How to Design a Common Telecom Infrastructure by Competitors Individually Rational and Collectively Optimal

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Abstract

The fast development of mobile networks calls for massive consumption of materials, land, and energy in building and maintaining infrastructures, which is always intensified by redundant constructions of competing network operators. To save this great investment for social benefit, one business solution, implemented in the Chinese telecom industry, is forming a joint venture responsible for building and maintaining common infrastructures. The novelty of this practice is that the joint venture is shared by the competing operators who also rent infrastructures from the joint venture. We note that such solution can be potentially generalized to different industries for saving resources. However, before generalization, an understanding to the pros and cons in economic perspective of the business model is urgently needed. In this paper, we study the market from a game theoretic approach. Our results show that by properly regulating the joint venture, the market can converge to equilibriums with desirable properties which can not be achieved without the joint venture. Further, we conduct experiments to study the investment reduction in the presence of the joint venture and verify our theoretical studies. Our numerical results show that under moderate user density, the investment on the infrastructures can be significantly reduced.

I. Introduction

In recent years, the telecom industry is experiencing an explosive growth of new technologies, applications and networks users. While it brings unprecedented benefit to people, it also leads to a huge consumption of natural resources. Aiming to save the scarce resources of our mother nature, an increasing number of green technologies and schemas have been proposed to improve the efficiency of resource utilization for telecommunication. However, to catch up the development of new green technologies and schemas, the basic telecom infrastructures have to be continuously updated or even reconstructed, which not only obstructs the adoption of new technologies but also induces tremendous resource depletion. Moreover, this problem is intensified for the areas served by multiple competing network operators where redundant construction of infrastructures is almost inevitable. This causes waste of resources from the perspective of the social benefit.

Sharing infrastructures among competing network operators is usually an effective way to reduce redundant constructions and alleviate inefficiency in utilizing infrastructure resources. Besides, by releasing from the burden of maintaining and updating the infrastructures, operators are able to focus their business on the end user service and to efficiently compete with the other opponents by concentrating on the quality of service. Following this idea, the Chinese government proposed and guided the establishment of a joint telecom infrastructure venture, China Tower, in 2014, responsible for building, maintaining and operating telecom infrastructures. According to the agreement, China Tower was co-invested and shared by three telecom giants in China, namely China Mobile, China Unicom and China Telecom, who claimed to no longer keep infrastructures by their own but to rent infrastructure services from Chine Tower[1]. Historically, the three telecom network operators built and operated their own infrastructures, e.g., tower networks, independently while competing at the end user market. As expected, reports[1] show that the presence of the joint venture makes real benefit: about one year after the establishment of China Tower, the three telecom companies had reduced tens of thousands of base stations and towers while saving millions square meter of land and billions of investment in CNY.[2]

In fact, resource sharing is not a totally new idea in the telecom industry. Specific to telecom infrastructures, for example, two UK mobile operators, Vodafone and Telefonica, announced the news of sharing their towers, masts, and radio equipment in 2012 that the two companies each would be running the infrastructural service in a particular region [2]. Similarly, in 2012, two Australian telecom companies, SingTel-Optus and Vodafone Hutchison Australia, agreed to share their respective towers by further individually upgrading and expanding their existing

[1] See http://www.cn-c114.net/576/a881107.html, http://it.sohu.com/20151016/n423413460.shtml

[2] A report (http://www.cn-c114.net/576/a881107.html) published on May 2rd, 2015 says by the joint venture, the three major telecom operators had reduced to build 29,000 base stations and 18,000 towers by co-construction and sharing, which directly saved CNY 6 billion in the investment. Another report (http://it.sohu.com/20151016/n423413460.shtml) published in Chinese on Oct 16th, 2015 shows that China Tower had achieved a reduction of about 200 thousand base stations, 40 billion CNY investment and 6 million square meter land resources for the three telecom giants.

networks [3]. In practice, however, an ideal goodwill cannot be realized without an appropriate business model that governs the business and operational decisions made by all parties. There are actually failed efforts on resource sharing in the telecom industry, though in a way different from founding an independent joint venture. For the above-mentioned case in Australia, the two companies had to renegotiate the contract in 2014 because Vodafone did not meet the committed progress [4]. Although this is just a single instance with possibly many complicated factors leading to the failure, and the agreement is also different from running an independent joint venture, questions will still naturally arise regarding the advantages and the limitations of a joint venture.

In this paper, we aim to analyze and understand the relationship between the joint venture and the participating member operators under different business models and explore the influence of the joint venture on the telecom market equilibrium and the resource consumption. Meanwhile, we aim to calculate the equilibrium capacity of the infrastructures, by considering the cost of building the infrastructures, i.e., what is the capacity that the joint venture should build. The direct assumption is that the joint venture has enough capital to build the equilibrium capacity, which seems to be restrictive. However, our model can also be applied to the situation with insufficient capital. In this case, the joint venture can raise additional capital, e.g., by taking a loan, at certain financial cost. This financial cost can be included in the cost of building the infrastructure.

Specifically, we consider a game system consisting of a joint venture established under the guidance of the government and several member operators. The joint venture is (partially) shared by network operators and leases its infrastructure services to the operators which run for their own profit. Instead of assuming the joint venture as a company running for its own profit, we are more interested in how the joint venture can achieve effective resource utilization from a social perspective because it is established under the guidance of the government. In particular, we consider two representative social objectives for the joint venture, aggregate profit maximization of the market and budget balance of the operators. The former is to maximize the sum of the profit of all players. The latter is to keep each member operator charging the end users at a relatively low price that covers its cost, which is usually the case for regulated natural monopoly (public utility) industry such as gas and water. The importance of the budget balance is that the end user can enjoy the service at a relatively low price instead of an unaffordable monopoly price that maximizes the revenue of the monopoly company.

The key decisions in the system include the shares of member operators on the joint venture, the price of renting infrastructure services from the joint venture, and the prices charged by the member network operators to the mobile end users. The sequence of making these decisions are critical to the system outcome. As in practice, the share composition of the joint venture and the price of using the infrastructures are decided first, then each network operator decides their service price to the end users. To model such interactions, we set up a leader-follower Stackelberg game [5]. In the Stackelberg game, the joint venture plays as the leader to determine the price of renting infrastructure services, and the member operators then decide their individual service prices to consumers when competing on the market. A subtle challenge here is that the operators may behave differently under competitions of different time scopes. To this end, we divide the competitions into three time scopes, one-shot competition, finite-round competition and long-run competition. For the long-run competition, we induce a time-discounted model [6], following the traditional game literature, where network operators compete for infinite rounds with an accumulated discount on the profit gained in later rounds. The cost of the infrastructures is constrained by both the infrastructure service coverage and capacity. Usually, different scales of infrastructures fit for different pairs of coverages and capacities. To avoid trivial and tedious exposition, we conduct numerical studies on the cost of necessary infrastructures to support the user demand as the outcome of the market competition.

Before summarizing the main results of our work, we first highlight one important special structure of our Stackelberg game. In the standard setting of a Stackelberg game, the leader and the follower(s) each has an absolutely independent objective function. When a follower makes a decision, he only needs to consider the profit directly earned by him. However, in the case of a joint venture, a follower (network operator) also has a share in the leader's (the joint venture's) business; as a result, a portion of the leader's profit will eventually become the wealth of a follower, which is a new factor that the follower has to take into account. This makes our game behaving quite differently from the classic Stackelberg game.

Our main contributions are summarized as follows.

- Motivated by the recent establishment of a joint telecom infrastructure venture in China, we study the relationship between the joint telecom infrastructure venture and member operators under different objectives of the joint venture and time scopes of the market competition. Our investigation reveals that the joint venture plays an active role in influencing the market equilibrium and the cost reduction on infrastructures.

- Our analysis reveals that there is a prisoners' dilemma embedded in the one-shot competition among operators where the operators cannot get expected profit as they seek to maximize their own profit simultaneously. This prisoners' dilemma leads to the inefficiency from the perspective of the aggregate profit of the market. However, by setting proper a price of renting infrastructure services and the share composition of the joint venture, this dilemma can be eliminated from the market such that the optimal aggregate profit can be achieved. Besides, by setting proper decisions of the joint venture, all of the operators of different scales can run at the price that keeps the budget balance. We further show that the market runs exactly the same when the operators fall in the finite-round competition.
- We show that when network operators engage in the long-run competition, they may automatically achieve the optimal aggregate profits while seeking to maximize their own long-run profit. Meanwhile, the influence of the decisions of the joint venture disappears unless there exists an outside investor of the joint venture other than the member companies. As to keep the member operators price at the budget balance price, the outside investment has to subside the member operators. As the operators keep the budget balance, the subsidy finally becomes part of the end user surplus.
- We further conduct numerical studies to investigate of the reduction of the construction of unnecessary infrastructures. Our numerical results show that the presence of the joint venture can significantly reduce the resources needed by constructing the infrastructures, especially in the area with moderate end user density.

