Obvious Strategy-Proof Mechanisms

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Overview of this talk

- Motivation
- Defining Obvious Strategy-Proofness (OSP) [Li]
- Demonstration and characterization
- Refinement of Obviousness [Pycia Troyan]
- Experimental evidence of limitation of theory [Breitmoser Schweighofer-Korditsch]

Dominant strategy

A strategy S_i is dominant for player i:

$$\forall S_i', S_{-i} \quad u_i(S_i, S_{-i}) \geq u_i(S'_i, S_{-i})$$

A game is dominant strategy incentive compatible (DSIC):

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Second price auction

- One item for sale
- Players bid $(b_1, \ldots, b_n)$
- Highest bidder wins
- Pays second highest bid

Dominant strategy: $b_i = v_i$

Ascending (clock) auction

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- Prices ascends slowly
- Players dropout until one player remains
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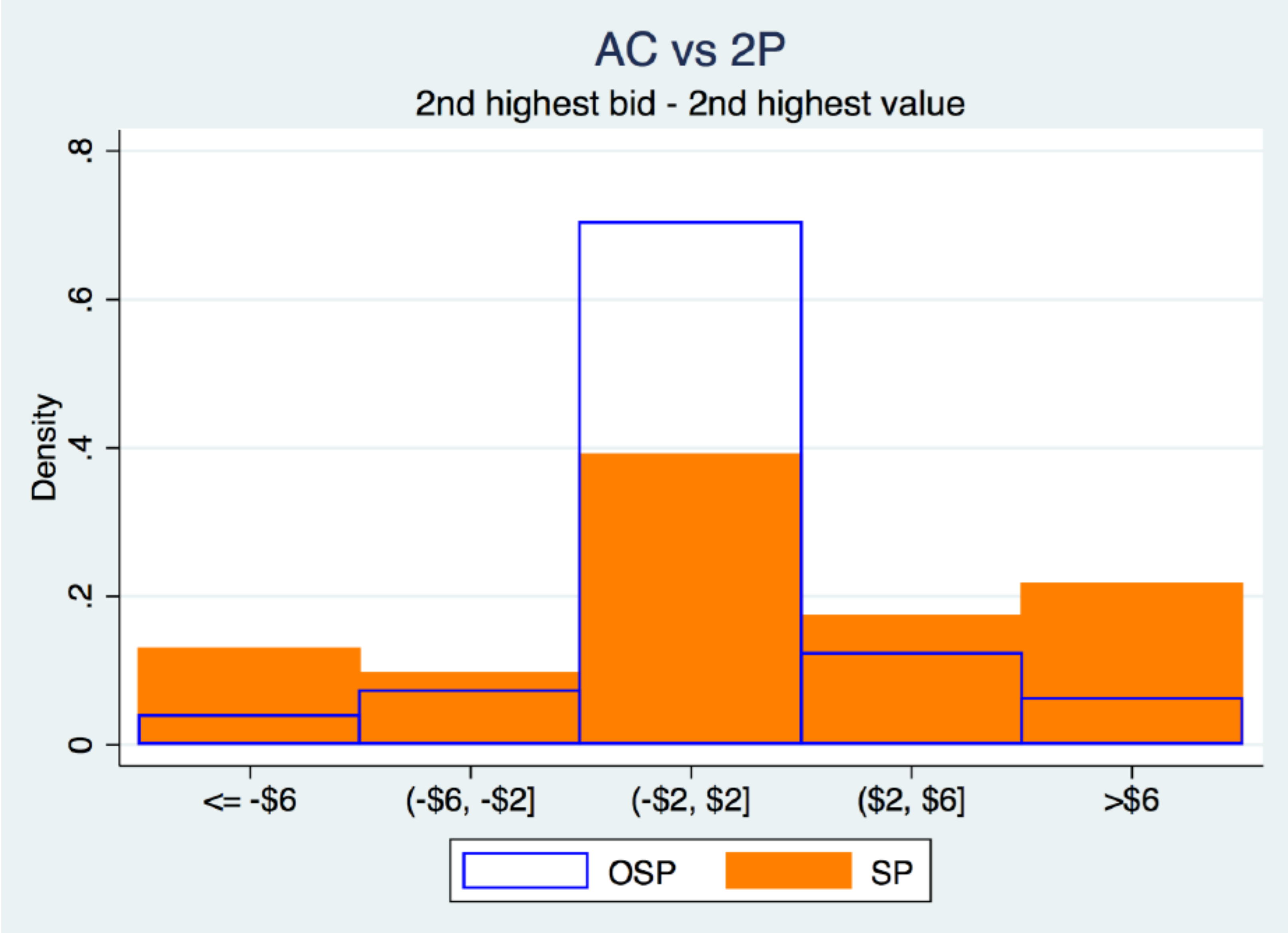
second price auctions are harder in practice [Kagel Harstad Levin '87]

Ascending auctions are easier

[Li]:



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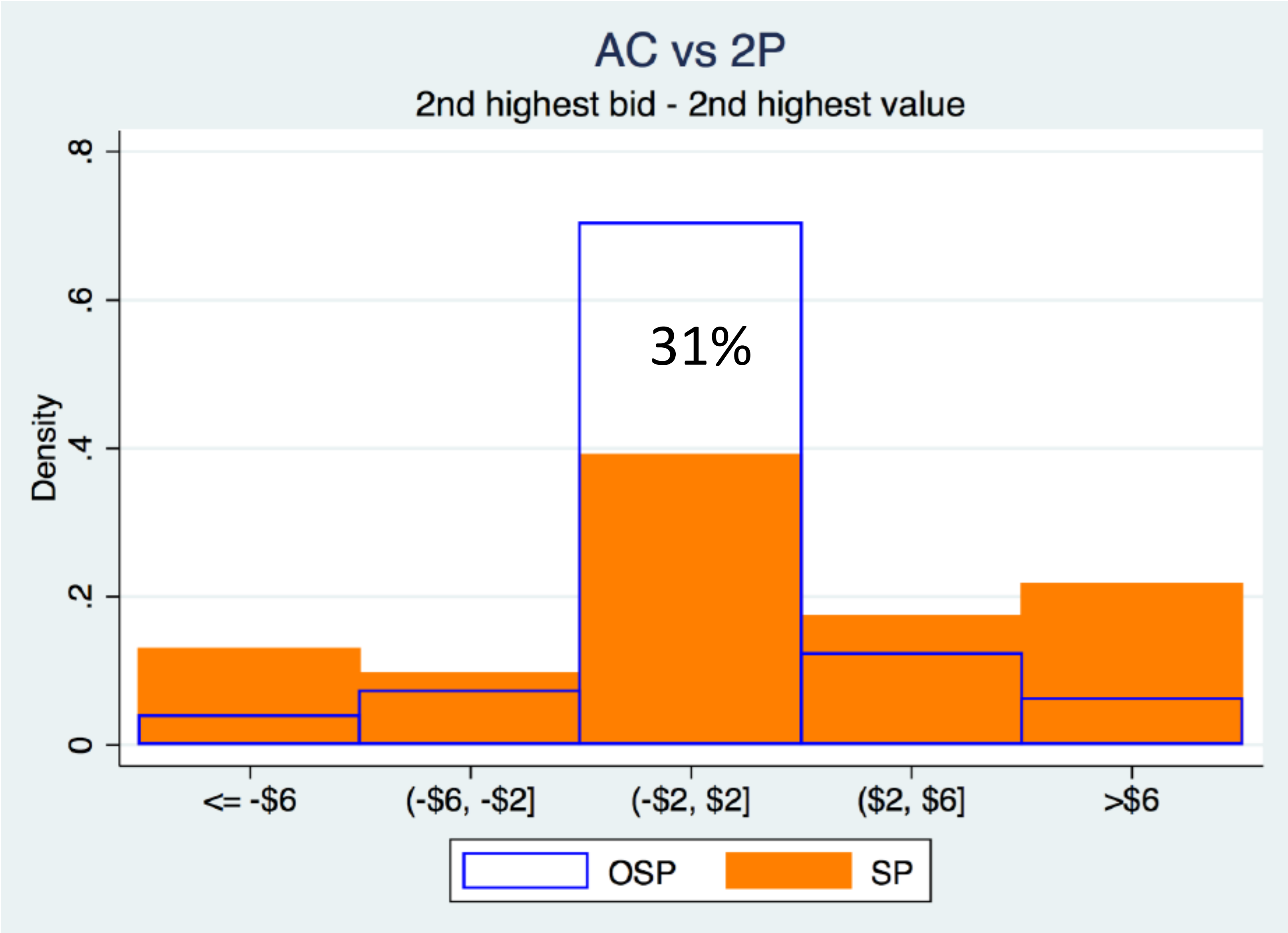
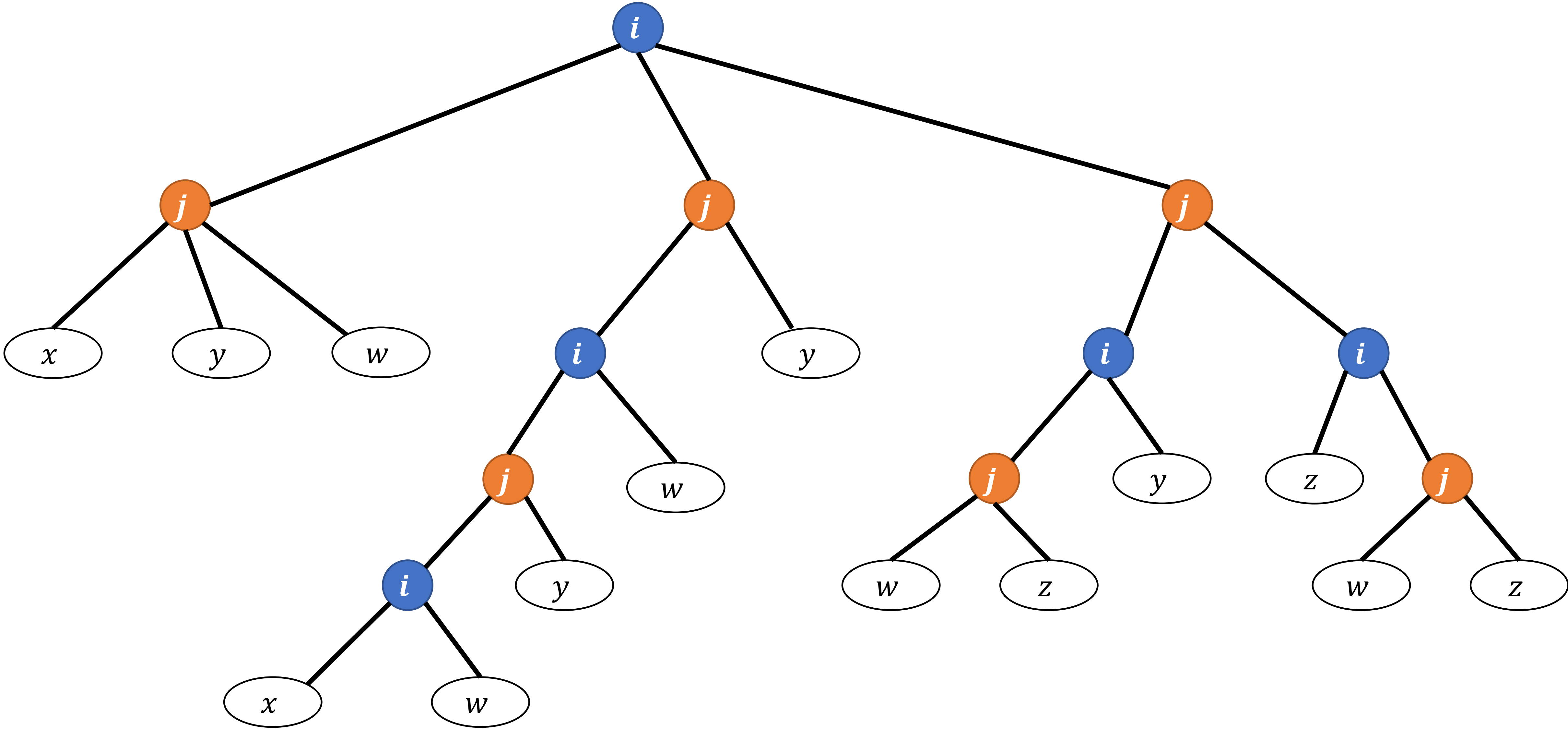


Extensive form games



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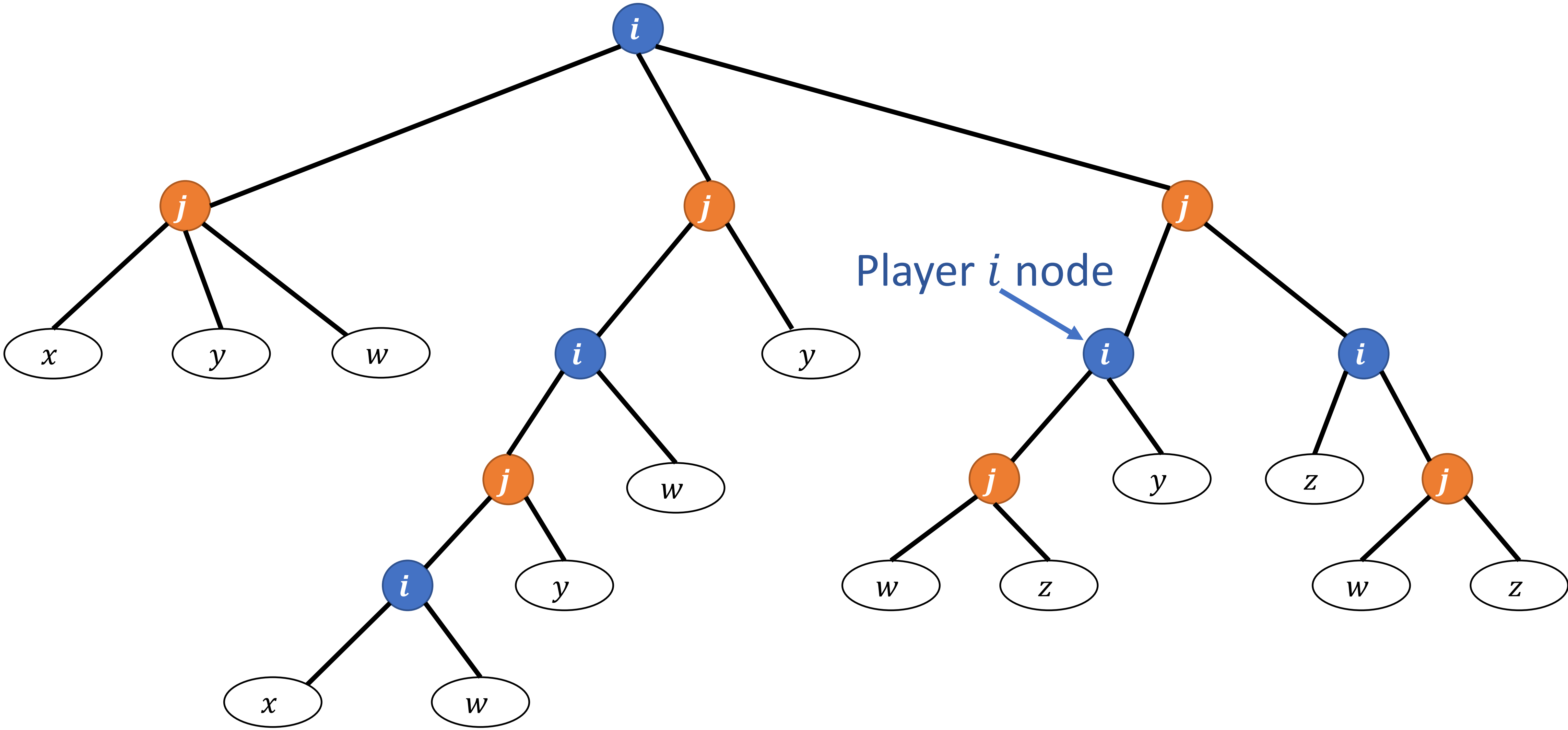

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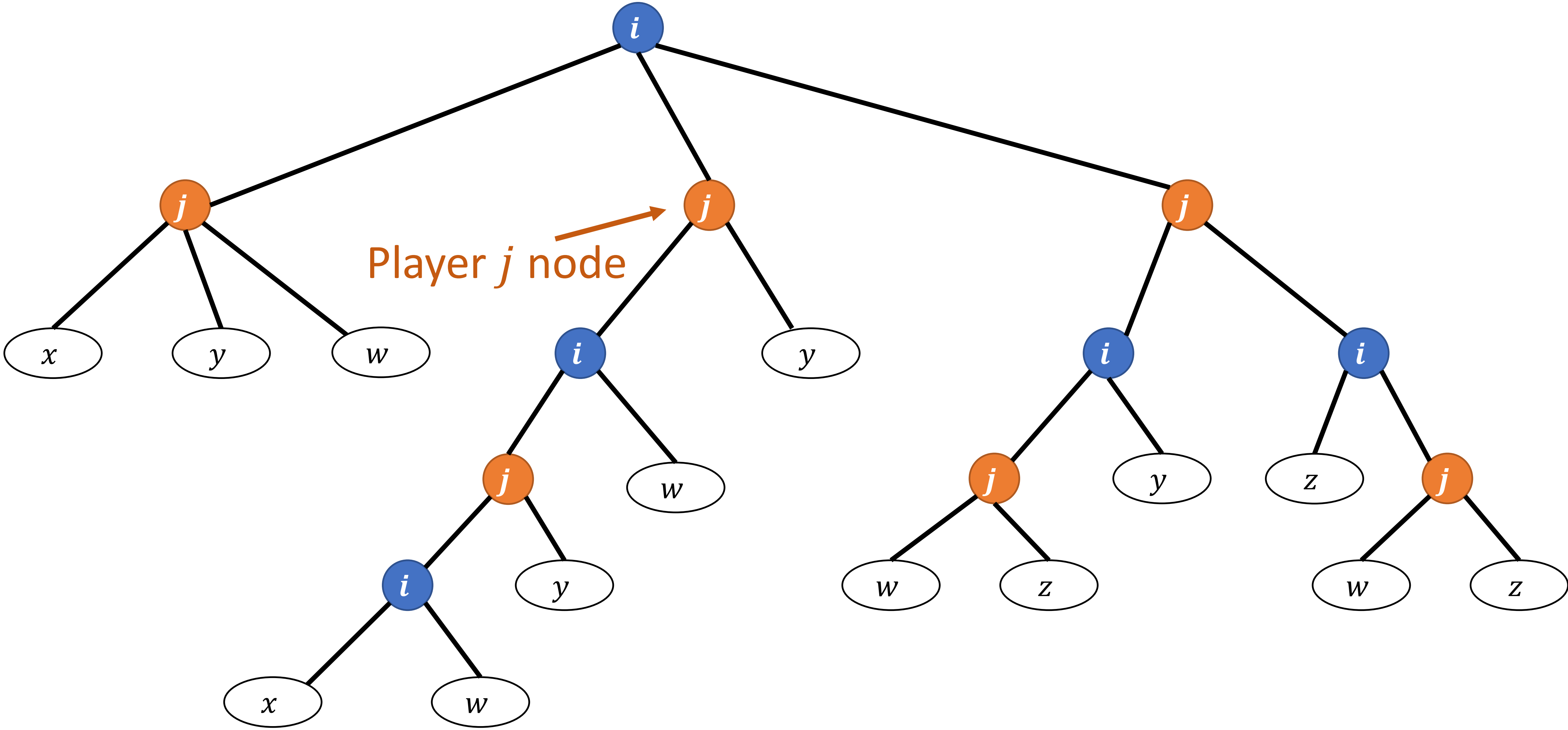

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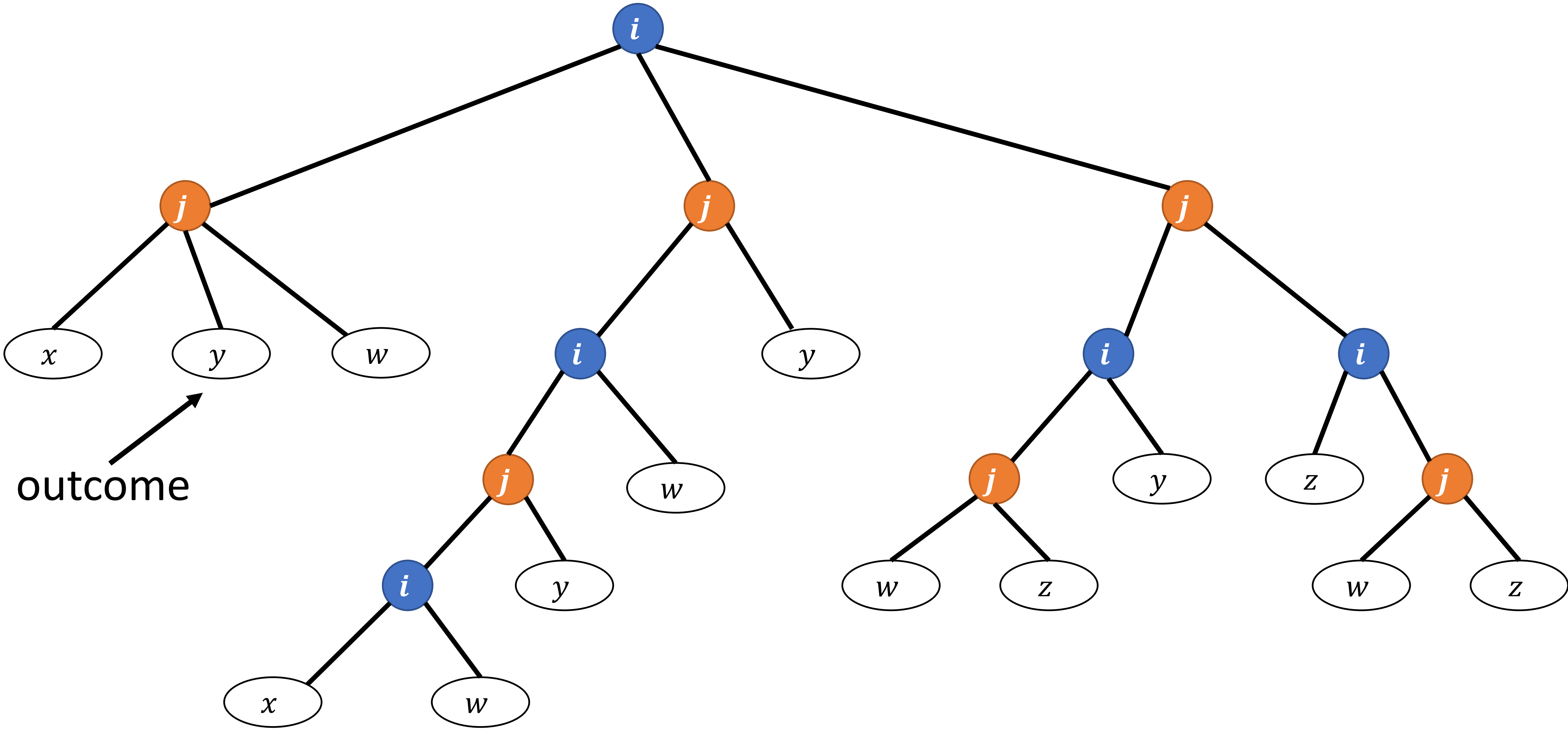

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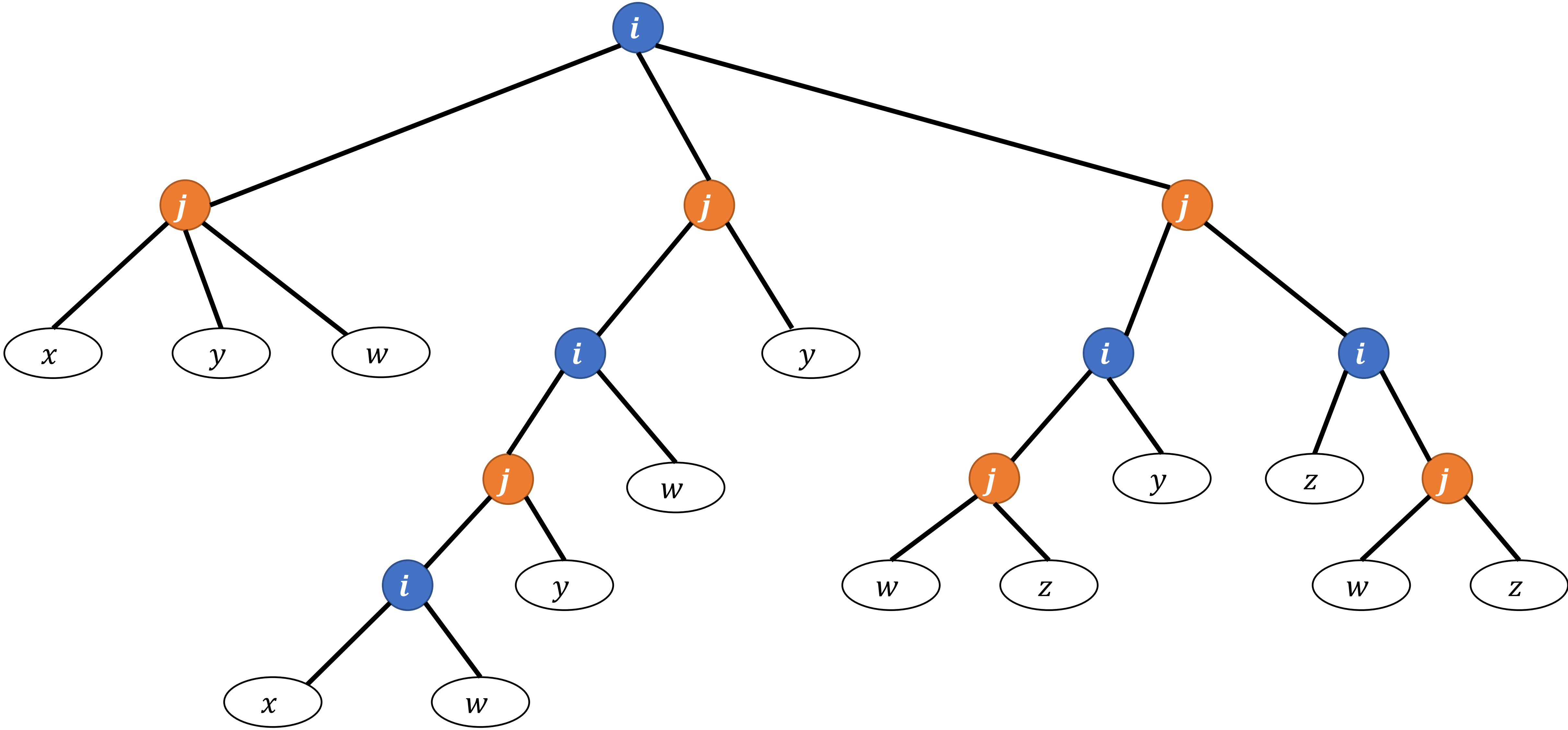

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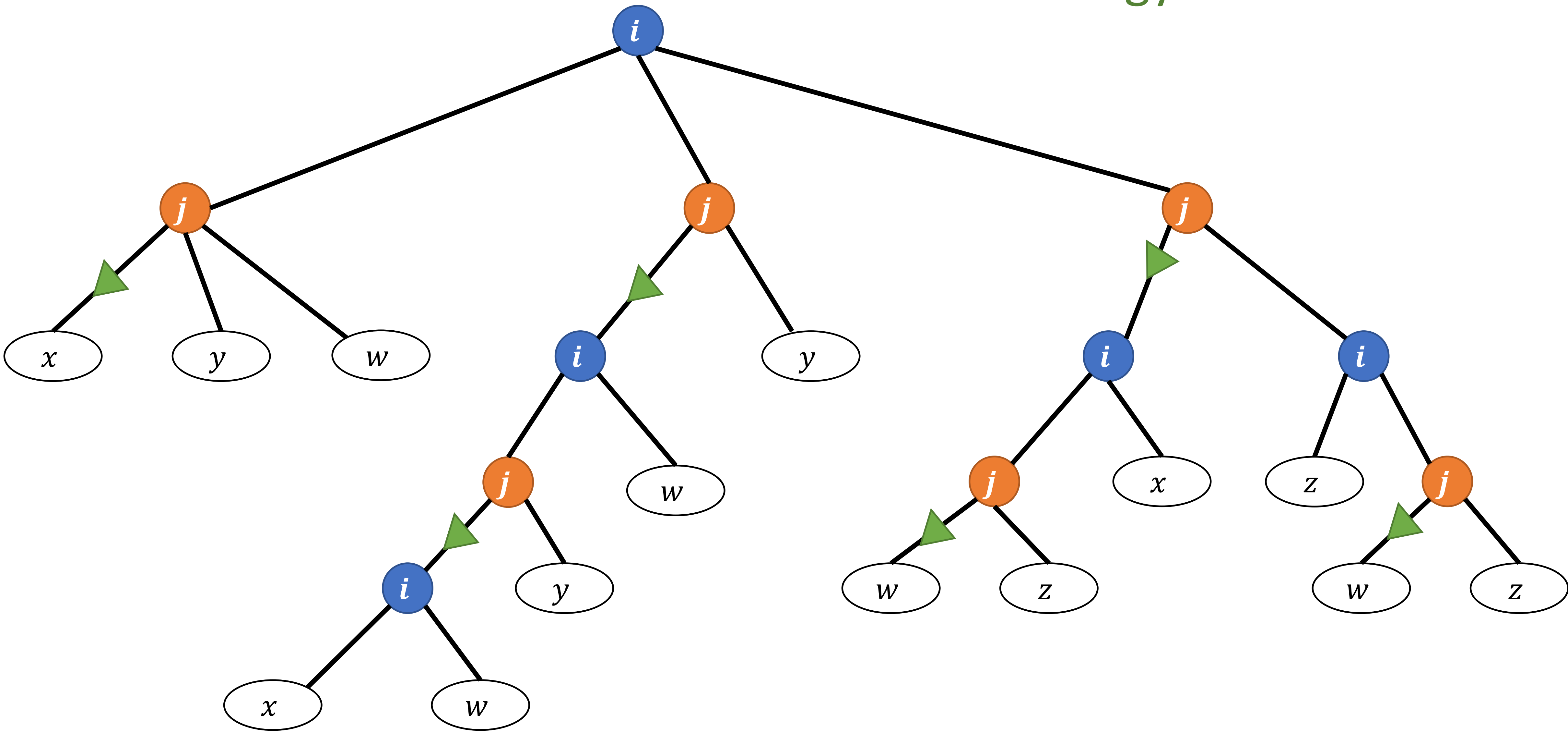
Strategy S



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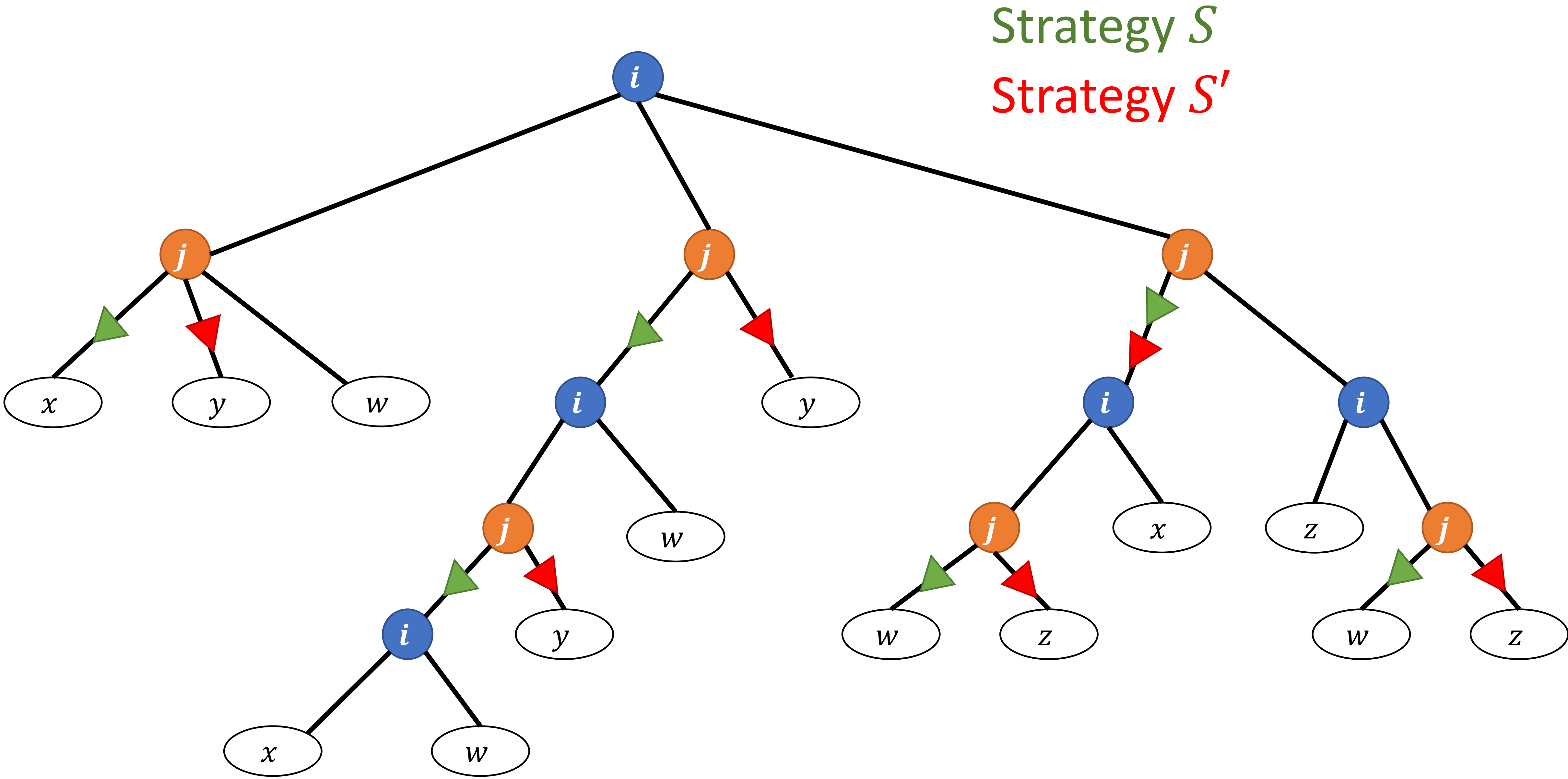

