w/ K. Costello 2201.02595, 2204.05301

local hol'c thy in 6d $\rightsquigarrow$ tree & 1-loop amplitudes in QCD

$$\left\{\begin{matrix}\text{correlators of} \\ \text{a certain 2d} \\ \text{chiral algebra}\end{matrix}\right\} \longleftrightarrow \left\{\begin{matrix}\underline{\text{form}}\ \underline{\text{factors}} \\ \text{in a 4d theory} \\ \rightsquigarrow \text{QCD scattering amplitudes}\end{matrix}\right\}$$

4d: self-dual YM $\quad \int \mathrm{tr}(BF(A)_-)$, $\quad B \in \Omega^2_-(\mathbb{R}^4, \mathfrak{g})$

$$\left\{+\frac{1}{2} g_{YM}^2 \int \mathrm{tr}(B^2)\right\}$$

SDYM in presence op. insertions $\mathrm{tr}(B^2)$, $\mathrm{tr}(B^2)(0)\,\mathrm{tr}(B^2)(x_1)\ldots$

l loops, n insertions $\rightsquigarrow n-l+1$ $(-)$-helicity

<u>twistor space</u>: $\mathbb{PT} \cong \mathbb{R}^4 \times \mathbb{CP}^1 \cong \mathcal{O}(1) \oplus \mathcal{O}(1)$

$$\begin{matrix} v_1 \quad \downarrow \quad v_2 \\ \mathbb{CP}^1_z \end{matrix}$$

$\mathbb{C}^4$, $\|x-y\|^2 = 0$

$z \in \mathbb{CP}^1$

$\bar{z} = \frac{v_2}{v_1}$

$$\left\{\begin{matrix}\text{massless helicity} \\ h \text{ particles on} \\ \mathbb{C}^4\end{matrix}\right\} \longleftrightarrow \left\{ H^{(0,1)}_{\bar\partial}(\mathbb{PT}, \underbrace{\mathcal{O}(2h-2)}) \right\}$$

hol'c BF thy $\qquad$ Penrose/Ward

$$\int_{\mathbb{PT}} \mathrm{tr}\left(\mathcal{B}\mathcal{F}^{(0,2)}(\mathcal{A})\right) \longmapsto \int_{\mathbb{R}^4} \mathrm{tr}\left(BF(A)_-\right)$$

$$\mathcal{A} \in \Omega^{(0,1)}(\mathbb{PT}, \mathfrak{g})$$
$$\mathcal{B} \in \Omega^{(3,1)}(\mathbb{PT}, \mathfrak{g})$$

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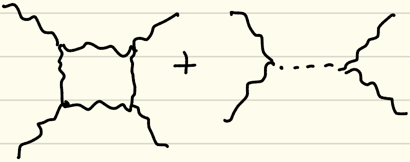
Can cancel gauge anomaly in 6d



$\mathfrak{g} = \mathfrak{su}(2), \mathfrak{su}(3), \mathfrak{so}(8)$, exceptional

6d: $\eta \in \Omega^{2,1}(\mathbb{PT}),\ \partial\eta = 0 \qquad \# \int \eta\, \mathrm{tr}(\mathcal{A}\partial\mathcal{A})$

$$\mathbb{R}^4: \int \mathrm{tr}(BF(A)_-) + \frac{1}{2}\int (\Delta\rho)^2 - \# \int \rho\left(F(A) \wedge F(A)\right)$$

4d thy SDYM + axion

	Generators	Spin
A	$J[m,n]$	$1-(m+n)/2$
B	$\tilde{J}[m,n]$	$-1-(m+n)/2$
η	$E[m,n]$	$-(m+n)/2$
	$F[m,n]$	$-(m+n)/2$