The rest of the paper is organized as follows. In Section II, we clarify the system model and formulate the infrastructure sharing problem as a Stackelberg game. In Section IV, we analyze the competition among the network operators and figure out the equilibrium strategies of them in both the short-term competition and the long-run competition, given the sharing scheme of the joint infrastructure venture. Then we figure out the optimal sharing schemes of the joint venture with respect to different objectives in Section V. We present computational studies in Section VI and conclude our work in Section VII.

II. Related Work

Resource sharing has been well studied in the communication field. One of the hot topics that are tightly combined with game theory is the spectrum sharing problem among the primary agents and the secondary agents. A variety of studies, e.g., [7], [8], [9], [10], [11], [12], exploit this topic under different scenarios and conditions. In [7], [8], the Nash equilibria of the spectrum sharing competition between the primary service providers and second service providers are exploited and interesting phenomenas are shown that in some scenarios with shared spectrum or with additional bandwidth, the welfare of the primary users or the whole welfare may decrease. In [9], [10], contracts and Nash bargaining processes are developed to improve the utility of all parties. [11] proposes a well-designed subsidization to motivate general service providers to share spectrum among themselves so as to enlarge the collective service coverage and to avoid monopolization. [12] studies the spectrum sharing among two service providers where one provider transfers its excess demand to the other with overflow capacity. The revenue generated by the excess demand is shared by both providers. The Nash equilibria of the capacities of both operator and the corresponding profits are analyzed so as to better guide the operators to well plan their infrastructure investment.

Specific to telecom infrastructure sharing, there are also a number of studies, e.g., [13], [14], [15], [16], [17]. [13] consider the infrastructure sharing from a technology perspective, where the spectrum sharing, the infrastructure sharing and a mixture of them are evaluated in the terms of the service coverage probability and the average user data rate. From an economic perspective, [14] studies the equilibrium investments of two competitive operators which hold telecom infrastructures in distinct areas and share the infrastructures via roaming. It derives the optimal level of investment on the shared infrastructures under different competition situations and roaming charges. [15] studies the scenario where two operators cover the same area and make decisions on whether to turn off its own base station and rent the other's in order to save energy and optimize their own profits. The results show that both the existence of the Nash equilibriums and the social efficiency of these equilibriums are sensitive to all the renting price, the capacity and the baseline energy cost. Further, [16] considers this problem under the constraint where the traffic is private information for each operator, and proposes an incentive-compatible mechanism by allowing money transfer to motivate each operator truthfully revealing its traffic to serve for the social efficiency. [17] study the base station sharing where each operator can control its price as well as redistribute its traffic among base stations.

Studies of infrastructure sharing in other fields include [18], [19], etc. [18] studies the optimal price of the shared Internet infrastructures which provides basic service to independent digit products. It shows that either pricing the infrastructures above the average cost or below may maximize the aggregate profit of the whole system under different properties of the cost function of infrastructures. [19] studies the fair cost sharing of the railway infrastructures among hetero-

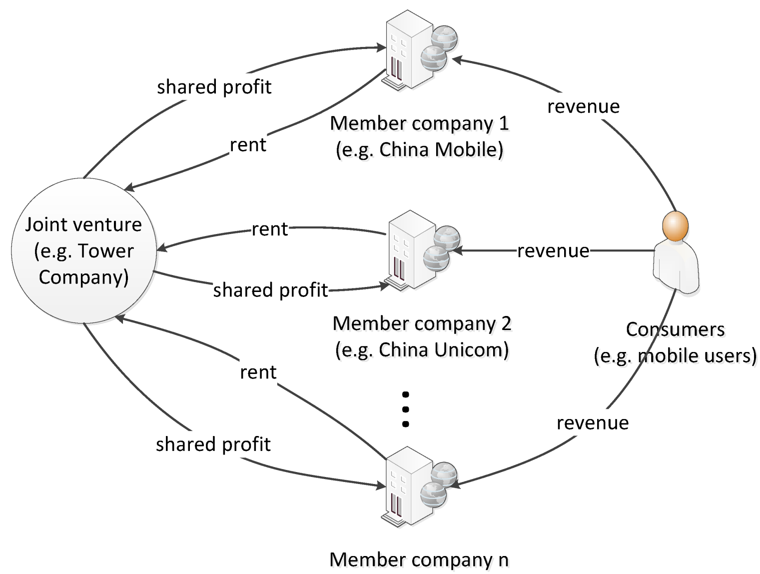

Figure 1. System model

geneous train companies in consideration of both the fixed cost and the variable maintenance cost of the infrastructures. However, all these works differ from ours in the specific setting of sharing infrastructures via a joint venture which not only charges to its downstream clients but is also shared by them.

The concept of founding a joint venture falls into the field of co-opetition [20], a term that describes how competitors may compete on one side and cooperate on the other side. In our scenario, the operators compete on the end user market but share the same profit on the infrastructure side. The idea of co-opetition has attracted some initial attention in telecom recently [21], [22], [23]. However, to the best of our knowledge, this specific form of co-opetition is new practice and has not been studied in the literature. Our proposed solution makes the first step towards to a systematic study of the joint venture making an approach to solve the resource sharing cooperation problem among the competitors.

III. System Model and Game Formulation

A. System Model

We consider a telecom market of a set $N = \{1, .., n\}$ of network operators attempting to serve the same unit area and competing on the end user market. A joint venture responsible for building and maitaining network infrastructures is set up in order to reduce the industrial cost of redundant construction and maintenance. The general system model is shown in Fig. 1. On one hand, the joint venture rents infrastructure services to all the network operators while the network operators no longer keep any infrastructure by themselves. On the other hand, the joint venture is shared by all network operators, which compete on the end user market. Without loss of generalizability of our analysis and results to industries other than telecom industry as well as to capture the nature of the joint venture, We assume a simple pricing model for the telecom market, for the generalizability of our analysis to other industries as well as to capture the nature of the joint venture, where the joint venture and the operators both charge fixed unit price for their services. The detailed system model is divided into three parts and introduced as follows:

Demand model: As the services from different operators are substitute goods for users in a degree, we adopt the classic substitute demand model in [24], where the price of one player has a positive linear influence on the demands of the other players, that

$$D_i = k_i - p_i + \sum_{j \in N, j \neq i} \beta_{ij} p_j, \, \forall i \in N. \tag{1}$$

D_i is the user demand referring to the demanded capacity per unit area. p_i is the fixed price charged for a unit of network service offered by operator i. k_i models operator i's initial user density, also referring to initially demanded capacity per unit area, and β_{ij} models the influence of operator j's price on operator i's user density. We have $k_i > 0$ and $\sum_{j \neq i} \beta_{ij} < 1$[3] for each operator $i \in N$. Let $K = \sum_{i \in N} k_i$.

Infrastructure capacity and cost: To support the network service over an area with certain capacity, an operator has to set up a number of base stations (infrastructures), which may consist of many small base stations or a few large base stations or a mixture of both types of base stations and may incur different fixed costs. We adopt the fixed cost model depicted in [25] which surveys main types of base stations with different maximal coverages and service capacities, and figures out the optimal fixed cost per square kilometer as a piecewise non-decreasing function of user density. Therefore, we denote the fixed cost of operator i as

$$g_i = h\left(k_i - p_i + \sum_{j \in N, j \neq i} \beta_{ij} p_j\right), \tag{2}$$

where $h(\cdot)$ represents the piecewise non-decreasing optimal fixed cost function. W.l.o.g., we also

[3] $\sum_{j \neq i} \beta_{ij} < 1, \forall i \in N$ guarantees that when the price of each operator goes to infinity, the demand of each operator won't still be positive. In other words, if $\sum_{j \neq i} \beta_{ij} < 1$ does not hold, e.g., $\beta_{ij} = \frac{1}{n-1}, \forall i, j \in N, i \neq j$, each operator can rise up their prices simultaneously to any positive value, i.e., $p_i = \rho > 0, \forall i \in N$ and the demand is still positive, i.e., $D_i = k_i, \forall i \in N$. This is impossible in reality.

consider a uniform unit operating and maintaining cost for infrastructures and denoted as c.

Joint venture price model and profit of each participant: Next, we introduce the price model of the joint venture and the profit of each participant in the system. Originally, each operator holds and maintains its own network infrastructures. So its profit consists of a selling profit given by

$$f_i = (p_i - c)(k_i - p_i + \sum_{j \in N, j \neq i} \beta_{ij} p_j), \tag{3}$$

which increases with operating period, and a fixed infrastructure cost given as Eq. (2). To guarantee the existence of a positive selling profit, we assume that $k_i > c, \forall i \in N$.

After the joint venture is set up and rents its infrastructure service to each operator at unit price p, the selling profit of the joint venture is given as

$$f_0 = (p - c) \sum_{i \in N} (k_i - p_i + \sum_{j \in N, j \neq i} \beta_{ij} p_j) \tag{4}$$

and the fixed infrastructure cost is given by

$$h(\sum_{i \in N} k_i - p_i + \sum_{j \in N, j \neq i} \beta_{ij} p_j). \tag{5}$$

Denote the share of operator i on the joint venture by α_i $(0 \leq \alpha_i \leq 1)$ and $\alpha = \sum_{i \in N} \alpha_i$, where $0 \leq \alpha \leq 1$ and $\alpha < 1$ means that there exists an outside investor, e.g., the government, sharing the joint venture. The selling profit of operator i is now given by

$$f_i = (p_i - p)(k_i - p_i + \sum_{j \in N, j \neq i} \beta_{ij} p_j) + \alpha_i f_0 \tag{6}$$

and the fixed cost shared by operator i is

$$g_i = \alpha_i g_0. \tag{7}$$

Finally, we denote the aggregate selling profit of all operators as F that $F = \sum_{i \in N} f_i$ and the aggregate selling profit of the whole market (including potential outside investors) as AP where $AP = F + (1 - \alpha) f_0$.

