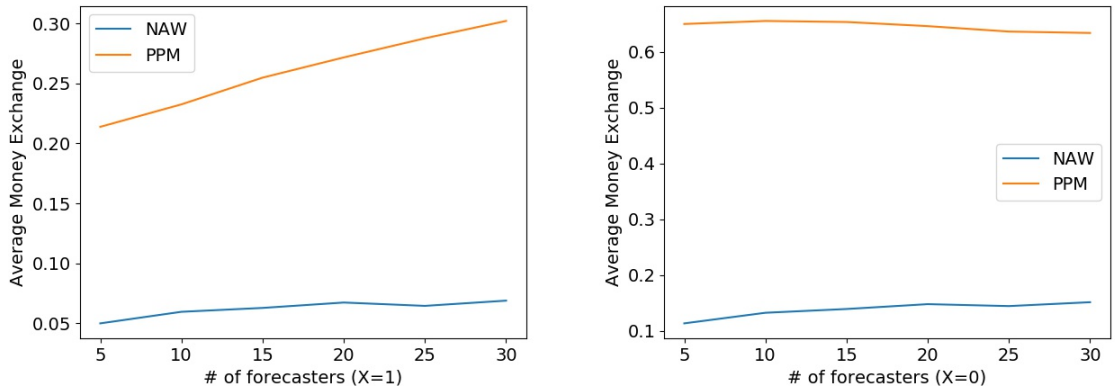

Figure 14: Average money exchange (Forecasts: **Beta(1,0.2)**, Wagers: **Pareto(1.16,1)**)

3.3 Average performances of different mechanisms with uniform wager

We ranged the number of forecasters for 5 to 50 with a step 5. For each number of forecasters, we ran 1000∼10000 experiments. We recorded the average, maximum and minimum money exchange during these 10000 experiments.

