A comparison of wagering mechanisms by simulation

1. Mechanism
   - Competitive scoring rule with Brier score (CSR)
   - No-arbitrage wagering mechanism (NAW)
   - Proxy pari-mutuel mechanism (PPM)
2. Metric and Evaluation Method
   - Money exchange - evaluated as the total amount of money lost by all losers after the future event is realized. When comparing, we normalize the money exchange according to the total amount of wagers.
     - For CSR: We assign each forecaster S_i as its wager. The net payoff of a forecaster is given by the competitive scoring rule

       $$f_i(p_i, x) = B(p_i, x) - \frac{\sum_{j \neq i} B(p_j, x)}{n-1}.$$

       The Brier score is defined as $B(p, x) = 2 - 2(1-p)^2$. We regard a point of score as one unit of money. The money exchange is independent of S_i for all i, but in order to guarantee that no forecaster will lose money more than its wager, we shall have $S_i \geq 2$. Therefore, we set $S_i = 2$ for all i.
     - For NAW: The total money lost by forecasters may exceed the total money won by forecasters. Therefore, we set the money exchange as the total amount of money lost by all losers.
     - For PPM: The total money lost by forecasters equals the total money won by forecasters.
3. Simulation Parameter and Result

   We ranged the number of forecasters for 5 to 50 with a step 5. For each number of forecasters, we ran 1000∼10000 experiments. We recorded the average, maximum and minimum money exchange during these 10000 experiments.

(a) Forecasts are generated by uniform distribution.