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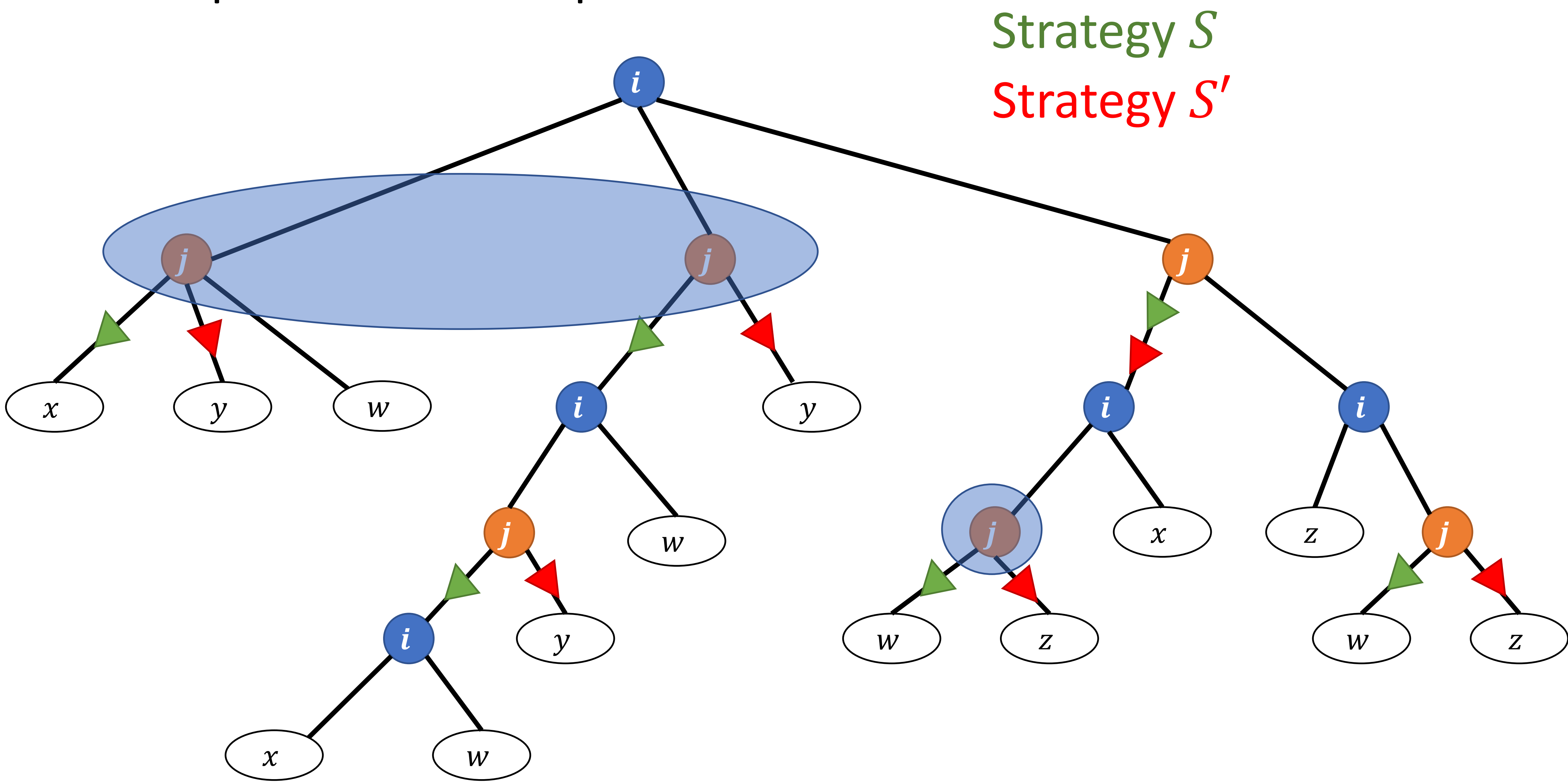

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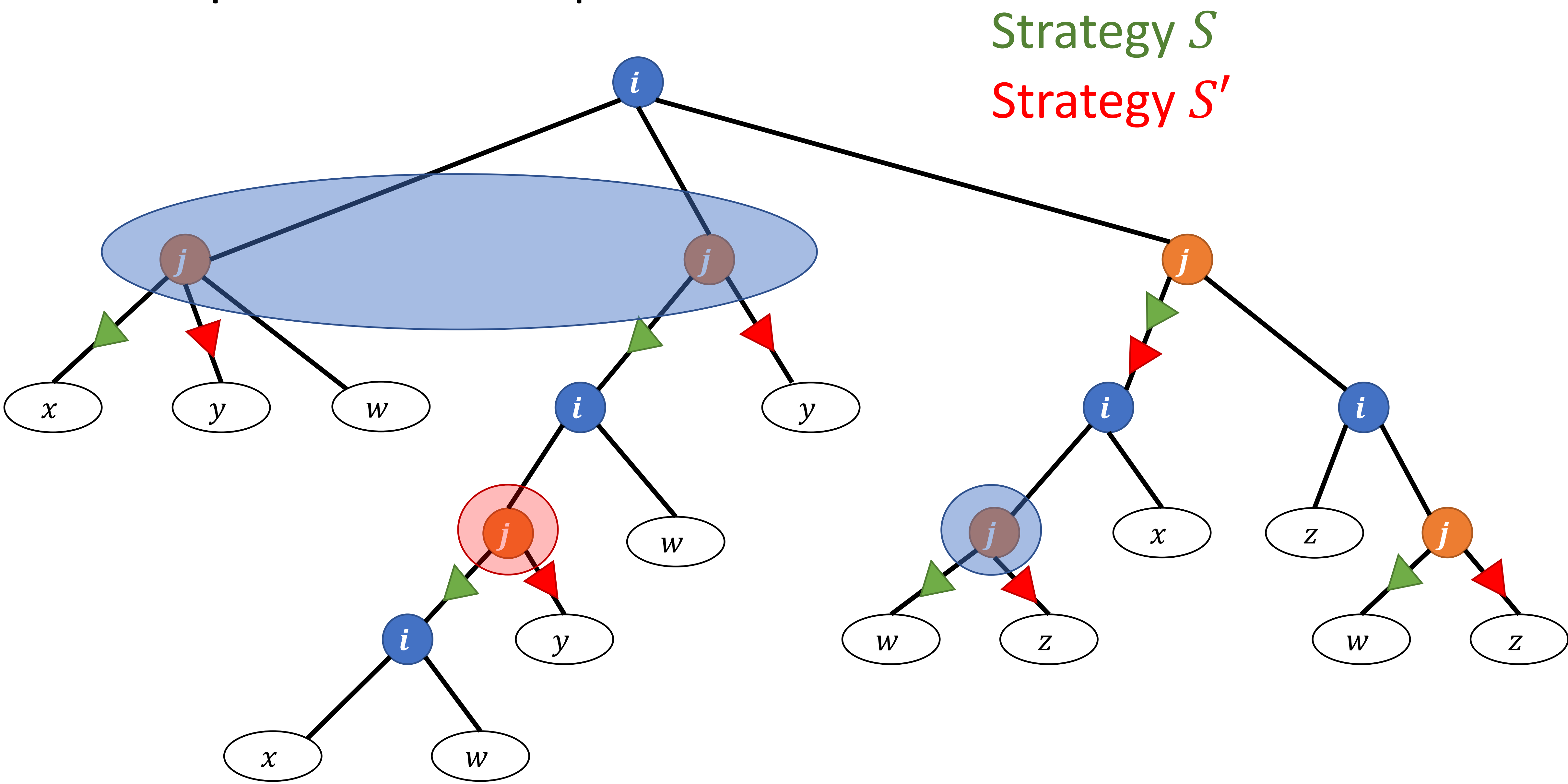

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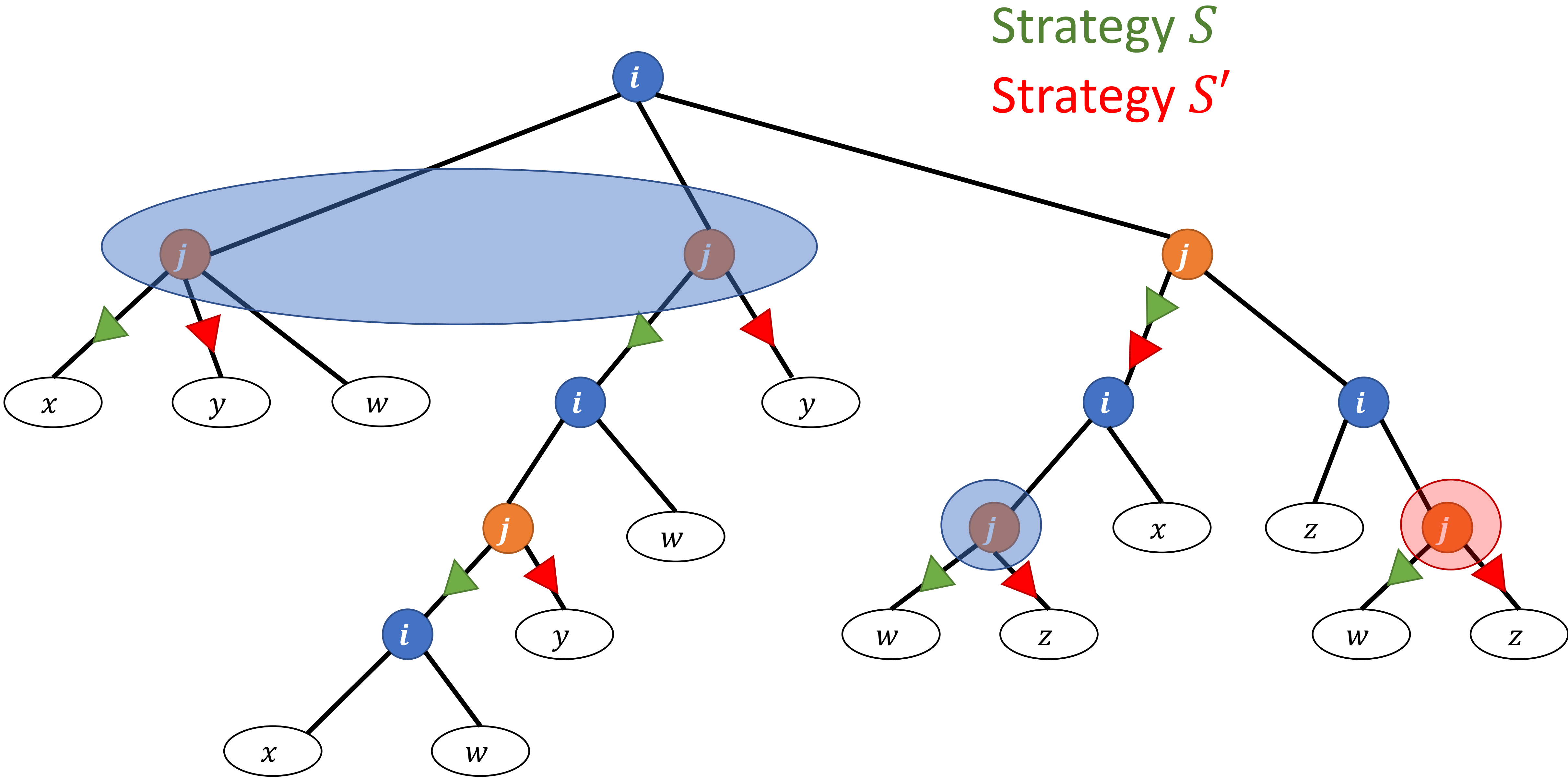
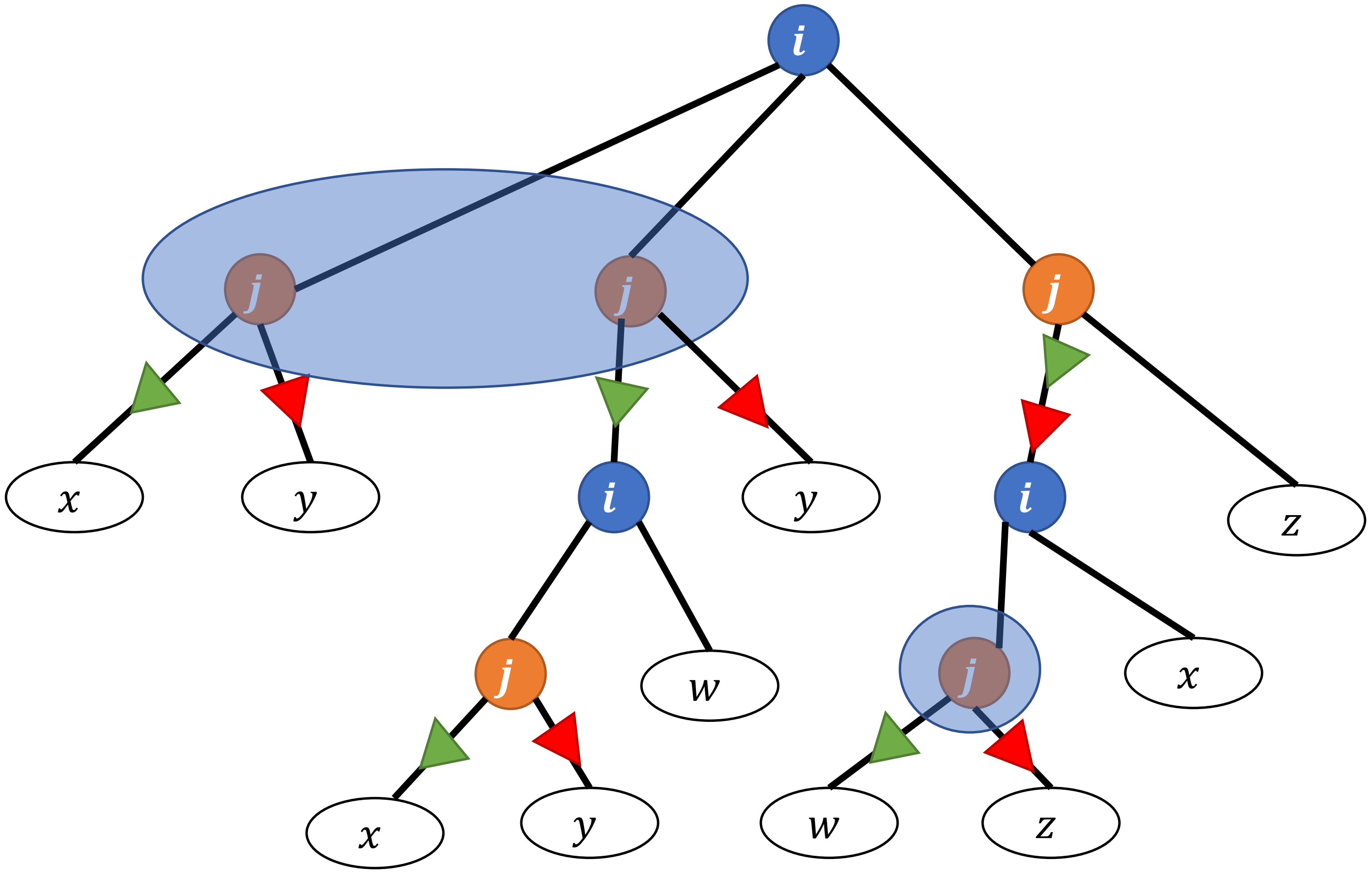


Obviously dominates [Li]

$x \succ_j w \succ_j y \succ_j z$



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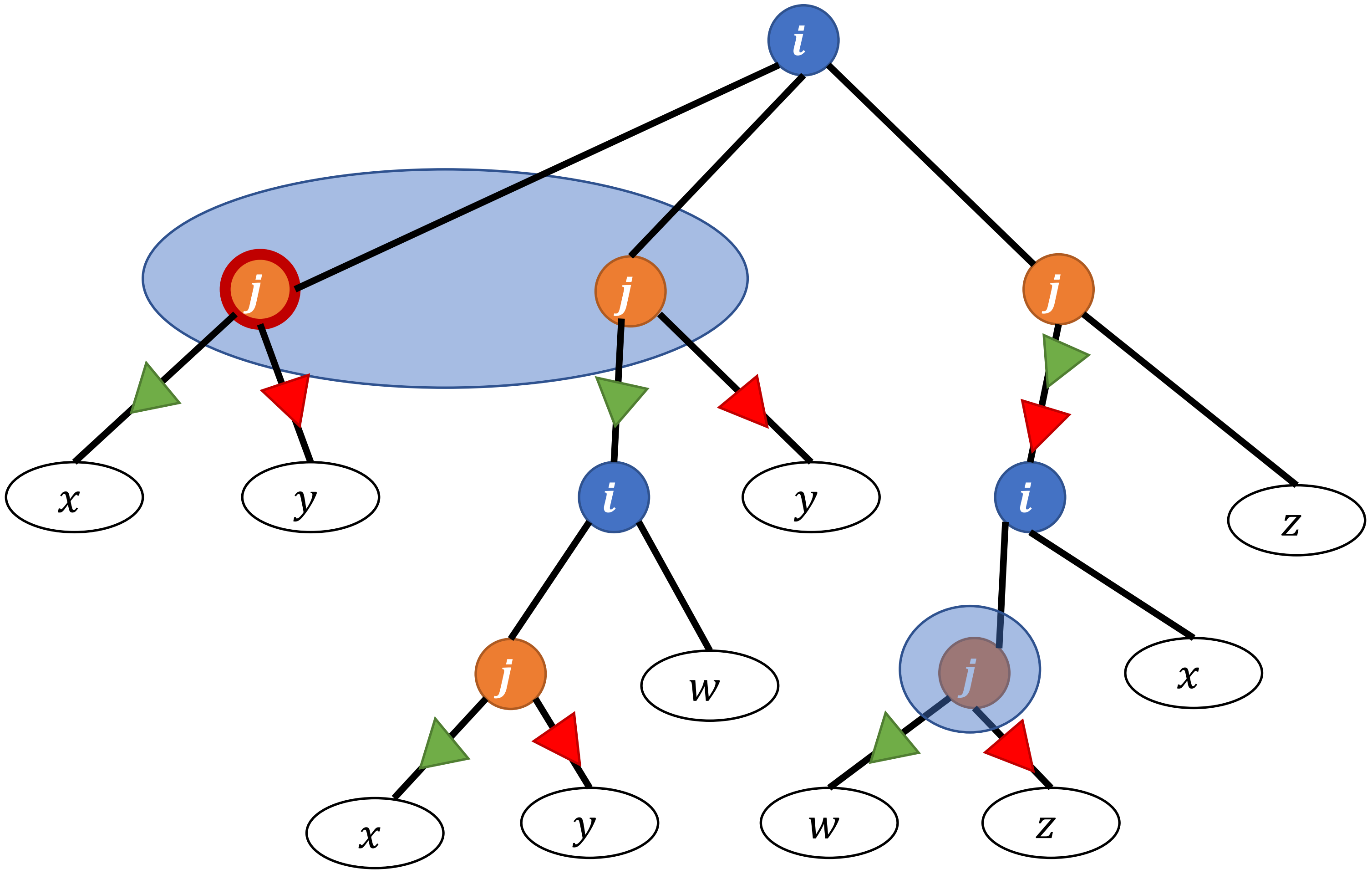

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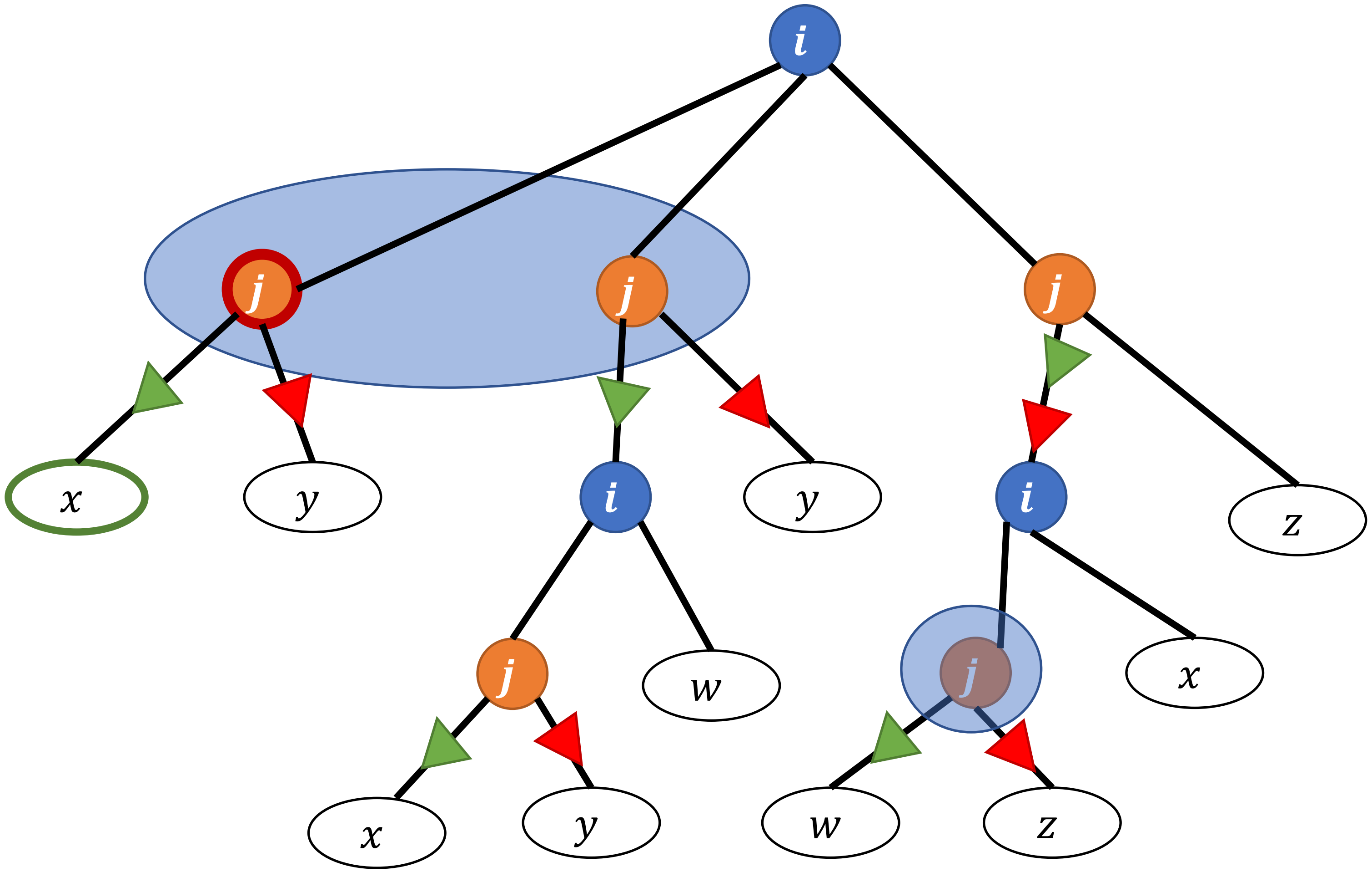

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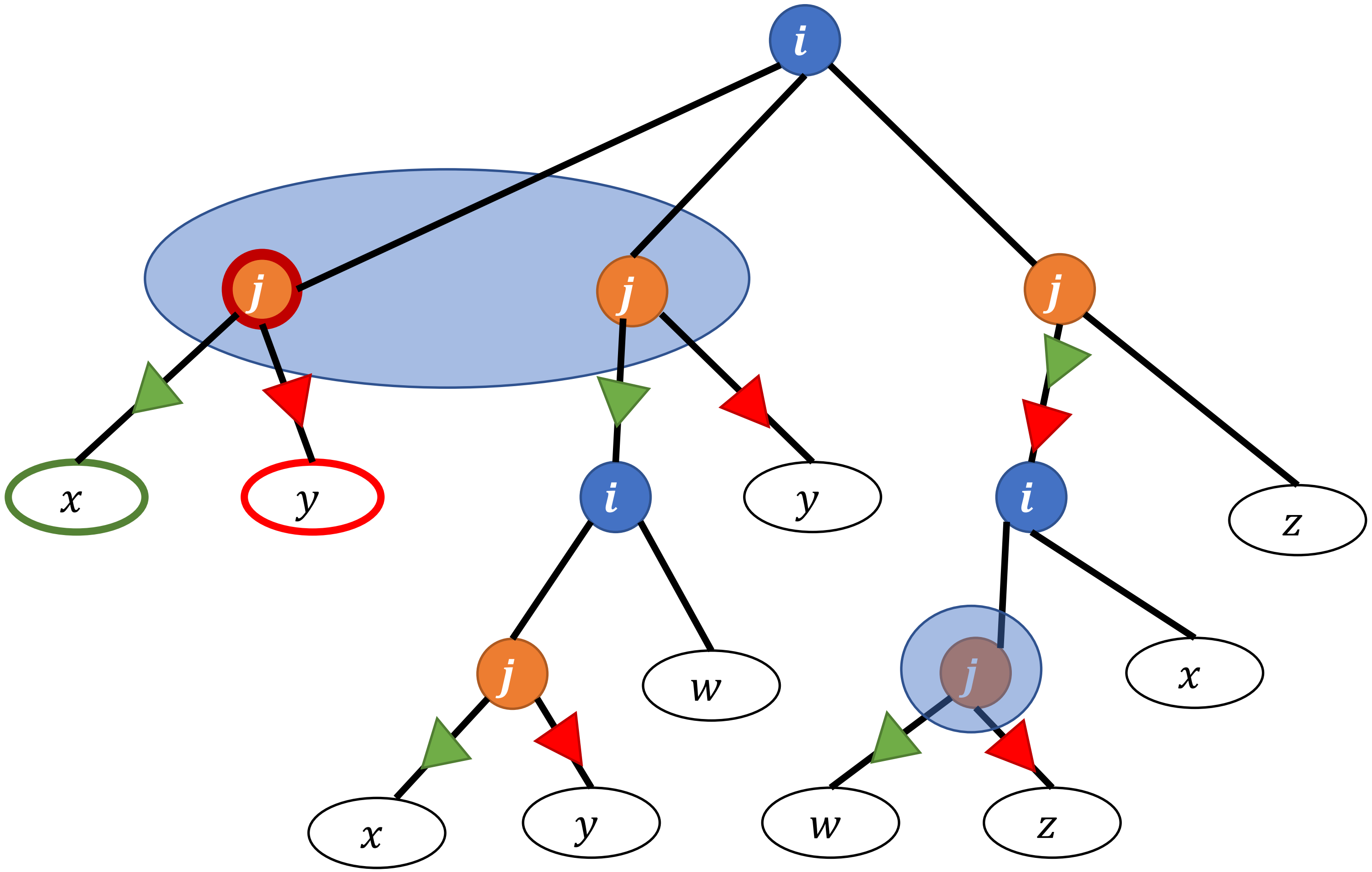

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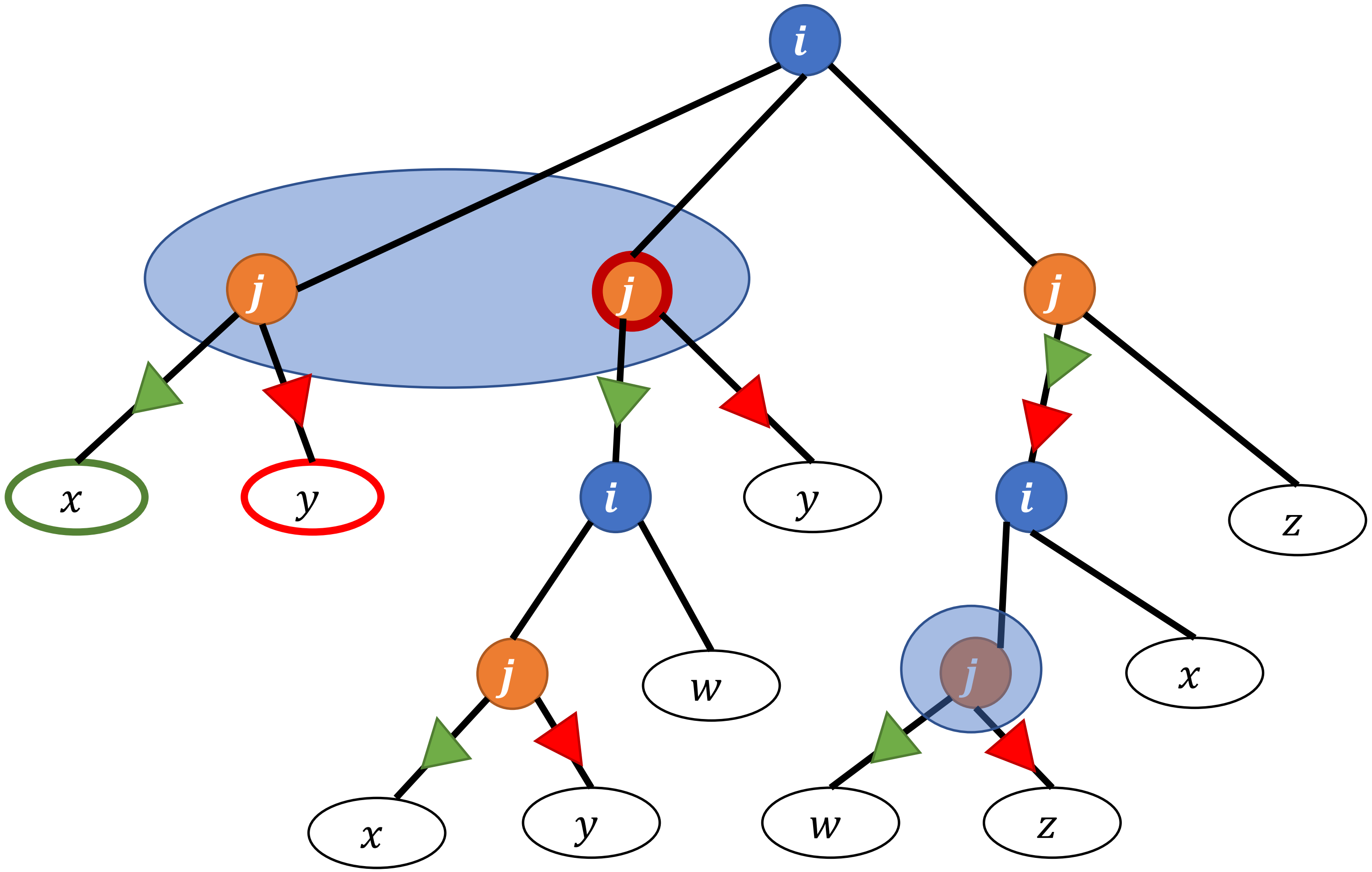

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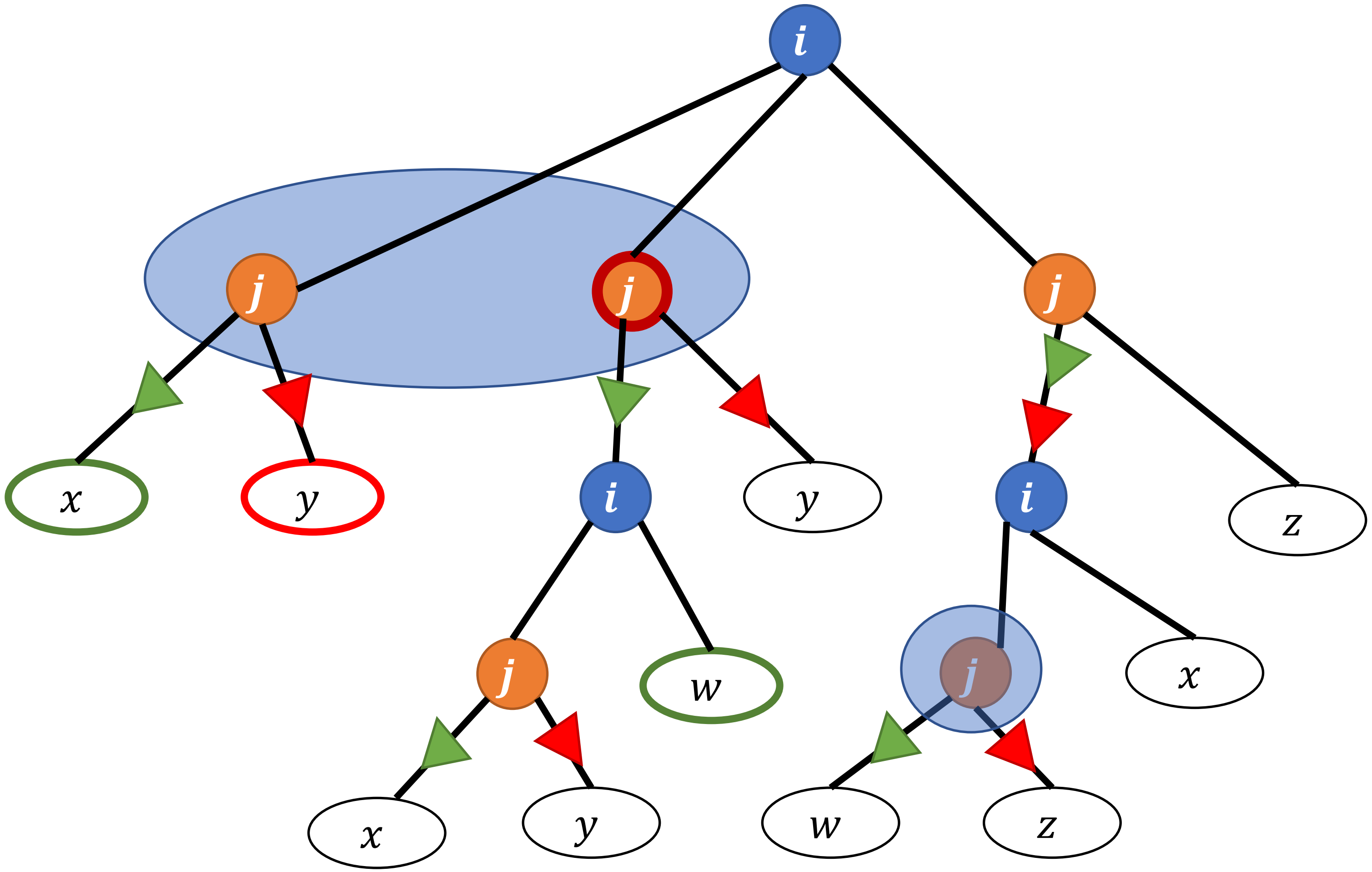