B. Game Formulation

We formulate the strategic interactions between the joint venture and the network operators as a leader-follower Stackelberg game. In brief, the Stackelberg game consists of two stages. In the first stage, the joint venture, as the leader, decides the price p of renting infrastructures and the share composition $\alpha_i, \forall i \in N$. In the second stage, the network operators, as the followers, take the share of each other and the price of renting infrastructure as input and decide their prices on the end user market[4]. After each parameter in the whole game is determined, the outcome of the game can be figured out and the fixed investment on infrastructures can thus be determined to satisfy the capacity requirement.

Next, we will introduce the game formulation in detail by clarifying the *game states* and the participants' *action* and *strategy spaces*. As the first stage, the whole game starts with the action of the joint venture. In this stage, the *action* as well as the *strategy* of the joint venture is to select certain renting price p and the share of each operator $\alpha_i, \forall i \in N$. Its action and strategy spaces consist of all the feasible tuples of price and the shares p and $\alpha_i, \forall i \in N$ satisfying that $p \in \mathbb{R}$, where the negative value of p means subsidizing the operators, and $0 \leq \alpha_i \leq 1, \forall i \in N$ as well as $\alpha \leq 1$. After taking the action, the *game state* becomes the determined tuple of the price of renting infrastructures and the shares of all operators. The state is known by all participants, and the game moves to the second stage.

The second stage is a market competition among the set N of network operators, the behaviors of which may vary dramatically with the time scope of the competition. To this end, we divide the competition into three different time scopes, the one-shot competition, the finite-round competition and the long-run competition, and analyze the game under each scope. In the one-shot competition, the operators simultaneously decide their service prices $p_i, i \in N$ to the end users for only once. The *action* of operator i is to select p_i from the action space $\mathbb{R}$. The *strategy* of operator i is a map from the previous game state, the tuple of p and $\alpha_i, \forall i \in N$ settled by the joint venture in the first stage, to the action space. After all operators determine the prices $p_i, \forall i \in N$ simultaneously, the game stops and the profits of all operators are directly given by $f_i, \forall i \in N$.

In the finite-round competition, the network operators repeat the one-shot game for finite rounds. In each round, the $action$ of operator p_i is still to select the price p_i from space $\mathbb{R}$ and after the actions are taken, the profits of all operators in this round are drawn and the *game*

[4]We note that the shares of network operators and the price of renting infrastructure from the joint venture can be also determined by negations among the joint venture and the network operators. However, inspired by China Tower, the joint telecom infrastructure company, which is established under the guidance of the government, we are more interested in how a regulator, e.g., the government, can utilize the joint venture to achieve social benefit via setting key decision variables of the joint venture. Thus, we consider that these variables are decided in the first stage by the joint venture and are given as inputs to the network operators.

Table I
MAIN NOTATIONS USED THROUGH OUT THIS PAPER

N	the set of operators with $\|N\| = n$	D_i	the demand of operator i
p	the renting price of the joint venture	p_i	the price of operator i charged to users
k_i	the initial user density of operator i	β_{ij}	the influence factor of p_i on D_j
$h(\cdot)$	the fixed infrastructure cost function	c	the unit infrastructure operating cost
g_0	the fixed cost of the joint venture	g_i	the fixed cost of each operator
f_0	the selling profit of the joint venture	f_i	the selling profit of operator i
α_i	the share of operator i with $\alpha = \sum_i \alpha_i$	γ	the time-discounted factor
F	the aggregate selling profit of all operators	AP	the aggregate selling profits of the whole market
P	$P = \sum_{i \in N} p_i$	β'	$\beta' = 1 - (n-1)\beta$
ne	the superscript for the value of variables in Nash equilibrium	co	the superscript for the value of variables in cooperation equalibrium

state becomes the tuple of all the prices $p_i, \forall i \in N$ set in this round. The $strategy$ of operator i in this round, however, is a map from all previous game states, i.e., p, $\alpha_i, \forall i$, and $p_i, \forall i$ in all previous rounds, to the action space and the $strategy$ of the whole second stage for operator i is the tuple of its strategies in every round. After the finite rounds, the game stops and each operate gets the aggregate profit over all the rounds. The long-run competition goes the same as the finite-round competition, except that the competition goes infinite rounds and the aggregate profit of each operator will increase towards the infinity. A standard technique to evaluate the strategies of the operators in the long-run competition is to discount the profit gained in each round by a constant time-discounted factor such that the aggregate profit of each operator over the infinite rounds will be bounded to finite and become comparable. To this end, we calculate the aggregate profit of operator i in the long-run competition as $\sum_{k=0}^{+\infty} \gamma^k f_i^k, i \in N$, where f_i^k is the selling profit of operator i at the kth round and $\gamma\,(0 < \gamma < 1)$ is the time-discounted factor.

Finally, we analyze the objective of each participant. We note that after building up the infrastructures, the fixed cost can hardly change. In the contrary, the selling profit will increase with the selling time. For this reason, we consider that the objective of each network operator is to maximize its selling profit. For the objective of the joint venture, we consider the objectives from a social perspective instead of maximizing its own profit, because the joint venture we consider is established under the guidance of government and is shared by all network operators. We consider two representative objectives, aggregate profit maximization of the market and budget balance of the whole industry. The former is to maximize the sum of the profits of all players

AP. The latter is to keep each network operator charging the end users at a relatively low price that covers its marginal cost. In addition, we consider the fixed cost of the infrastructure as a necessary condition to support the user demand of each operator and numerically compare the difference in the fixed cost of the infrastructure in the presence or the absence of the joint venture. We summarize our notations used throughout this paper in Table I.

IV. Competition Equilibrium Analysis

To analyze the Stackelberg game of the telecom market with the joint venture, we conduct a back induction analysis. In this section, we first analyze the competition of the network operators in the second stage of the game. We take the share of each operator and the price of renting infrastructure services from the joint venture as given and analyze the equilibrium prices of the network operators in the competitions under aforementioned three time scopes.

A. The one-shot competition equilibrium

In the one-shot competition, all operators simultaneously determine their prices $p_i, i \in N$ to the end users at one shot with the objective to maximize their own selling profits $f_i, i \in N$ after the prices are revealed. To facilitate analysis, we first present a closed-form equilibrium expression for the case with all identical $\beta_{ij} = \beta, \forall i, j \in N$, then develop the matrix form solution of the equilibrium for the general case with arbitrary β_{ij}. We use superscript ne to indicate the value of corresponding variables at the one-shot equilibrium.

Given p and $\alpha_i, \forall i \in N$, each operator aims to maximize

$$f_i = (p_i - p)(k_i - p_i + \beta \sum_{j \in N \setminus \{i\}} p_j) + \alpha_i (p - c)(\sum_{j \in N} k_j - \beta' \sum_{j \in N} p_j)$$

where $\beta' \equiv 1 - (n-1)\beta$. At equilibrium, no operator can change its price individually to improve its profit. Thus, we have $\frac{\partial f_i}{\partial p_i} = 0, \forall i \in N$, i.e.,

$$2p_i - \beta \sum_{j \in N \setminus \{i\}} p_j = k_i + p - \alpha_i \beta' (p - c), \forall i \in N.$$

Summing up these equations, we get the one-shot equilibrium price for operator i that

$$p_i^{ne} = \frac{1}{2+\beta}(k_i + \beta P^{ne} + p - \alpha_i \beta' (p - c)) \tag{8}$$

where

$$P^{ne} \equiv \sum_i p_i^{ne} = \frac{1}{1+\beta'} \left(K + np - \alpha \beta' (p - c) \right).$$

For the general case of arbitrary β_{ij}, we show that there exists a unique equilibrium in the market and the equilibrium price can be efficiently computed.