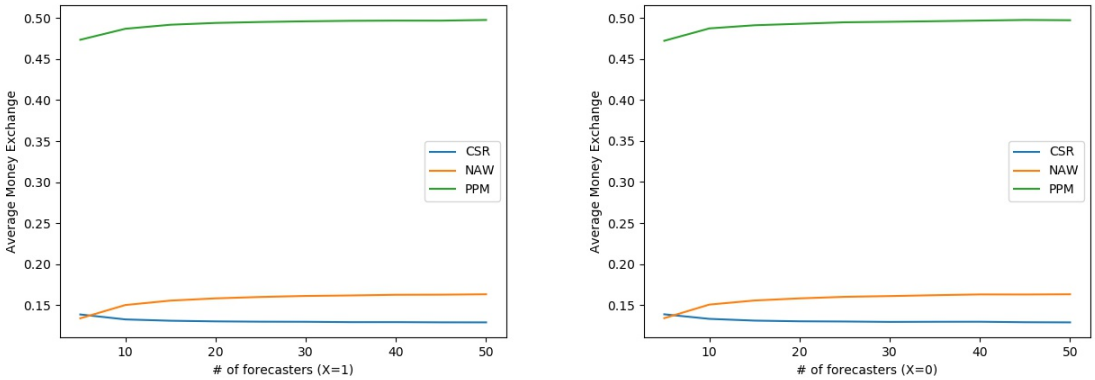

Figure 1: Average money exchange over 10000 runs

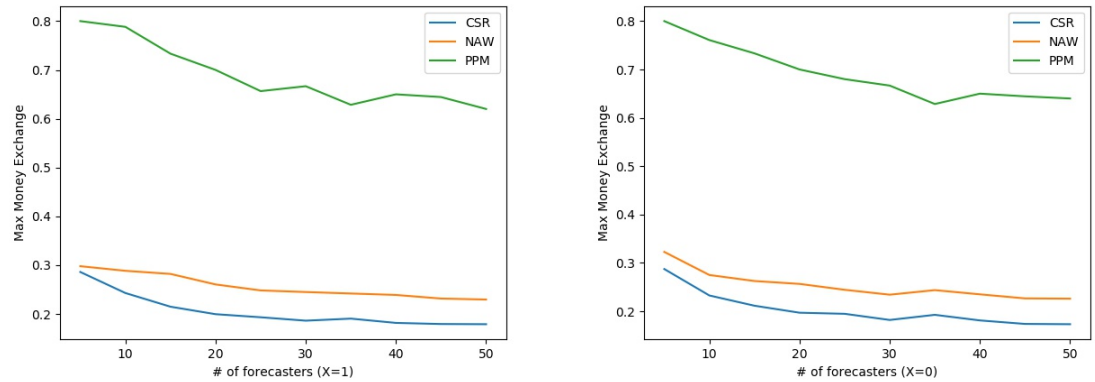

Figure 2: Maximum money exchange over 10000 runs

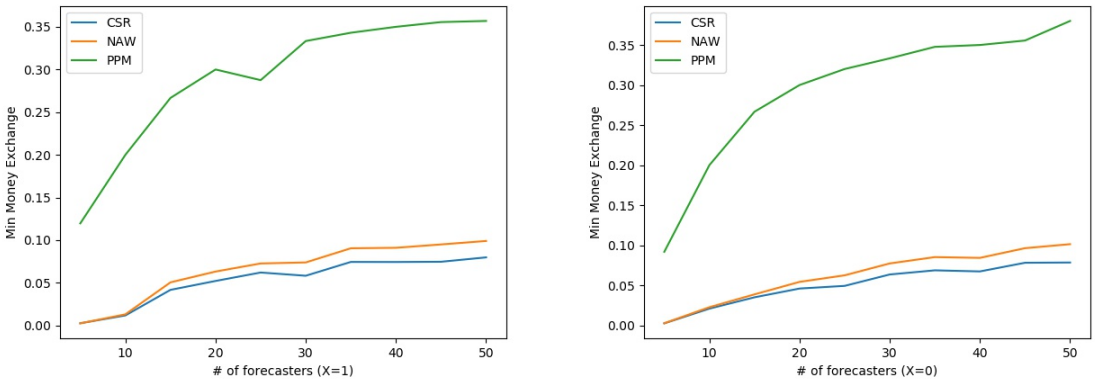

Figure 3: Minimum money exchange over 10000 runs

(b) Forecasts are generated by distribution $Beta(0.3, 0.3)$, which puts most probability mass on the two ends, i.e., $Pr(p > 0.95) = 0.23, Pr(p < 0.05) = 0.23$