1. Forecasts are generated by uniform distribution.

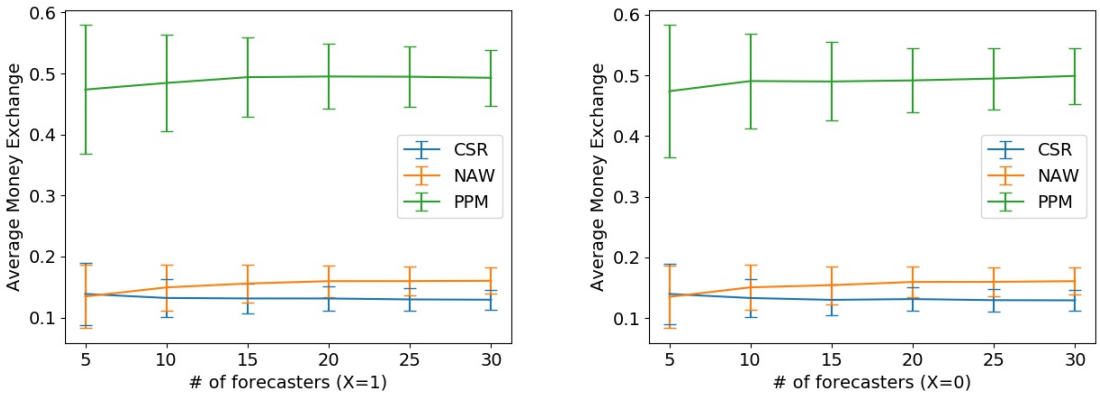

Figure 15: Average money exchange over 1000 runs

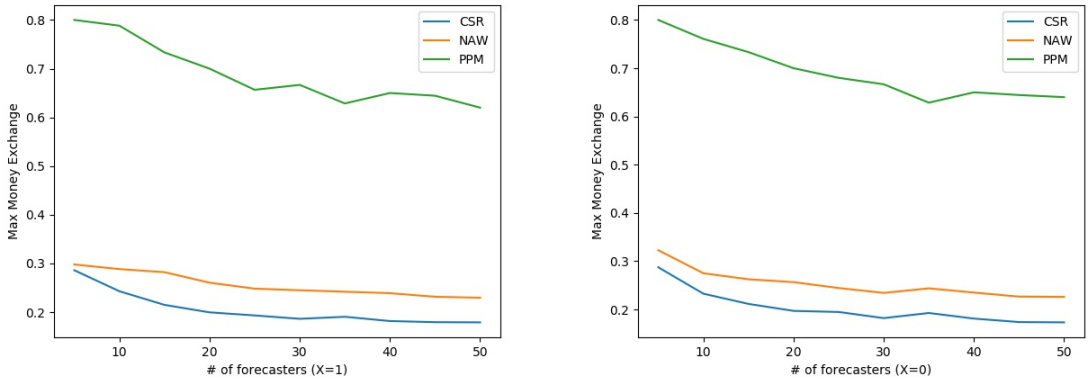

Figure 16: Maximum money exchange over 10000 runs

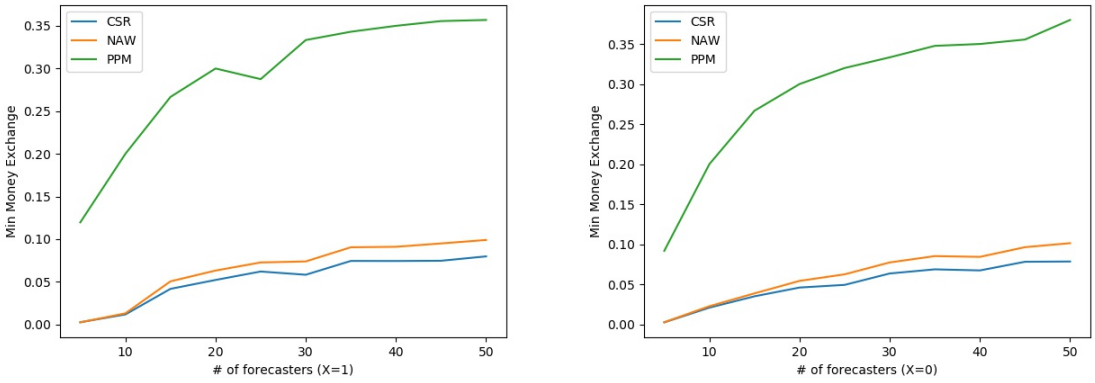

Figure 17: Minimum money exchange over 10000 runs

2. Forecasts are generated by distribution $Beta(0.3, 0.3)$, which puts most probability mass on the two ends, i.e., $Pr(p > 0.95) = 0.23, Pr(p < 0.05) = 0.23$