Theorem 1. *Let $B^{n\times n} = (b_{ij})_{ij}$ where $b_{ij} = -\beta_{ij}$, $\forall i, j, i \neq j$ and $b_{ij} = 2$, $\forall i, j, i = j$. Let $\boldsymbol{u} = (k_i + c\alpha_i(1 - \sum_{j\neq i}\beta_{ji}))_i$, and $\boldsymbol{v} = (1 - \alpha_i(1 - \sum_{j\neq i}\beta_{ji}))_i$. B is invertible and in the one-shot competition, the market equilibrium is unique and the equilibrium price vector, $\boldsymbol{p}^{ne} = (p_i^{ne})_{i\in N}$, can be computed as $\boldsymbol{p}^{ne} = B^{-1}(\boldsymbol{u} + p\boldsymbol{v})$.*

Proof. Given p and $\alpha_i, \forall i \in N$, the equilibrium is given by $\frac{\partial f_i}{\partial p_i} = 0$, $i \in N$, i.e.,

$$2p_i - \sum_{j\neq i}\beta_{ij}p_j = k_i + p - p\alpha_i(1 - \sum_{j\neq i}\beta_{ji}) + c\alpha_i(1 - \sum_{j\neq i}\beta_{ji}) \tag{9}$$

for all $i \in N$. Put the above equations in matrix form, we get $B\boldsymbol{p}^{ne} = \boldsymbol{u} + p\boldsymbol{v}$. To complete our proof, we only need to show that B is invertible. By assumption $\beta_{ij} \ll 1$, $\forall i \neq j$, we have $\sum_{j\neq i}\beta_{ij} < 1, \forall i \in N$, which implies $b_{ii} = 2 > 1 > |-\sum_{j\neq i}\beta_{ij}| = |\sum_{j\neq i} b_{ij}|$. B satisfies that each diagnose element is larger than the absolute sum of other elements in the same row. By Geršgorin theorem, such matrix is invertible.

□

Following the classic backward induction argument [26], we can generalize the results of the one-shot competition into the finite-round competition.

Corollary 1. *In the finite round competition where the same one-shot competition is repeated for finite rounds and each operator aims to maximize the aggregate profit of all rounds, the market equilibrium is unique and the equilibrium strategy for each operator is to charge its end users at the one-shot equilibrium price in all rounds.*

By above analysis, we get the following implications about strategic behaviors of the network operations in the one-shot competition (and the finite-round competition):

1) Since $\frac{\partial p_i^{ne}}{\partial p} = \frac{\beta(n-\alpha\beta')}{(1+\beta')(2+\beta)} + \frac{1-\alpha_i\beta'}{2+\beta} > 0, \forall i \in N$, the rise and fall in the price of renting infrastructure services from the joint venture will induce the change of the one-shot equilibrium price of each operator in the same direction. In addition, by substituting p_i^{ne} by Eq. (8), we can rewrite f_i as a concave quadratic function of p, indicating that the profit of an operator at the one-shot equilibrium will first increase with the price of renting infrastructure services, reaching the maximum, and will then decrease with p rising to the positive infinity.

2) Since $\frac{\partial p_i^{ne}}{\partial \alpha_i} = \frac{\beta'(1+\beta+\beta')(c-p)}{(1+\beta')(2+\beta)} \propto (c-p), \forall i \in N$, when the price p of renting infrastructure is larger than the marginal cost c, the one-shot equilibrium price of any operator decreases with its share of the joint venture. In contrast, when the price p is lower than the marginal cost, the one-shot equilibrium price of any operator increases with its share. The interpretation is that when the joint venture benefits from providing the infrastructure services, i.e., $p > c$, the more share an operator holds, the more incentive the operator has to improve the profit of the joint venture, and consequently, the operator will enlarge its demand by reducing its end-user price.
3) Since $0 < \frac{\partial p_i^{ne}}{\partial k_j} < \frac{\partial p_i^{ne}}{\partial k_i}, \forall i, j \in N, i \neq j$, the change in the initial user density of an operator will cause a greater change in its own one-shot equilibrium price in the same direction and will cause slighter changes in the other operators' one-shot equilibrium prices in the opposite direction.

B. The long-run competition equilibriums

Before we analyze the long-run competition equilibriums, we present the observation of the structure of the prisoners' dilemma in the one-shot competition, which is the base to understand the long-run competition equilibriums. The prisoners' dilemma depicts a situation where all players would get higher profits if they cooperate to maximize their aggregate profit instead of reaching the equilibrium that maximizes their own profits with respect to each other's price strategy. The prisoners' dilemma always exists in the oligopoly market which consists of a few competing companies with similar market power, e.g. our telecom market without the joint venture, and leads to the inefficiency from the perspective of the market profit. To our surprise, in the telecom market with the joint venture, we find the absence of the prisoners' dilemma under a proper combination of the price of renting infrastructure and the share composition of the joint venture.

For illustration, we consider a market in the one-shot competition with identical operators, i.e., $k_i = k_1, \forall i \in N$ and $\beta_{ij} = \beta$, $\forall i, j \in N$. Each operator shares $\frac{1}{n}$ fraction of the joint venture. According to Eq. (8), the equilibrium price of any operator is given by

$$p_i^{ne} = \frac{1}{1+\beta'}(k_1 + p - \frac{1}{n}\beta'(p-c)) \tag{10}$$

Now, we consider the price of each operator that maximizes their aggregate profit F. By solving

$\frac{\partial F}{\partial p_i} = 0, \forall i \in N$, we get the price of operator i, denoted as p_i^{opt}, that

$$p_i^{opt} = \frac{1}{2\beta'}(k_1 + \beta' c). \tag{11}$$

When $p \neq c + \frac{\beta n(n-1)(k_1-\beta' c)}{2\beta'(n-\beta')}$, we have $p_i^{ne} \neq p_i^{opt}$ and the profit of each operator under price profile $p_i^{opt}, \forall i \in N$, denoted as f_i^{opt}, is strictly higher than that of the one-shot equilibrium, i.e., $f_i^{opt} = \frac{F^{opt}}{n} > \frac{F^{ne}}{n} = f_i^{ne}, \forall i \in N$ where $F^{opt} \equiv \sum_{i \in N} f_i^{opt}$. However, in the one-shot competition, p_i^{opt} is not the best price for operator i when the other operators price their network service at p_j^{opt} for $j \in N, j \neq i$. Therefore, each operator has incentive to derive from p_i^{opt} and finally price at p_i^{ne}, trapped by the prisoners' dilemma.

The difference of the long-run competition from the one-shot competition is that each operator can observe the historical decisions of the others and then make its decisions in the current round. We formulate the long-run competition as an infinite-round time-discounted game with discount factor γ. With this help of the price history and sufficiently large discount factor γ, the operators can reach an equilibrium with higher profit at each round than that in the one-shot equilibrium enbedded with the structure of prisoners' dilemma, or even reach an equilibrium that achieves maximal aggregate profit. The following theorem gives a characterization of the long-run equilibriums.

Theorem 2. *Given the price of renting infrastructure p and the share composition $\alpha_i, \forall i \in N$ of the joint venture, for any profit profile $f_i, \forall i \in N$ of the one-shot price competition such that there is a price profile achieving this profit profile and $f_i \geq f_i^{ne}, \forall i \in N$, there exists an equilibrium in the long-run competition with sufficiently large discount factor γ that achieves profit $\gamma^{k-1} f_i$ at the kth round for any operator i, $i \in N$ and round k, $k \in \mathbb{N}$.*

Proof. Let $p_i, \forall i \in N$ be the price profile that achieves the profit profile $f_i, \forall i \in N$. Consider following pricing strategy for operator $i, \forall i \in N$:

Charge the end users at price p_i in each round as long as operator j price at p_j for all $j \in N, j \neq i$; if any other operator starts to charge the end users at any other price, then set the price as p_i^{ne} forever.

Next, we show that these pricing strategies form a Nash equilibrium of the infinite-round time-discounted game with sufficiently large discount factor γ. If all operators follow this strategy, they will each get total profit

$$\sum_{k=1}^{\infty} f_i^k = f_i + \gamma f_i + \gamma^2 f_i + ... = \frac{f_i}{1-\gamma}.$$

If any operator deviates the strategy, even just once, then it will get total profit (w.l.o.g. suppose the operator deviate at the first round and let f_i' be the maximum profit it can get in the first round by derivation),

$$\sum_{k=1}^{\infty} f_i^k \leq f_i' + \gamma f_i^{ne} + \gamma^2 f_i^{ne} + ... = f_i' + \frac{\gamma f_i^{ne}}{1-\gamma}.$$

The first total profit is larger if $\gamma > \frac{f_i'-f_i}{f_i'-f_i^{ne}}$, where $\frac{f_i'-f_i}{f_i'-f_i^{ne}} < 1$ since $f_i > f_i^{ne}$. Therefore, at an appropriate discount rate, no operator is willing to deviate from the aforementioned strategy individually and these pricing strategies form a Nash equilibrium[5]. □

The price profile that maximizes the aggregate profit of the operators is usually an equilibrium price profile in the long-run competition of operators with similar initial user density as $f_i \approx \frac{F}{n} > \frac{F^{ne}}{n} \approx f_i^{ne}, \forall i \in N$. Therefore, we consider this price profile as a typical long-run equilibrium price profile[6] and denote the values of the variables at this equilibrium with superscript co. Next, we present the closed-form expression of this long-run equilibrium price profile under $\beta_{ij} = \beta, \forall i, j \in N$. By solving $\frac{\partial F}{\partial p_i} = 0, \forall i \in N$, we get

$$p_i^{co} = \frac{1}{2}\left(c + (1-\alpha)(p-c) + \frac{\beta' k_i + \beta K}{\beta'(1+\beta)}\right) \tag{12}$$

For general β_{ij}, this price profile can be computed in matrix form in a similar way as described in Theorem 1.