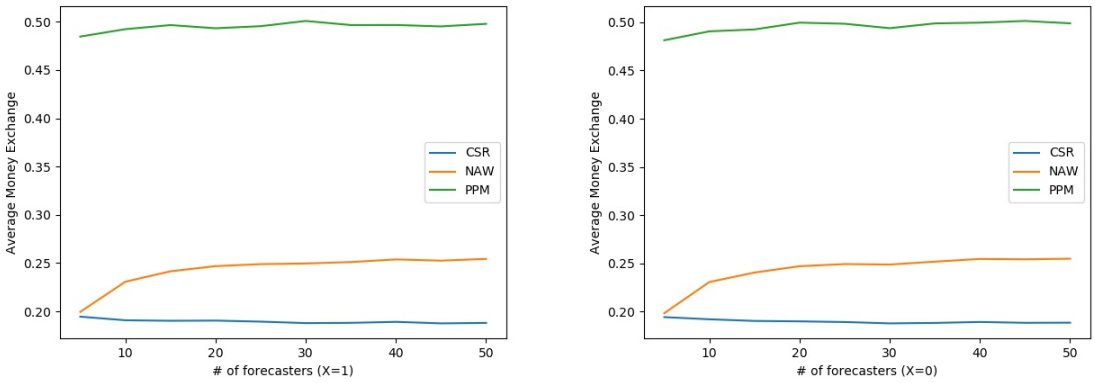

Figure 4: Average money exchange over 1000 runs

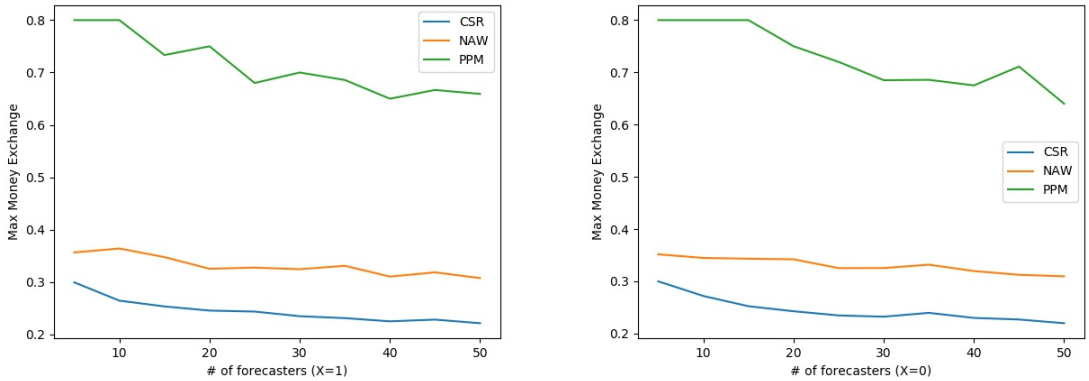

Figure 5: Maximum money exchange over 1000 runs

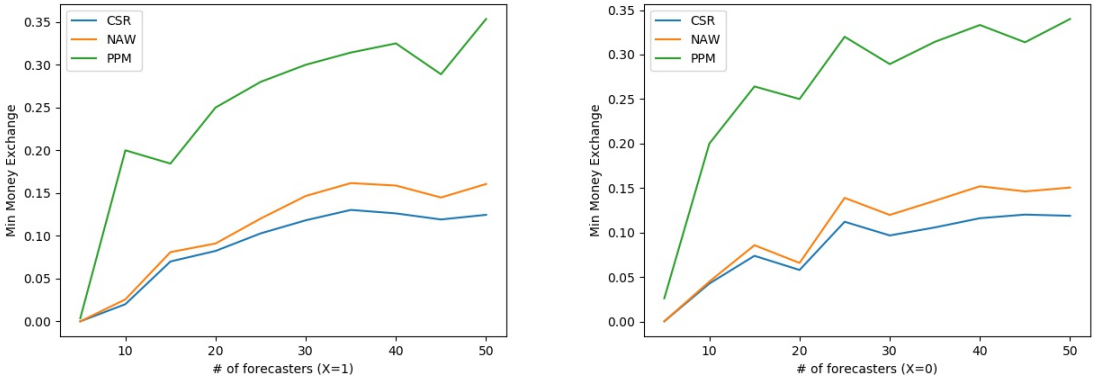

Figure 6: Minimum money exchange over 1000 runs

(c) Forecasts are generated by distribution $Beta(0.2, 1)$, which is a long-tail-like distribution over [0,1], i.e., $Pr(p > 0.95) = 0.55, Pr(p < 0.5) = 0.13, Pr(P < 0.05) = 0.01$.