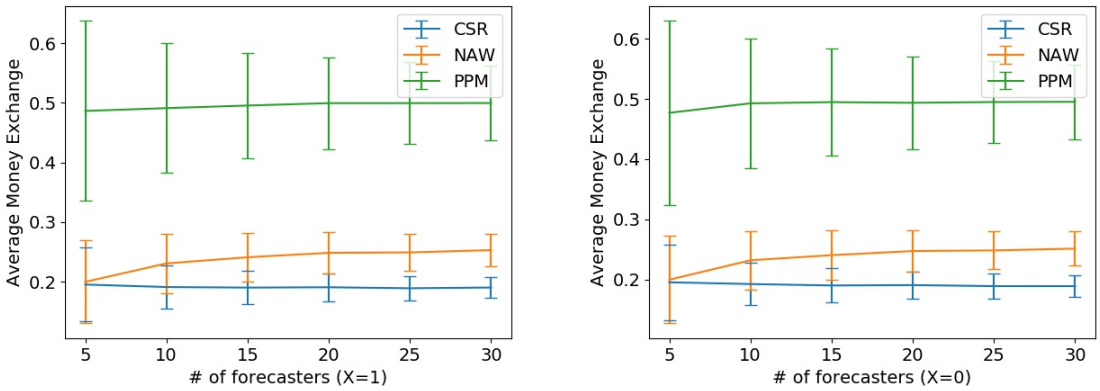

Figure 18: Average money exchange over 1000 runs

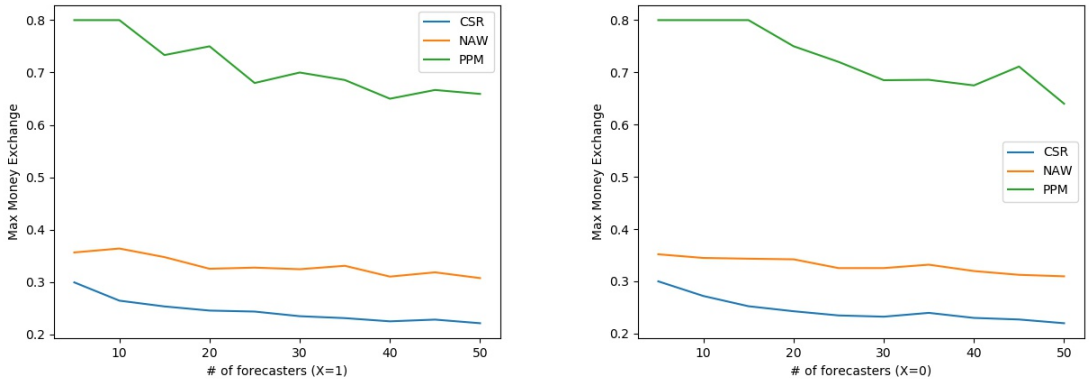

Figure 19: Maximum money exchange over 1000 runs

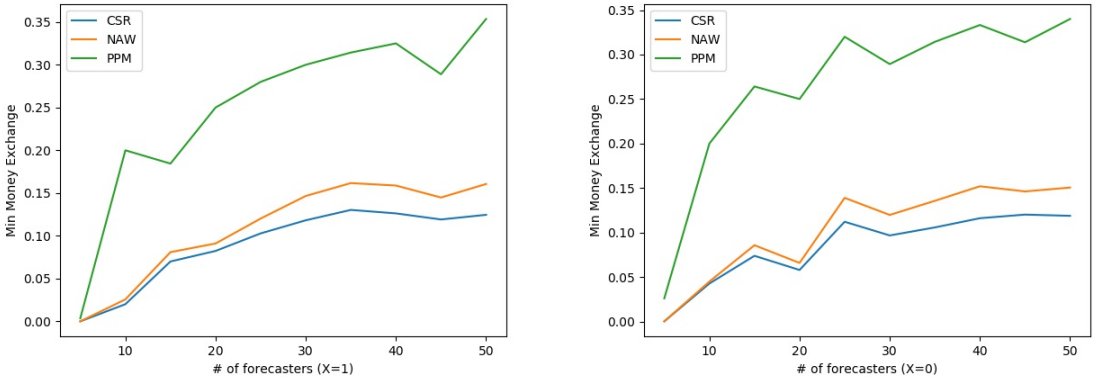

Figure 20: Minimum money exchange over 1000 runs

3. Forecasts are generated by distribution $Beta(1, 0.2)$, which is a long-tail-like distribution over [0,1], i.e., $Pr(p > 0.95) = 0.55, Pr(p < 0.5) = 0.13, Pr(P < 0.05) = 0.01$.