Meanwhile, we notice that there could be a large gap of the aggregate profit between the equilibrium that maximizes the aggregate profit and the worst counterpart. By the Folk theorem, the worst equilibrium is the equilibrium achieving $f_i^{worst} = \min_{p_j, j\neq i} \max_{p_i} f_i$ where the minimization is constrained by $f_j \geq 0, \forall j \neq i$. The constraint $f_j \geq 0, \forall j \neq i$ is a piecewise quadratic function since the demand could not be negative. For simplicity of the expression, we conduct the numerical studies on the gap in Section VI-C instead of presenting the closed analysis. The results reveal that the gap can be extremely large in certain conditions but can also be eliminated

[5]This proof can be found in traditional game literatures. We present it for completeness.

[6]We conduct numerical experiments to study the condition of initial user density under which such a price profile is exactly a long-run equilibrium in Section VI.

from the market of identical operators by determining p and $\alpha_i, i \in N$ according to equation $p_i^{ne} = p_i^{co}$ such that the Nash equilibrium strategies coincide with the global optimal strategies and the embedded prisoners' dilemma is eliminated.

By above analysis, we get the following implications about the long-run equilibrium prices:

1) In the long-run competition, the telecom market may have multiple equilibriums that achieve better profit for each operator w.r.t. the equilibrium of the one-shot competition in each round. If the initial user densities of the network operators are close enough, pricing at the price that maximizes the aggregate profit of all operators will be one of the equilibrium price for each operator.
2) If $\alpha < 1$, $\frac{\partial p_i^{co}}{\partial p} > 0$, indicating that when there exists an outside investor investing the joint venture, the rise and fall in the price of renting infrastructure from the joint venture will cause the changes of the long-run equilibrium prices of all operators $p_i^{co}, \forall i \in N$ in the same direction. If $\alpha = 1$, $\frac{\partial p_i^{co}}{\partial p} = 0$, indicating that the long-run equilibrium prices of all operators $p_i^{co}, \forall i \in N$ are independent with the price of renting infrastructure from the joint venture when there is no outside investor.
3) If $p > c$, $\frac{\partial p_i^{co}}{\partial \alpha} < 0$, indicating that when the joint venture earns a positive profit, the long-run equilibrium prices of all operators $p_i^{co}, \forall i \in N$ change with the share of the outside investor in the same direction. The contrary happens if $p < c$. In addition, when α is fixed, $\frac{\partial p_i^{co}}{\partial \alpha_i} = 0$, indicating that when the total share of all operators is determined, the long-run equilibrium prices $p_i^{co}, \forall i \in N$ are independent with the specific share composition of the joint venture.
4) Since $0 < \frac{\partial p_i^{co}}{\partial k_j} < \frac{\partial p_i^{co}}{\partial k_i}, \forall i, j \in N, i \neq j$, the change in the initial user density of an operator will cause a greater change in its long-run equilibrium price p_i^{co} in the same direction and cause slighter changes in the other operators' long-run equilibrium prices $p_j^{co}, \forall j \in N, j \neq i$ in the opposite direction.

The intuition behind the second and the third points is that when the outside investor shares more positive profit for each unit of the sales (p is larger), the operators would prefer to reduce their sales such that the outside investor will grab less profit from their sales, otherwise, the operators would prefer to expand their sales.

V. Optimization via the Joint Venture

In this section, we explore the first stage of the Stackelberg game. We settle the decisions of the joint venture in the first stage w.r.t two objectives, aggregate profit maximization of the market and budget balance of the whole industry. Similarly, we consider different responses of the operators in the competitions of different time scopes in the second stage. As the equilibrium of the finite-round competition exactly coincides with the one-shot competition, we only consider the one-shot competition and the long-run competition. For the one-shot competition, the equilibrium price of each operator is uniquely determined by Eq. (8). For the long-run competition, we consider the equilibrium price profile given by Eq. (12) which maximizes the aggregate profit of all operators. For mathematical simplicity and without loss of too much generality, we assume $\beta_{ij} = \beta, \forall i, j \in N, i \neq j$.

A. The market aggregate profit maximization

The aggregate profit of the market is given by

$$\begin{aligned} AP &= \sum_{i \in N} f_i + (1-\alpha) f_0 \\ &= \sum_{i \in N} (p_i - c)(k_i - p_i + \beta \sum_{j \in N, j \neq i} p_j) \end{aligned} \tag{13}$$

For the one-shot competition: By solving $\frac{\partial AP}{\partial p_i} = 0, \forall i \in N$, we get that AP is maximized at

$$p_i = \frac{c}{2} + \frac{\beta' k_i + \beta K}{2\beta'(\beta + 1)} \tag{14}$$

for all $i \in N$. Therefore, if we can determine the p and $\alpha_i, \forall i \in N$ such that the one-shot equilibrium prices coincide with the prices given by Eq. (14), the aggregate profit of the market is then maximized. By solving the combined equation of Eq. (8) and Eq. (14) for all $i \in N$ together with $\sum_{i \in N} \alpha_i = \alpha$, we get the desired shares and the price of renting infrastructure, denoted as $\alpha_i^{opt}, i \in N$ and p^{opt}, that

$$p^{opt} = c + \frac{\beta(n-1)(K - n\beta' c)}{2\beta'(n - \beta'\alpha)} \tag{15}$$

$$\alpha_i^{opt} = \frac{\alpha}{n} + \frac{(n - \beta'\alpha)(k_i - \overline{k})}{(n-1)(1+\beta)(\sum_j k_j - n\beta' c)} \tag{16}$$

To satisfy $\alpha_i^{opt} \in [0, 1]$, we get the constraints on α and $k_i, \forall i \in N$ that

$$k_i \geq \frac{1-\alpha}{1+\beta\alpha} \overline{k}_{-i} + \frac{(1+\beta)\beta'\alpha}{1+\beta\alpha} c \tag{17}$$

where $\overline{k}_{-i} = \frac{1}{n-1} \sum_{j \in N \setminus \{i\}} k_j$. When $\alpha = 1$, the Ineq. (17) always holds for any $k_i > c, \forall i \in N$. The formulas indicate that if the network operators fall in the one-shot competition, the price of renting infrastructure that maximizes the aggregate profit of the market with the joint venture should be always large than the marginal cost, and the difference between the share of each operator and the average share should be positively proportional to the difference between the initial user density of the operator and the average initial user density. Meanwhile, the total share of all operators does not influence the maximum of the aggregate profit of the market with the joint venture.

For the long-run competition: In the long-run competition, the aggregate profit of the market is given by Eq. (13) with an extra coefficient $\frac{1}{1-\gamma}$. Therefore, by solving combined equation of Eq. (12) and Eq. (14), we get that either setting $\alpha = 1$ or $p = c$ can maximize the aggregate profit of the whole market.

B. The budget balance for each operator

At the budget balance of the industry, the price to the end users of each operator is equal to the marginal cost, i.e.,

$$p_i = c, \forall i \in N.$$

For the one-shot competition: By solving the combined equation of Eq. (8) and $p_i = c$ for all $i \in N$ together with $\sum_{i \in N} \alpha_i = \alpha$, we get the corresponding price of renting infrastructure, denoted as p^{bb}, and the share of each operator, denoted as α^{bb},

$$p^{bb} = c - \frac{K - n\beta' c}{n - \beta' \alpha} \tag{18}$$

$$\alpha_i^{bb} = \frac{\alpha}{n} - \frac{(n - \beta'\alpha)(k_i - \overline{k})}{\beta'(\sum_j k_j - n\beta' c)} \tag{19}$$

for all $i \in N$, where $\overline{k} = \frac{1}{n}\sum_{i \in N} k_i$. To satisfy $\alpha_i^{bb} \in [0, 1]$, we get the constraints on α and $k_i, \forall i \in N$ that

$$\frac{(n-1)^2\beta\overline{k}_{-i} + (n-\alpha)\beta'^2 c}{(n-1)^2\beta + (n-\alpha)\beta'} \le k_i \le \frac{(n-1)\overline{k}_{-i} - \alpha\beta'^2 c}{n - 1 - \alpha\beta'} \tag{20}$$

The formulas indicate that when the operators fall in the one-shot competition, to keep the budget balance of each operator, the price of renting infrastructure should be always smaller than the marginal cost, and the difference of the share between each operator and the average share should be negatively proportional to the difference between the initial user density of the operator and the average initial user density.