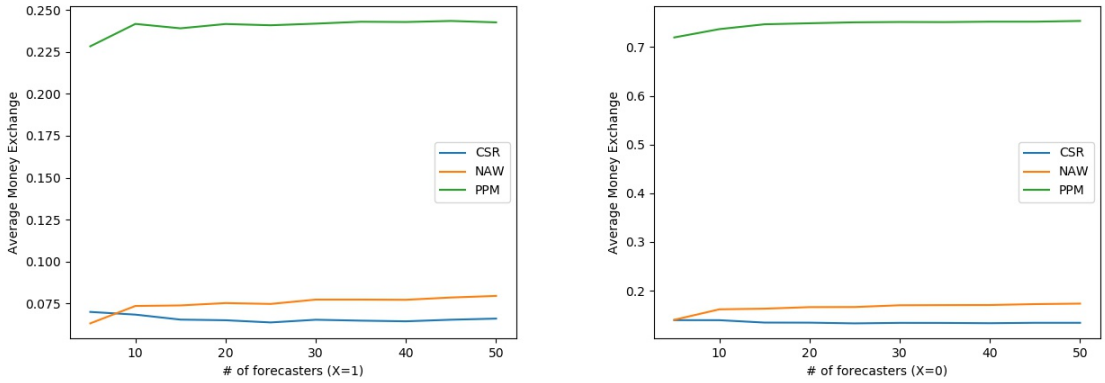

Figure 7: Average money exchange over 1000 runs

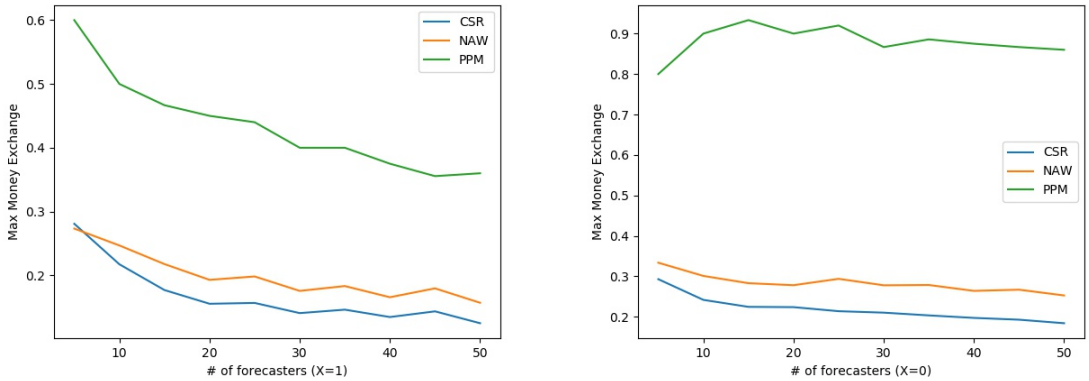

Figure 8: Maximum money exchange over 1000 runs

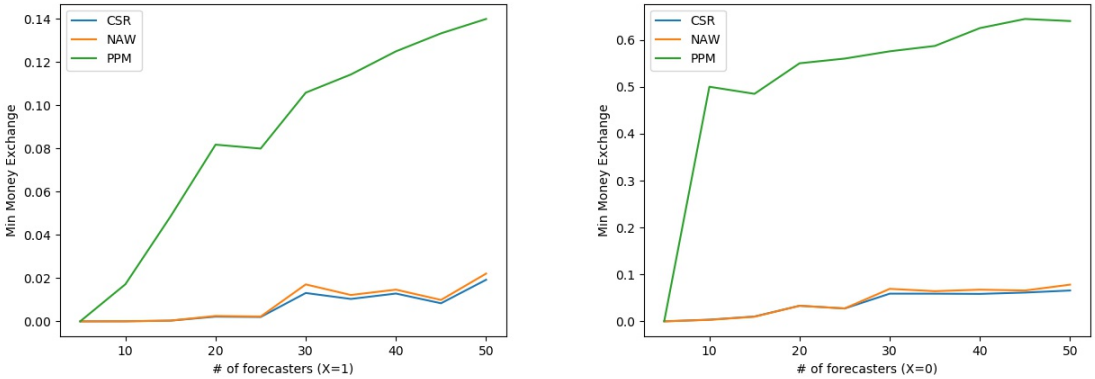

Figure 9: Minimum money exchange over 1000 runs

(d) Forecasts are generated by distribution $Beta(100, 100)$, which is a normal-distribution-like distribution, putting all probability mass over [0.35,0.65] with expectation 0.5.