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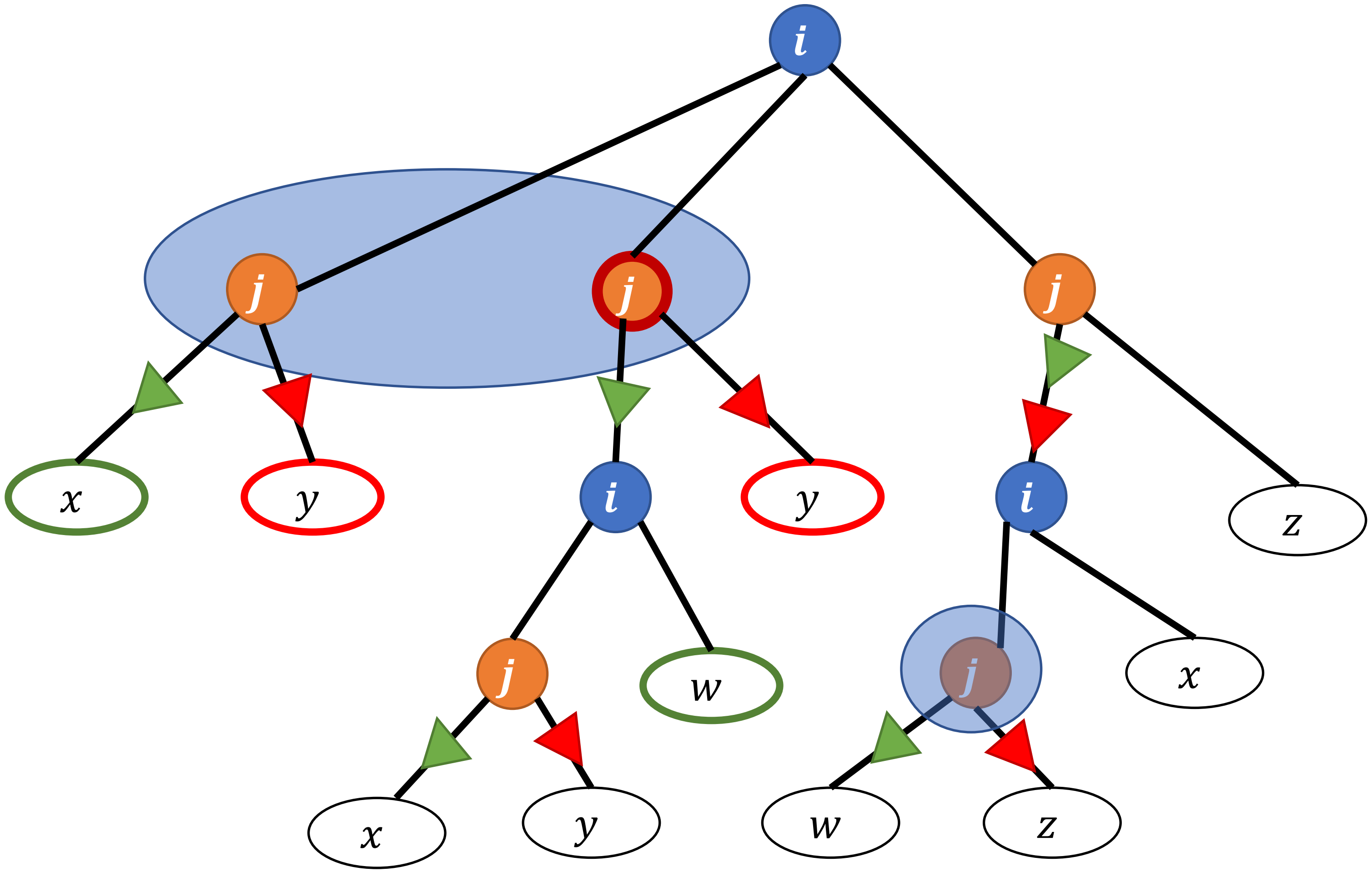

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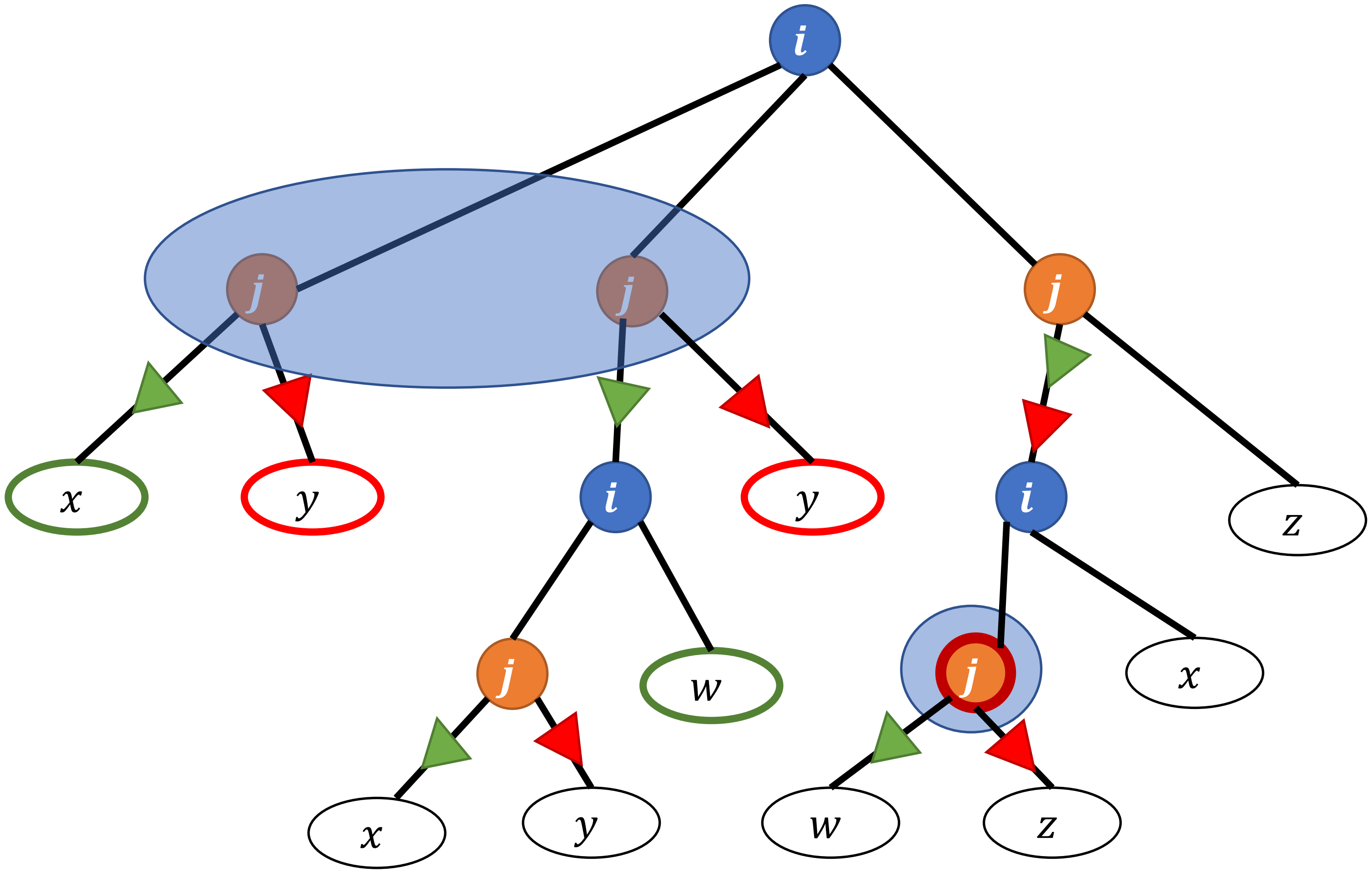

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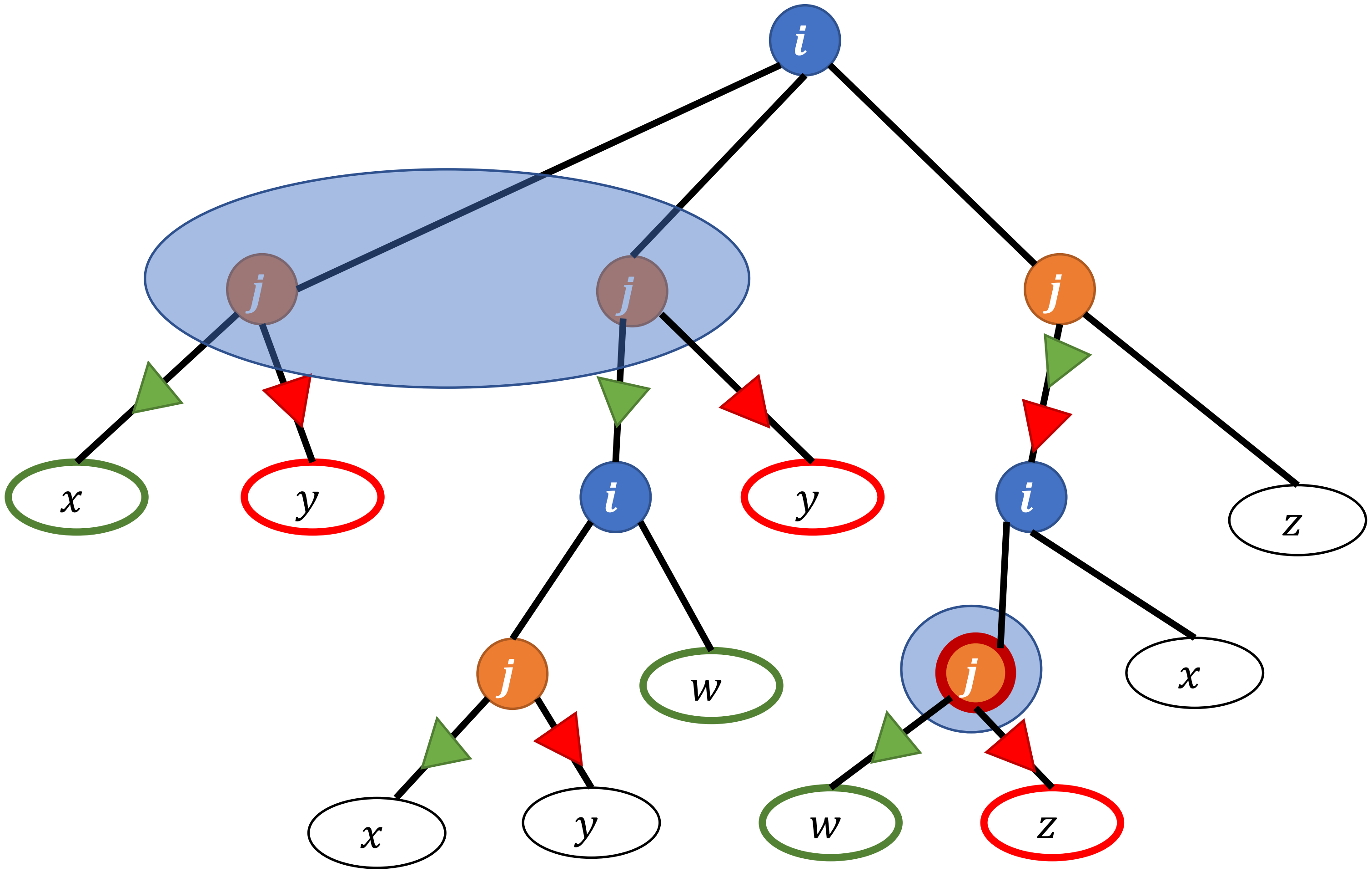

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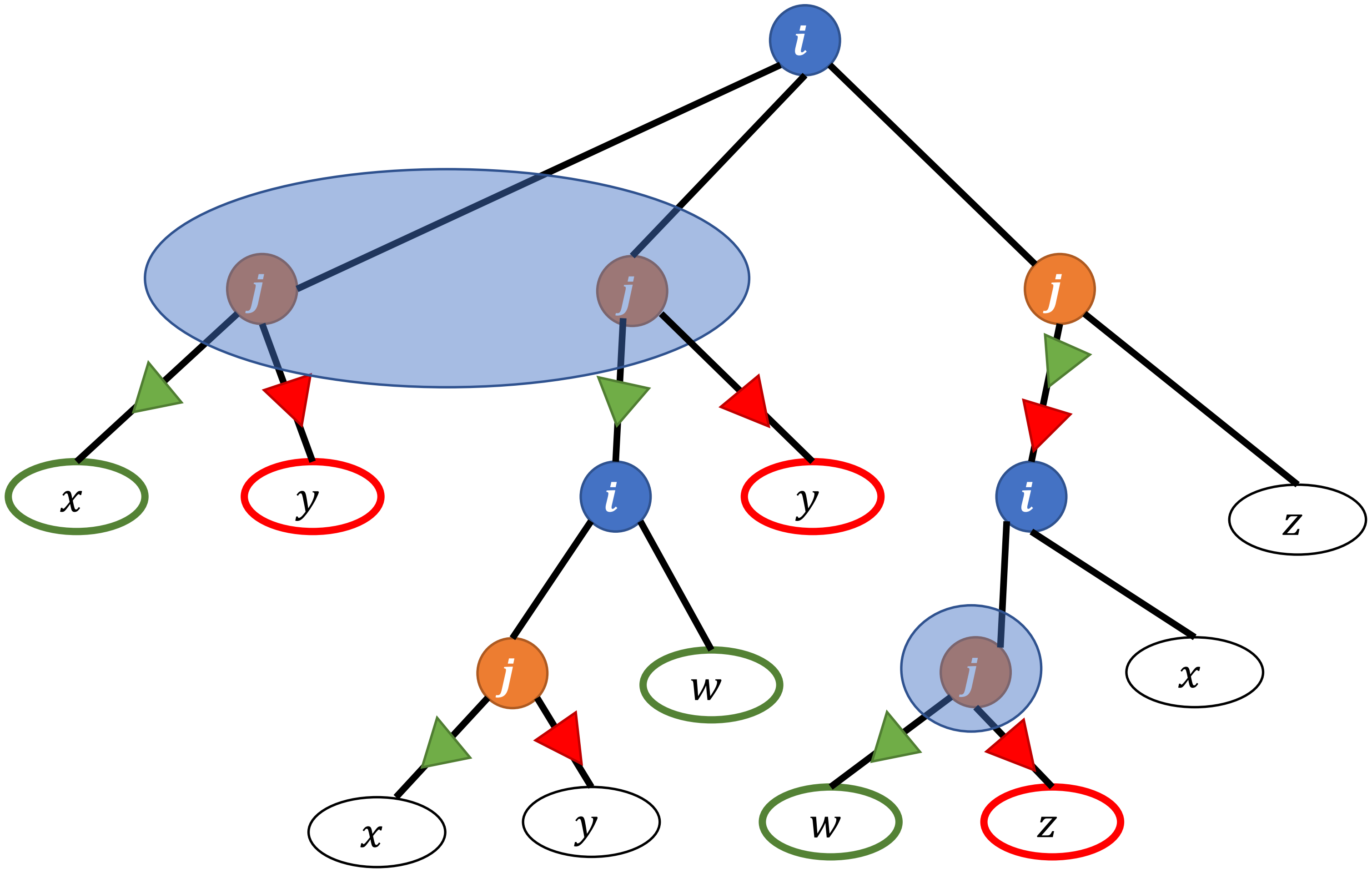

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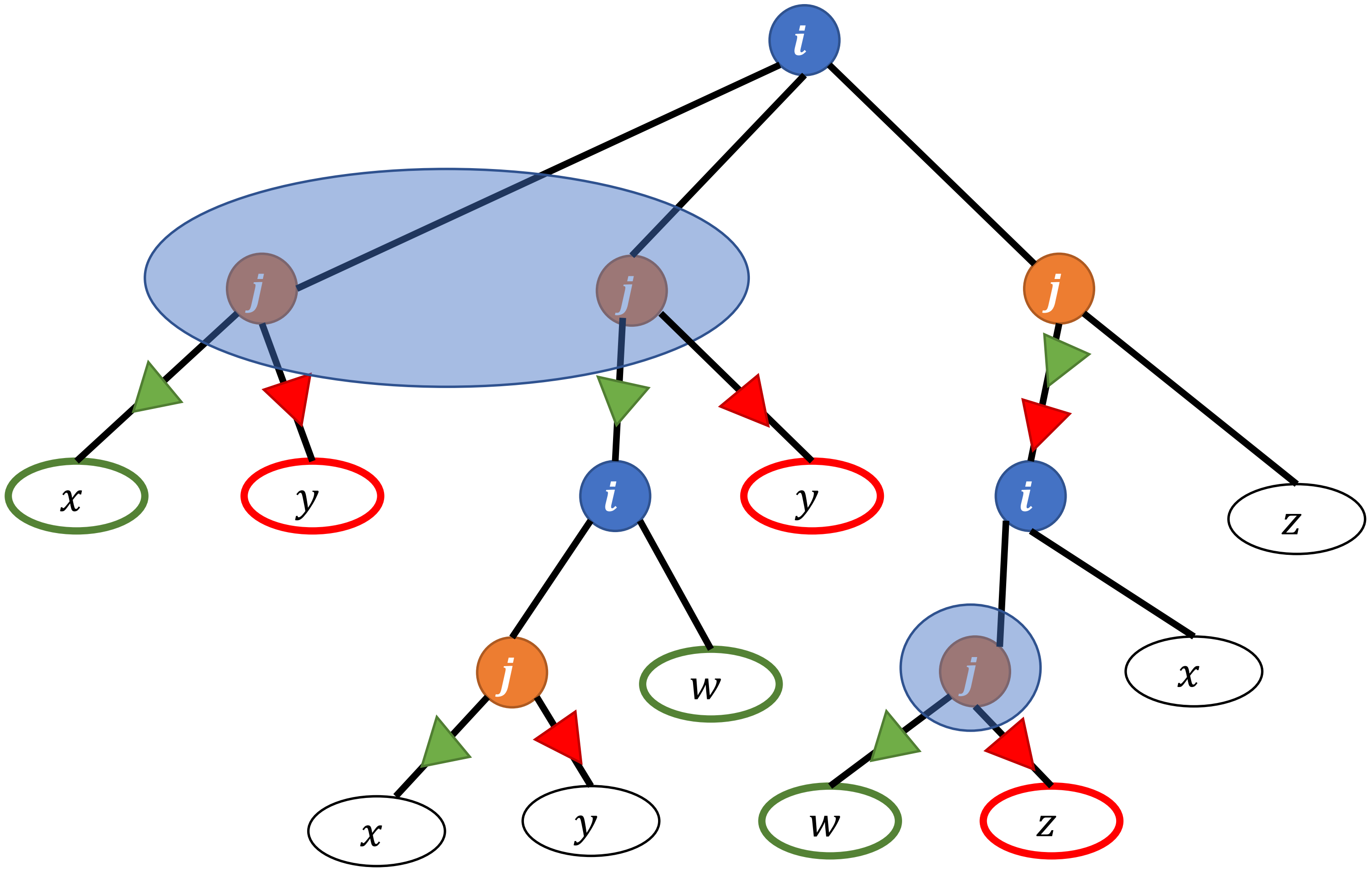

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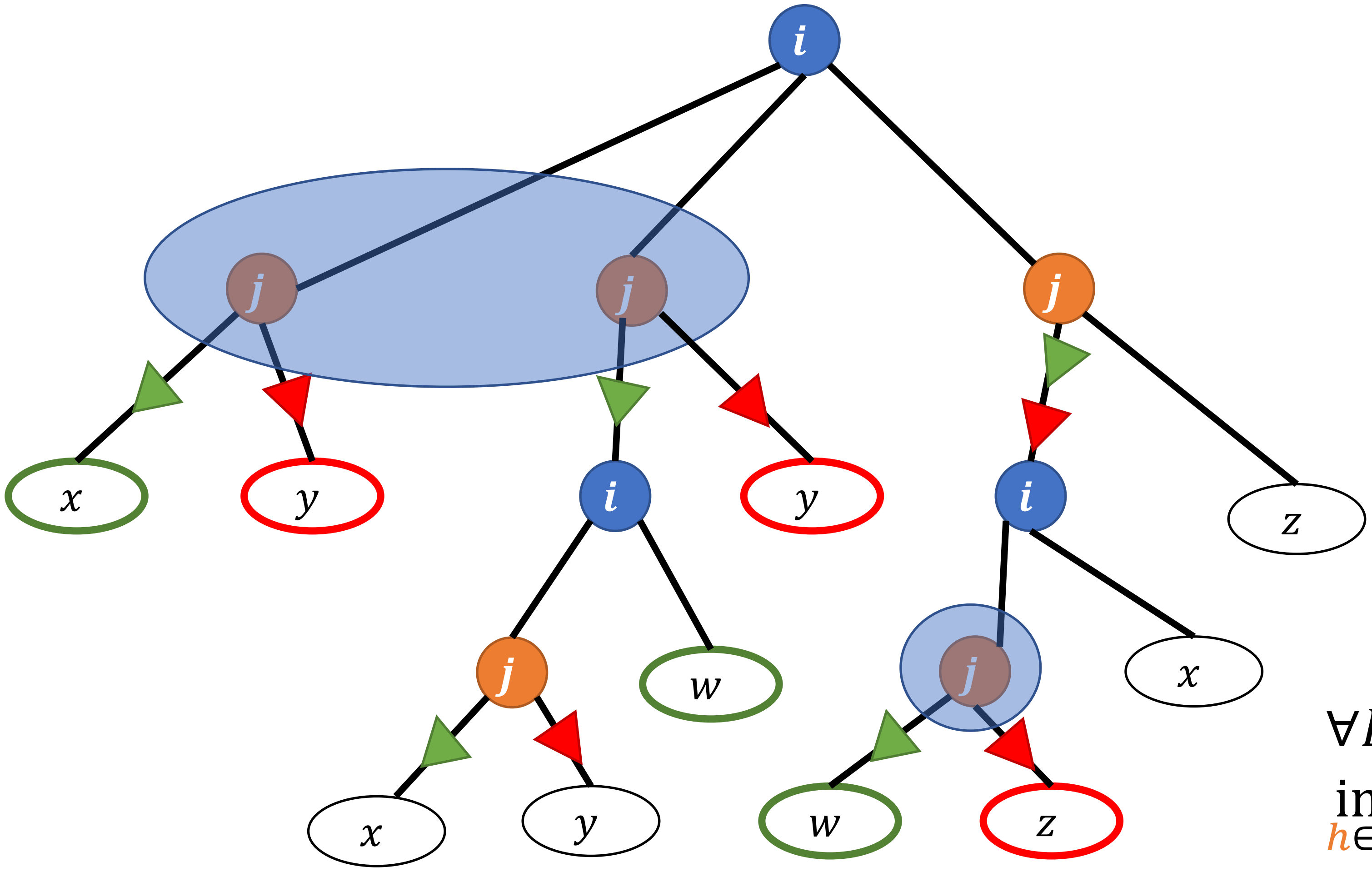

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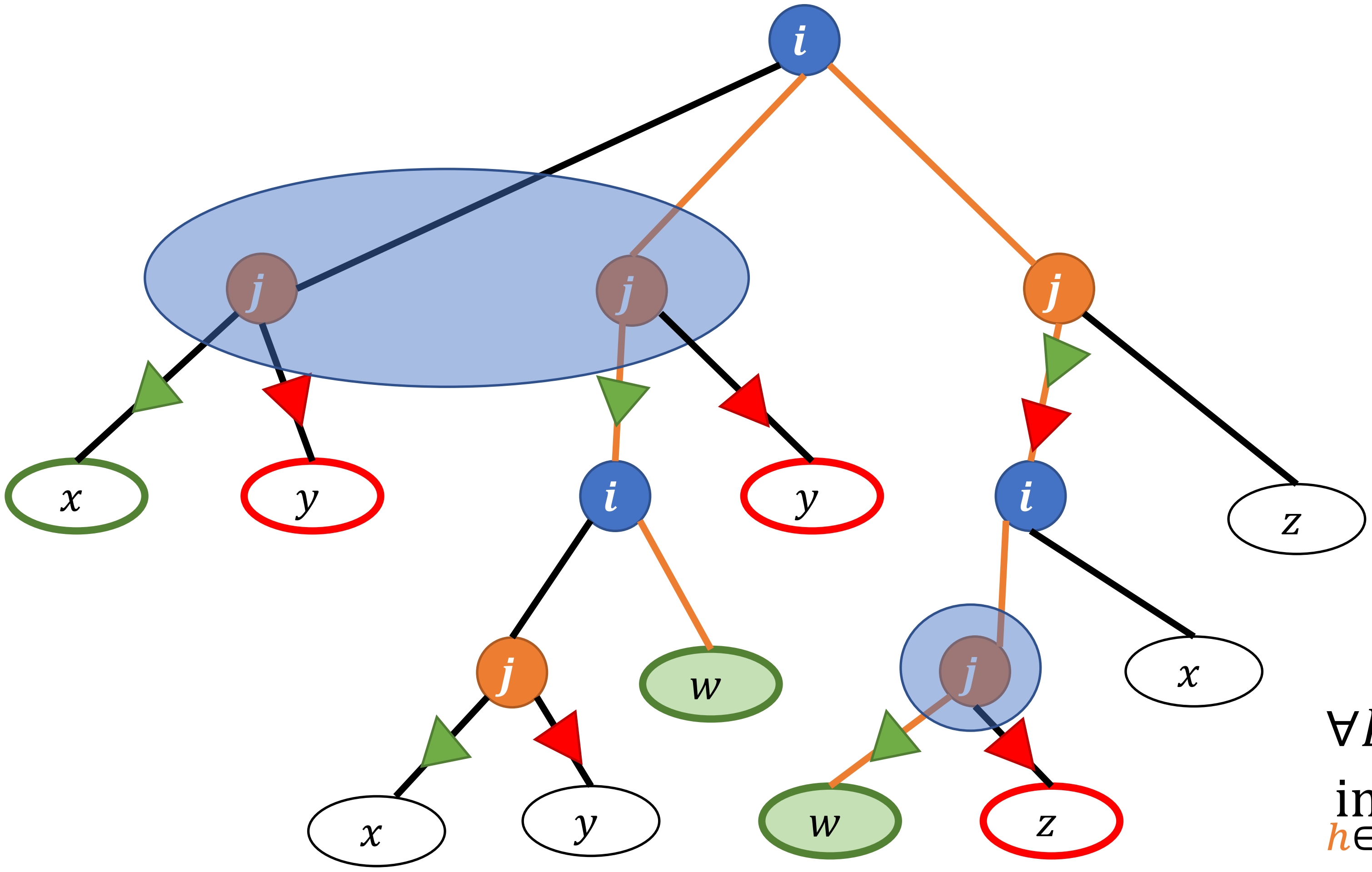

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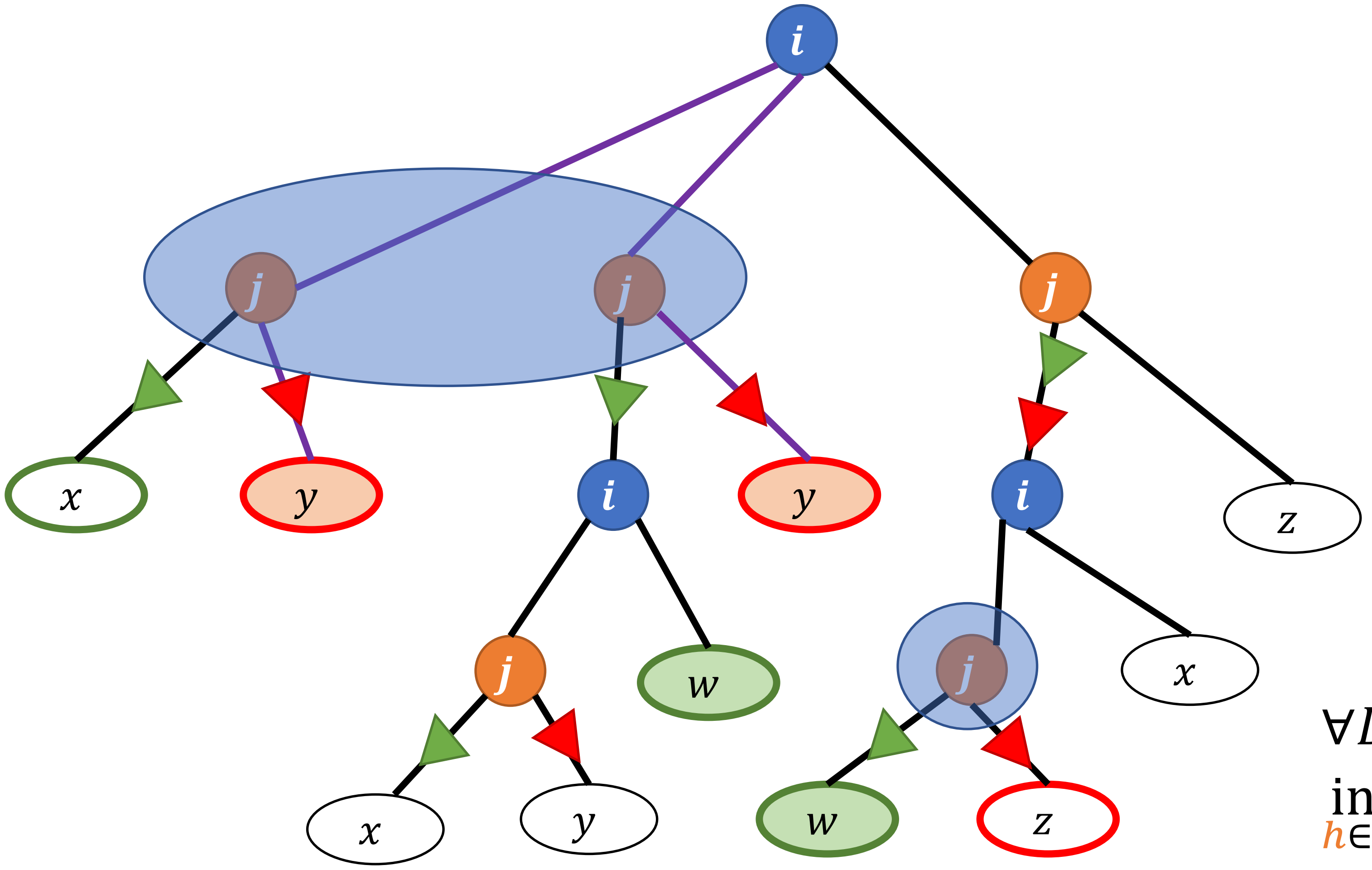

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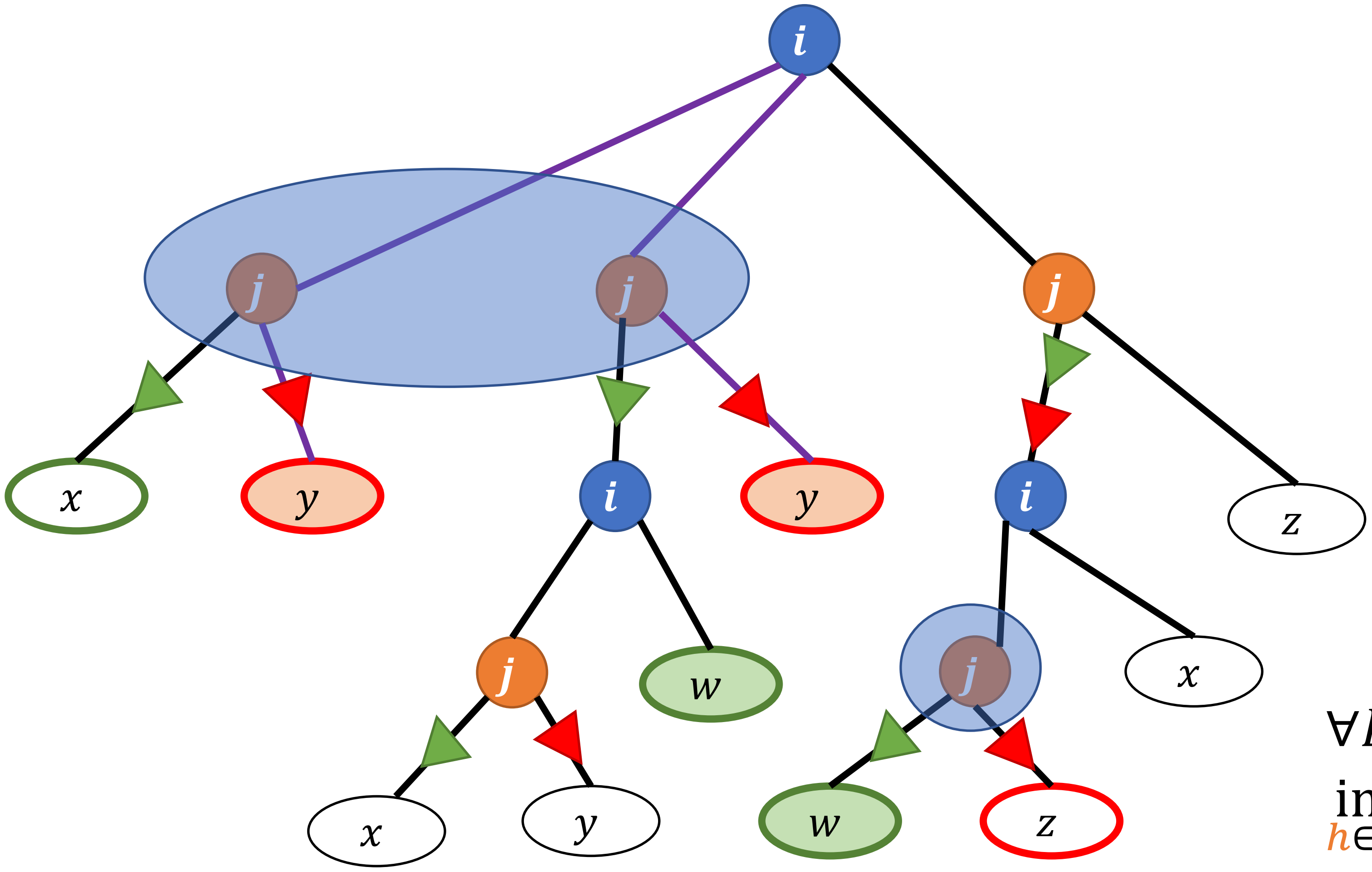