For the long-run competition: We consider identical operators first, i.e., $k_i = k_1, \forall i \in N$. By solving the combined equation of Eq. (12) and $p_i = c$, we get that to keep each operator operating on the budget balance price, the joint venture shall sustain the necessary and sufficient condition

$$(1-\alpha)(p-c) = -\frac{k_1 - \beta' c}{\beta'}. \tag{21}$$

Since $k_1 \geq c$, we get that $(1-\alpha)(p-c) = -\frac{k_1-\beta' c}{\beta'} < 0$ and therefore, we have $\alpha < 1$ and $p < c$. This indicates that to drive these selfish operators to price at the marginal cost c in order to benefit the public as well as to keep the whole industry budget balance, there should be an outside investor, e.g., the government, investing the joint venture. Since $p < c$, the joint venture operates at a negative profit, implying that the outside investor actually subsidizes the operators via holding part of the shares of the joint venture. Note that the total subsidy is given by $(1-\alpha)|f_0| = (1-\alpha)(c-p)D(\boldsymbol{p})$, so $(1-\alpha)(c-p) = \frac{k_1-\beta' c}{\beta'}$ is actually the subsidy per unit of the sales. The implication is that in order to make the telecom industry, which consists of selfish operators, a public utility industry, which shall benefit a large base of public by pricing its service at a near-cost price, an outside investment is demanded to motivate the operator to enlarge its service quantity to cover part of the low value end users. Similar subsidy policy had been implemented in the agriculture in America in the form of target price-deficiency payments program [27], [28]. In this program, the American government subsidized farmers by paying the deficiency between the market price and the target price for each unit of cotton, wheat, and feed grains sold so as to increase the productivity. We suggest that in the telecom market, such subsidy can come from either public tax or a fixed fee charged to the network operators for the permission of entering the telecom market.

For non-identical operators, we find that it is unable to drive each operator to keep the budget balance, no matter what p and α are adopted. Instead, we set the objective to minimize the maximal gap between the prices of the operators to the marginal cost c, i.e., $\min_{p,\alpha} \max_{i\in N} |p_i - c|$. To achieve this objective, we have

$$(1-\alpha)(p-c) = -(\frac{k_{max} + k_{min}}{2\beta'} - c)$$

where $k_{max} = \max_{i\in N} k_i$ and $k_{min} = \min_{i\in N} k_i$.

VI. NUMERICAL RESULTS

In this section, we report our numerical results. In the first subsection, we conduct numerical computation to visualize our theoretical results about the influence of joint venture decisions and market parameters on the final market equilibriums from both the one-shot and the long-run perspective. In the second subsection, we conduct simulations to complement our theoretical analysis where closed form solutions are too complex to present. As these experiments are designed to verify the theoretical results, we ignore the physical unit of each variable in these experiments. In the last subsection, we report the comparison of the fixed costs of the infrastructures in the presence or the absence of the joint venture based on the real data from [25]

A. The influence of joint venture decisions and market parameters on market equilibriums

In this section, we conduct numerical studies to verify our theoretical results. We first invest the one-shot competition scenario and then, the long-run scenario, under linear demand model where we carried out the theoretical analysis. Then, we extend the simulations of both scenarios to non-linear demand setting to show the robustness of our analysis and results.

For the one-shot competition scenario, we conduct four sets of experiments in a telecom market with three operators. The first three sets of experiments show the influence of joint venture respectively on the equilibrium price, the equilibrium profit of an operator and the aggregate profit of the whole market at equilibrium. W.l.o.g, in these three experiment sets, we set $k_i = 8, i \in \{1, 2, 3\}$, $c = 4$, $\beta_{ij} = 0.3, \forall i, j \in \{1, 2, 3\}, i \neq, j$. As for the decisions of the joint venture, p ranges from 2 to 14 with an increment of 0.08 and α_1 ranges from 0.2 to 0.6 with an increment of 0.01, while α_2 and α_3 always keep a value of 0.2. We confirm the equilibrium computation by checking the equilibrium condition that no operator can improve its own profit by changing its price individually and that each operator has a positive demand at equilibrium.

Fig. 2 shows the variation of the unit service price, p_1, of operator 1 charging to the end users at the one-shot Nash equilibrium. When the other parameters are fixed, e.g., as the blue line in Fig. 2 shows, p_1 increases linearly with the unit price p of renting infrastructures from the joint venture. On the other hand, the influence of the share of operator 1, α_1, on price p_1 depends on the difference between the renting price p and the marginal cost c. When $p = c = 4$, the share of operator 1 does not influence the equilibrium price of the operator as shown in Fig. 2 by the red line, which is parallel to the p-α_1 plane. When $p > c$, the equilibrium price p_1 decreases linearly with share α_1, while if $p < c$, p_1 increases linearly with α_1. The result shows that when

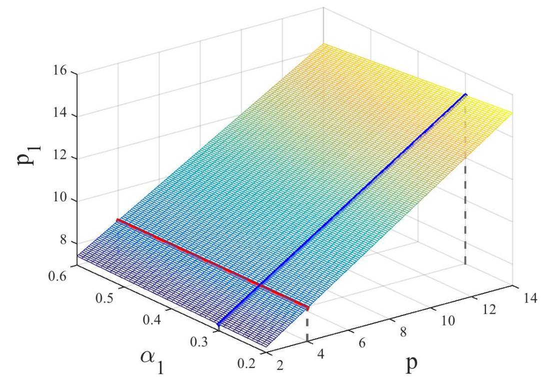

Figure 2. The influence of joint venture decisions on the one-shot equilibrium price under linear demand

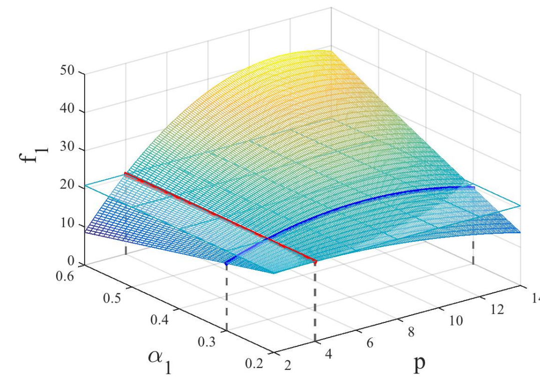

Figure 3. The influence of joint venture decisions on the one-shot equilibrium profit of an operator under linear demand

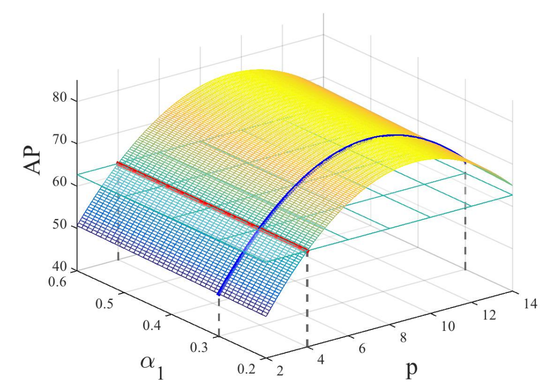

Figure 4. The influence of joint venture decisions on the market aggregate profits at the one-shot equilibrium under linear demand

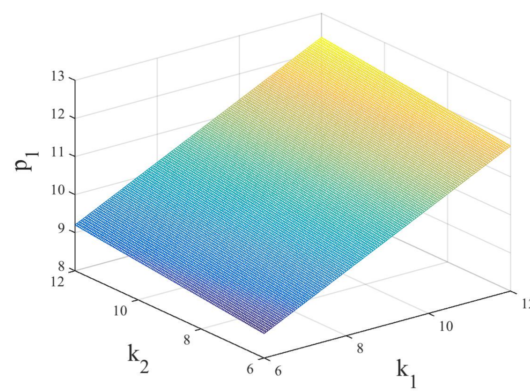

Figure 5. The influence of initial user densities on the one-shot equilibrium price under linear demand

the joint venture earns a positive profit per unit of demand, the more an operator shares, the more the operator is willing to decrease its price in order to increase the demand and the profit of the joint venture.

Fig. 3 shows the variation of the profit, f_1, of operator 1 when the renting price p and its share α_1 change. The profit surface intersects with any plane vertical to α_1-axis by a downward parabola, e.g., the blue curve in Fig. 3, and intersects with any plane vertical to p-axis by a straight line, e.g., the red lines in Fig. 3. It shows that the profit of operator 1 first rises and then decreases with the renting price p of the joint venture. The share of operator influences its profit f_1 similarly as it influences its equilibrium price p_1. The marked plane parallel to the p-α_1 plane

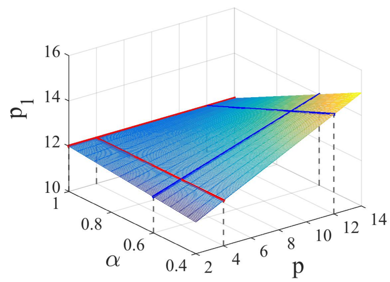

Figure 6. The influence of joint venture decisions on the long-run equilibrium price under linear demand

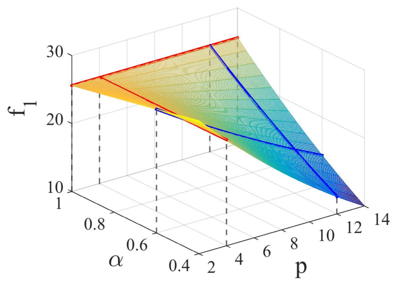

Figure 7. The influence of joint venture decisions on the long-run equilibrium profit of an operator under linear demand

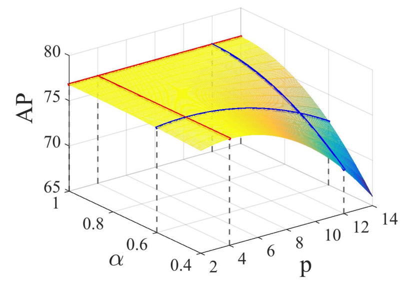

Figure 8. The influence of joint venture decisions on the market aggregate profits at the long-run equilibrium under linear demand

shows the one-shot equilibrium profit of operator 1 before the joint venture is established. The part of the profit surface lower to the marked plane shows, without properly setting the renting price of the joint venture and the shares, the profit of each operator could be even worse than not to establish the joint venture. As shown by Fig. 4 and by the corresponding blue curve and red line, the joint venture decisions affect the aggregate profit of the whole market AP in a similar way as it influences the equilibrium profit f_1 of operator 1, except that when p is slightly larger than c, the aggregate profit slightly and linearly increases with the share α_1, while when p is much larger than c, the aggregate profit linearly decreases with the share α_1.