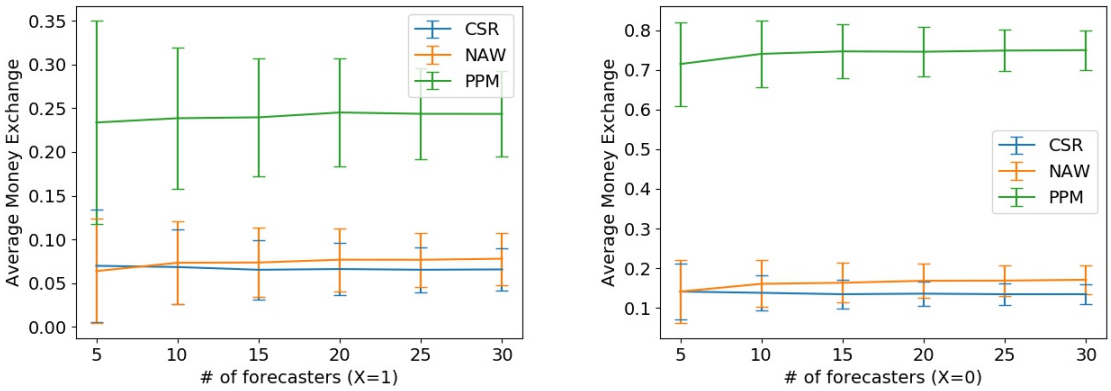

Figure 21: Average money exchange over 1000 runs

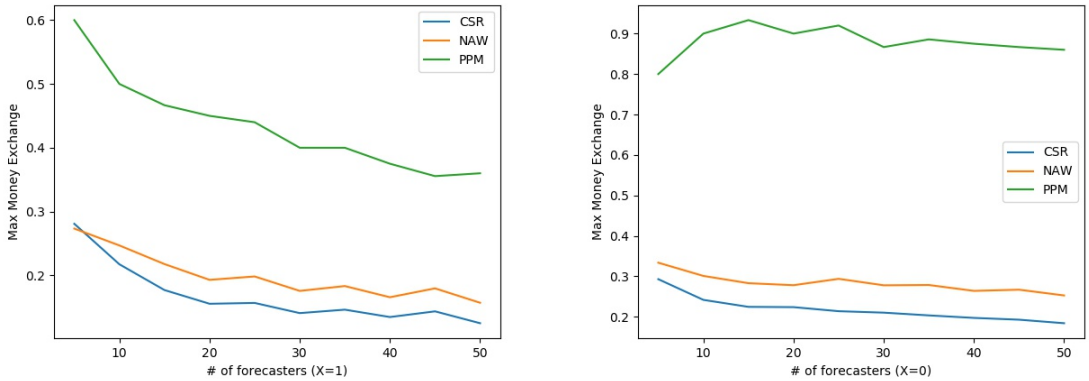

Figure 22: Maximum money exchange over 1000 runs

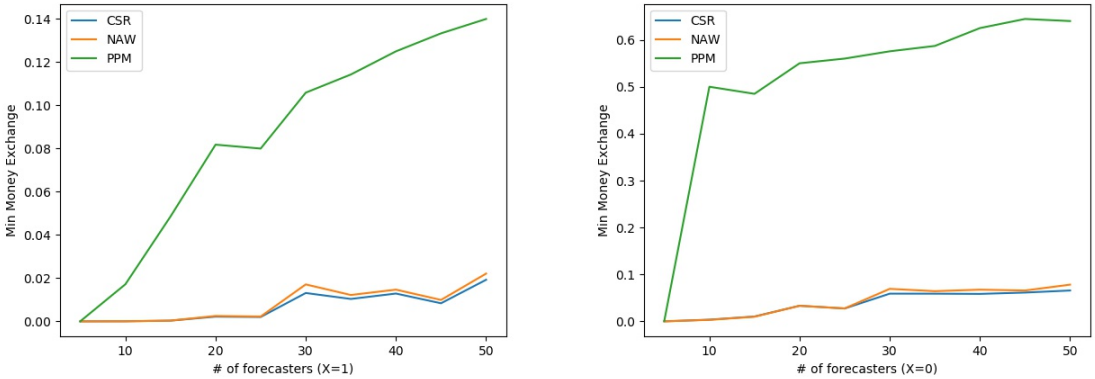

Figure 23: Minimum money exchange over 1000 runs

4. Forecasts are generated by distribution $Beta(100, 100)$, which is a normal-distribution-like distribution, putting all probability mass over [0.35,0.65] with expectation 0.5.