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best outcome made possible by dev $\leq$ worst outcome by not dev

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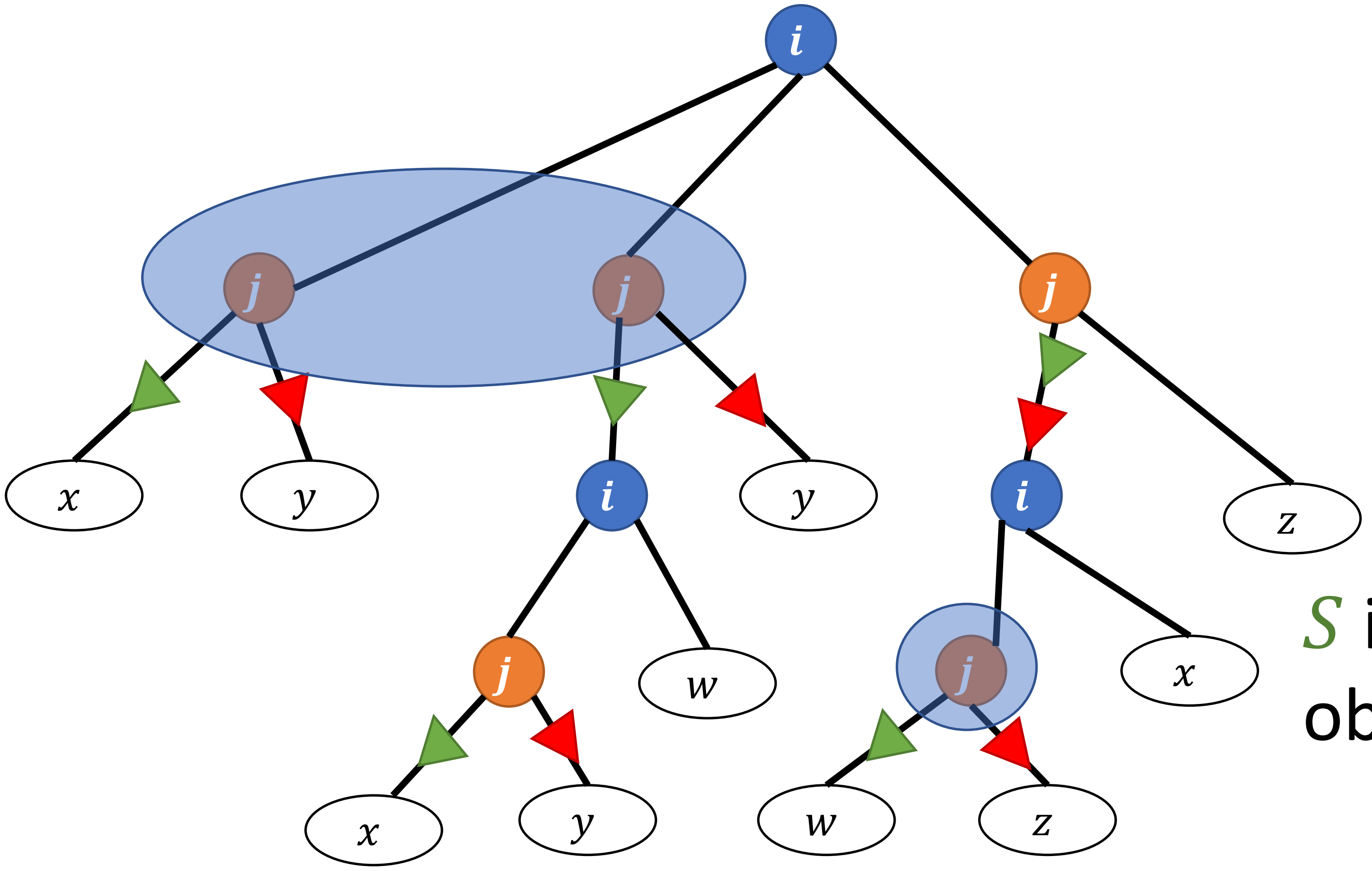

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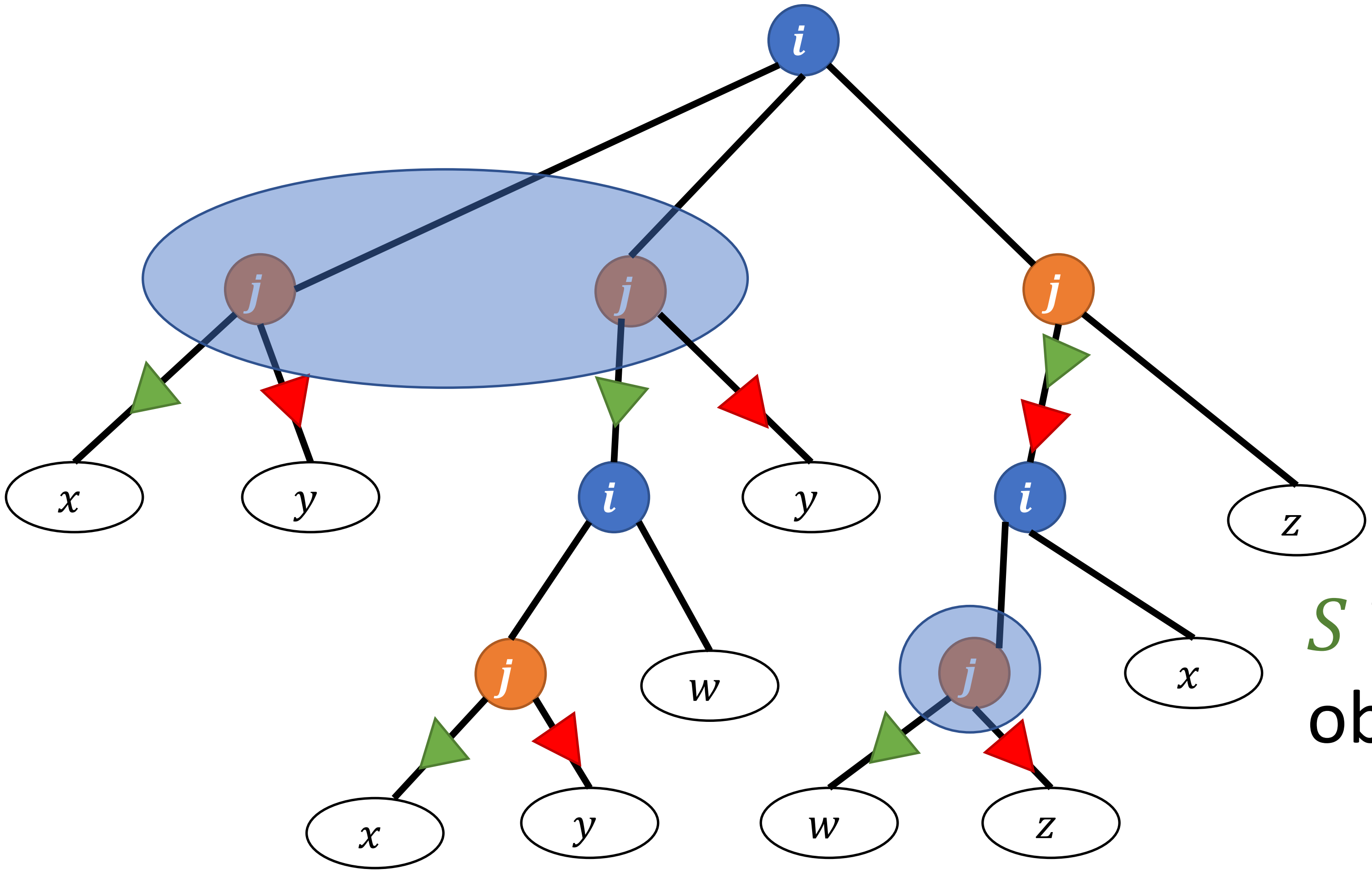

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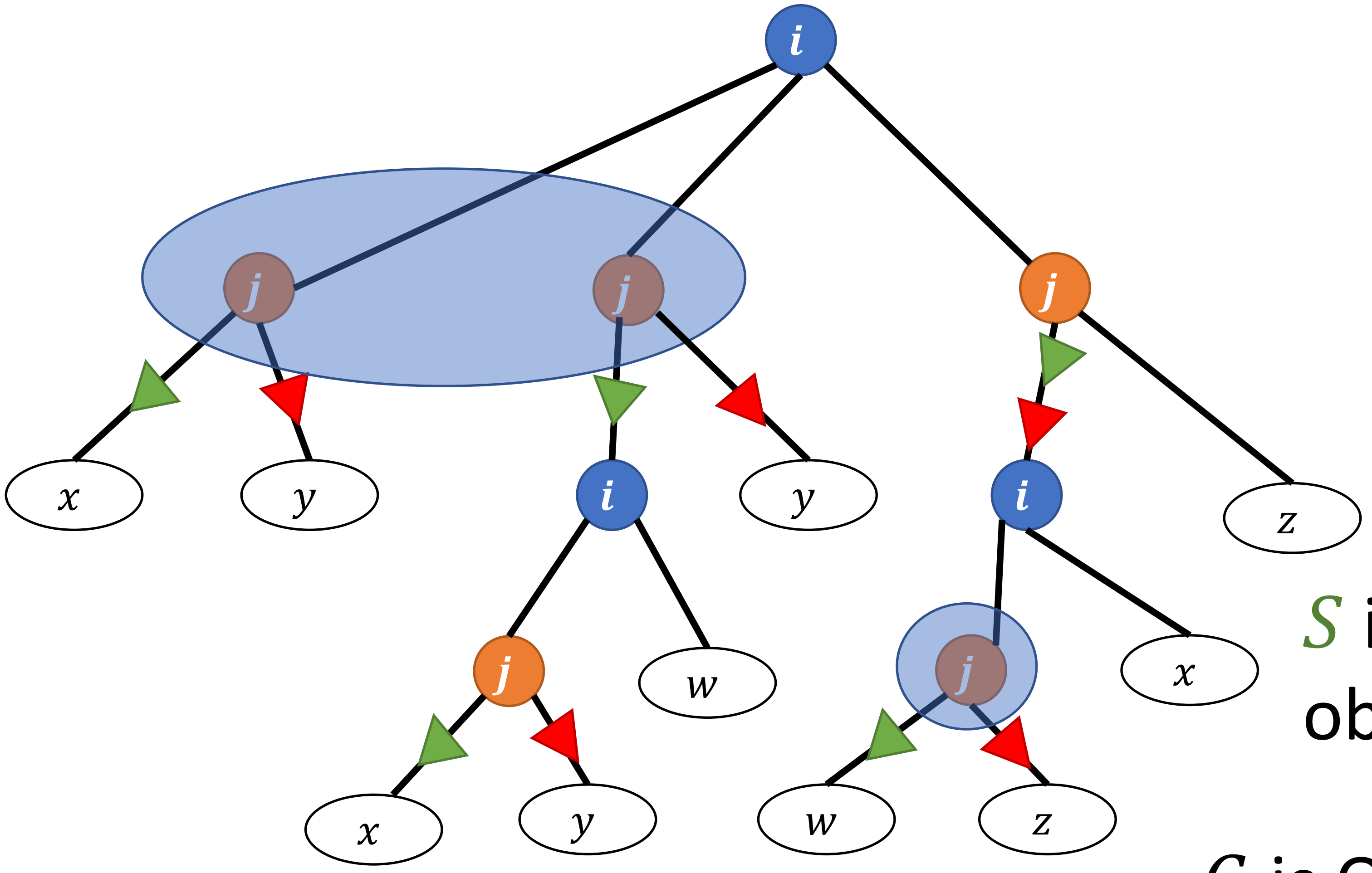

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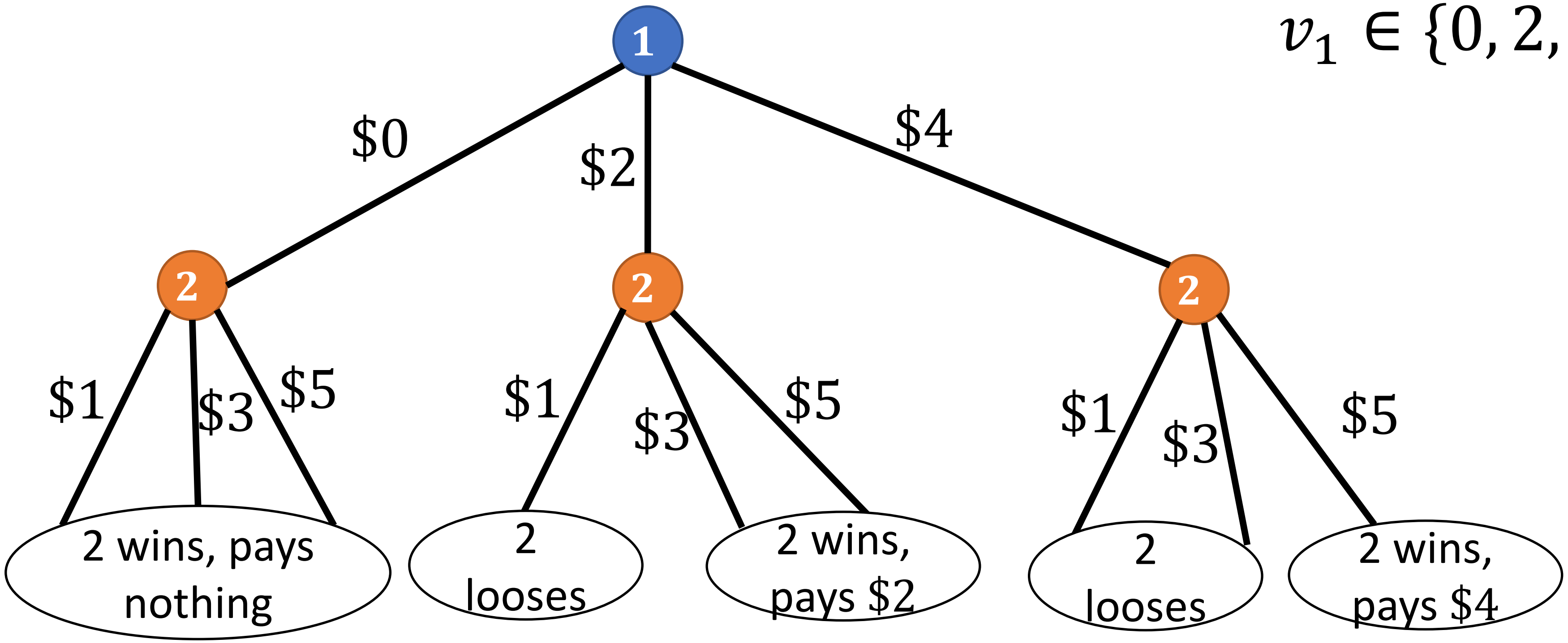
G is OSP if every player has an obviously dominant strategy

Second price auction is not OSP

$v_1 \in \{0, 2, 4\}$ $v_2 \in \{1, 3, 5\}$



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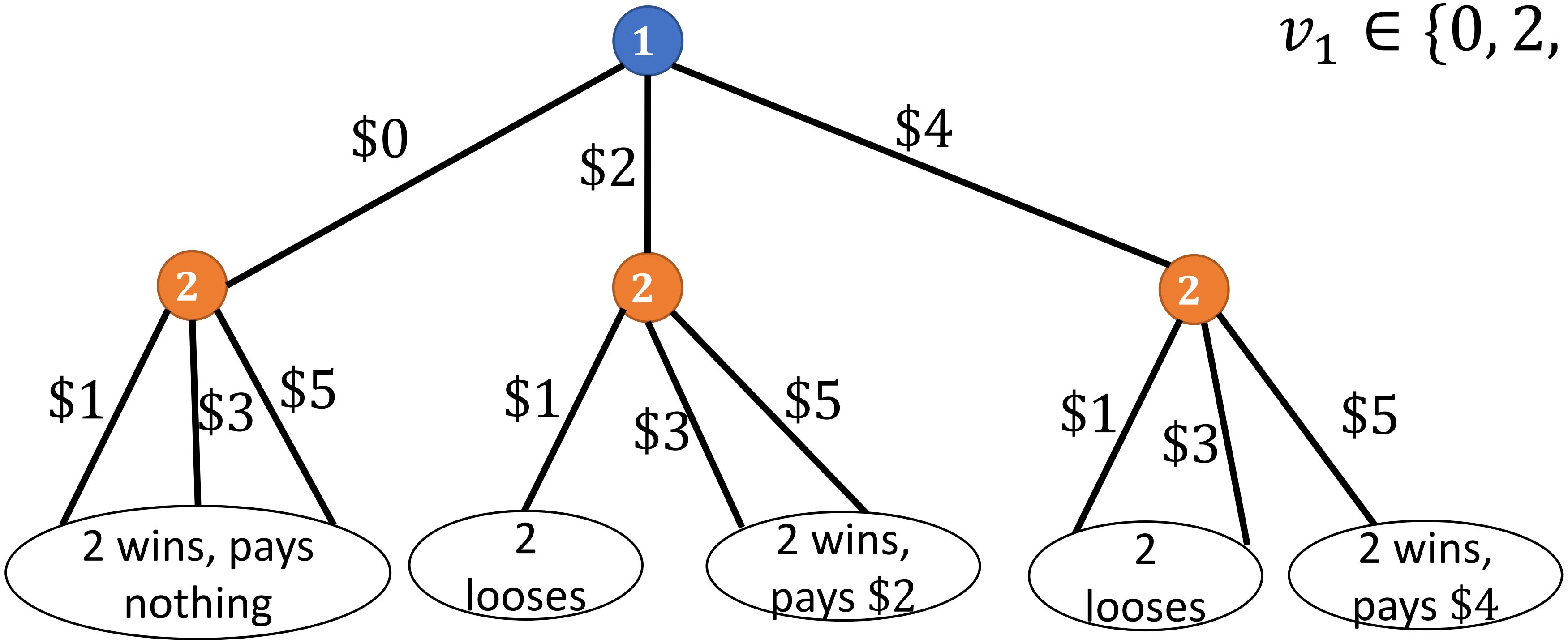

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Suppose $v_2 = 3$

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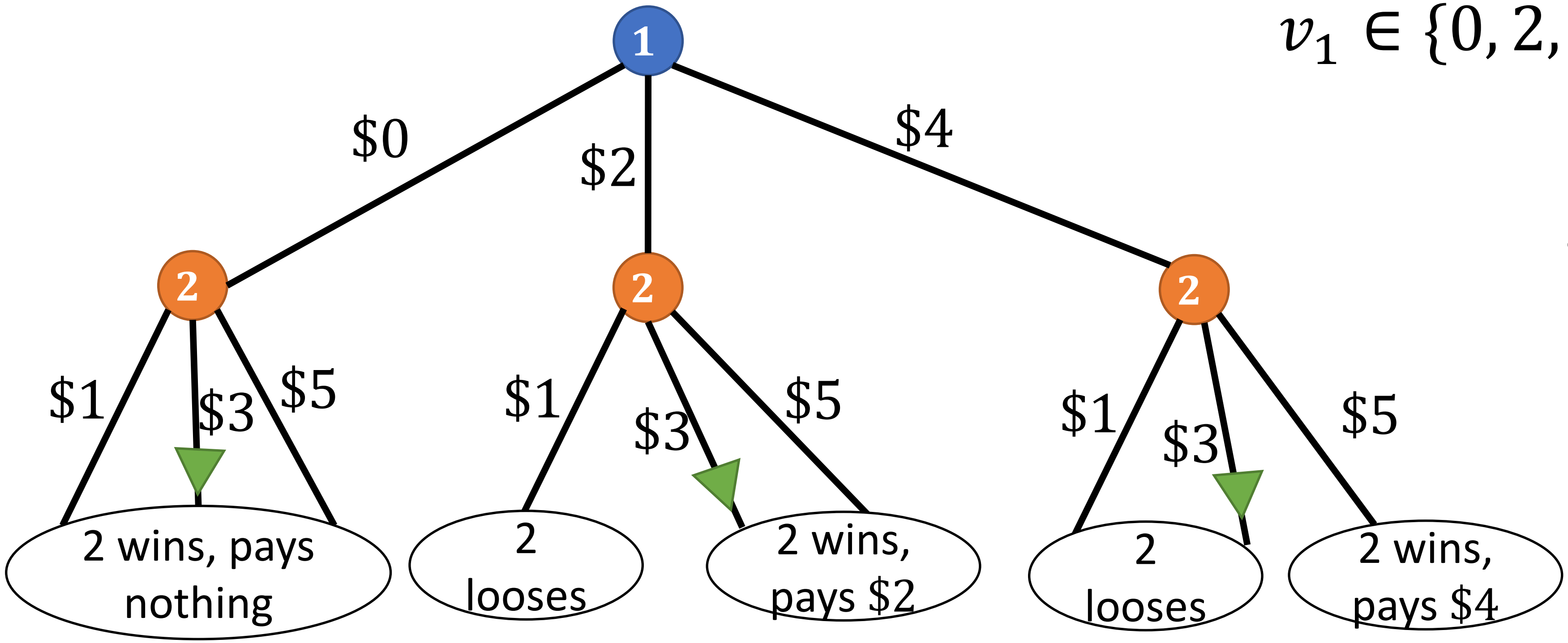

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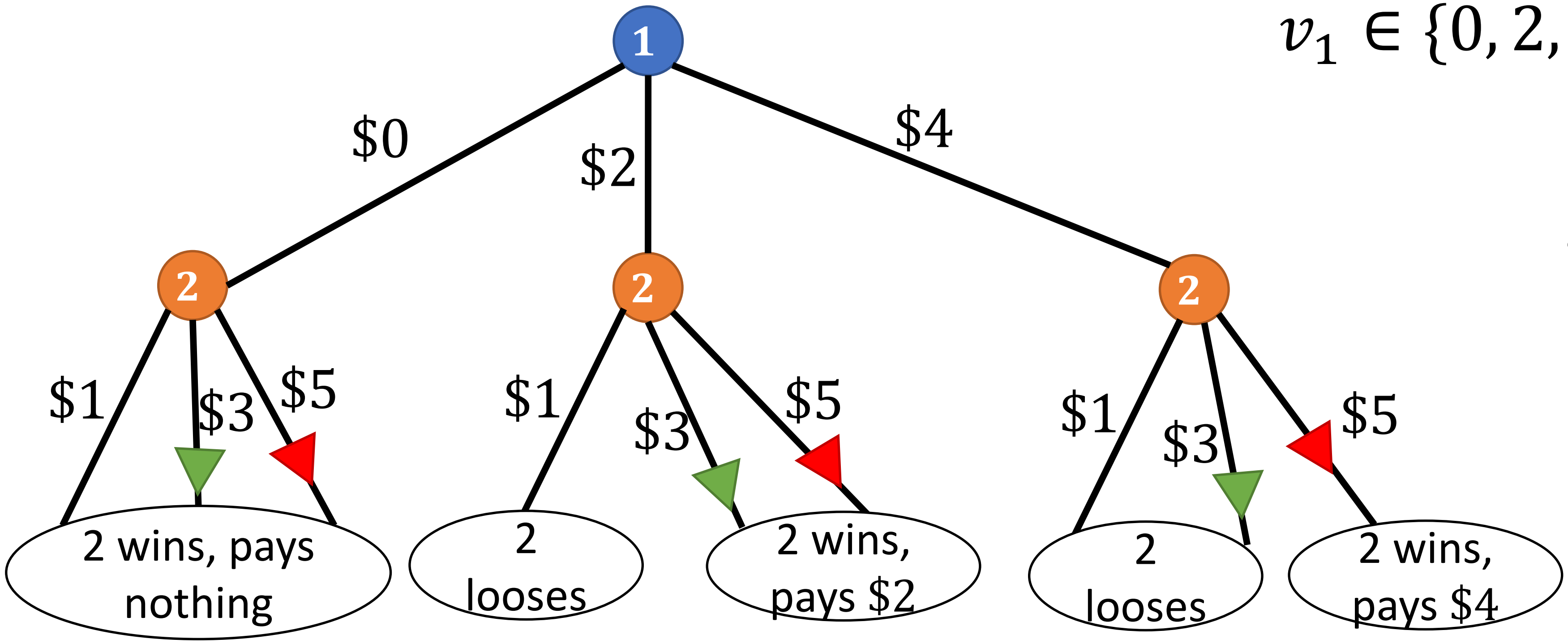

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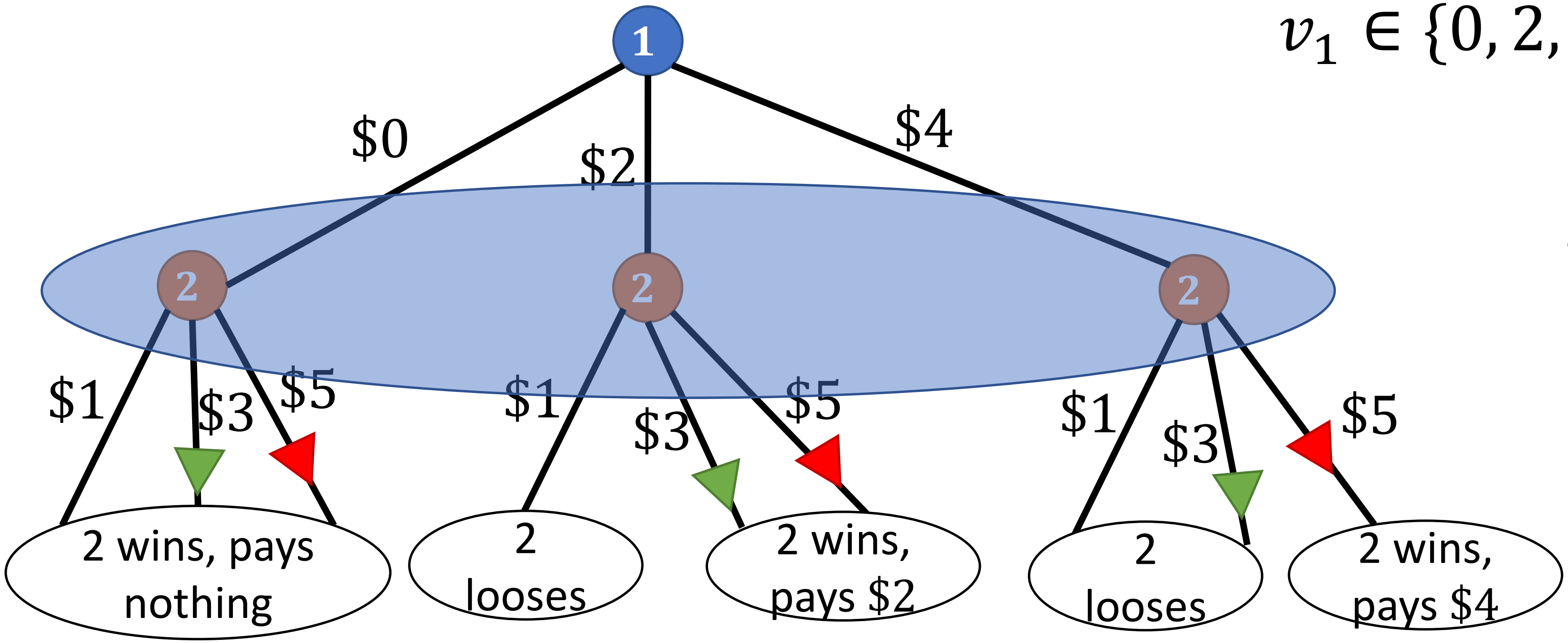

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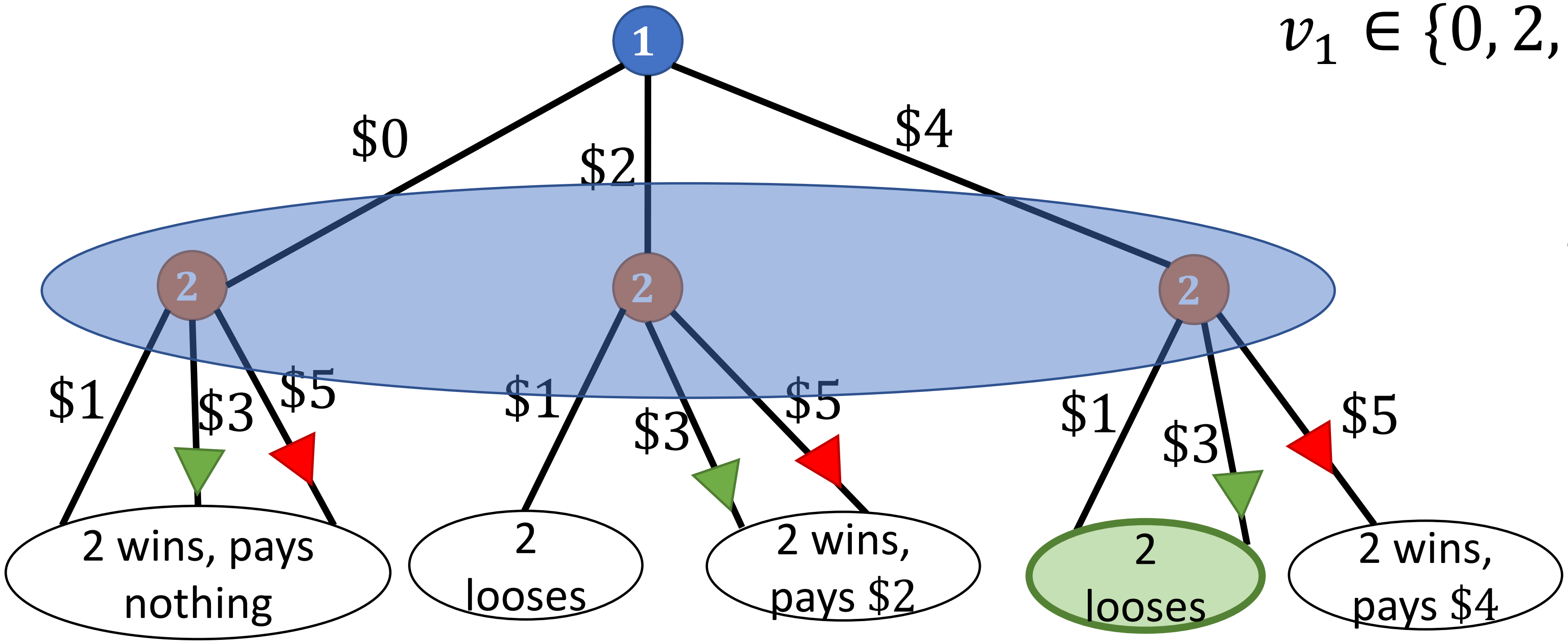

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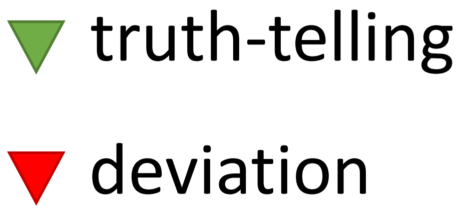

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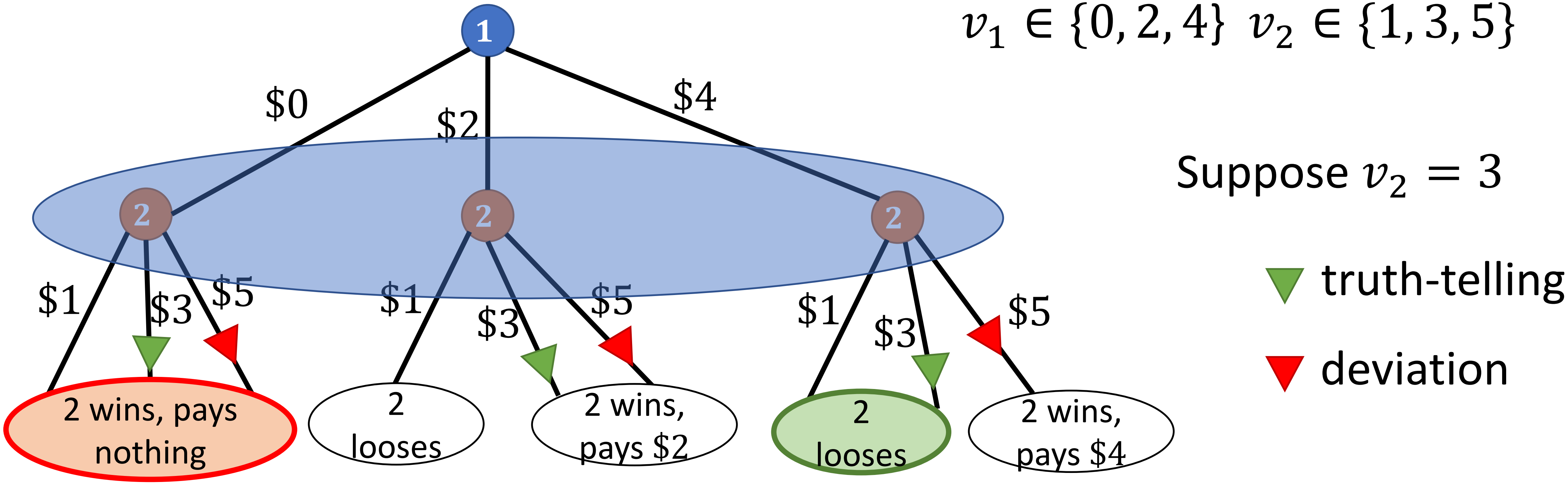