The fourth set of experiments shows the influence of initial user densities of different operators on the one-shot equilibrium price of an operator. In this set, we set $p = 6$, $c = 4$, $\alpha_i = 0.2, i = 1, 2, 3$ and $\beta_{ij} = 0.3, \forall i, j \in \{1, 2, 3\}, i \neq, j$. The initial user densities of operator 1, k_1, and operator 2, k_2 both range from 6 to 12 with an increment of 0.05. The initial use density k_3 keeps 8. Fig. 5 shows the variation of the equilibrium price, $p1$, of operator 1 when the initial user densities of operator 1 and operator 2 change while that of operator 3 keeps unchanged. The equilibrium price surface is a parallelogram, meaning that the influence of initial user densities of different operators on the equilibrium price is linearly additive. At the same time, we can observe that the influence of initial user densities of operator 1 and operator 2 both affect positively to the equilibrium price of operator 1, while the influence of the initial user density of operator 1 outweighs the influence of the other.

For the long-run scenario, we also consider three operators and conduct three sets of experiments, which aim to show the influence of joint venture respectively on the price and the profit of an operator and the aggregate profit of the whole market at the long-run equilibrium. Since there are multiple equilibriums, we focus on the long-run equilibrium where the aggregate profit of all operators is maximized. Besides, we focus on the influence induced by the aggregate share α instead of that induced by any specific share α_i, because the equilibrium prices and the aggregate profit of the market at the equilibrium are independent with the specific shares but the sum of them. W.l.o.g, we set the initial user density of each operator as $k_i = 8, i = 1, 2, 3$, $c = 4$, $\beta_{ij} = 0.3, \forall i, j \in \{1, 2, 3\}, i \neq, j$. As for the decisions of the joint venture, p ranges from 2 to 14 with an increment of 0.08 and α ranges from 0.4 to 1 with an increment of 0.01, while each operator shares one-third of the aggregate share. Similarly, we guarantee that in our results, each operator has a positive demand at the equilibrium.

Fig. 6 shows the variation of the equilibrium price, p_1, of operator 1. As the two blue lines in Fig. 2 show, the price varies linearly both with the price p of renting infrastructures and with the aggregate share α. While p_1 increases with p, p_1 decreases with α when $p > c = 4$ and increases with α if $p < c$. The two red lines, which are parallel to the p-α plane, show that either $p = c$ or $\alpha = 1$ makes the equilibrium price independent with the other variable.

Fig. 7 shows the variation of the profit, f_1, of operator 1 when the renting price p and the aggregate share α change. As the two blue curves in Fig. 7 show, the equilibrium profit varies in a quadratic fashion with either p or α. In fact, f_1 decreases with price p, following an upward parabola. When $p < c$, f_1 increases with α, following an upward parabola, and when $p > c$, f_1 decreases with α, following a downward parabola. The two red lines also show when the influence of the joint venture decisions on the profit f_1 disappears. Fig. 8 shows the variation of the aggregate profit of the whole market, AP, when the renting price p and the aggregate share α change. The AP varies with either p or α following a downward parabola. The two blues curves show two examples for this. The peak of any parabola locates in one of the two red lines, which indicates that when $p = c$ or $\alpha = 1$, the aggregate profit of the market is maximized, which agrees with our theoretical analysis.

Finally, we redo the aforementioned sets of experiments under non-linear demand setting. W.l.o.g., we consider only two operators. The demand of each operator is in a non-linear form that $D_i = k_i(p_i - \sum_{j \in N, j \neq i} \beta_{ij} p_j)^{1-\rho}$ for any operator $i \in N$[7]. We adopt $\rho = 2.3$ in our experiments. In each set of experiments, p varies from 2 to 20 and other parameters follow the setting of each counterpart experiment in the aforementioned linear demand setting. Fig. 9∼Fig. 14 show the

[7]This form of demand can be derived by assuming that the valuation of telecom service to end user follows a power distribution [29]. In most practice scenarios, which follow power law distribution, ρ is usually estimated within range $(2, 3)$.

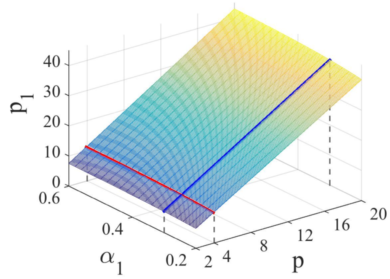

Figure 9. The influence of joint venture decisions on the one-shot equilibrium price under non-linear demand

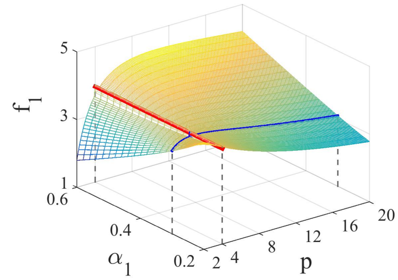

Figure 10. The influence of joint venture decisions on the one-shot equilibrium profit of an operator under non-linear demand

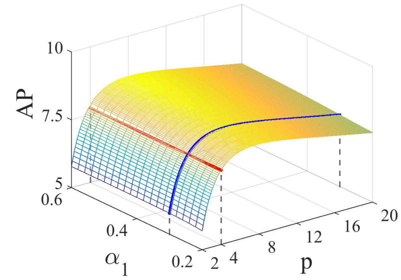

Figure 11. The influence of joint venture decisions on the market aggregate profits at the one-shot equilibrium under non-linear demand

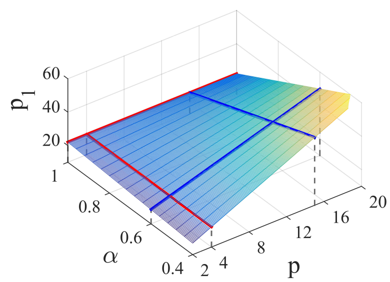

Figure 12. The influence of joint venture decisions on the long-run equilibrium price under non-linear demand

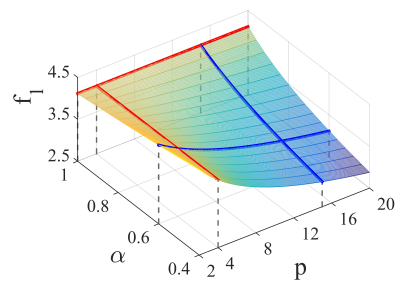

Figure 13. The influence of joint venture decisions on the long-run equilibrium profit of an operator under non-linear demand

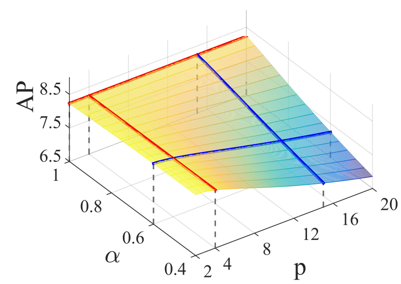

Figure 14. The influence of joint venture decisions on the market aggregate profits at the long-run equilibrium under non-linear demand

numerical results of each set of experiments respectively. From these figures, we can see that even though the exact form of demand differs, the variation patterns of the equilibrium price, profit of an operator, and the aggregate profit of the whole market at equilibrium coincide with each counterpart in the linear demand setting, no matter whether under the one-shot competition or the long-run competition. It shows that our conclusions drawn from the theoretical analysis are not limited to our specific assumption of the linear demand model, but general in a degree.

B. Visualization of the prisoners' dilemma and its critical condition

In this section, we conduction two sets of experiments about the prisoners' dilemma in the second stage of the Stackelberg game. In the first set of experiments, we visualize the prisoners' dilemma in the one-shot competition with identical players. In the second one, we numerically study the condition of the initial user densities of the operators where each of them can get a higher profit by cooperatively maximizing the aggregate profit of them instead of staying at

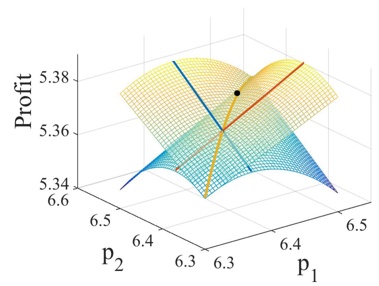

Figure 15. The profit surface of prisoners' dilemma

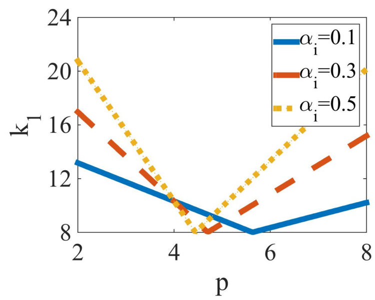

Figure 16. The variation of the critical value of k_1 with the price of renting infrastructure

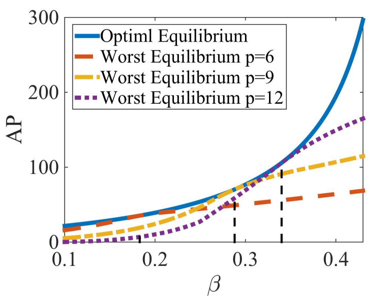

Figure 17. The optimal and worst equilibrium profits among long-run equilibriums

the equilibrium of the prisoners' dilemma. In the two sets of experiments, we assume $\beta_{ij} = 0.1, \forall i, j \in N$ and the marginal cost $c = 4$.

