About the reading group

- 4~5 meetings the rest of the semester
- Topics:
  - Obviously strategy-proof mechanisms (Alon, next Wednesday)
  - Gali’s series of works (Auction and regret quantal response)
  - level-k reasoning
  - Endowment Effect
  - Planning
- We are open to any suggestions. Whoever wants to lead a session is very invited (and can bring her/his own topic).

Expectation-based Reference-dependent Utility

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Outline

- Paradoxes under EU model
- Reference-dependent models
- **Expectations-based models and solution concepts**
- Applications
  - **Overbid in Auction**

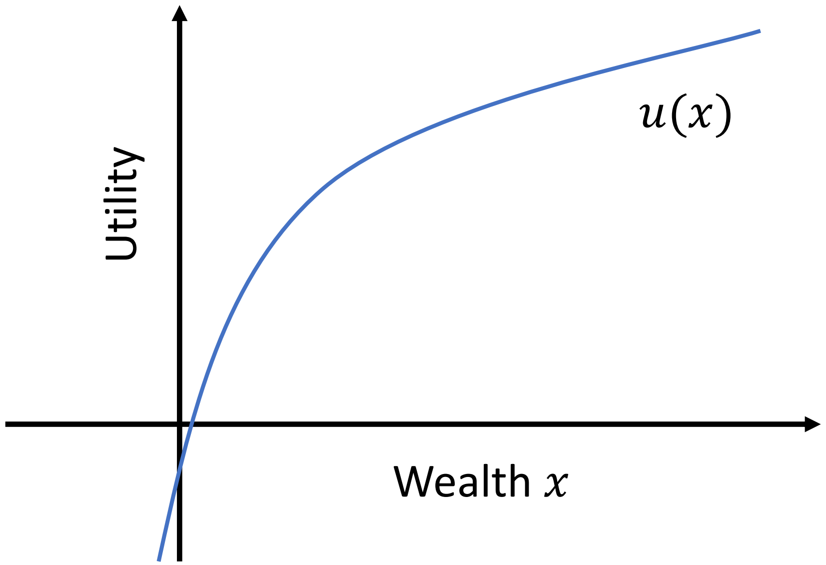
Preliminaries

- Payoff: a lottery $L = (x_1, p_1; x_2, p_2; \ldots; x_n, p_n)$
- A decision problem: choice one out of a set of lotteries

Preliminaries

- Payoff: a lottery $L = (x_1, p_1; x_2, p_2; \ldots; x_n, p_n)$
- A decision problem: choice one out of a set of lotteries
- Expected Utility model (EU):
  - $U(L) = \sum_{i=1}^{n} p_i u(x_i)$
  - Completeness, continuity, transitivity and **independence** axioms

Risk-averse:
$u(x)$ is concave



Paradoxes under Expected Utility Model

1. Small-stake risk aversion implies absurd risk aversion at large stake
    - Turn down (\$110, 0.5; −\$100, 0.5) at all wealth levels ⇒ Turn down (+\$∞, 0.5; \$-1000, 0.5) Rabin (2000)

2. Violation of independence axiom
    - Axiom: $L \gtrsim L' \Leftrightarrow \alpha L + (1-\alpha)L'' \gtrsim \alpha L' + (1-\alpha)L'', \forall L, L', L'', \alpha \in (0,1)$

    - Samuelson (1963) anecdote:
        - A colleague will turn down $L =$(\$200, 0.5; -\$100, 0.5) but will accept 100 independently draw L.
    - Allais' paradoxes, Allais (1954)

Reference-dependent Models

- Prospect Theory (Kahneman and Tversky, 1979)
  - $U(L|r) = \sum_{i=1}^{n} p_i v(x_i - r)$, solving Paradox 1
  - Probability weighing $U(L|r) = \sum_{i=1}^{n} w(p_i) v(x_i - r)$, solving both paradoxes

Exogenous reference: “status quo”

- Expectations-based reference-dependent model (EBRD, Koszegi and Rabin 2006, 2007, 2009)

Endogenous reference: “expectation”

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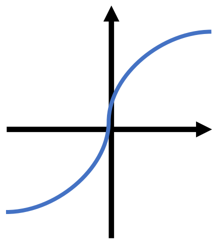
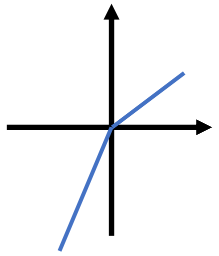


- $v(x_i - r) = \begin{cases} v(0) = 0 \\ \mathit{Diminishing\ sensitivity}: v''(x) \cdot x < 0 \\ \mathit{Loss\ aversion}:\ v(x) < -v(-x); v'(x) < v'(-x), \forall x > 0 \end{cases}$



Why use expectation as reference point

- When facing a lottery choice problem, one forms a prior expectation of its outcome, and then after the uncertainty is resolved one experiences elation or disappointment ("Disappointment aversion", Loomes and Sugden, 1986).