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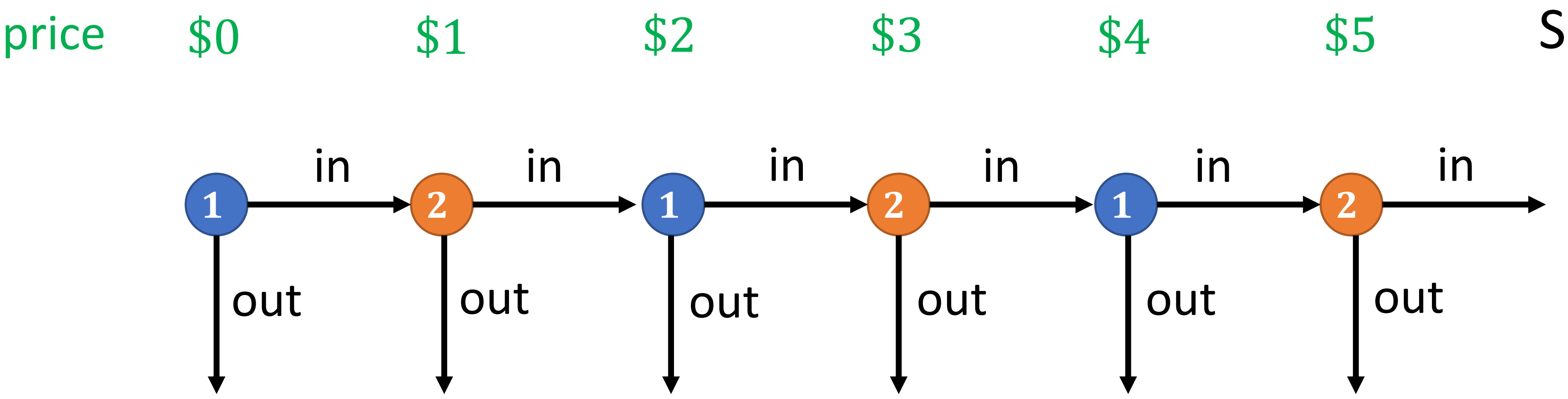

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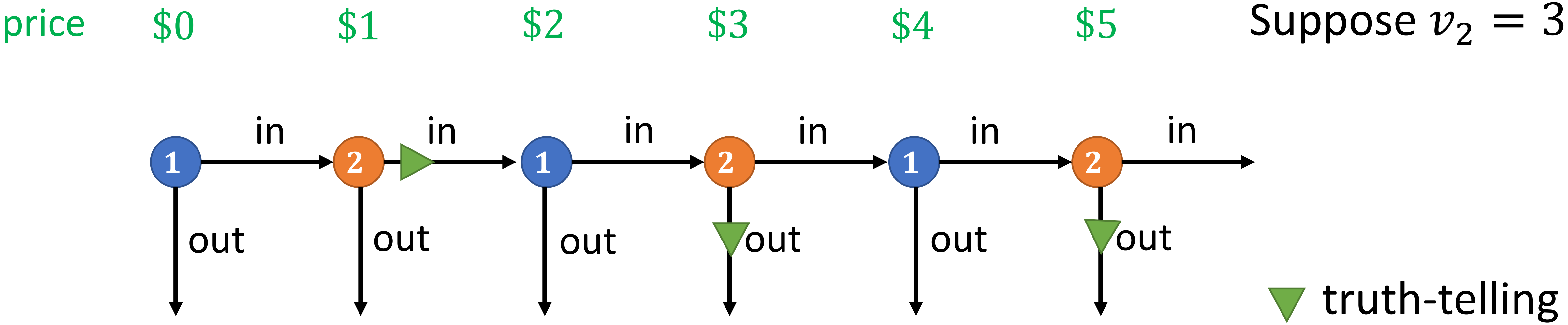

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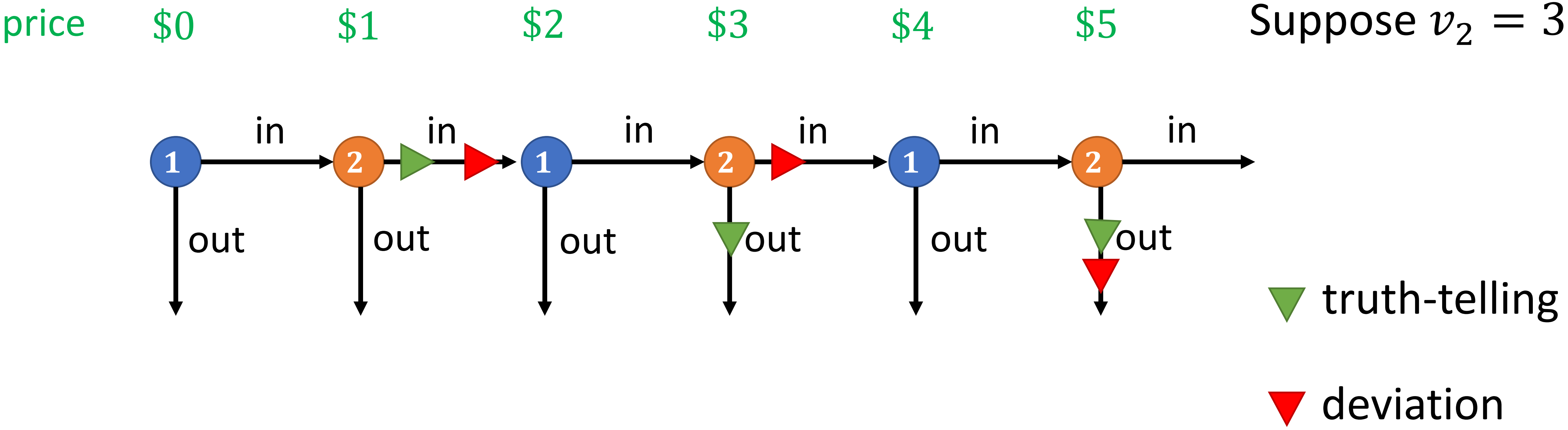

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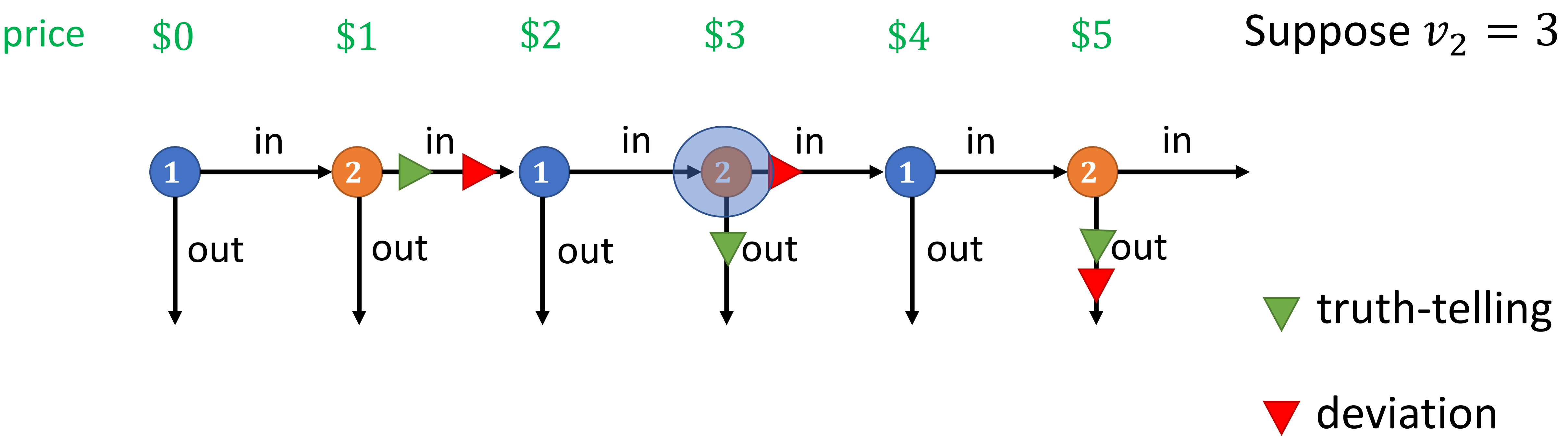

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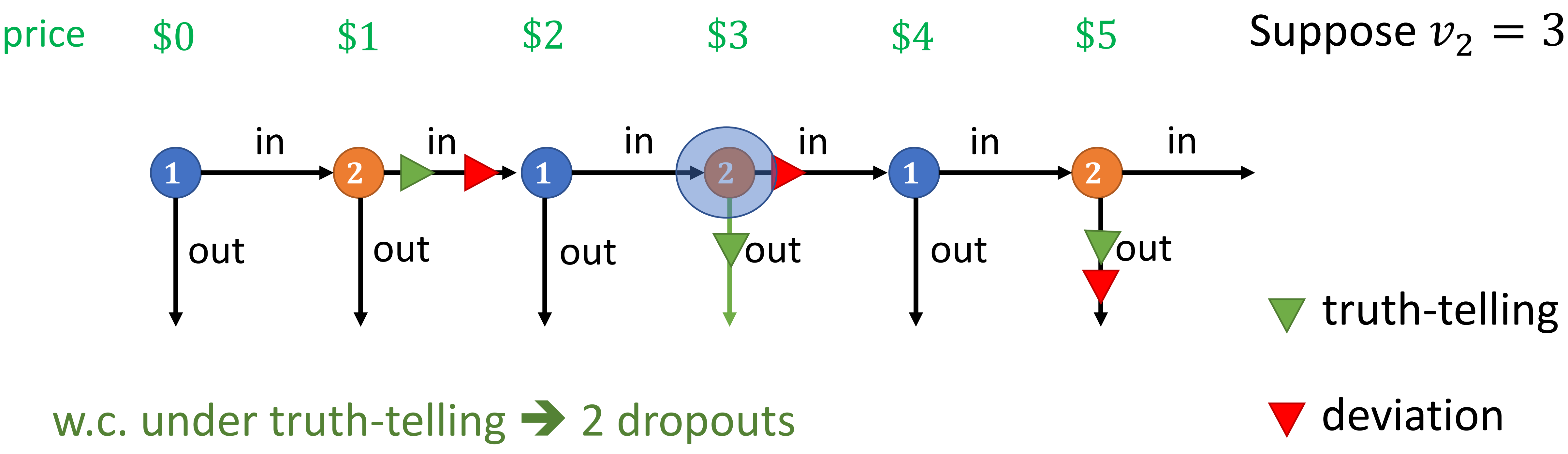

w.c. under truth-telling ➔ 2 dropouts

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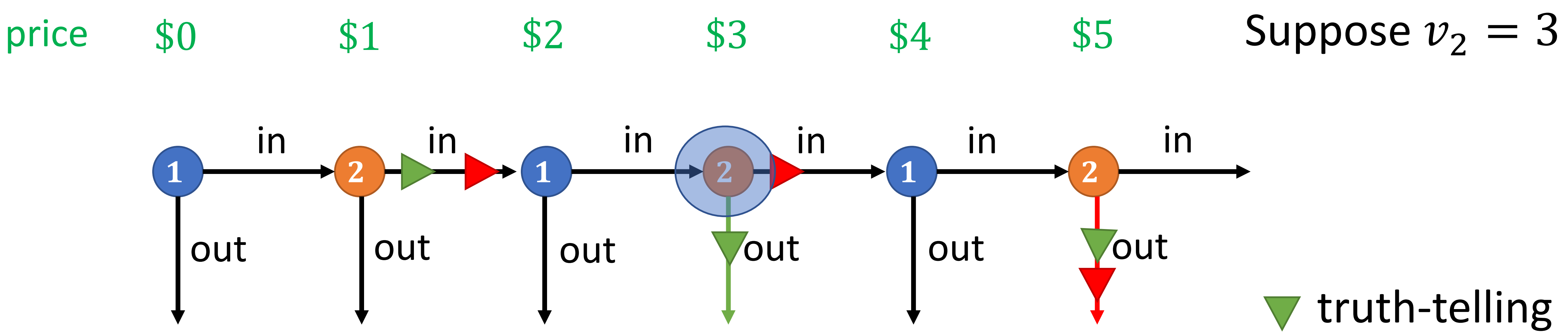

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b.c. under deviation ➔ 2 dropouts

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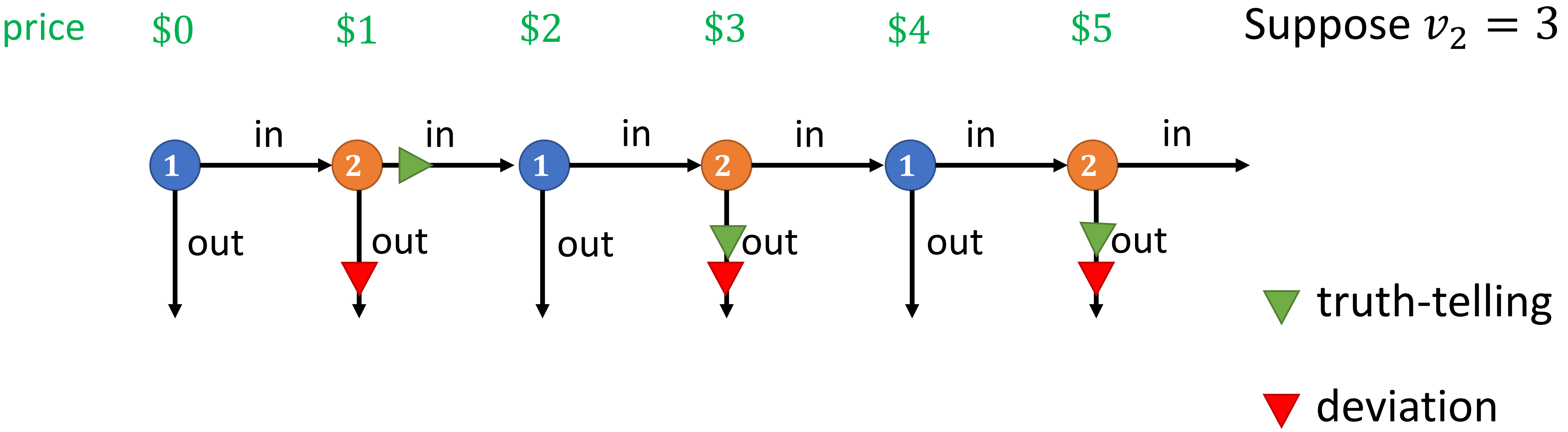

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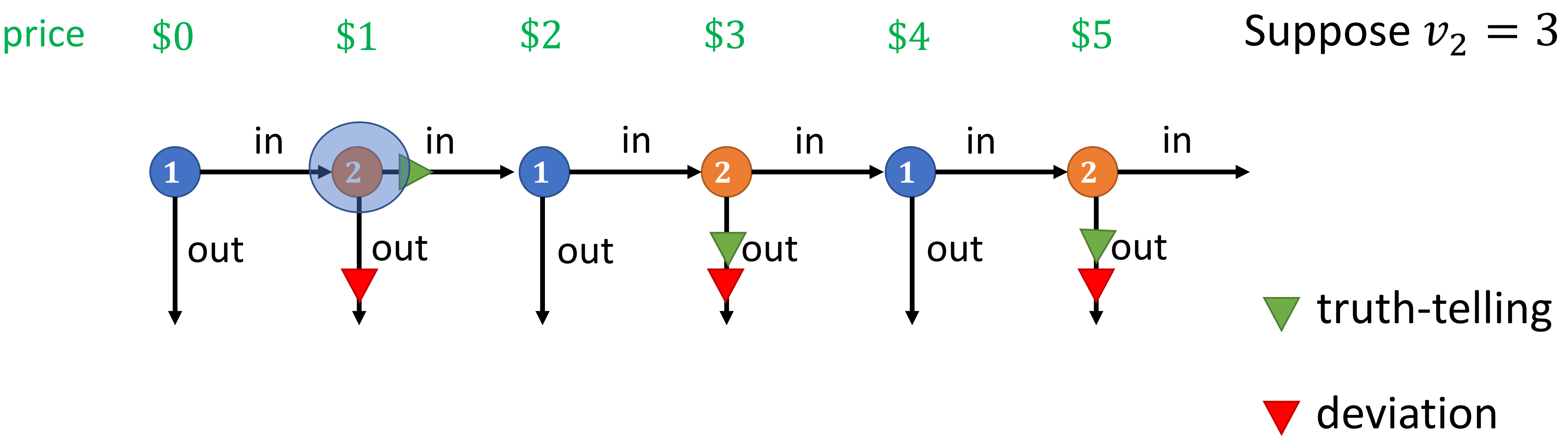

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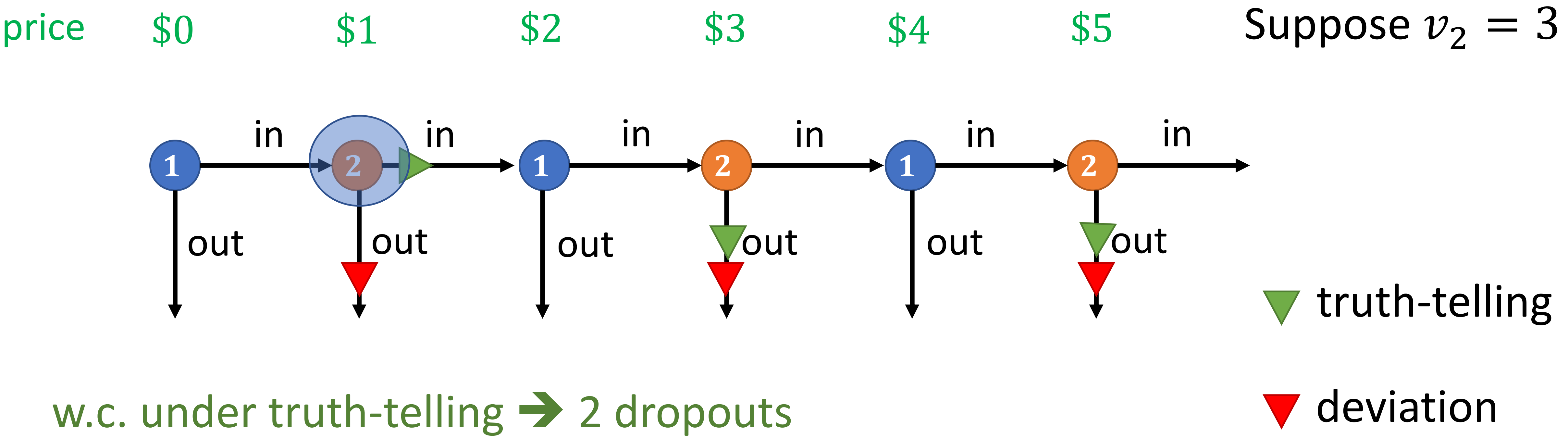

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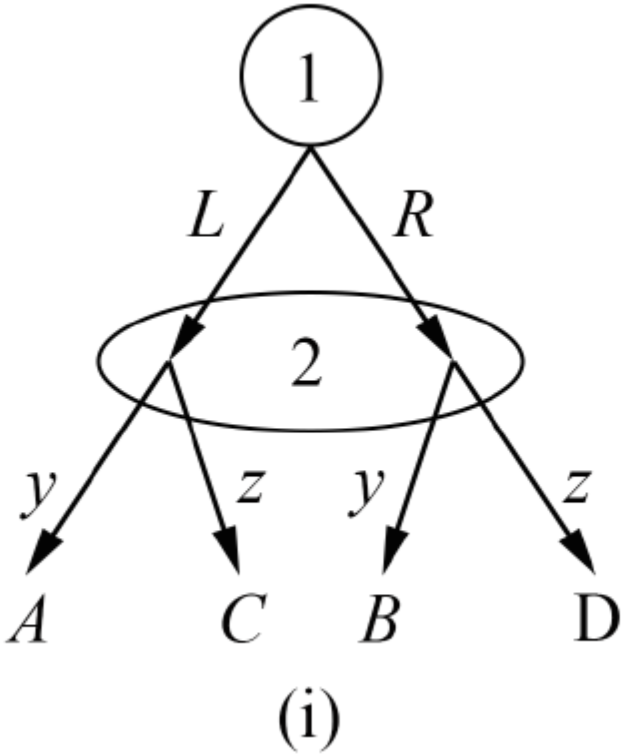
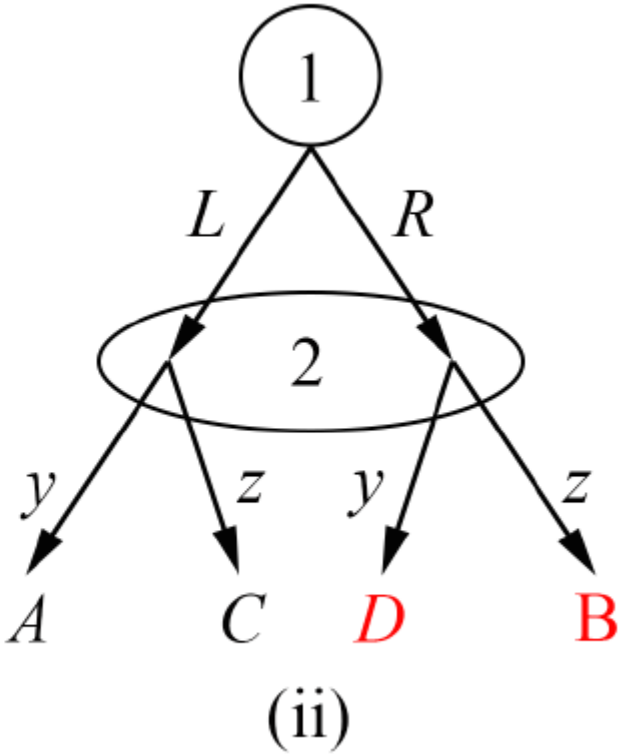
[Li] people are more prone to in RSD when preferences are given in advance

Experience

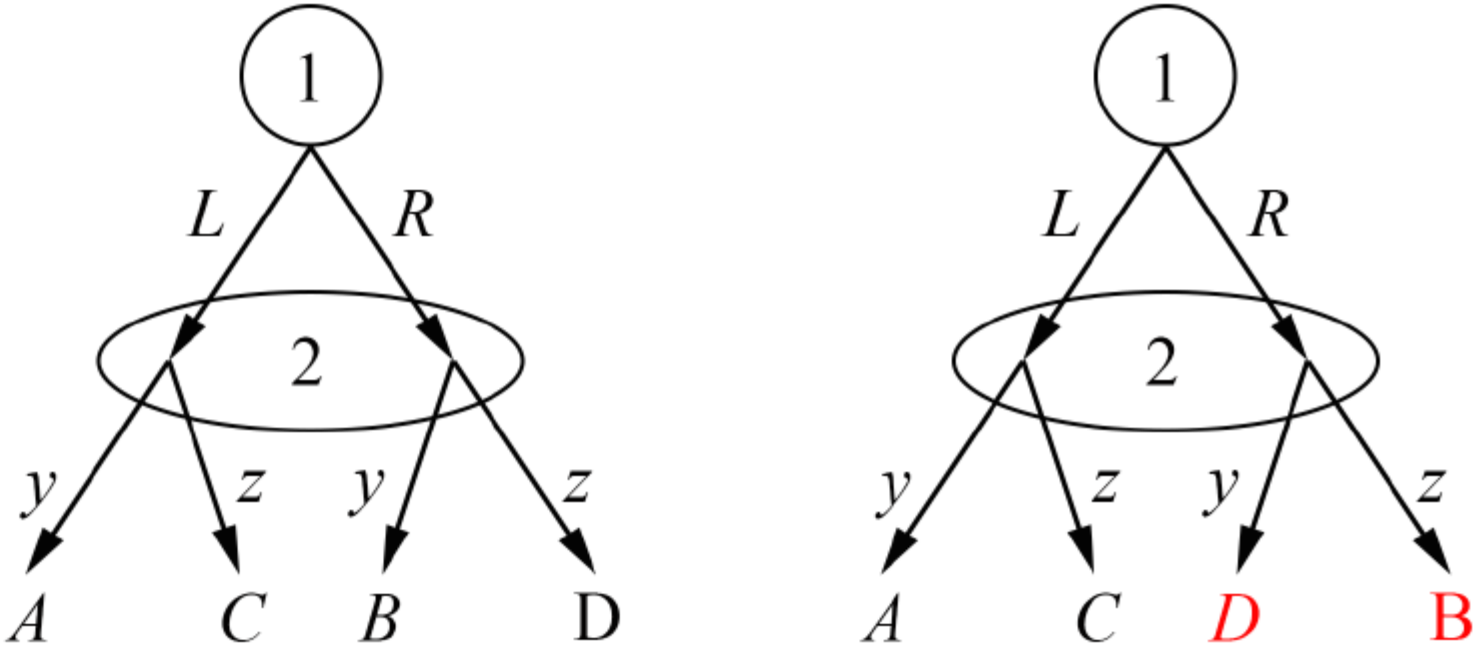




$A \succ_1 B \succ_1 C \succ_1 D$

Playing L is a dominant strategy at (i) but not at (ii)

Experience



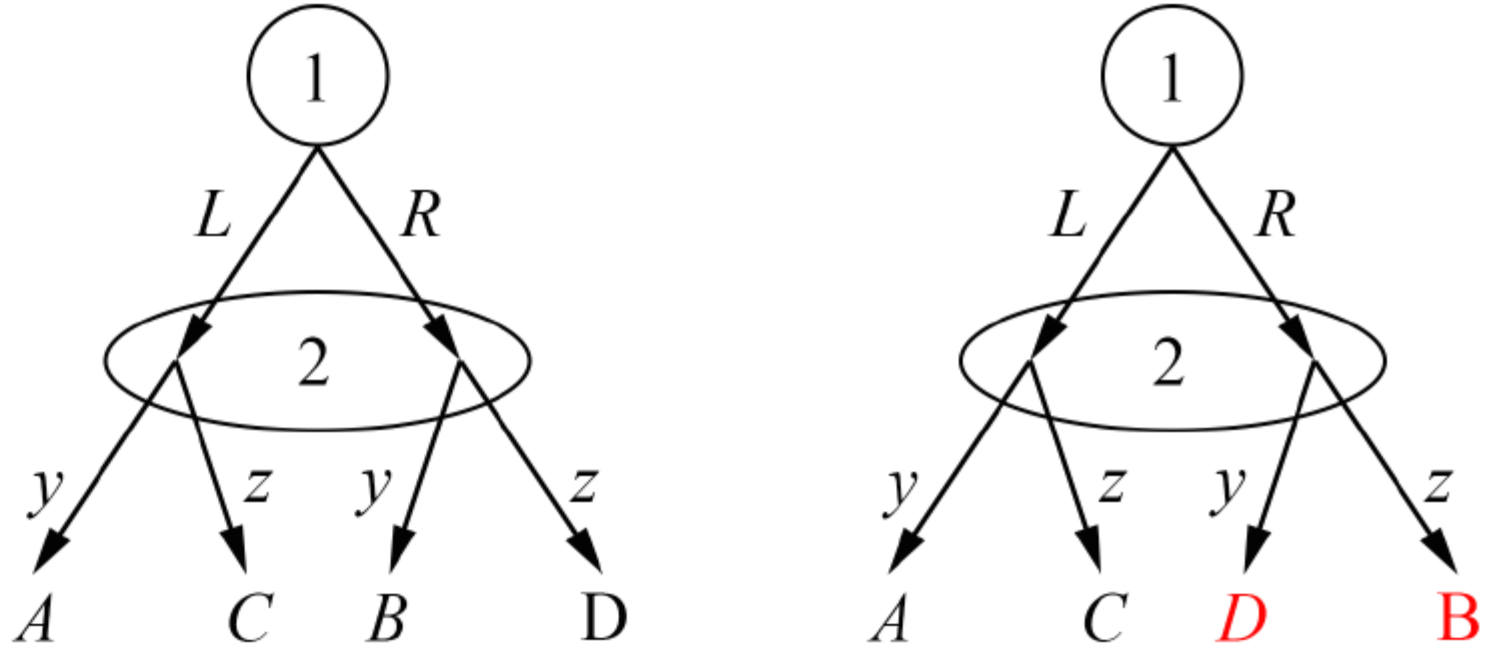
(i) (ii)

$A \succ_1 B \succ_1 C \succ_1 D$

Playing L is a dominant strategy at (i) but not at (ii)

Player 1's experience is very similar

Experience

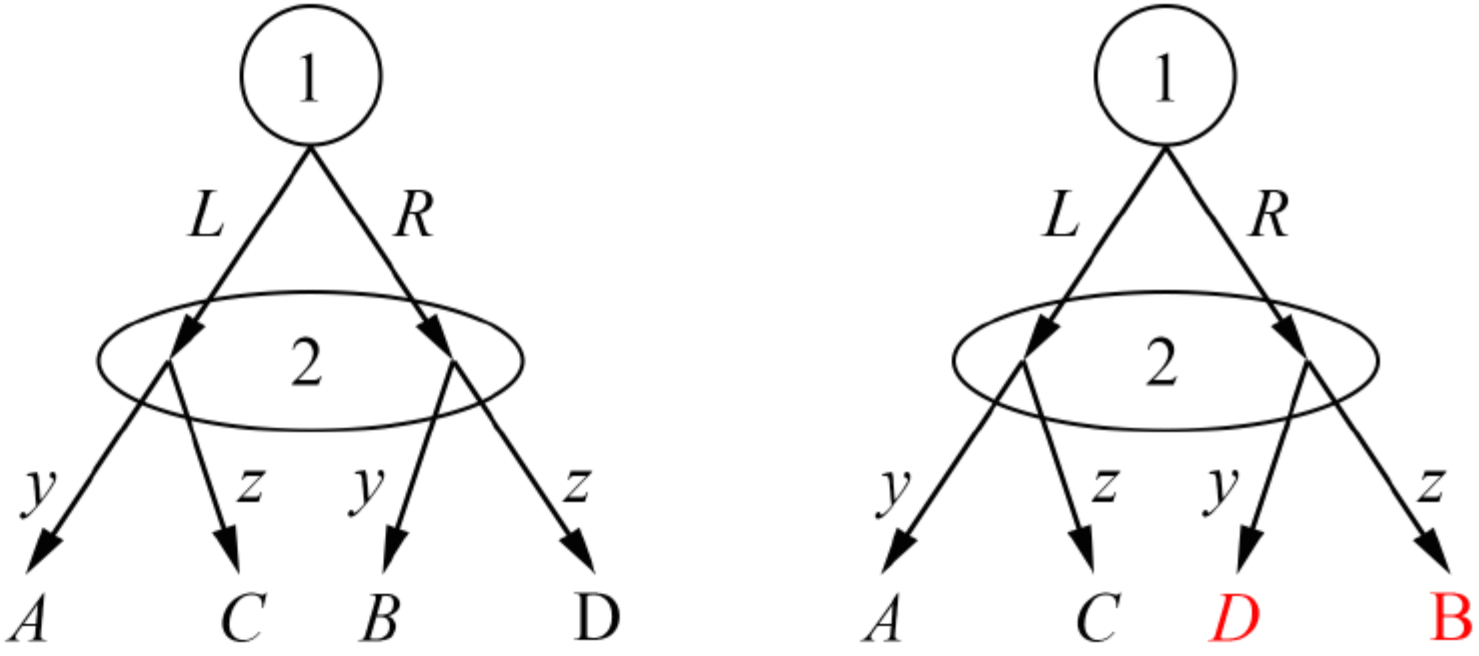


(i) (ii)

$A \succ_1 B \succ_1 C \succ_1 D$

Experience ψ_1	Associated Outcomes
$\{I_1\}$	$\emptyset$
$\{I_1, L\}$	A, C
$\{I_1, R\}$	B, D

Experience



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$A \succ_1 B \succ_1 C \succ_1 D$

Experience ψ_1	Associated Outcomes
$\{I_1\}$	$\emptyset$
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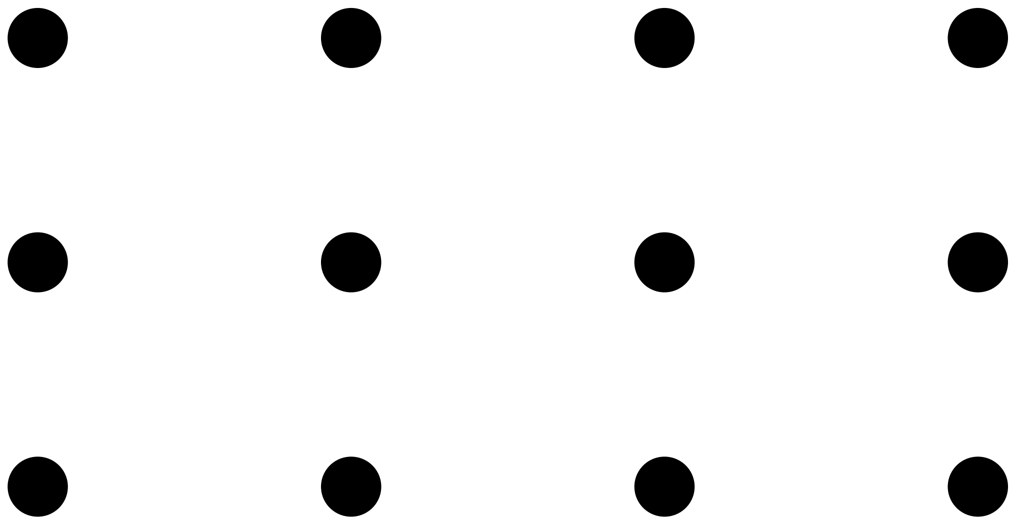
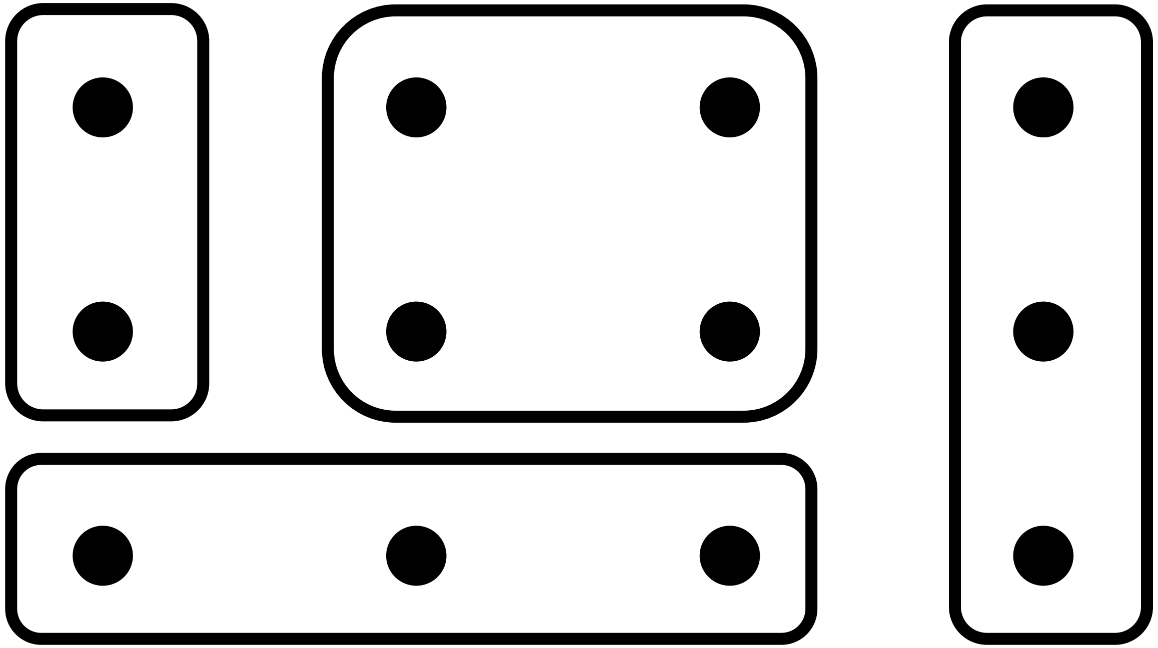
G and G' are i-indistinguishable
if they generate the same experience
for i