First, we visualize the prisoners' dilemma in the one-shot competition in the second stage of our Stackelberg game. We consider a telecom market with two identical operators with $k_1 = k_2 = 8$ and $\alpha_1 = \alpha_2 = 0.5$. We set the price of renting infrastructure as $p = 4.2$. In the experiment, we let the prices of both operators p_1 and p_2 vary from 6.3 to 6.5 with an increment 0.001. For each price profile (p_1, p_2), the corresponding profits of operator 1 (f_1) and operator 2 f_2 are shown in Fig. 15, which form two intersecting surfaces. The surface of f_1 crosses the blue curve and the surface of f_2 crosses the red curve, where the blue curve consists of the best response prices of operator 1 given the price of operator 2 varying from 6.3 to 6.5 and the red curve consists of the best response prices of operator 2. These two colored curves intersect at the point $(6.374, 6.374)$, which is the Nash equilibrium price profile for the one-shot competition. However, the two operators can further improve their own profit by raising up the price to the cooperative price equilibrium, marked by the black point $(6.444, 6.444)$, which is the maximum of the intersection curve (the yellow curve) of surface f_1 and surface f_2. These features depict the prisoners' dilemma.

Second, we conduct numerical experiments to investigate the condition where cooperating to maximize the aggregate profit of all the operators can improve each operator's profit w.r.t. the profits under the one-shot equilibrium. We also consider a market with two operators where operator 2's initial user density is fixed at $k_2 = 8$. We compute the critical value of the initial user density of operator 1 under different price p and shares $\alpha_i, i = 1, 2$ such that at least one operator cannot benefit from cooperation. W.l.o.g, we assume $k_1 > k_2$. We let p vary from 2 to 8 with an increment 0.01 and let α_1 and α_2 simultaneously vary from 0.1 to 0.5 with an increment

0.2. Fig. 16 shows the critical value of k_1. When k_1 is less than the critical value, cooperating to maximize the aggregate profit of both operators can benefit both operator 1 and 2, otherwise, at least one of them cannot be favored. We can see that the each line in Fig. 16 intersects with the horizontal axis $k_1 = 8$. These intersections indicate the cases where the one-shot equilibrium price profile happens to maximize the aggregate profit of both operators. Along both sides of these intersections, the critical value increases linearly towards the corresponding end of the horizontal axis, showing an increasing range of initial user densities for two operators to achieve cooperation.

C. The profit gap of the long-run equilibriums

The long-run competition in the second stage of the Stackelberg game may contain multiple equilibriums. In this section, we conduct numerical studies to analyze the gap of aggregate profit of the whole market between the optimal long-run equilibrium and the worst long-run equilibrium. W.l.o.g, we consider three identical operators sharing the whole joint venture where the aggregate profit of the whole market is equal to the aggregate profit of the operators. We select the parameters that $k_i = 8, \alpha_i = \frac{1}{3}, i \in \{1,2,3\}, c = 4$ and $\beta_{ij} = \beta, \forall i,j \in \{1,2,3\}, i \neq j$, where β varies from 0.1 to 0.4 with an increment of 0.01, while p is chosen from $\{6, 9, 12\}$. As $\alpha = 1$, the aggregate profit of the whole market at the optimal equilibriums under different values of p are the same, computed by Eq. (12), and are shown as a single solid curve in Fig. 17. The worst profits at equilibriums under different values of p, are computed numerically with precision 0.01 according to the Folk theorem. Fig. 17 shows the whole result. As β approaches to the limitation 0.5, the optimal aggregate profit increases super-linearly while the worst aggregate profits under different p increase linearly, dramatically enlarging the gap. At the same time, the two kinds of profits coincide with each other at certain combinations of p and β, which are shown as the intersections between the solid curve and the other three curves representing the worst case, marked by the black dash lines. These combinations right belong to the solution set of equation $p_i^{ne} = p_i^{co}$ given the other parameters. At the solution sets, the prisoners' dilemma is eliminated from the system.

D. Comparison of the fixed cost of the infrastructures

We will show two sets of experiments to investigate the fixed infrastructure cost reduction in per km^2 in the presence of the joint venture. We assume that the fixed infrastructure cost function

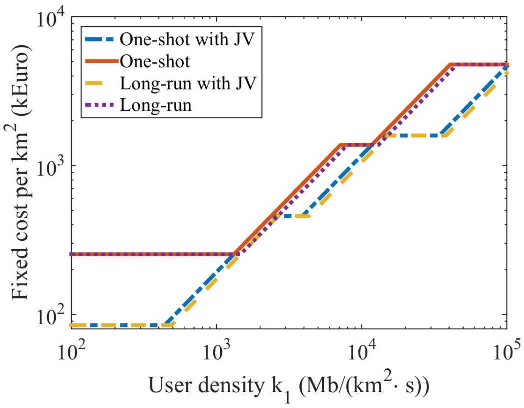

Figure 18. The fixed infrastructure cost of different user densities

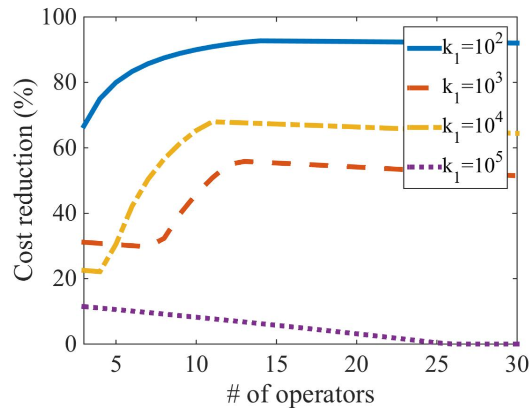

Figure 19. The fixed cost reduction with different numbers of operators

$h(\cdot)$ is given as the minimal fixed cost for all three types of base stations with corresponding user density in [25]. W.l.o.g. we assume $c = 0$k€.

In the first set of experiments, we exam how the fixed cost varies with the initial demand of the operators. We consider three identical operators with $\alpha_i = \frac{1}{3}, \beta_{ij} = 0.1, \forall i, j \in \{1, 2, 3\}$. We consider two time scopes of market competition, the one-shot competition and the long-run competition. In both cases, we assume that the joint venture will set $p = c$ such that the operators pricing strategies will not differ from the strategies when there is no joint venture in the market. Fig. 18 shows the result. From the figure, we find that in both cases, when the user density is small, two-thirds of the fixed cost is saved with the presence of the joint venture. The intuition is clear. Without a joint venture, each operator has to build a set of infrastructures to cover the same service area. When the user density is small, by sharing infrastructure via the joint venture, all operators can use only one set of infrastructures, reducing the redundant construction of the other two sets. On the contrary, when the aggregate user density excesses the capacity of one set of infrastructures, the presence of the joint venture may bring no cost reduction because the joint venture needs to build more infrastructures to satisfy the capacity requirements.

In the second set of experiments, we exam how the cost reduction varies with the number of the operators. We consider identical operators with $k_i = k_1, \forall i \in N$ and $\beta_{ij} = 0.01, \forall i, j$. We consider four levels of the initial user density that $k_1 = 10^2, 10^3, 10^4$ or 10^5 users/km^{2}s. All the shares on the joint venture are uniformly distributed among these identical operators. We select the one-shot competition for demonstration, and similar observations hold for the long-run competition. We also assume $p = c$. Fig. 19 shows the result. We can see that with moderate initial user density, the cost reduction ratio first generally increases with the number of the operators, where a set of infrastructures can support the aggregate user density. The ratio then decreases with the number of operators, showing that with increasing aggregate user density, more sets of infrastructures need to be constructed. In contrast, when the initial user density of operators is extremely high, the cost reduction continuously decreases with the number of operators, showing the limitation of cost reduction by sharing infrastructure when the number of the operators and total user density are saturated.

VII. Discussion of Our Results and Remarks on Future Work

In this paper, we present a new framework for companies to develop a joint effort to battle costly infrastructure construction. It has the advantage of building it into an individually rational and collectively optimal game theoretical setting for such technology shifting at the marketplace. Such a solution is highly desirable in our era of globalization and fast technology advancement in the IT and communication field. We expect further works and studies to extend this framework to other similar settings and application areas.

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