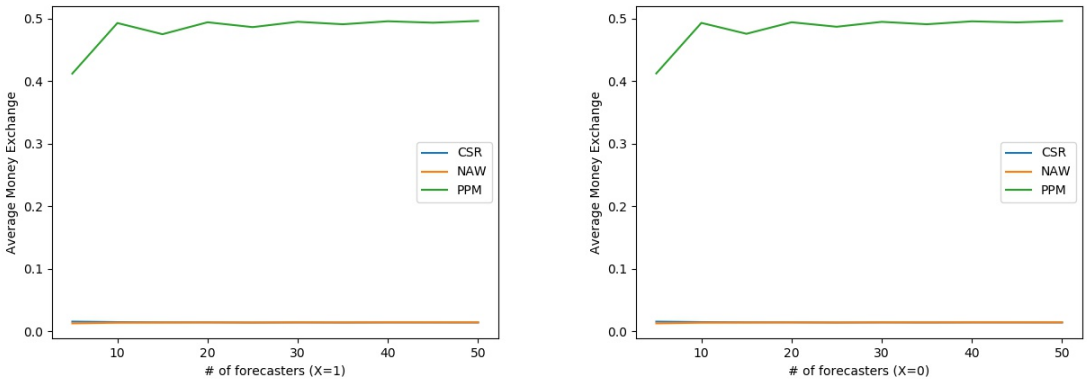

Figure 10: Average money exchange over 1000 runs

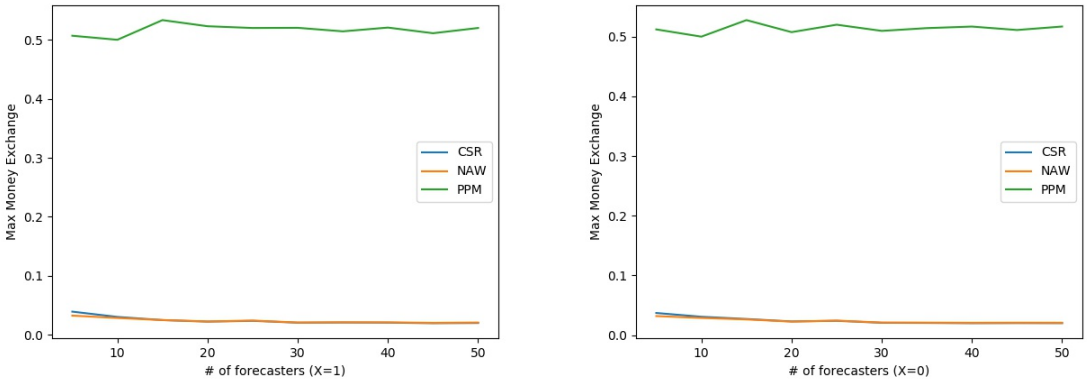

Figure 11: Maximum money exchange over 1000 runs

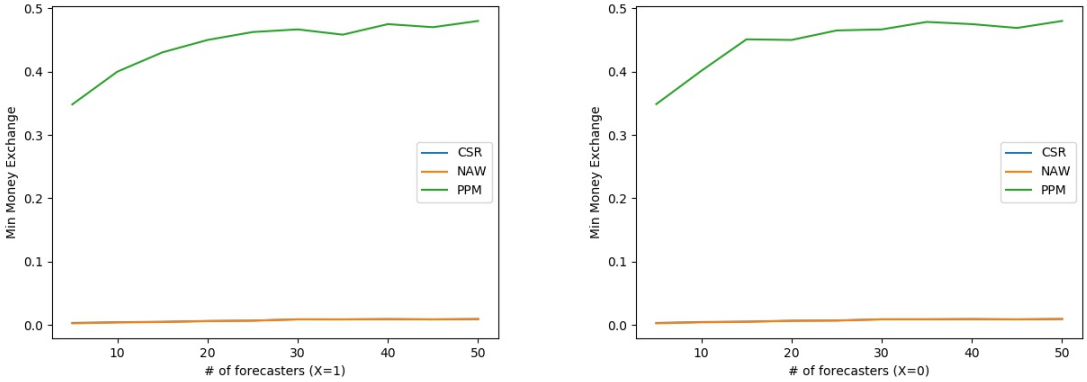

Figure 12: Minimum money exchange over 1000 runs