Equivalence classes

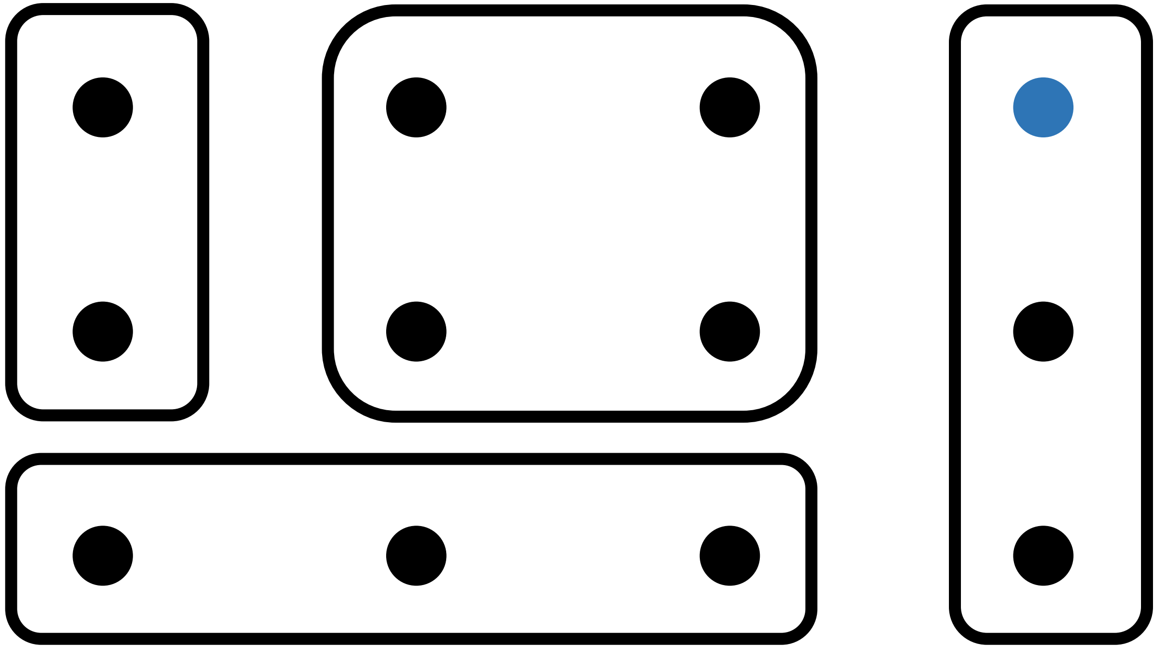


Equivalence classes



games that are i-indistinguishable

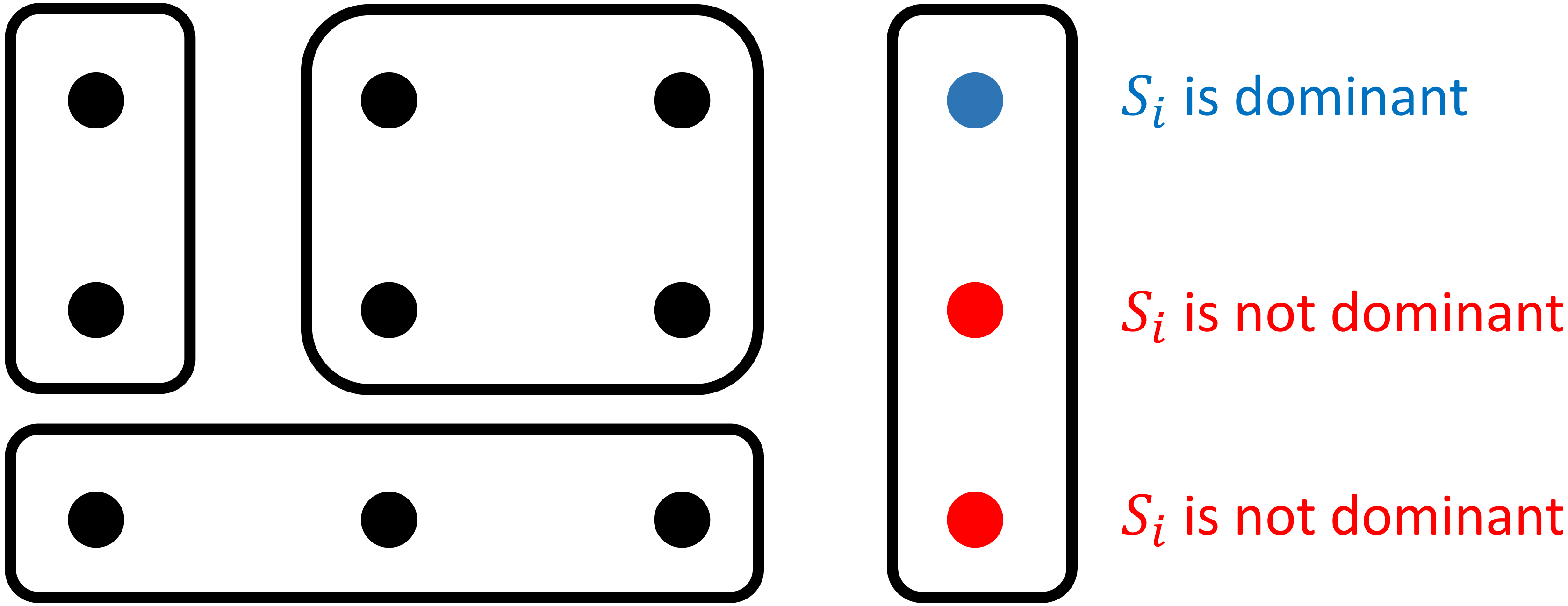
Equivalence classes



S_i is dominant

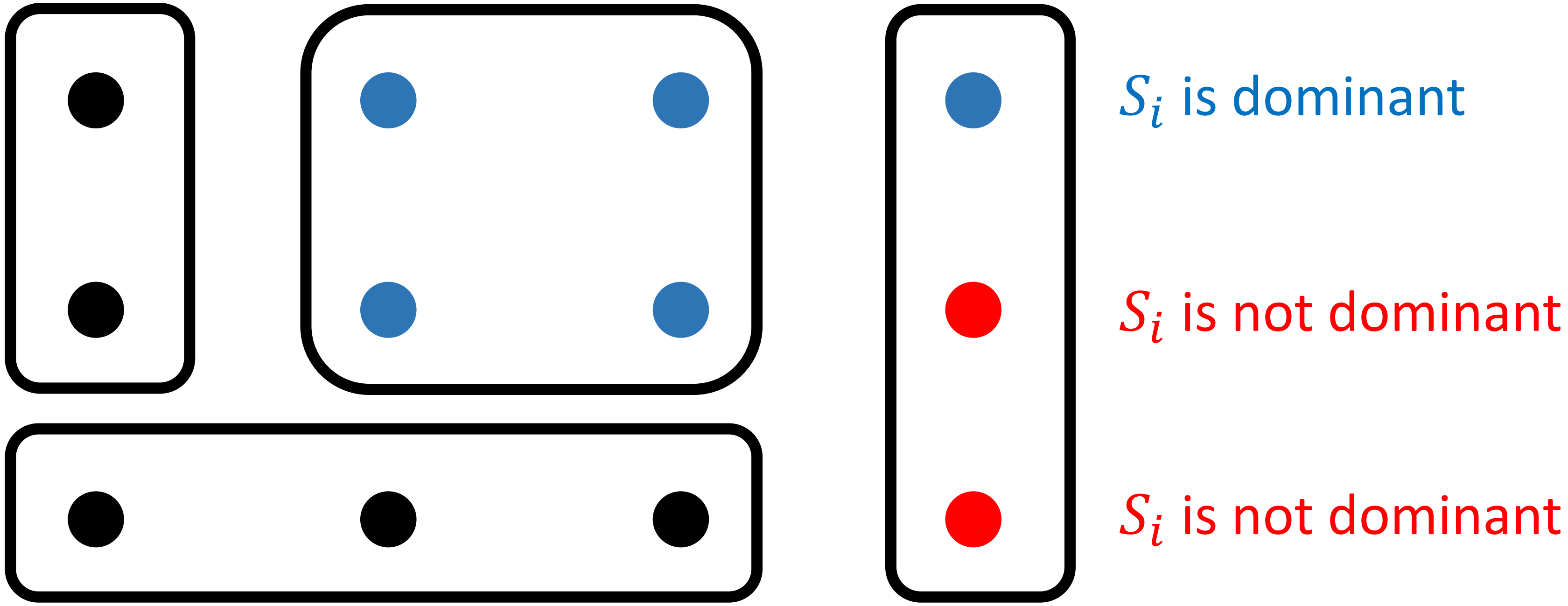
games that are i-indistinguishable

Equivalence classes



games that are i-indistinguishable

Equivalence classes



games that are
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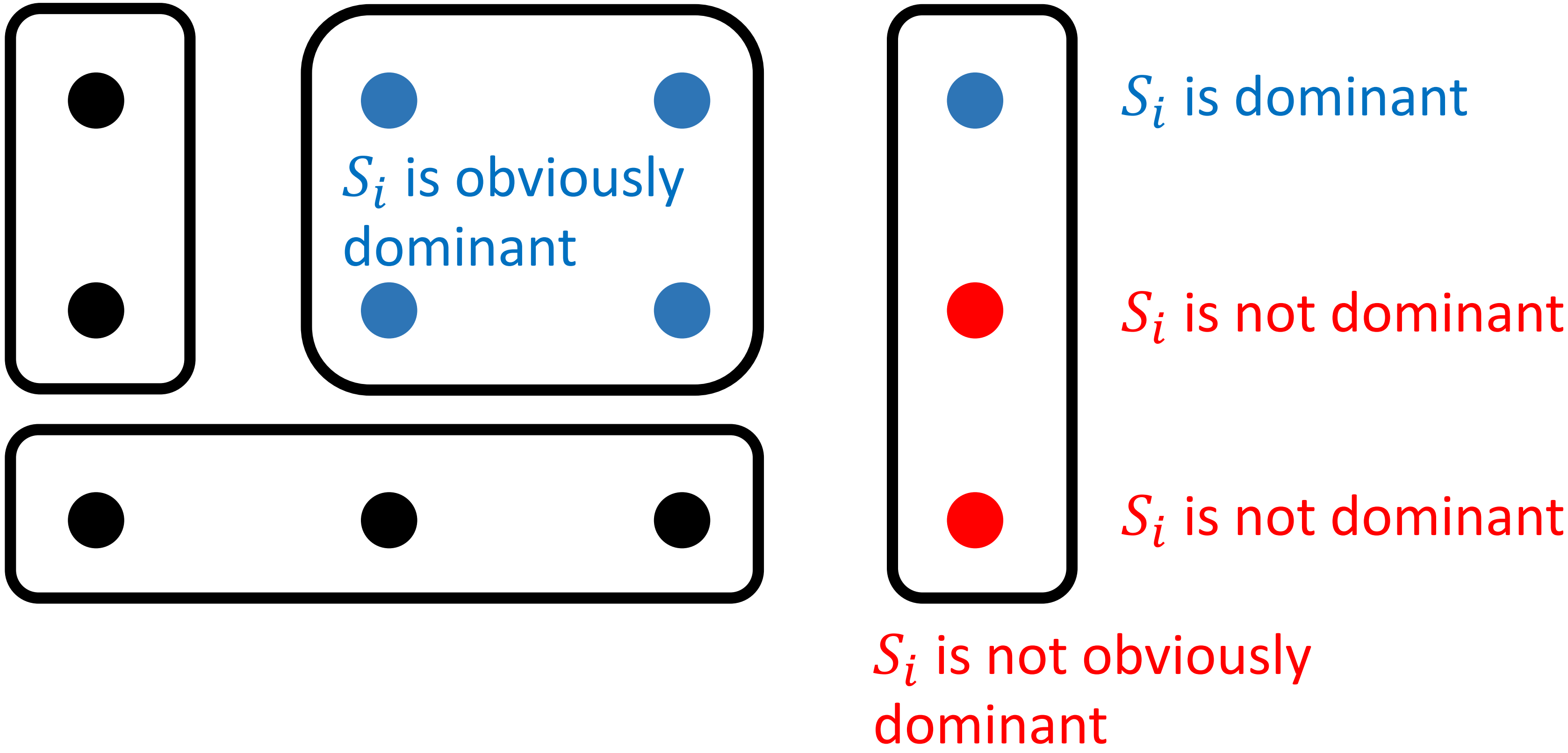
Equivalence classes



games that are
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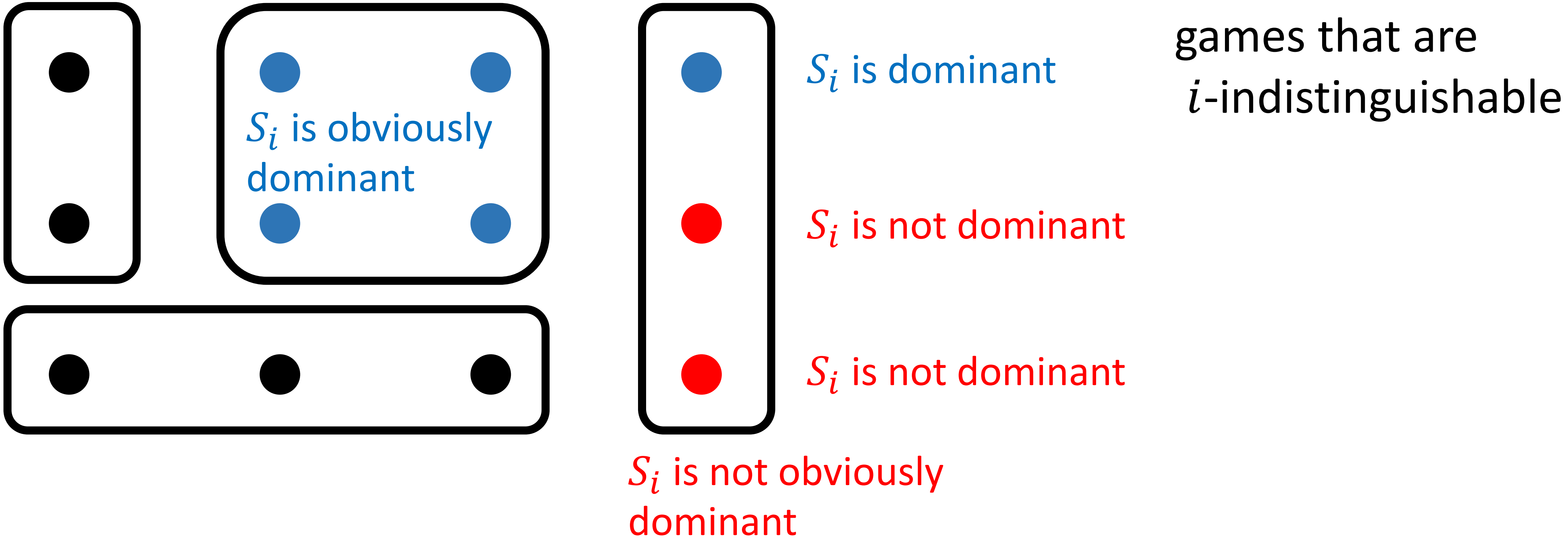
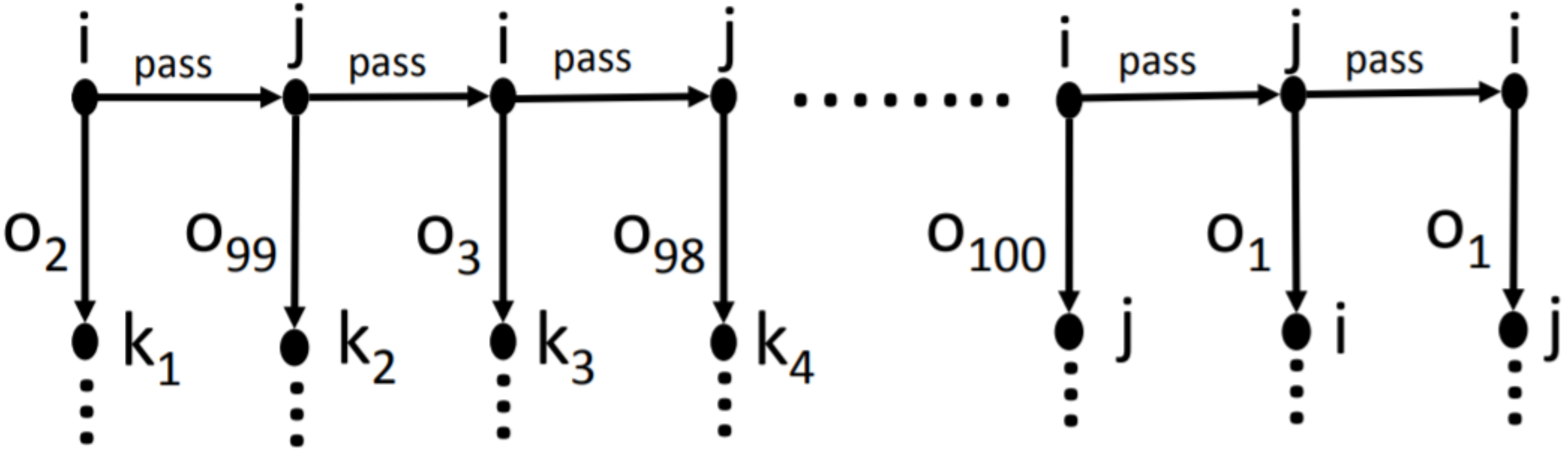
Equivalence classes



[Li] Thm. S_i is obviously dominant for G iff it is dominant in any G' in its equivalence class

Is this game "obvious"? [Pycia Troyan]

$$o_1 \succ_i o_2 \succ_i \ldots \succ_i o_{99} \succ_i o_{100}$$



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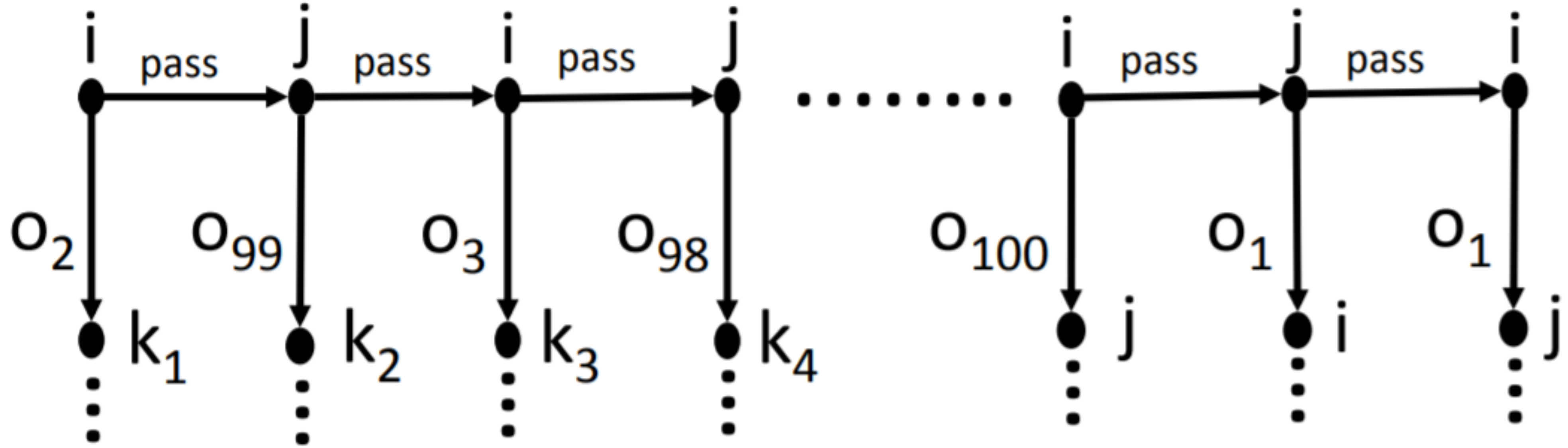

Passing is obviously dominant for agent i

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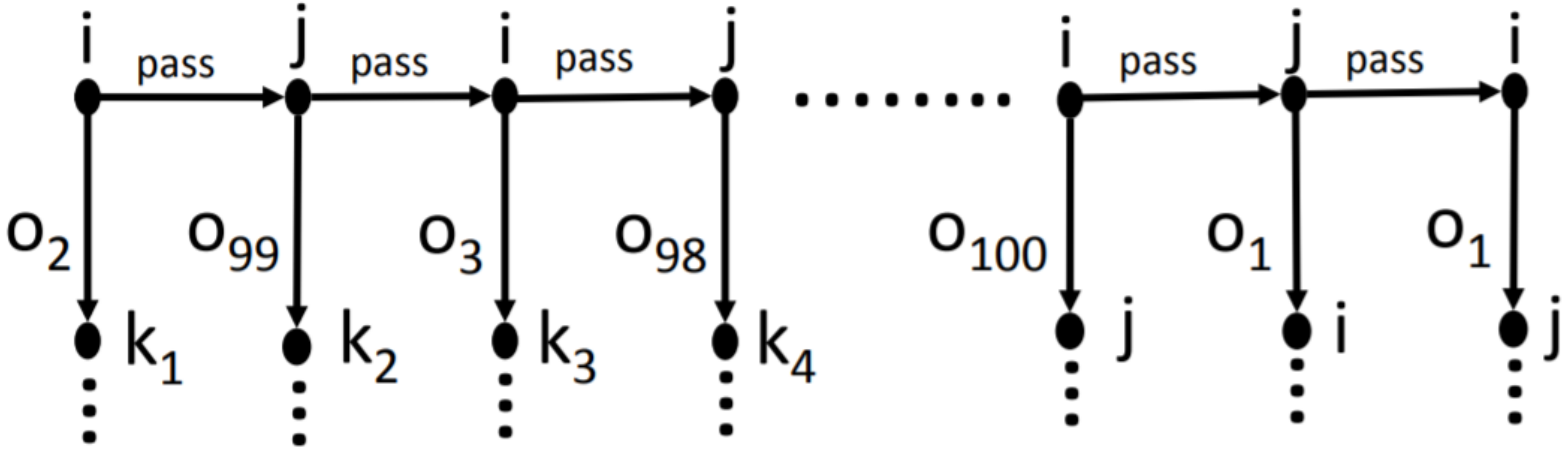

Passing is obviously dominant for agent i

Is chess obvious to the white player?

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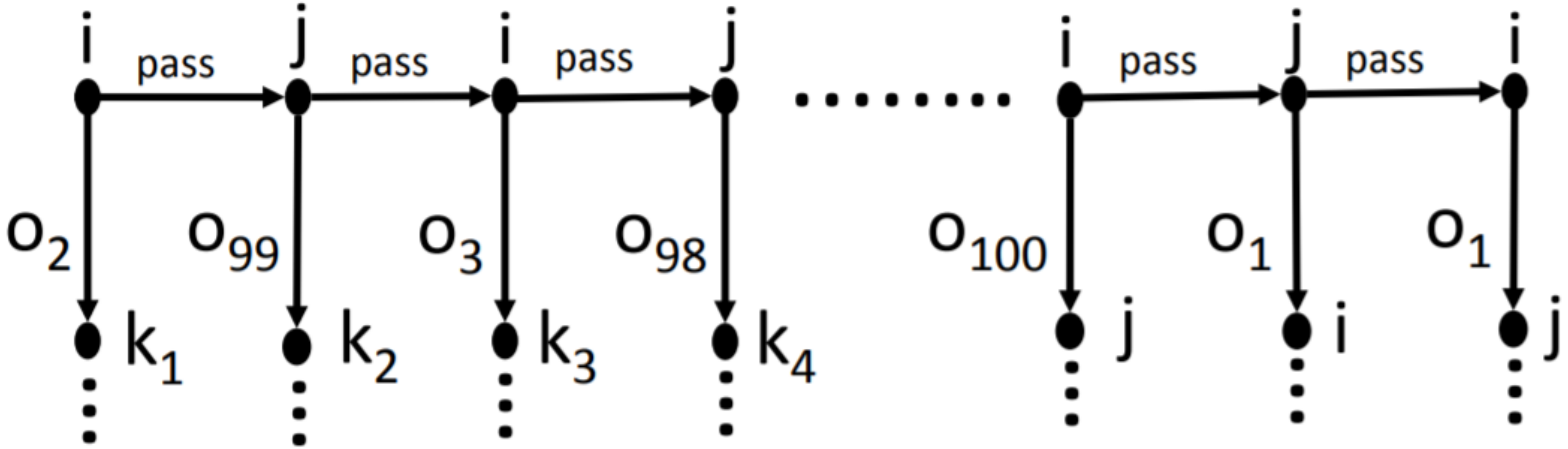
$o_1 \succ_i o_2 \succ_i \dots \succ_i o_{99} \succ_i o_{100}$

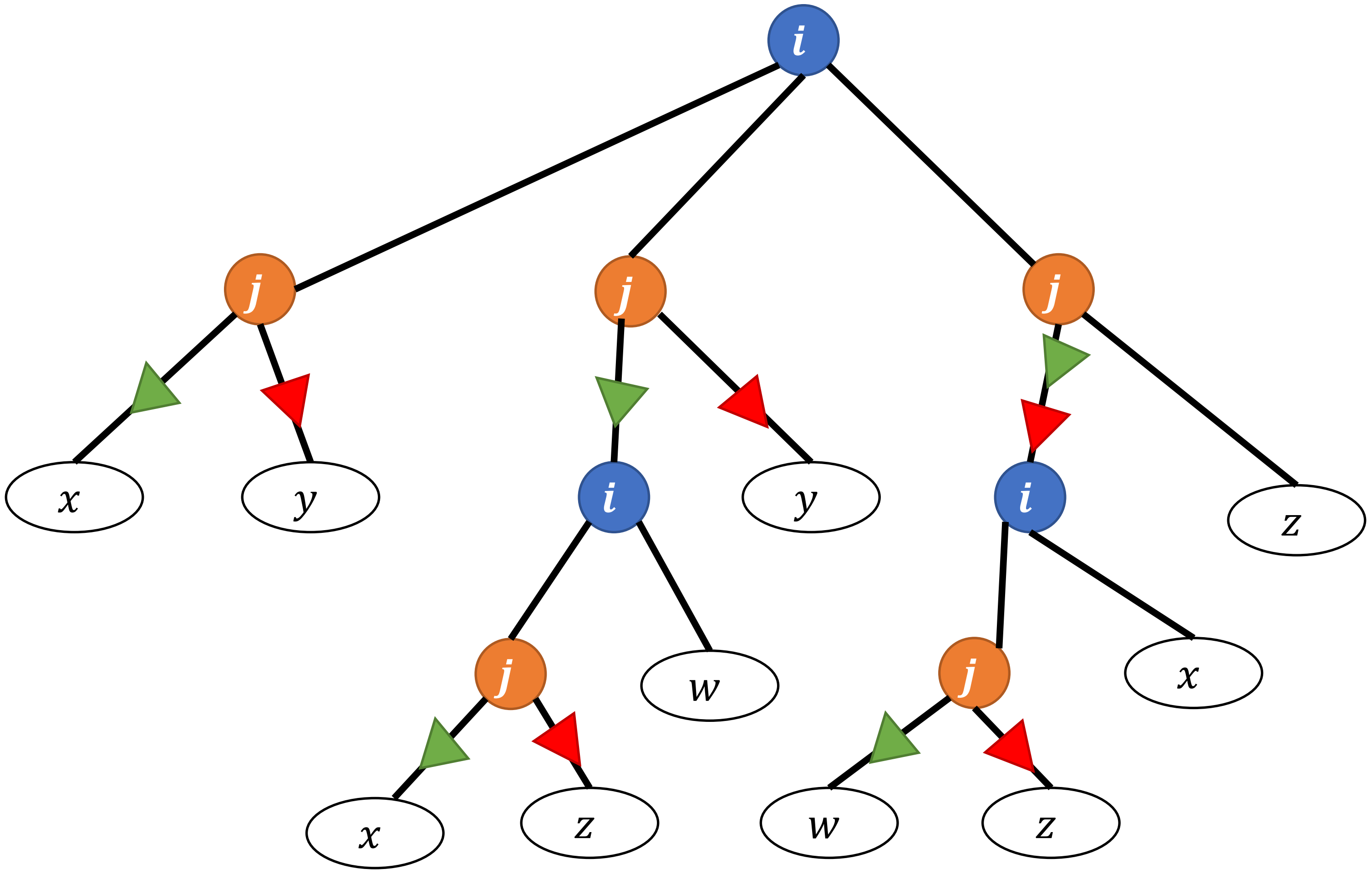


Passing is obviously dominant for agent i

Is chess obvious to the white player?