$SL_2(\mathbb{R}) \circlearrowright z$

$\mathbb{CP}^1_z$

$\mathbb{C}^2 \to \mathfrak{g}$

$$J[r,s,k] = \oint J[r,s]\, z^{k-1}\, dz$$

$$\begin{cases} J^a[r,s](0)\, J^b[t,u](z) \sim \frac{1}{z} f^{ab}_{c}\, J^c[r+t, s+u](0) \\ J^a[r,s](0)\, \tilde{J}^b[t,u](z) \sim \frac{1}{z} f^{ab}_{c}\, \tilde{J}^c[r+t, s+u](0) \end{cases}$$

2d | 4d (on-shell gauge thy) | Penrose-Ward | 6d

{generators of chiral alg.} $\longleftrightarrow$ {basis of conformal primary states in 4d of neg. wt} $\longleftrightarrow$ {Conf. primary states on twistor}

$$\underbrace{J[r,s](z_0)}_{\text{Koszul dual vertex algebra}} \longleftrightarrow \underline{A = \delta_{z=z_0} v_1^r v_2^s}$$

[{$\mathcal{O}(0)$ 4d} $\longleftrightarrow$ {conf. block in 2d $\langle \mathcal{O}|$}] $\langle \mathcal{O} | V_1(z_1) \dots V_n(z_n) \rangle$

Why?

– rational fn. of z_i

4d form factors (scattering amp. in presence of $\mathcal{O}$)

2d chiral	4d theory
conf. primary	single-particle states
OPEs	collinear limits
conf. blocks	local operators
correlation fns.	form factors

6d

$\mathbb{CP}^1$ $\quad$ $\mathbb{PT}' \cong \underbrace{S^3 \times \overset{\zeta}{\mathbb{CP}^1} \times \mathbb{R}_{>0}}$, S^3 KK reduction

4d

$\underline{\mathbb{R}^4 \setminus \{0\}} \cong \mathbb{R}_{>0} \times S^3$ $\qquad$ $\mathbb{R}_{>0} \times \underbrace{\mathbb{CP}^1}_{\mathbb{C}_z}$ 3d thy

<u>3d theory</u> top'l-hol'c

$\mathcal{H}(S^3 \times \mathbb{CP}^1) \cong \mathcal{H}(S^3) \cong \mathcal{H}(\mathbb{CP}^1)$

- chiral alg. $\mathcal{S}$ is bdy alg. of 3d on $\mathbb{R}_{>0,r} \times \mathbb{CP}^1_z$
- <u>define</u> a conf. block $\mathcal{H}(\mathbb{CP}^1_z)$
- $\mathcal{S}$ b.c. @ $r = \infty$

MHV amplitudes

$\mathcal{O}(0) = \mathrm{Tr}(B^2)(0) \qquad \langle \mathrm{tr}(B^2) | \tilde{J}(z_1) \tilde{J}(z_2) J(z_3) \dots J(z_n) \rangle$

(1) $\langle \mathrm{tr}(B^2) | \tilde{J}^{a_1}(z_1) \tilde{J}^{a_2}(z_2) \rangle \qquad \tilde{J}(z) = \tilde{J}[0,0](z)$

$B \rightsquigarrow \mathcal{B}$ $\qquad = K^{a_1 a_2} (z_1 - z_2)^2$ $\quad$ <u>symmetry</u>

(2) $\langle \mathrm{tr}(B^2) | \tilde{J}^a \tilde{J}^b J^c \rangle = f^{cb}_{\ d} \frac{1}{z_2 - z_3} \langle \mathrm{tr}(B^2) | \tilde{J}^a(z_1) \tilde{J}^d(z_2) \rangle$

tree-level OPE $\qquad + f^{ca}_{\ d} \frac{1}{z_1 - z_3} \langle \mathrm{tr}(B^2) | \tilde{J}^d(z_1) \tilde{J}^b(z_2) \rangle$

$$= \frac{(z_1 - z_2)^3}{(z_1 - z_3)(z_2 - z_3)} f^{abc}$$

induct on n

$$\rightsquigarrow \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle} \mathrm{tr}(t_{a_1} \dots t_{a_n}) + \text{perms.}$$

Parke-Taylor