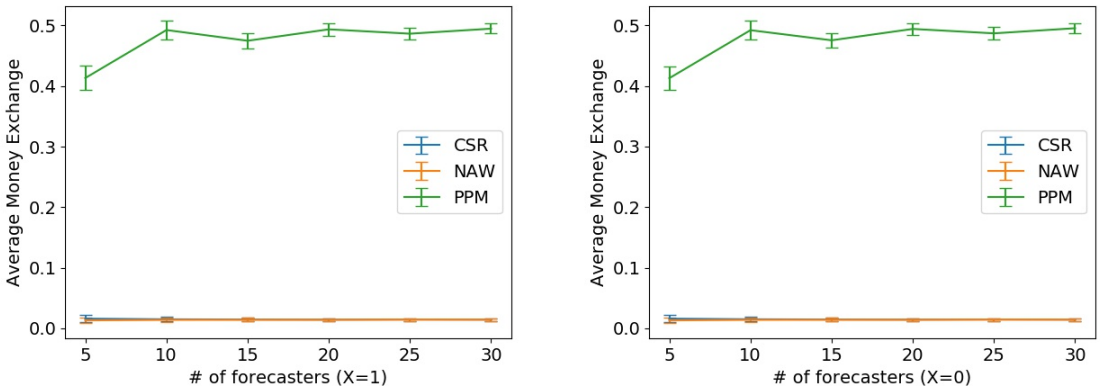

Figure 24: Average money exchange over 1000 runs

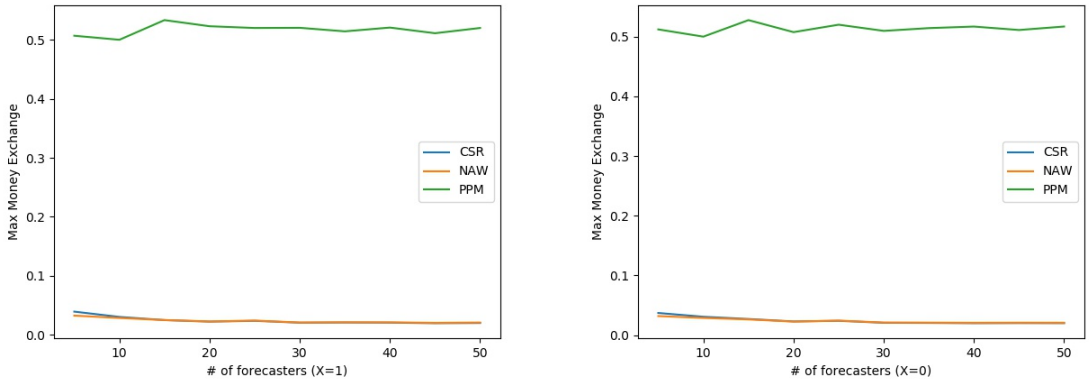

Figure 25: Maximum money exchange over 1000 runs

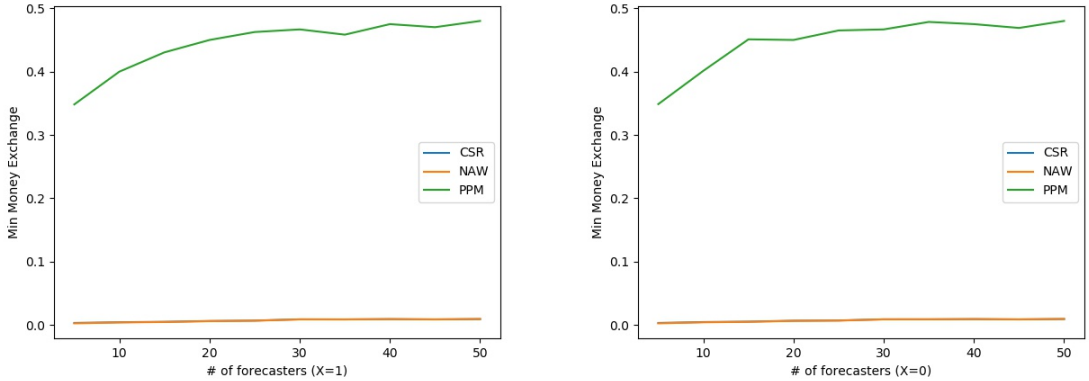

Figure 26: Minimum money exchange over 1000 runs