Some complexity notion is not captured by OSP theory

Strong OSP [Pycia Troyan]

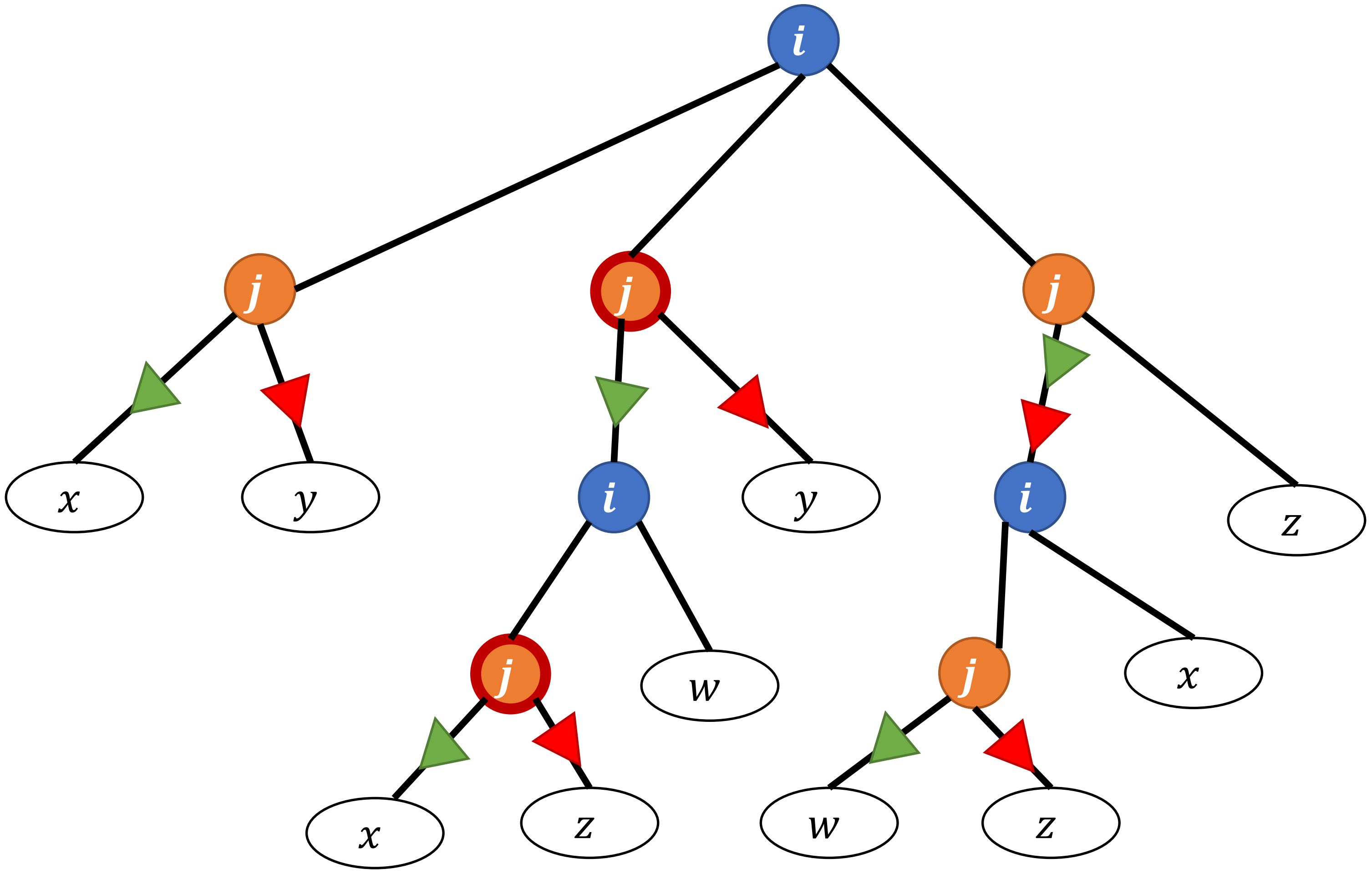


OSP:

the player has a **complete** strategic plan

$x \succ_j w \succ_j y \succ_j z$

Strong OSP [Pycia Troyan]



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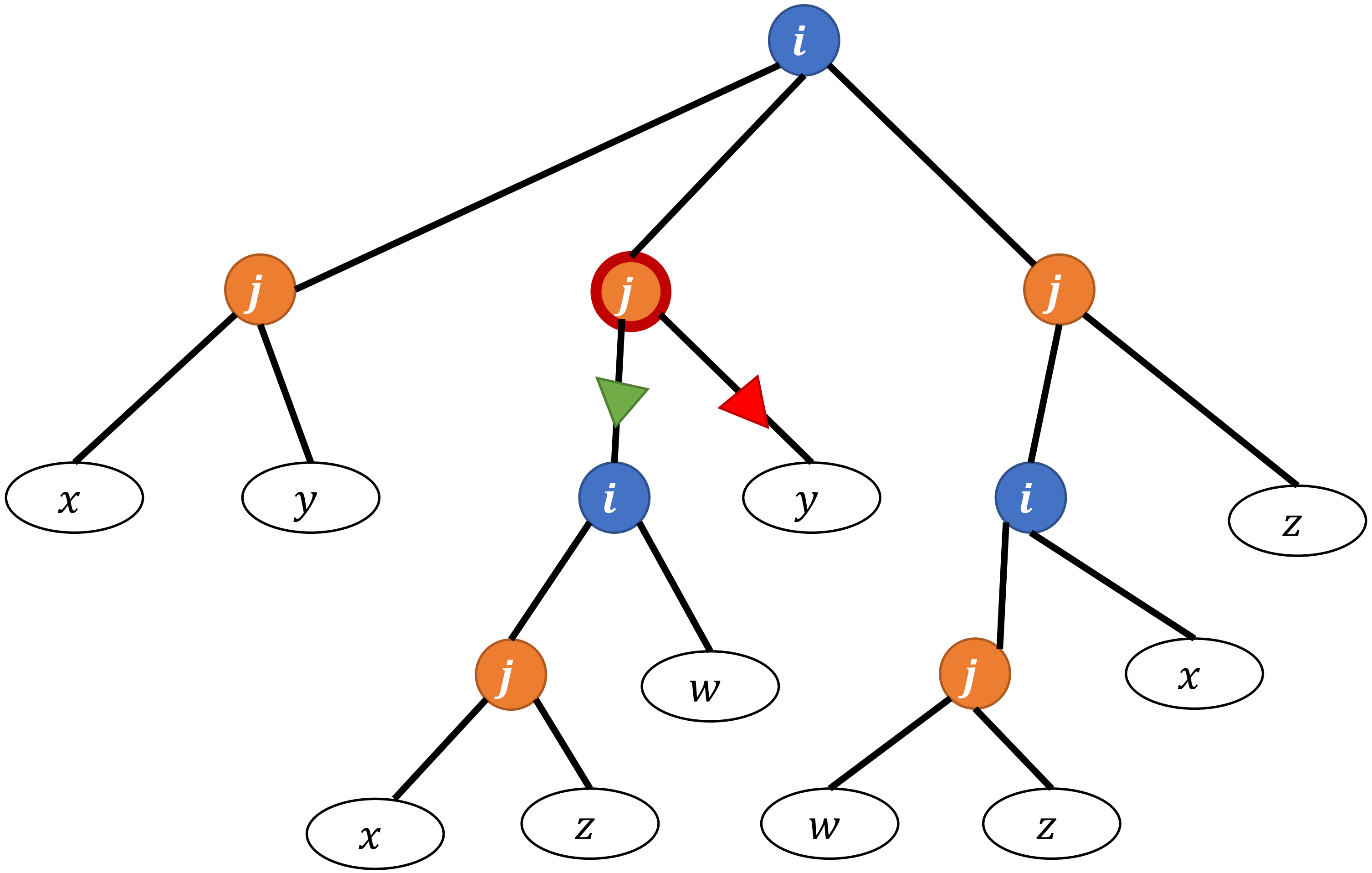

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Player cannot plan ahead

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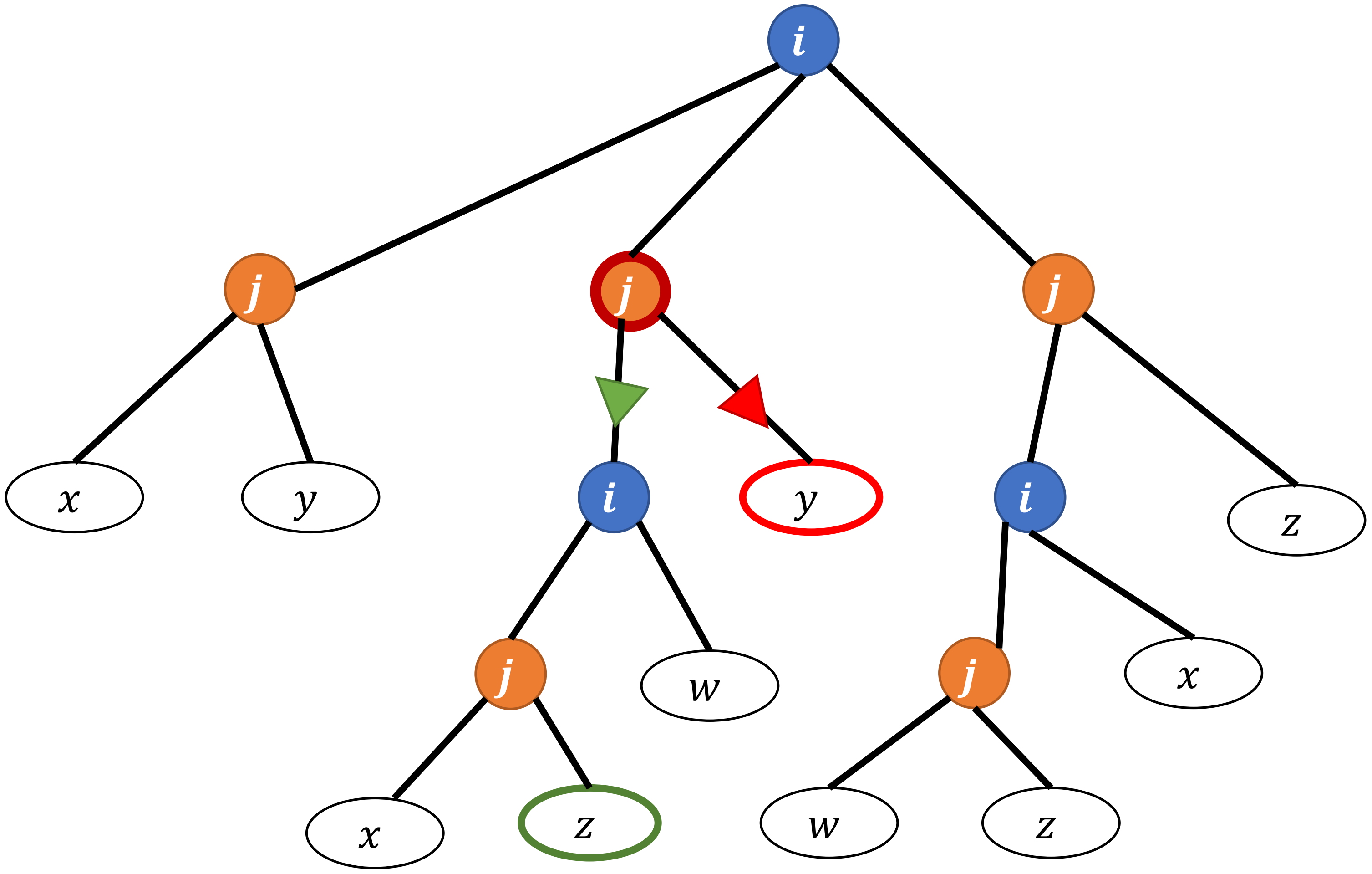

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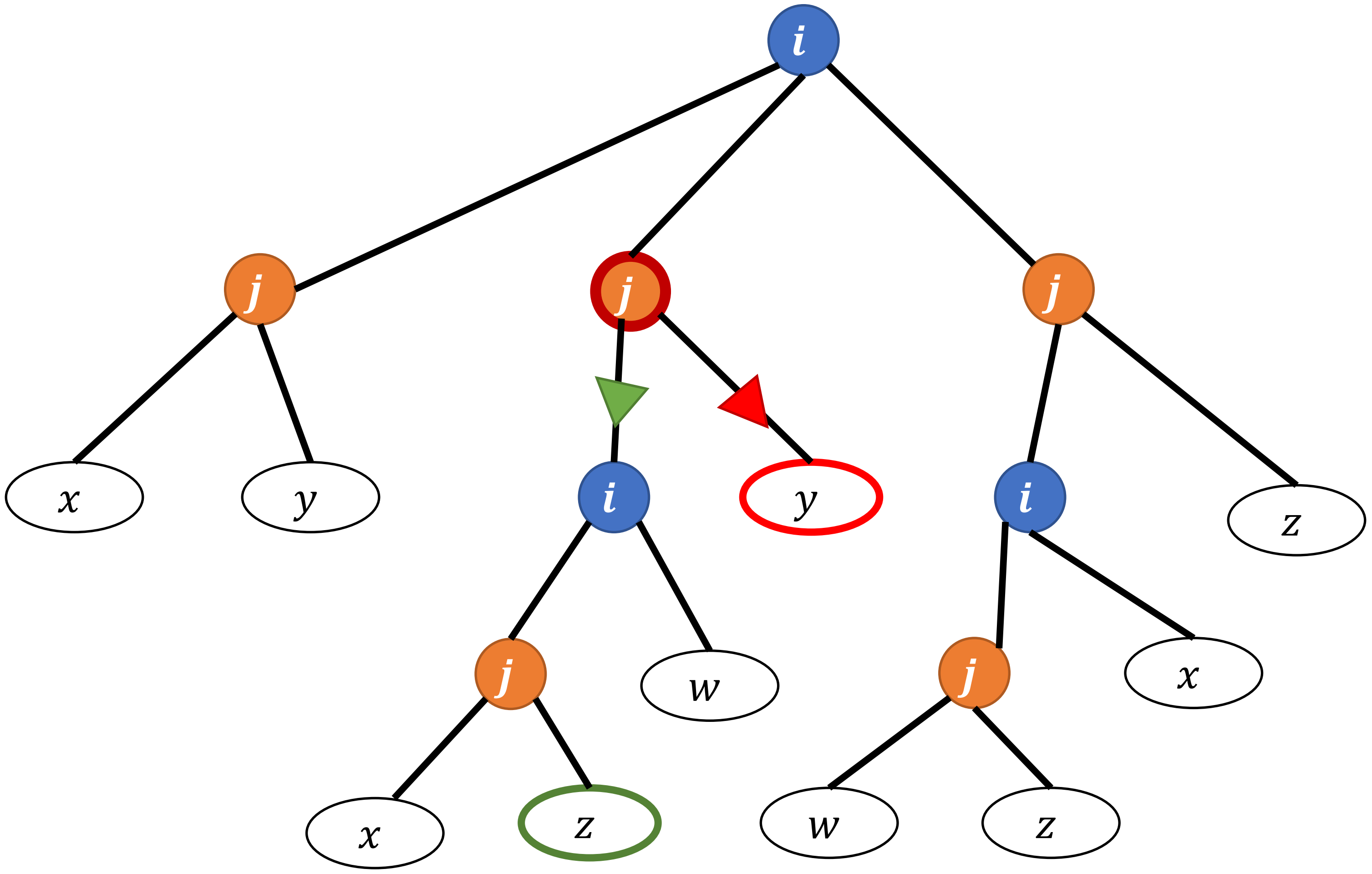


SOSP:

Player cannot plan ahead

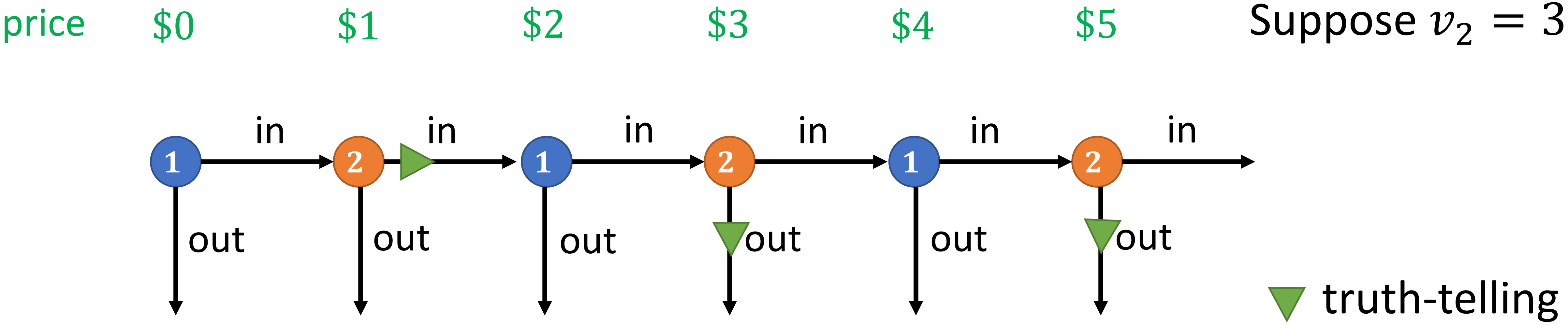
Important special cases:

- Random serial dictatorship
- Posted price mechanisms

$x \succ_j w \succ_j y \succ_j z$

Ascending price auctions are not SOSP

Suppose $v_2 = 3$



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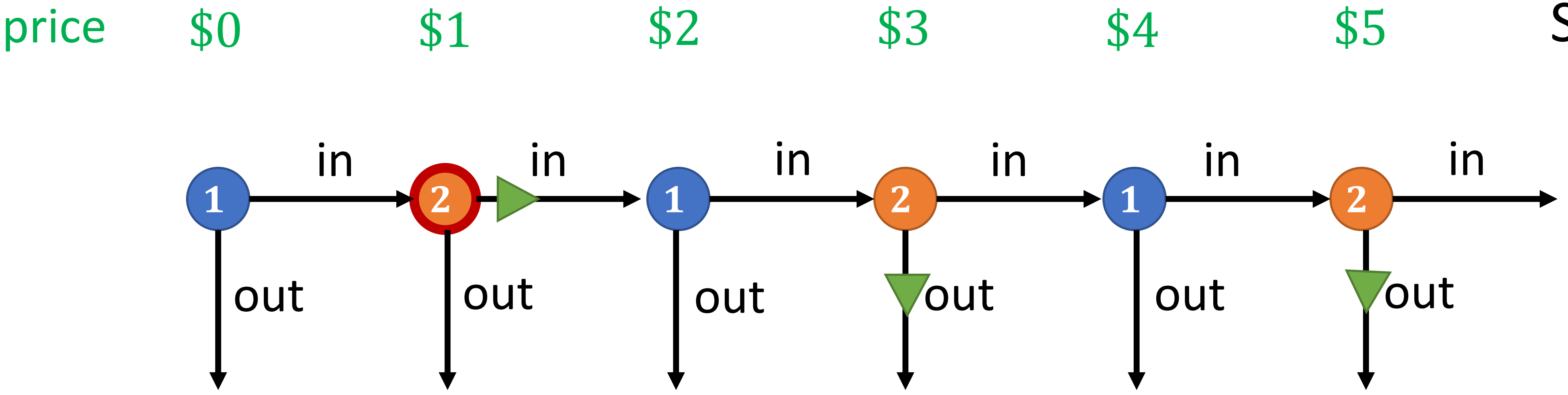
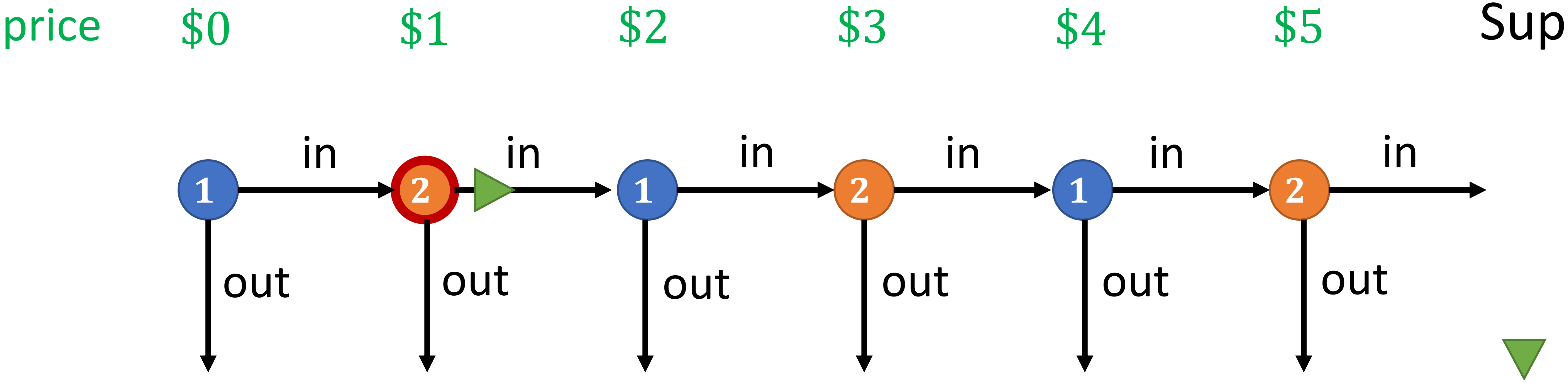


Suppose $v_2 = 3$

truth-telling

Ascending price auctions are not SOSP



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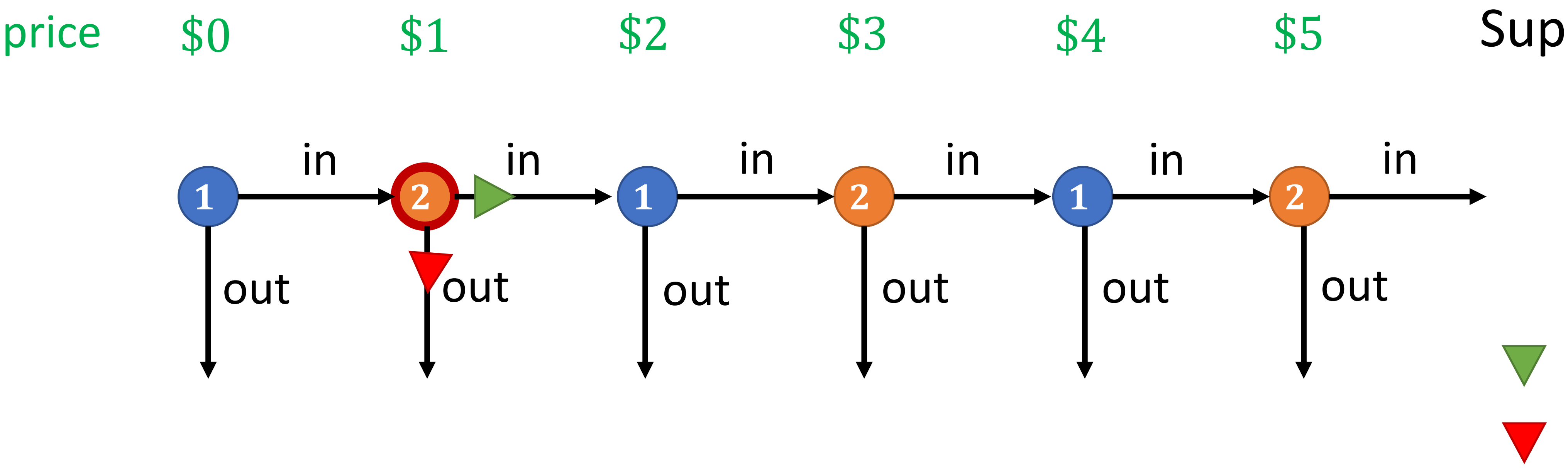

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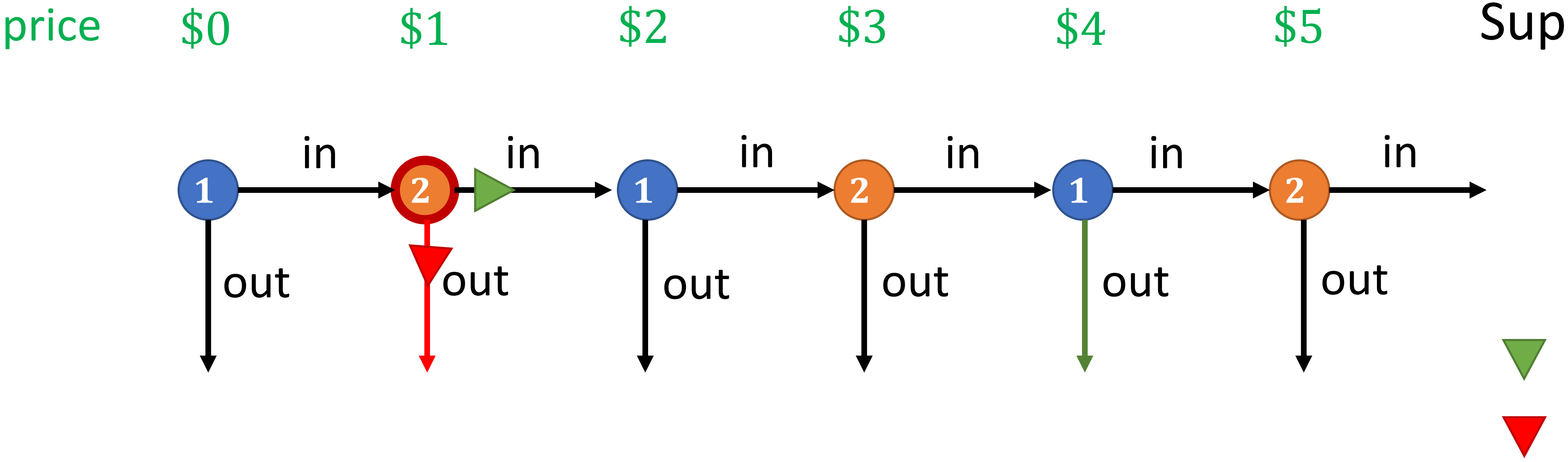

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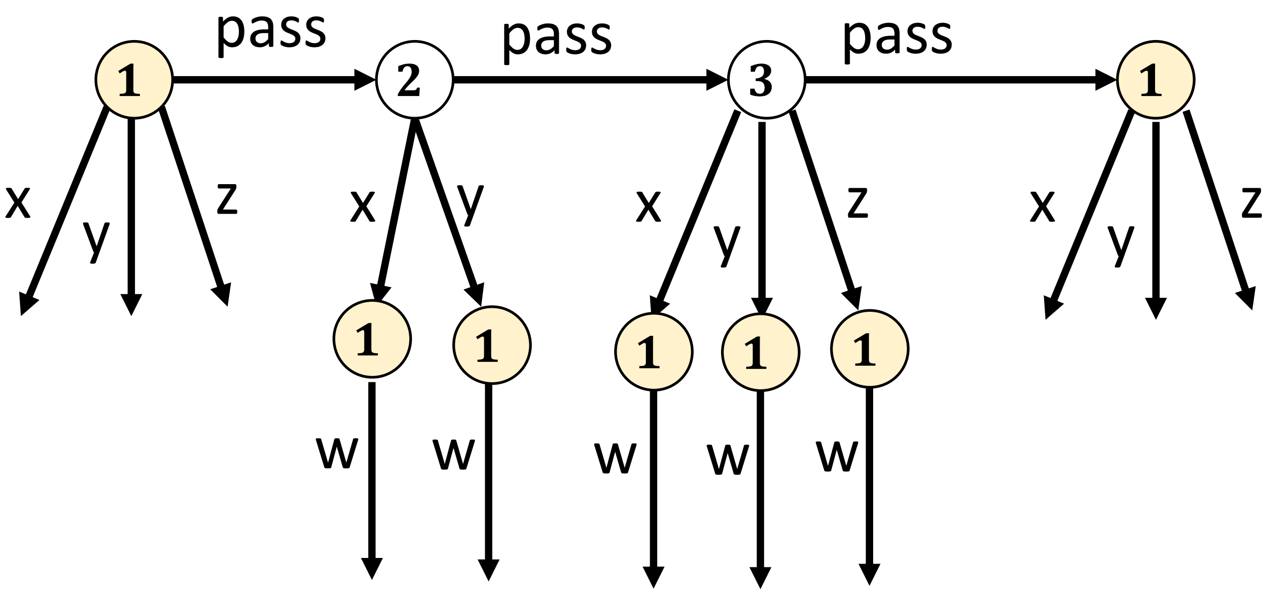
One Step Foresight (OSF) [Pycia Troyan]

[Bade Gonczarowski]



$w \succ_1 x \succ_1 y \succ_1 z$

OSF:

Players are able to plan one step ahead

i.e. have a strategic plan for the current and next node

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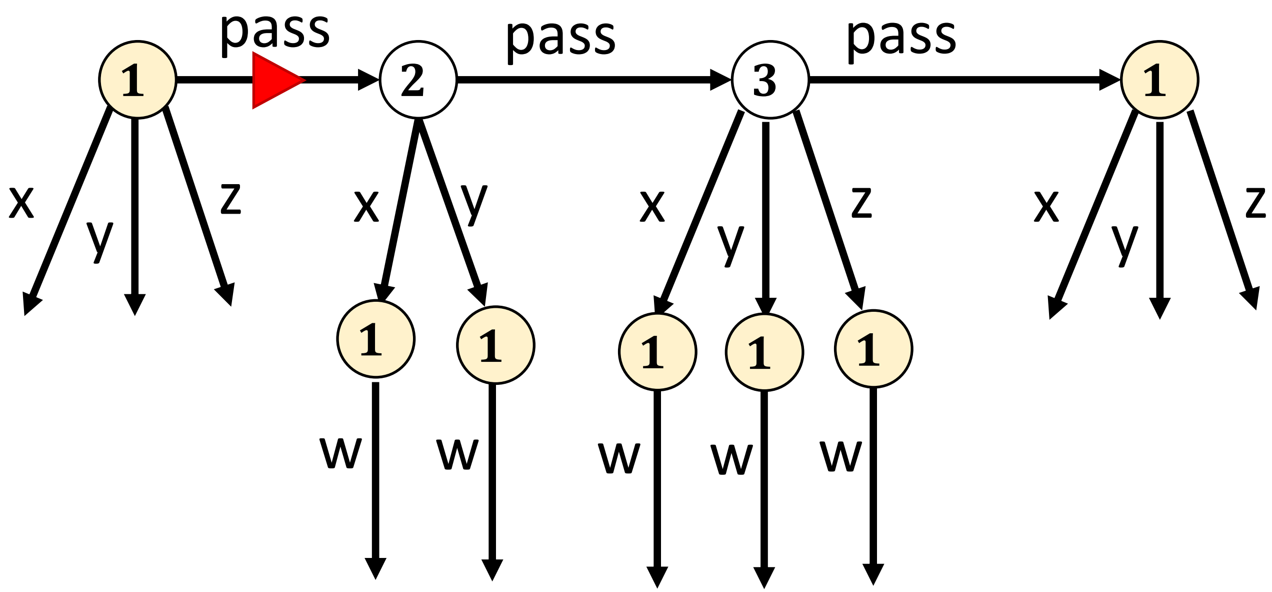

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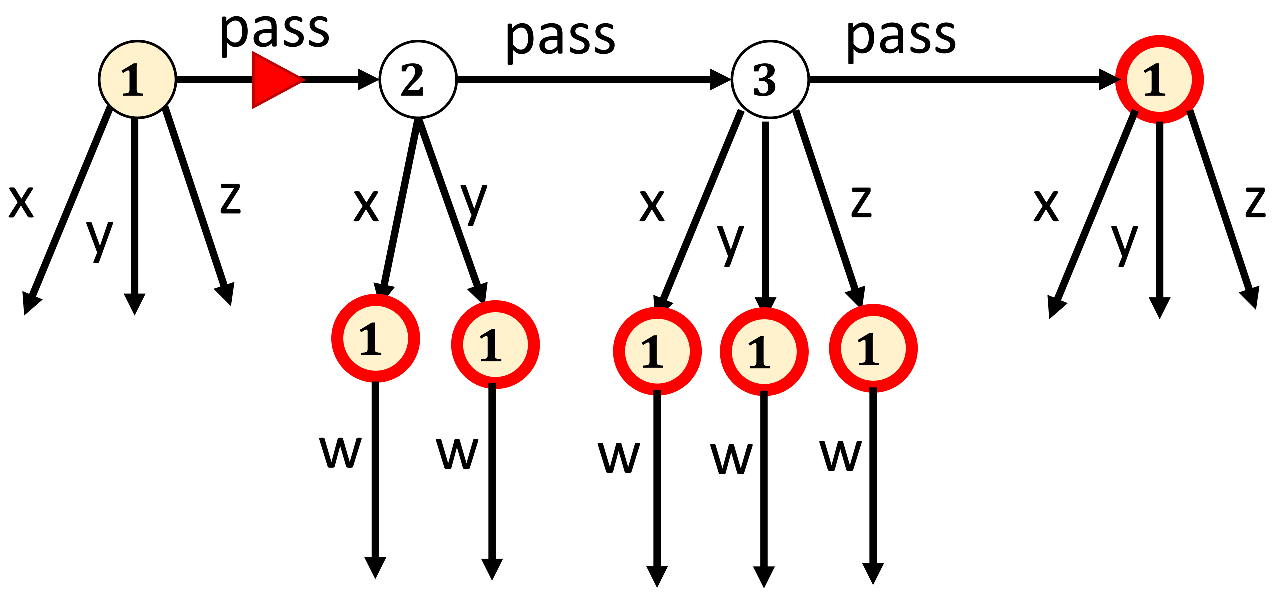

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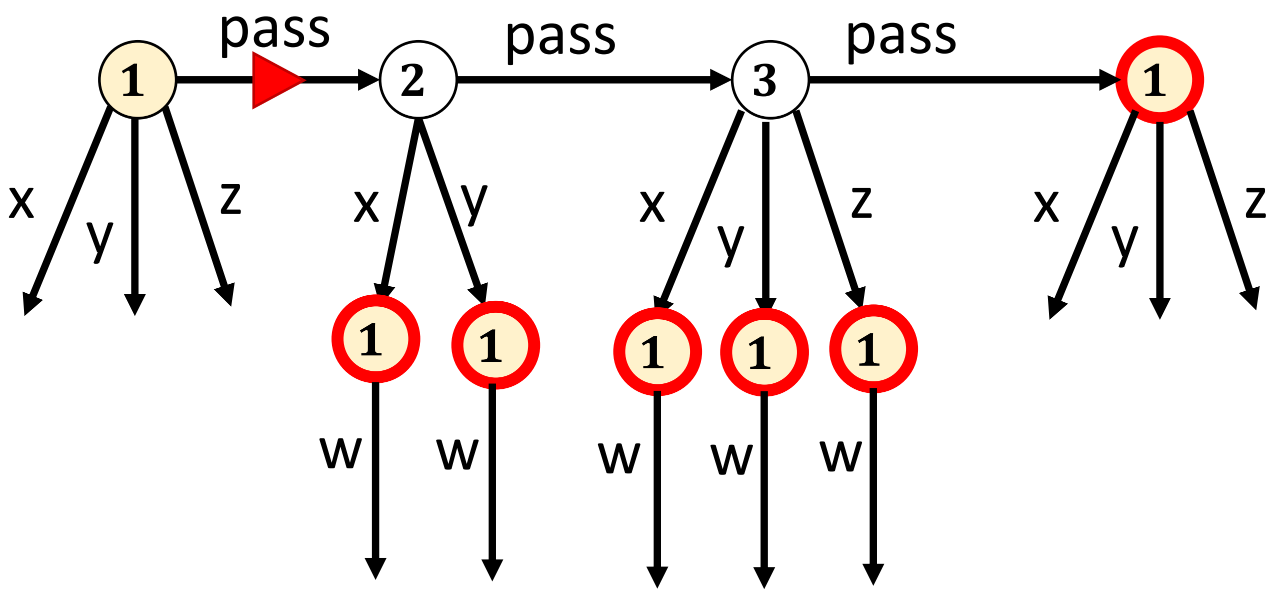

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OSF:

Players are able to plan one step ahead

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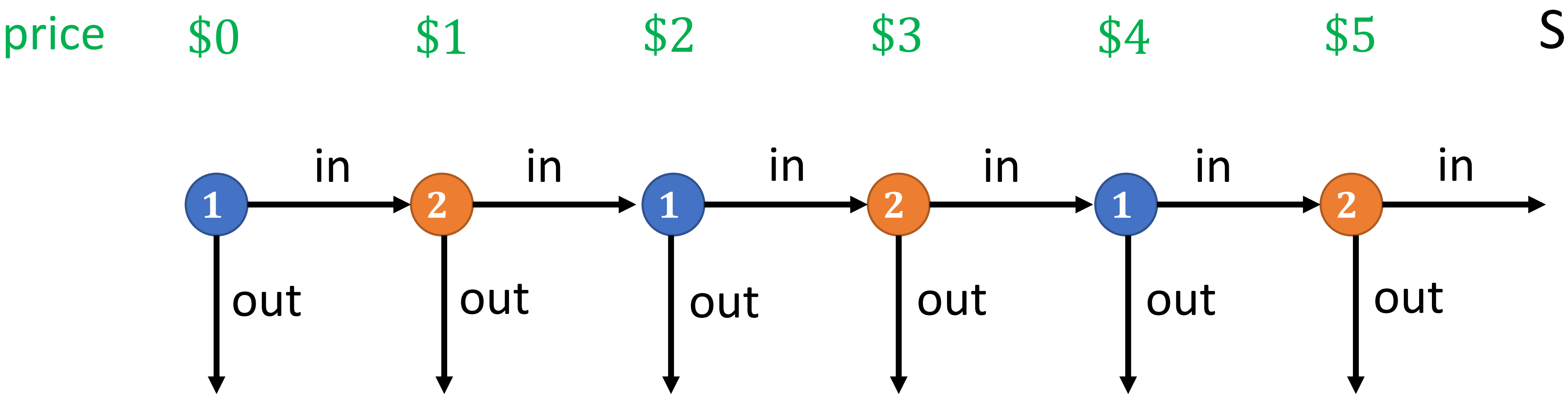
Pycia and Troyan enable varying the amount of foresight of an agent