- Impose some discipline on models of reference dependence.

EBRD Framework

(This slide: given the reference, next slide: how to set the reference)

- Reference point: $R = (r_1, q_1; r, q_2; \dots; r_m, q_m)$
- Utility:

$$U(L|R) = EU(L) + V(L|R)$$

Monetary/Intrinsic/
Instrumental utility

Gain-loss utility

- Gain-loss utility: $V(L|R) = \sum_{i=1}^{n} p_i v(x_i|R)$
  - **KR approach:** $v(x_i|R) = \sum_{j=1}^{m} q_j\, v\big(u(x_i) - u(r_j)\big)$
  - DA approach: $v(x_i|R) = v\big(u(x_i) - EU(R)\big)$

Solution Concepts

- Given a lottery set $S = \{L, L', L'', \dots\}$, which one will be chosen?

- Choice-acclimating Personal Equilibrium (CPE, KR 2007):
$$L^* = \mathrm{argmax}_{L \in S}\{U(L|L)\}$$
Most tractable one!

- Personal Equilibrium (PE, KR 2006)
$$L^* \in \{L \in S | U(L|L) \geq U(L'|L), \forall L' \in S\}$$

- Preferred Personal Equilibrium (PPE, KR 2007)
$$L^* = \mathrm{argmax}_{L \in \mathrm{PE}(S)}\{U(L|L)\}$$

CPE

- $v(x) = \begin{cases} \eta x, & x \geq 0 \\ -\eta\lambda x, & x < 0 \end{cases}$ $\lambda > 1$ (Loss aversion)
- Maximize

$$U(L|L) = \sum_{i=1}^{n} p_i u_i + \sum_{i=1}^{n} \sum_{j=1}^{n} p_i p_j v\left(u_i - u_j\right)$$
$$= \sum_{i=1}^{n} p_i u_i - \sum_{i=1}^{n} \sum_{j=i+1}^{n} p_i p_j \, \Lambda(u_j - u_i)$$

$$\Lambda = \eta(\lambda - 1)$$

- For two outcomes: $U(L|L) = EU - \Lambda p_1(1 - p_1)(u_2 - u_1)$

CPE

- $v(x) = \begin{cases} \eta x, & x \geq 0 \\ -\eta\lambda x, & x < 0 \end{cases}$ $\lambda > 1$ (Loss aversion)
- Maximize

$$\begin{aligned} U(L|L) &= \sum_{i=1}^{n} p_i u_i + \sum_{i=1}^{n} \sum_{j=1}^{n} p_i p_j v\left(u_i - u_j\right) \\ &= \sum_{i=1}^{n} p_i u_i - \sum_{i=1}^{n} \sum_{j=i+1}^{n} p_i p_j \, \Lambda(u_j - u_i) \\ &= \sum_{i=1}^{n} p_i u_i - \sum_{i=1}^{n} \sum_{j=1}^{n} p_i p_j \frac{\Lambda}{2} |u_j - u_i| \end{aligned}$$

Var = $\sum_{i=1}^{n} \sum_{j=1}^{n} p_i p_j \frac{1}{2} \left(u_j - u_i\right)^2$

- [Solved] Samuelson (1963) anecdote “A colleague will turn down L =($200, 0.5; -$100, 0.5) but will accept 100 independently draw L.”

CPE

- $v(x) = \begin{cases} \eta x, & x \geq 0 \\ -\eta\lambda x, & x < 0 \end{cases}$ $\lambda > 1$ (Loss aversion)
- Maximize

$$\begin{aligned} U(L|L) &= \textstyle\sum_{i=1}^{n} p_i u_i + \sum_{i=1}^{n} \sum_{j=1}^{n} p_i p_j v\left(u_i - u_j\right) \\ &= \textstyle\sum_{i=1}^{n} p_i u_i - \sum_{i=1}^{n} \sum_{j=i+1}^{n} p_i p_j \, \Lambda(u_j - u_i) \\ &= \textstyle\sum_{i=1}^{n} p_i u_i - \sum_{i=1}^{n} \sum_{j=1}^{n} p_i p_j \, \dfrac{\Lambda}{2} |u_j - u_i| \end{aligned}$$

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- [Solved] Samuelson (1963) anecdote “A colleague will turn down L =($200, 0.5; -$100, 0.5) but will accept 100 independently draw L.”