Ascending price auctions OSF obvious

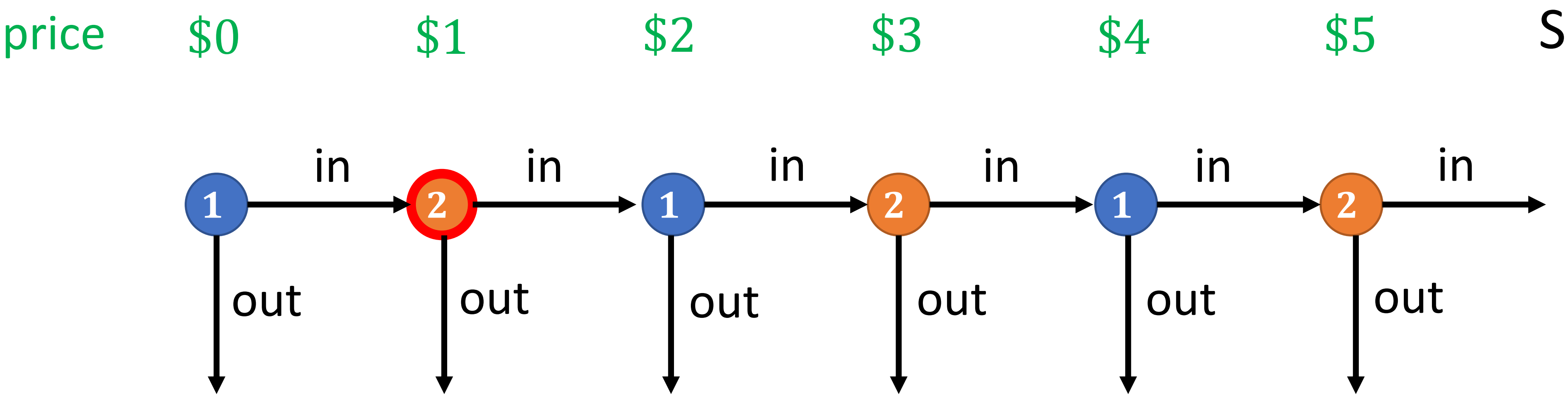
Suppose $v_2 = 5$



One Step strategy plan:

- If price < v: stay in, drop out next turn
- If price >= v: drop out

Ascending price auctions OSF obvious



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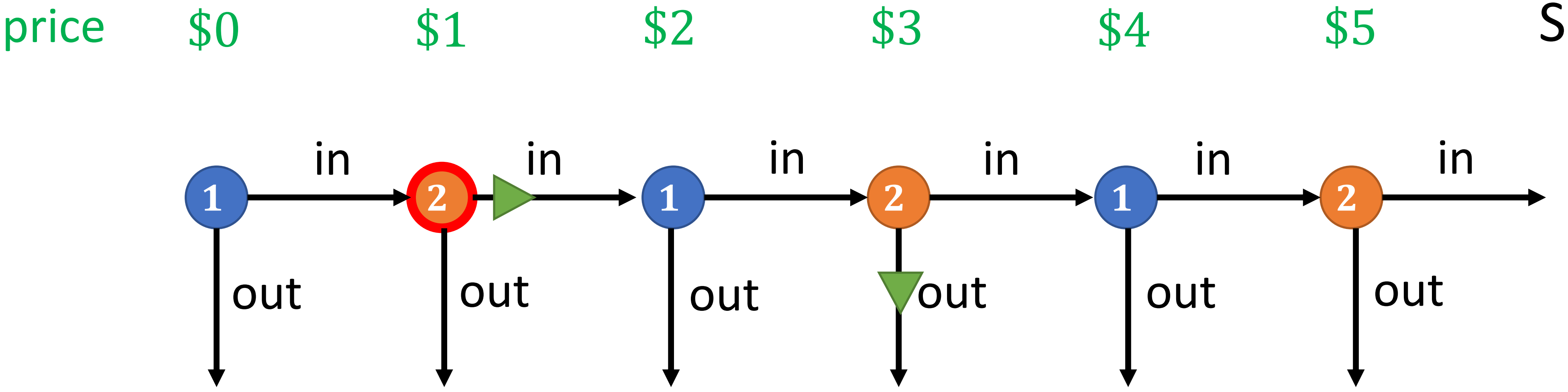

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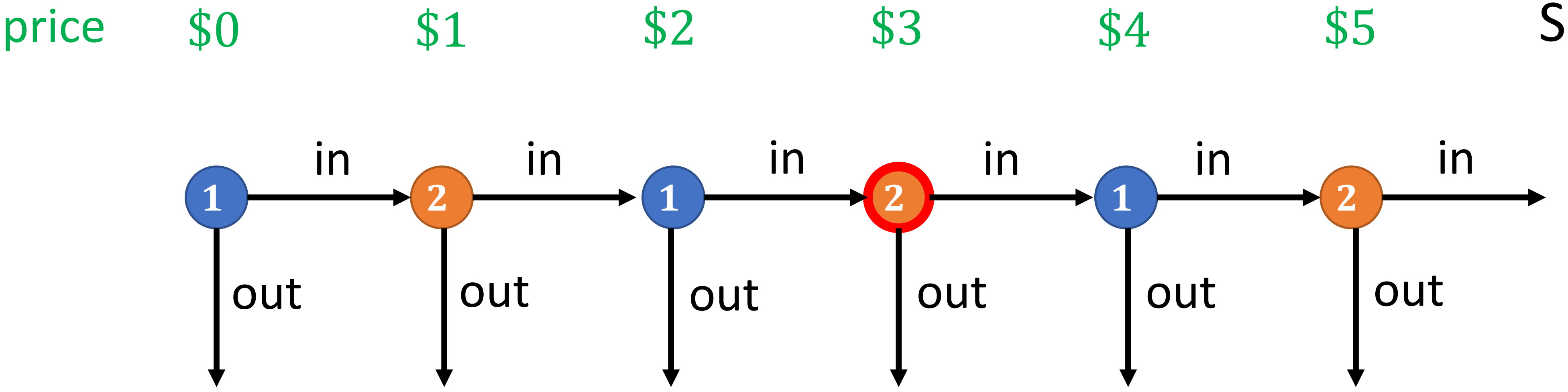

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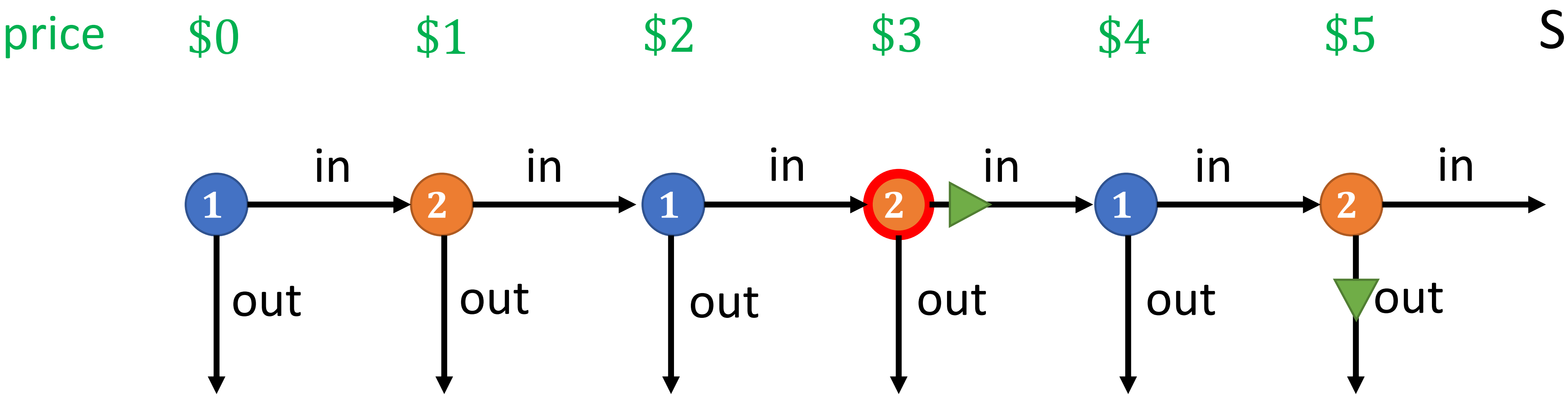

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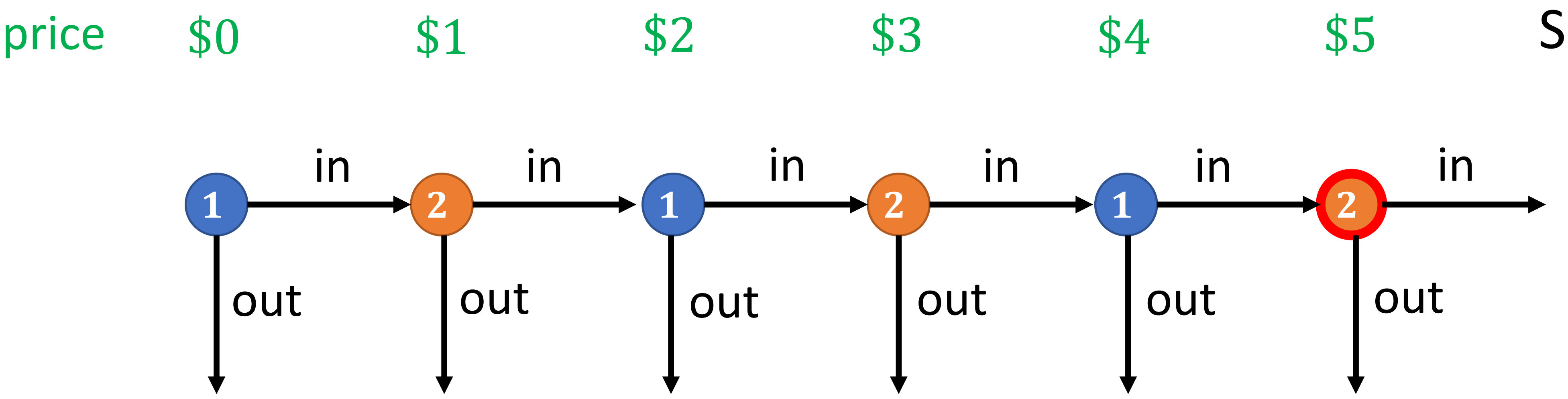

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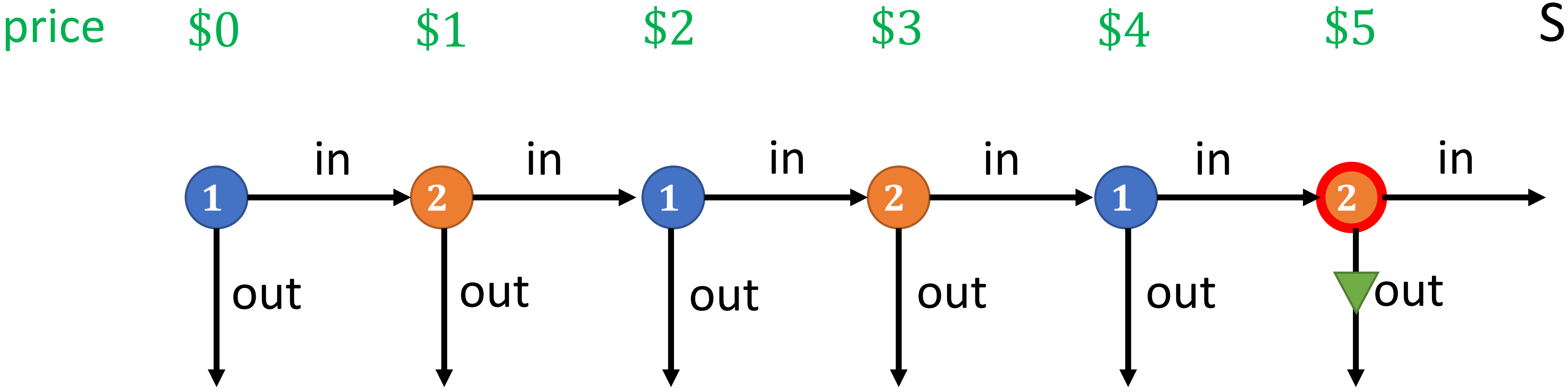

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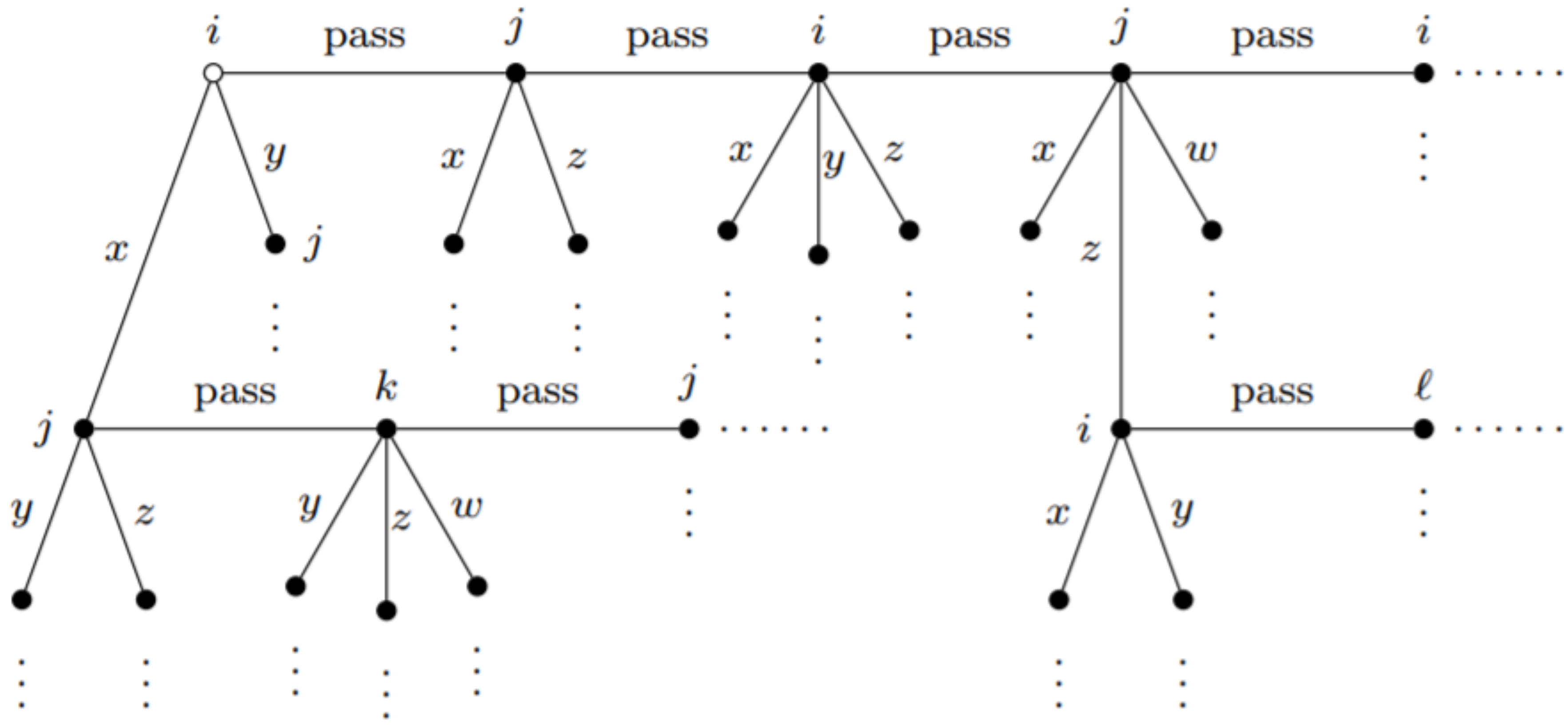
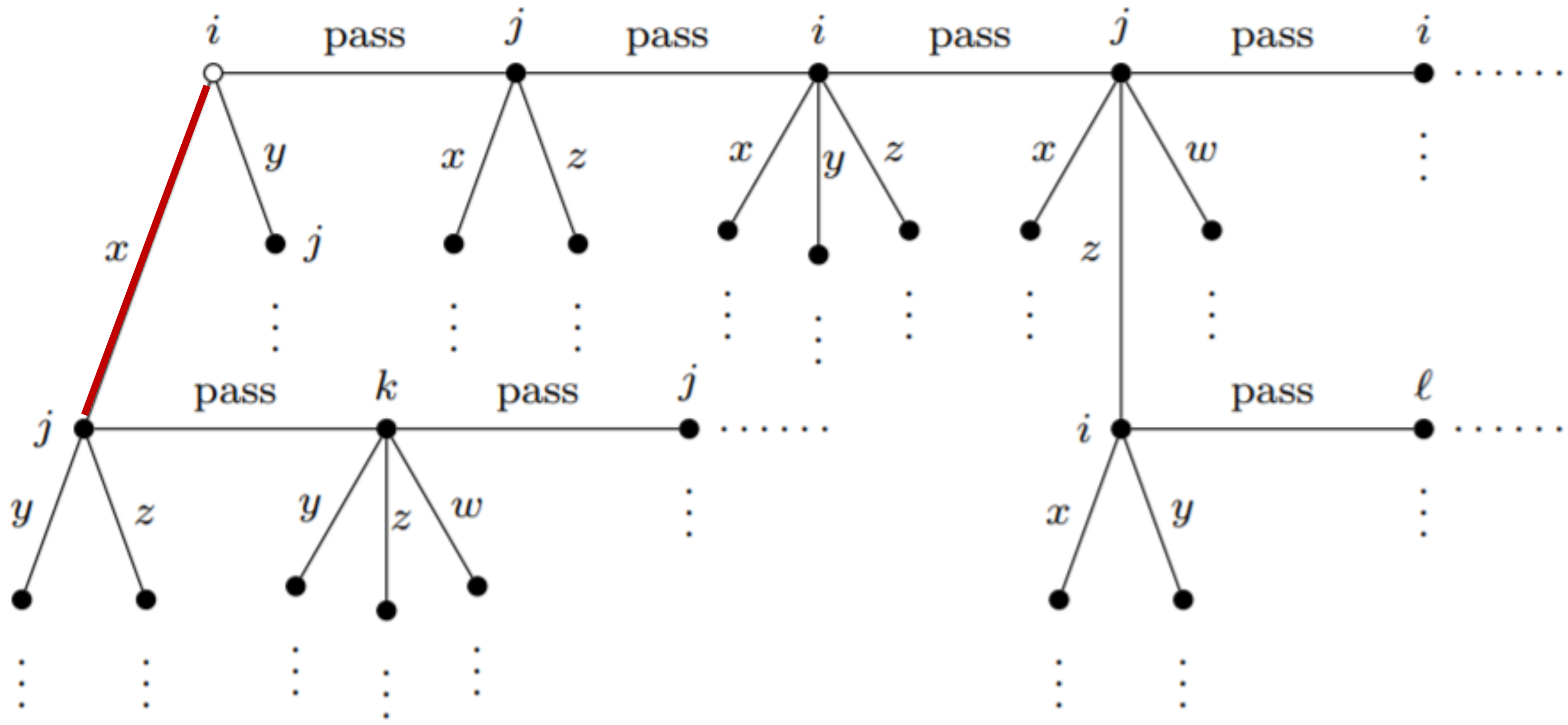
Millipede games



Each player can:

- Clinch one of several options, and leave the game
- Pass, and may play again

Millipede games



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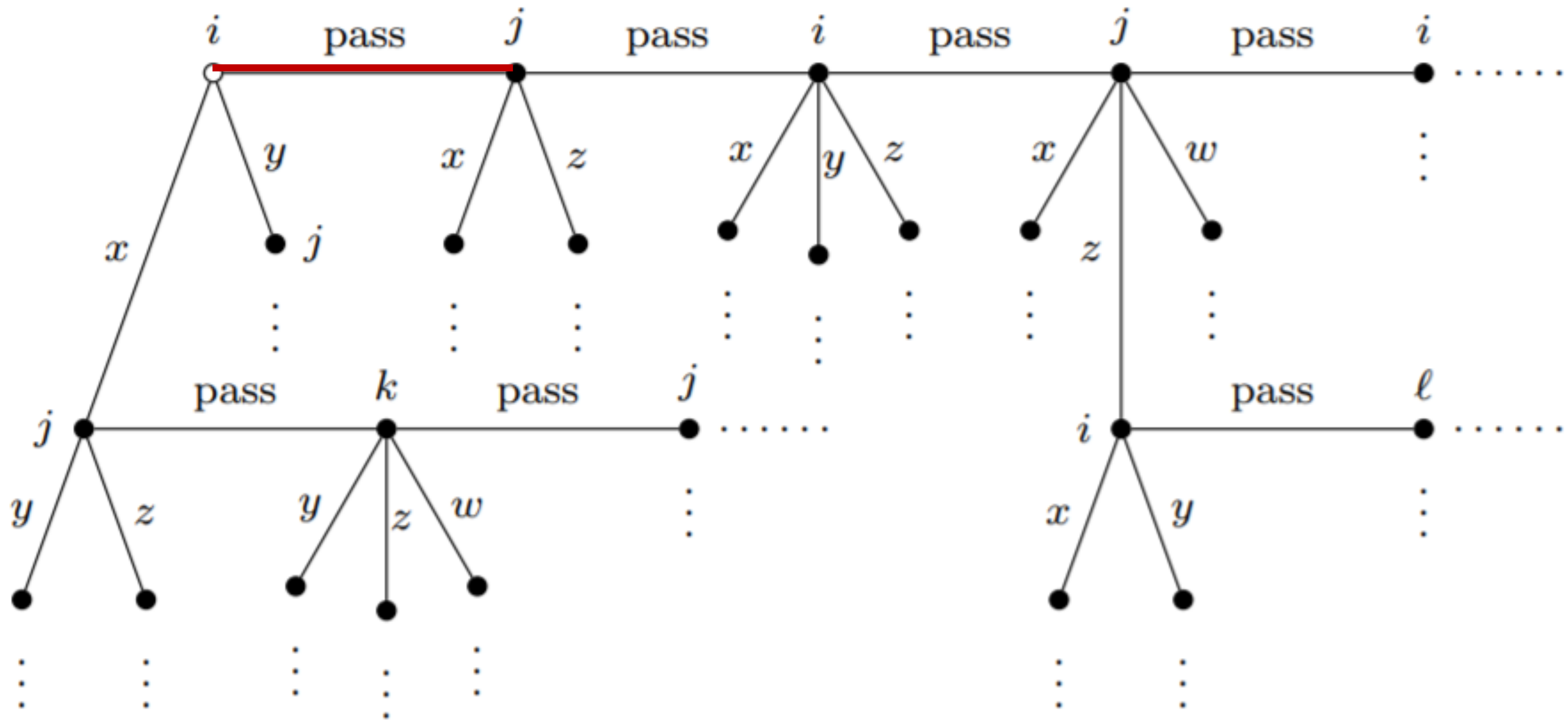

Each player can:

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Millipede games cont.

Each player can:

- Clinch one of several options, and leave the game
- Pass, and may play again

After a pass:

- If an outcome that was possible for i disappears, i is offered everything that was clinchable for i
- If something that was clinchable disappears, i is offered everything that was previously possible for i

Equivalence [Pycia Troyan]

Thm. a game with no transfers is OSP iff it is equivalent to a millipede game

Not all about the normal form

[Breitmoser Schweighofer-Kodritsch] compare 5 conditions:

1. 2P auction
2. 2P auction + simulation of ascending auction w/o dropout info
3. 2P + simulation of ascending auction w. dropout info
4. Ascending auction w/o dropout info
5. Ascending auction w. dropout info

Not all about the normal form

[Breitmoser Schweighofer-Kodritsch] compare 5 conditions:

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Not simple

4. Ascending auction w/o dropout info
5. Ascending auction w. dropout info

simple

Not all about the normal form

[Breitmoser Schweighofer-Kodritsch] compare 5 conditions:

1. 2P auction ⟵ Worst performance
2. 2P auction + simulation of ascending auction w/o dropout info
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4. Ascending auction w/o dropout info
5. Ascending auction w. dropout info ⟵ Best performance

Not all about the normal form

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1. 2P auction ← Worst performance
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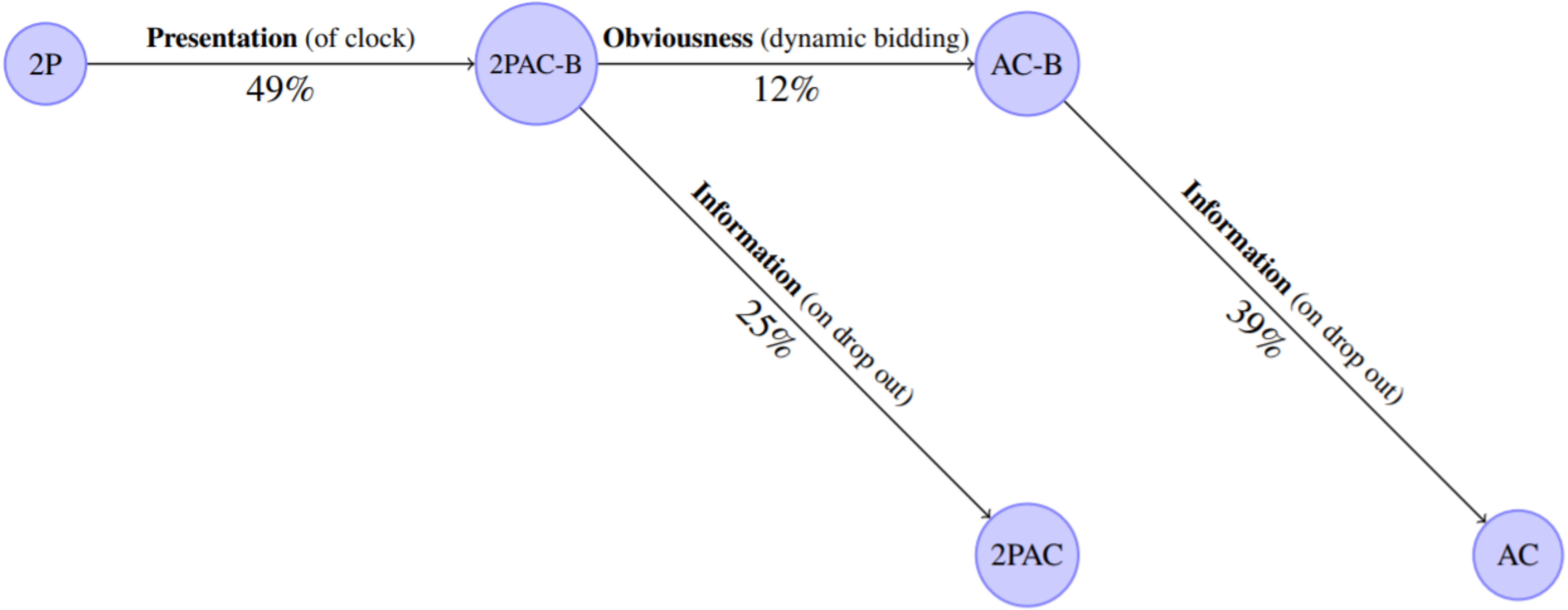
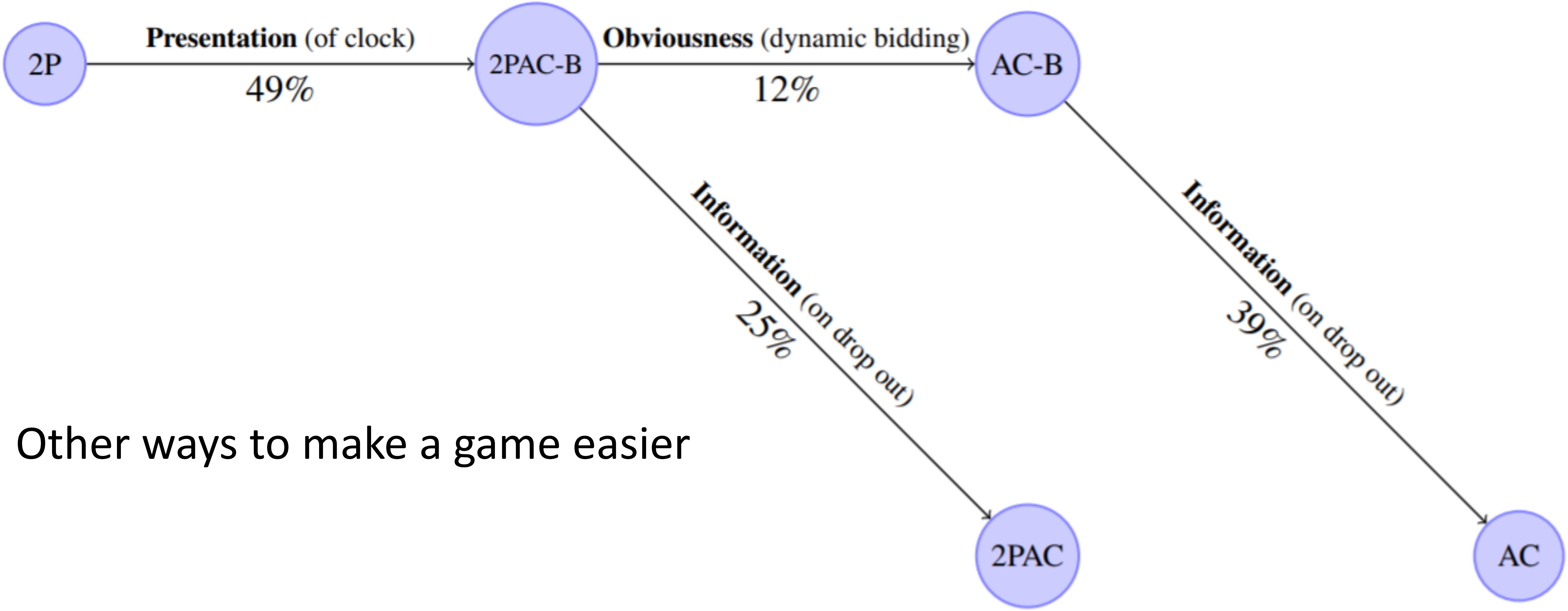
?

[Breitmoser Schweighofer-Kodritsch] results

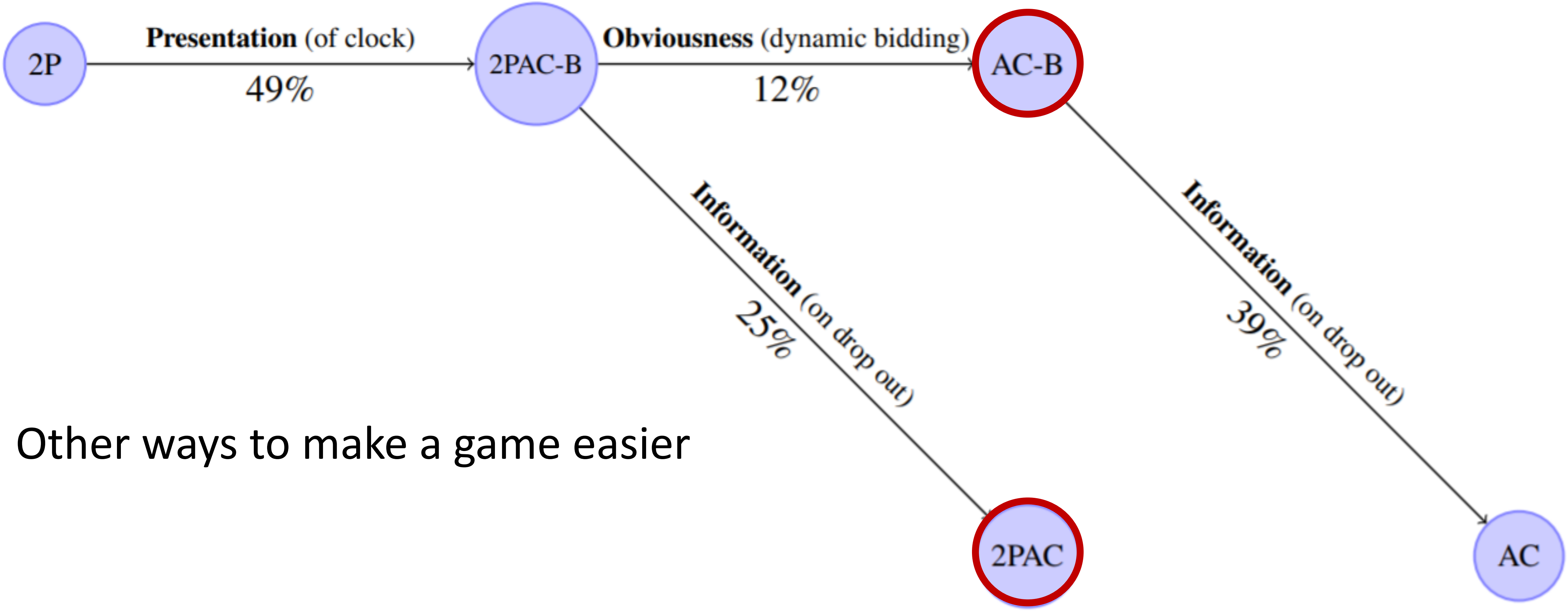


[Breitmoser Schweighofer-Kodritsch] results



Other ways to make a game easier

[Breitmoser Schweighofer-Kodritsch] results



Other ways to make a game easier