CPE and first-order stochastic dominance

- First-order stochastic dominance (FOSD):
  - $L' = (\$10, 0.9, \$11, 0.1) \succsim L = (\$10, 1)$
- CPE violates FOSD, but EU, PE and PPE not.
- Experimental result: people violate FOSD when it is not obvious.

Applications

- Overbid in auction (Theory: Lange and Ratan 2010; Experiments: Banerji and Gupta)

First-price auction

- One item, n agents
- Intrinsic valuation: CDF $H: [\underline{w}, \overline{w}] \rightarrow [0,1]$
- Gain-loss utility function:
  - Two dimensions: $v_k(x) = \begin{cases} 0, \ x \geq 0 \\ -\lambda_k x, \ x < 0 \end{cases}$, $k = 0$ for money, 1 for item
  - $v = v_0 + v_1$
- Solution concept: Symmetric increasing Bayesian Nash equilibrium under EBRU

First-price auction

- U(L|L):

$$\Pi^{1,\text{CA}}(b^i, w^i) = f(b^i)(w^i - b^i) - f(b^i)(1 - f(b^i))[\lambda_0 b^i + \lambda_1 w^i]$$

Intrinsic utility Gain-loss utility

$f(b^i)$ -- Winning probability under bid b^i

Solving the equilibrium

- First-order derivative equation:

$$f'(b^i)\left[w^i - b^i - (1 - 2f^i)(\lambda_0 b^i + \lambda_1 w^i)\right] = f^i\left[1 + (1 - f^i)\lambda_0\right]$$

  -variable $b(w^i)$

  -$f\left(b(w^i)\right) = H^{n-1}\left(b^{-1}(b(w^i))\right)$

- Meanwhile, at the equilibrium, $b^{-1}(b(w^i)) = w^i$
- Therefore,$f'(b^i) = (H^{n-1}(w^i))'/b'(w^i)$

$$(H^{n-1})'(w^i)w^i\left[1 - (1 - 2H^{n-1}(w^i))\lambda_1\right] = \left[H^{n-1}(w^i)(1 + (1 - H^{n-1}(w^i))\lambda_0)b(w^i)\right]'$$

Equilibrium Bid

Proposition 1 *(Commodity first-price auction – CA). The unique monotonically increasing symmetric Bayesian Nash equilibrium bidding function for commodity auctions is given by*

$$b^{1,\mathrm{CA}}(w) = \begin{cases} w\frac{1-\lambda_1(1-H^{n-1}(w))}{1+\lambda_0(1-H^{n-1}(w))} - \frac{\int_{w_L}^{w} H^{n-1}(z)[1-\lambda_1(1-H^{n-1}(z))]dz}{H^{n-1}(w)[1+\lambda_0(1-H^{n-1}(w))]} & \textit{if } w \geqslant w_L^{\mathrm{CA}} \\ 0 & \textit{if } w < w_L^{\mathrm{CA}} \end{cases} \tag{7}$$

- For large w, an agent overbids w.r.t. BNE
- For small w (some w with winning probability < 0.5), an agent underbids w.r.t. BNE.
- Intuition: People tend to reduce the uncertainty.

Equilibrium Bid (induced-value)

- One-dimension gain-loss utility v

Corollary 1 *(Induced-value first-price auction – IV). The unique continuous monotonically increasing symmetric Bayesian Nash equilibrium bidding function for induced-value auctions is given by*

$$b^{1,\mathrm{IV}}(w) = \begin{cases} w - \frac{\int_{w_L^{\mathrm{IV}}}^{w} H^{n-1}(z)[1-\lambda_0(1-H^{n-1}(z))]dz}{H^{n-1}(w)[1-\lambda_0(1-H^{n-1}(w))]} & \text{if } w \geqslant w_L^{\mathrm{IV}} \\ w & \text{if } w < w_L^{\mathrm{IV}} \end{cases} \tag{9}$$

- Induced-value, lab experiments
- People should always overbid w.r.t. BNE

Second-price auction

- Commodity auction:
  - For small w (some w with winning probability < 0.5), an agent underbids w.r.t. BNE.
- Induced-value auction:
  - Bid truthfully (=BNE)
- Support by (Banerji and Gupta, 2014)
- Might contradict with (Noti et al 2014): Position auction, induced-value auction, small w agent overbids.

Applications

- Endowment Effects
- Labor Supply (Farber, 2005)
- …

Q&A